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On Assessing the Accuracy of Offshore Wind Turbine Reliability-based Design Loads from the Environmental Contour Method

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We discuss the use of the environmental contour method to derive design loads for an active stall-regulated offshore wind turbine. Two different Danish offshore environments, Rødsand and Horns Rev, are considered for the locations of the turbine. The accuracy of the derived design loads is assessed by comparing them with exact solutions derived using full integration over an accurate description of the failure domain. The error in estimating design loads is introduced because 2 key assumptions of the method are violated: first, the limit state surface, especially in the operating range of the turbine, is not well-approximated by a tangent hyperplane at the design point; and second, failure in any of the possible turbine states (e.g., operating or parked states are discussed here) needs to be considered in computing accurate failure probabilities. It is recommended that environmental contours and iso-response curves be plotted and interpreted before establishing design load levels.

INTRODUCTION

Inverse reliability techniques are commonly used when there is interest in establishing design levels associated with a specified reliability of probability of failure. The heuristically appealing Inverse First-Order Reliability Method (Inverse FORM), also called the environmental contour method, is an example of an inverse reliability technique that has been applied to estimate design loads in many applications, including offshore platforms (Winterstein et al., 1993) and onshore wind turbines (Fitzwater et al., 2003; Saranyasoontorn and Manuel, 2004). For offshore wind turbine applications, Christensen and Arnbjerg-Nielsen (2000) utilized the environmental contour method to derive design levels for shear and overturning moment at the seabed for an active stall-regulated wind turbine located at 2 different Danish offshore sites. Even though application of the environmental contour approach to deriving design loads for wind turbine structures has been well established (see, for example, Saranyasoontorn and Manuel, 2004), it has not been clearly explained how the treatment of multiple turbine states (or accounting for the possibility of failure under different conditions, such as *operating* or *parked*) typical for wind turbine applications can affect the accuracy of the derived design loads. Errors in deriving design loads may be introduced if certain assumptions inherent in the environmental contour method are violated. Two of the most important assumptions of the method are as follows:

• that the failure probability is associated with a single governing failure mode (or turbine state) alone, and any probability of failure associated with other states is neglected in the calculations;

• that a local linearized limit state surface can serve as an approximation of the true limit state surface by virtue of using a tangent hyperplane at the *design point*, and that the desired (target) probability of failure or reliability may be associated with the tangent hyperplane.

We shall see that errors may be large in practical situations when the true limit state surface is highly nonlinear, or when it has no well-defined shape that can be reasonably approximated by a tangent hyperplane, and/or when failure in secondary turbine states can become reasonably likely compared to primary turbine states. Both of these sources of error are common when considering wind turbines where failure might result, for example, under either operating or parked conditions (turbine states). In this paper's illustrations using offshore wind turbines, the different regimes associated with the machine's power production require us to consider the possibility of failure in either operating or parked states. Additionally, as we shall see, the transition from operating to parked conditions, which takes place abruptly at the machine's cut-out wind speed, introduces highly irregularly-shaped failure surfaces to which a single tangent hyperplane would be a poor approximation. We also discuss how, in some situations, due to the error sources mentioned above, design loads based on the environmental contour method can be wrongly interpreted, ascribing more importance to derived environmental conditions of one state than is found when the design load is computed using exact full integration approaches.

The objective of this study is to investigate the various error-related issues more closely in order to make recommendations regarding the use of environmental contour methods to derive design loads for onshore and offshore wind turbines. For the sake of illustration, we consider an offshore wind turbine located at 2 different sites. The characterization of the random wind and wave environment and dependence on loads is of interest here. We will study failure due to extreme shear or overturning moment at the base of the offshore wind turbine. For these extreme/ultimate limit states, failure in either of 2 states is considered: When the turbine is operating and the corresponding hub-height wind speed is between the cut-in and cut-out wind speeds, or when the turbine is parked, and the wind speed is higher than the cut-out wind speed. Although the turbine and environmental conditions used in this study are the same as those analyzed and described at great length in the previous study by Christensen and Arnbjerg-Nielsen (2000), our goals in the present study are very different. Our focus is not on simply deriving design loads. Rather, we will attempt to explain and interpret results by highlighting the role of each turbine state as it influences the derived design load for a specified return period. We also compare the predicted design

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Site	Random Variable		Distribution	Parameters
Rødsand	H	Weibull	$F_H(h) = 1 - \exp\left(-\left(\frac{h}{h_0}\right)^{\beta_h}\right)$	$h_0 = 0.863$ m $\beta_h = 1.817$ (median of $H = 0.71$ m)
	$V\|H$	Lognormal	$F_{V\|H}(v; h) = \Phi\left(\frac{\ln(v) - \alpha_v(h)}{\beta_v(h)}\right)$ $\beta_v(h) = \sqrt{\ln((\sigma_V/\mu_V(h))^2 + 1)}$ $\alpha_v(h) = \ln(\mu_V(h)) - \frac{1}{2}\beta_v(h)^2$	$\mu_V(h) = 10.30h + 3.32$ m/s, $\sigma_V = 1.72$ m/s at a height of 70 m above sea level. (median of $V\|H = \exp(\ln(\mu_V(h)) - 0.5\beta_v^2)$ m/s)
Horns Rev	V	Weibull	$F_V(v) = 1 - \exp\left(-\left(\frac{v}{a}\right)^k\right)$	$a = 11$ m/s, $k = 1.8$ at a height of 60 m above sea level (median of $V = 8.97$ m/s)
	$H\|V$	Normal	$F_{H\|V}(h; v) = \Phi\left(\frac{h - \mu_H(v)}{\sigma_H}\right)$	$\mu_H(v) = 0.13v$ $\sigma_H = 0.24$ m (median of $H\|V = 0.13v$ m)

Table 1 Distributions and parameters for environmental variables at 2 offshore sites (from Christensen and Arnbjerg-Nielsen, 2000)

loads and associated failure probability values for the 2 offshore environments with *exact* results obtained from direct integration over the failure region, and using the joint probability distribution of the environmental random variables. Finally, with the help of examples, we will explain how predicted design loads based on the environmental contour method can sometimes be wrongly interpreted.

OFFSHORE ENVIRONMENT AT 2 SITES

Two different offshore sites in Denmark are chosen for analysis in this study; one is at Rødsand, the other at Horns Rev. At each of these sites, the environment that influences wind turbine loads there is assumed to be mainly characterized by 2 random variables: the 10-min mean wind speed, V, and the 1-h significant wave height, H. Based on fits to experimental data, probability distributions and associated parameters for these random variables at the 2 sites are summarized in Table 1 (Christensen and Arnbjerg-Nielsen, 2000). In order to highlight the different environmental conditions at these sites, we also provide median values of the 2 random variables (H and V) for both sites in the table. It is seen that, when the joint median levels of both random variables are considered, the Horns Rev site is seen to have a larger median 1-h significant wave height equal to 1.17 m compared to 0.71 m at the Rødsand site. The Horns Rev site also has the higher median 10-min wind speed than the Rødsand site. For a more detailed environmental description at the 2 sites, the reader is referred to the study of Christensen and Arnbjerg-Nielsen (2000). Based on these probabilistic models, joint probability density functions for the 2 random variables at both sites can be sketched, as is done in Figs. 1 and 2 for the Rødsand and Horns Rev sites, respectively. Note that the wave period is not explicitly treated as a random variable here.

WIND TURBINE AND RESPONSE DESCRIPTION

The wind turbine and characterization of the extreme response used in this study are taken from studies by Tarp-Johansen and Frandsen (2000) and Christensen and Arnbjerg-Nielsen (2000). The machine selected is an active stall-regulated wind turbine with a hub height of 63.5 m and a rotor-swept area of 3160 m^2. The cut-out wind speed for this machine is taken to be 25 m/s. For wind speeds higher than the cut-out speed, the machine is assumed to be parked, so as to limit the extreme loads that might result. The Horns Rev site has a mean water depth of 9 m and, for the sake of comparison, the same water depth is also assumed for the Rødsand site.

The extreme response variables selected for the reliability-based design studies here include the shear force and overturning moment at the seabed. Plots of the extreme response as a function of both V and H, adapted from Christensen and Arnbjerg-Nielsen (2000), are shown in Figs. 3a and b, respectively, for shear and overturning moment at the base of the gravity-type foundation. It may be noted from the figures that the shear force at the seabed is influenced noticeably by both wind and wave, while the overturn-

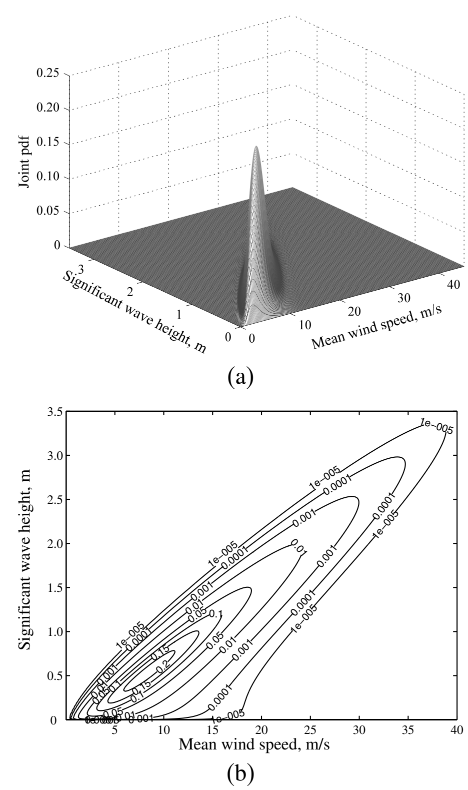

Fig. 1 (a) Joint probability density function, and (b) its contour for 10-min wind speed, V, and 1-h significant wave height, H, random variables at Rødsand site (based on Christensen and Arnbjerg-Nielsen, 2000)

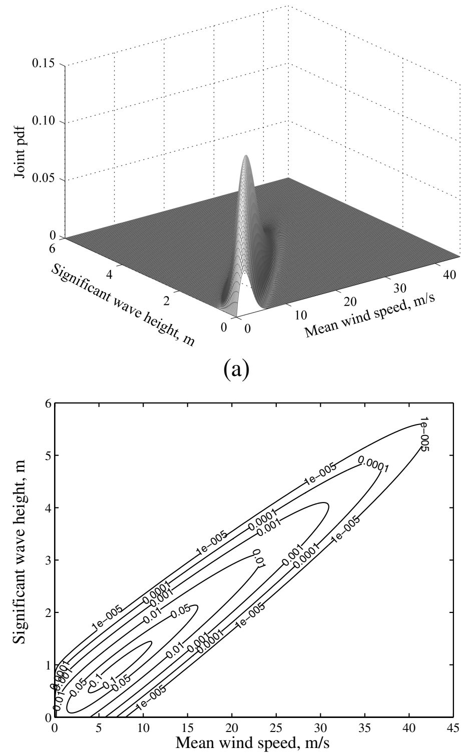

Fig. 2 (a) Joint probability density function, and (b) its contour for 10-min wind speed, V, and 1-h significant wave height, H, random variables at Horns Rev site (based on Christensen and Arnbjerg-Nielsen, 2000)

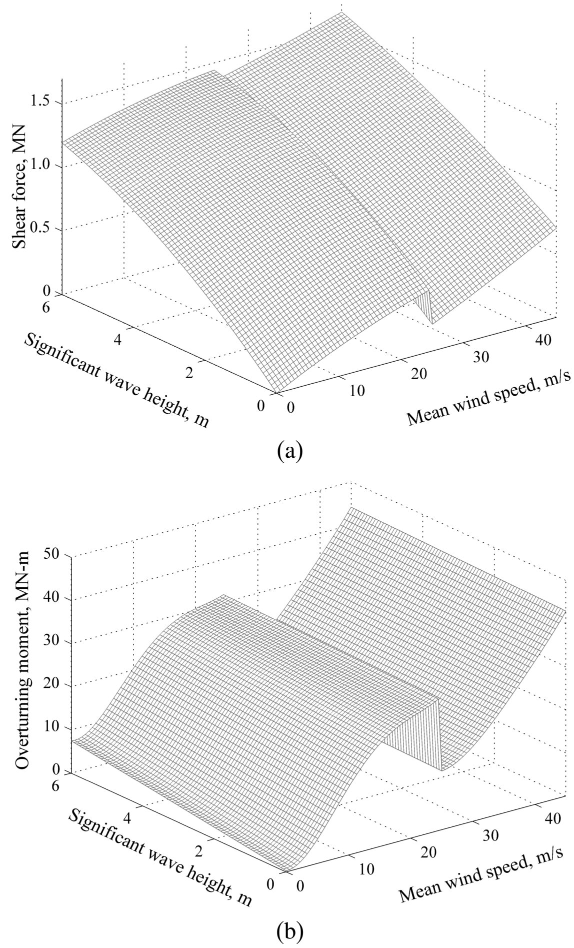

Fig. 3 (a) Shear force, and (b) overturning moment of stall-regulated wind turbine conditional on 10-min wind speed, V, and 1-h significant wave height, H (adapted from Christensen and Arnbjerg-Nielsen, 2000)

ing moment is predominantly influenced by wind. This is because overturning moment is associated with forces and moment arms; the wind loads at the level of the rotor are associated with greater moment arms than the wave loads that are mostly at the bottom. Shear forces at the seabed do not disproportionately weight wave and wind loads.

NUMERICAL STUDIES

We are interested first in deriving design shear forces and overturning moments at the seabed for the selected active stall-regulated wind turbine at the 2 offshore sites utilizing the environmental contour method. Then, these derived design loads will be compared with *exact* solutions obtained by full integration that involves use of the extreme response surface (conditional on wind and wave conditions as described in Fig. 3) and the joint probability density function of the environmental random variables as described in Figs. 1 and 2 for the 2 sites. Typical design lives for turbines are closer to 20–25 years. Here, we assume however that our interest is in establishing 50-year characteristic loads to be used for the design of the turbine that can fail either under operating conditions (when wind speed is below the cut-out wind speed), or under parked conditions (when the wind speed is greater than the cut-out wind speed). Based on the prescribed target return period of 50 years and accounting for the temporal correlation structure of wind/waves that limits the number of independent/stationary environmental events over design life, reliability indices, β, used with the environmental contour method, are 3.71 and 3.74, respectively, for the Rødsand and Horns Rev sites (Christensen and Arnbjerg-Nielsen, 2000). The single-event target failure probability levels, P_T, are 1.05×10^{-4} due to 9,522 independent environmental events of 46-h duration over 50 years at Rødsand, and 9.13×10^{-5} due to 10,950 events of 40-h duration at Horns Rev.

Environmental Contour Method

Here we briefly discuss the background behind the environmental contour method. Additional details may be found in the work of Winterstein et al. (1993). For a known reliability index β associated with a prescribed return period or failure probability, P_T, one can easily construct an environmental contour such that the probability on the side of a tangent hyperplane away from the origin at any point on the contour is the same and equal to the P_T. This is based on an assumption that the limit state function is linear at the design point. In an uncorrelated standard normal U space, when 2 environmental random variables are involved, this contour is a circle with a radius equal to β. Thus, we have:

$$u_1 = \beta \cos\phi \quad \text{and} \quad u_2 = \beta \sin\phi \qquad \text{for } -\pi \le \phi \le \pi \tag{1}$$

where u_1 and u_2 are the environmental random variables appropriately defined in the U space. Although associated with the same probability level, each point along the contour is associated with a *different* response level that is exceeded by that probability level. To obtain the design point, one needs to search the entire circle to find the largest response or load level. We refer to this largest level as the design load, L_{des} in the following. On applying the Rosenblatt transformation (Rosenblatt, 1952), the design point in the $\boldsymbol{X}$ space of the physical random variables can be obtained as follows:

$$x_1 = F_{X_1}^{-1}[\Phi(\beta \cos \phi)] \quad \text{and} \quad x_2 = F_{X_2|X_1}^{-1}[\Phi(\beta \sin \phi)]. \tag{2}$$

where X_1 and X_2 are the 2 environmental random variables; F_{X_1} and $F_{X_2|X_1}$ are the cumulative probability distribution functions for X_1 and X_2 given X_1, respectively; and $\Phi()$ is the standard normal cumulative distribution function. Also, the design point can be obtained by plotting separate iso-response curves (for specified environmental conditions) together with the environmental contour in physical $\boldsymbol{X}$ space. One can then locate the iso-response curve of the highest response level that intersects the environmental contour. Note that the implication of this method is that only a single governing failure mode (turbine state, here) associated with the target reliability index, β, is considered when computing the failure probability. The probability of failure at a specified response level, due to any other failure modes (or arising from other turbine states, here), if they exist, is ignored in the calculations. We discuss next numerical results summarizing the design loads for both Rødsand and Horns Rev sites.

Rødsand

It is shown in Fig. 4a, where the 50-year environmental contour (corresponding to a reliability index of 3.71) and the iso-shear force curves are plotted together, that the design shear force derived by the environmental contour method is 1.02 MN. This design shear force is associated with a 10-min mean wind speed of 25 m/s and a significant wave height of 2.45 m. This suggests that the machine at Rødsand is more likely to fail under operating conditions (the governing turbine state, here) than under parked conditions. The corresponding design overturning moment is 31.40 MN-m, which is also associated with a 10-min mean wind speed of 25 m/s and a significant wave height of 2.45 m (Fig. 4b). Again, the environmental contour method predicts that the machine will more likely fail under operating than parked conditions when considering the possibility of extreme overturning moment.

Horns Rev

The same procedure was employed to arrive at design points for base shear and overturning moment for the wind turbine at the Horns Rev site. Environmental contours in $\boldsymbol{X}$ space (for a reliability index of 3.74) and iso-response shear force and moment curves are shown in Figs. 5a and b, respectively. The 50-year design values are 1.39 MN and 31.71 MN-m for shear force and overturning moment, respectively. For this site, the environmental contour method predicts that parked conditions ($V = 37.8$ m/s, $H = 5.01$ m) control design against shear failure while operating conditions ($V = 25.0$ m/s, $H = 3.97$ m) control design against overturning moment failure.

True Failure Probability

The failure probability, P_f, for a specified load capacity can be determined analytically by the method of direct integration (here referred to as full integration). This method involves combining the description of the extreme load, L (given wind and wave conditions), with the joint density function of the environmental variables. For specified extreme load levels, conditional on

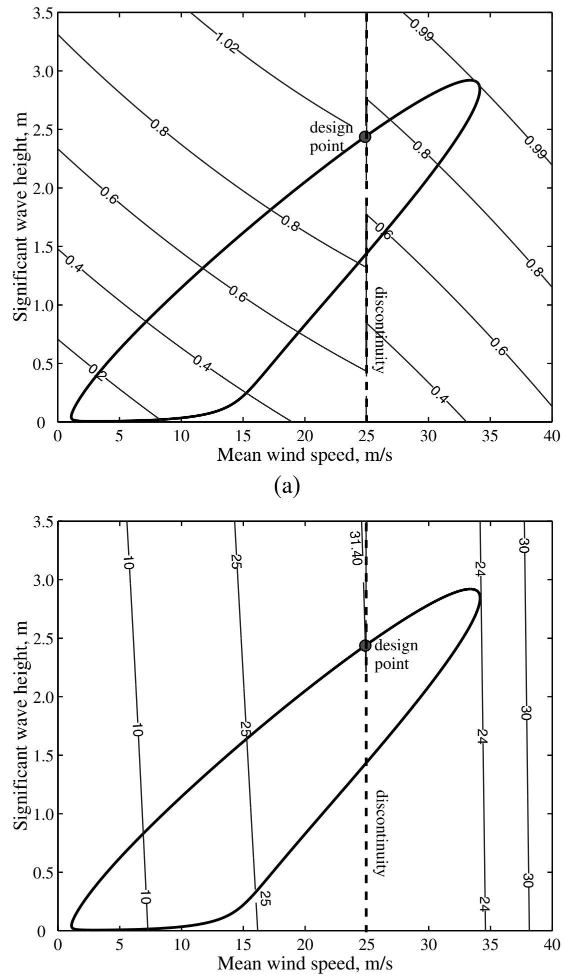

Fig. 4 Environmental contour (in physical space, $\boldsymbol{X}$) associated with 50-year return period at Rødsand site and iso-response curve for (a) shear force and (b) overturning moment at seabed. Design shear force and bending moment values are 1.02 MN and 31.40 MN-m, respectively.

the environmental variables, as are available here, the true probability of failure associated with a given load capacity, L_{des}, can be expressed as follows:

$$P_f = P[L > L_{des}] = \int\int_{V,H} I[g < 0] f_{V,H}(v,h)\, dv\, dh \tag{3}$$

where $f_{V,H}(v, h)$ is the joint probability density function of V and H, and g denotes the limit state function. The indicator function, $I[\ ]$, is equal to unity whenever the limit state function, $g = L_{des} - L(v, h)$, is less than zero (i.e., in states of failure for a given pair of V and H values), and is zero otherwise (i.e., in safe states). It is important to state that material properties and hence resistance are not treated as random here.

Note that the exact limit state surface is used in this approach. Note also that Eq. 3, unlike the environmental contour method, because of the generality in its formulation, yields the probability of failure by including the likelihood of failure in all turbine states under consideration. Specifically for a situation involving consideration of 2 turbine states, as is the case here, the total probability of failure for any capacity, L_{des}, results from the sum of the failure probabilities in each state. Thus, when failure for the offshore

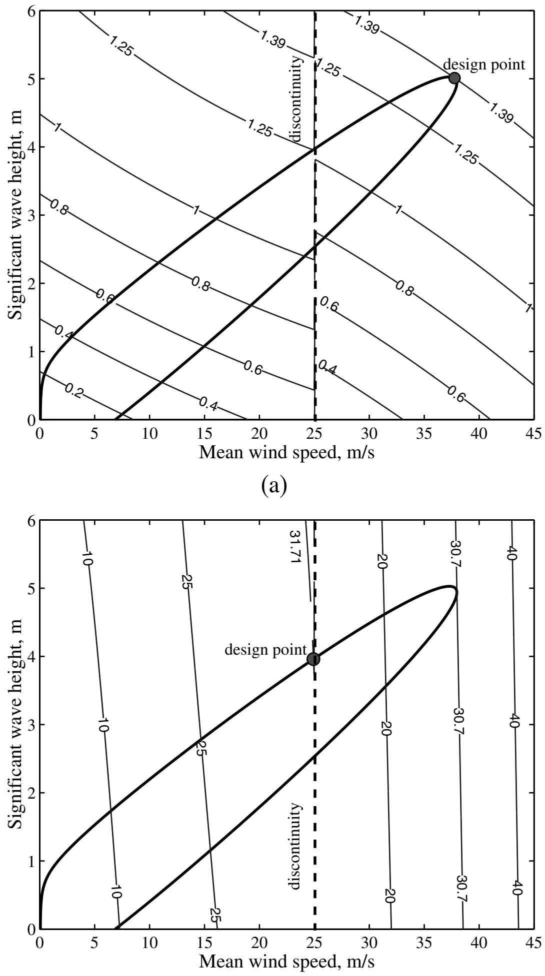

Fig. 5 Environmental contour (in physical space, $\boldsymbol{X}$) associated with 50-year return period at Horns Rev site and iso-response curve for (a) shear force and (b) overturning moment at seabed

wind turbines is considered in either operating or parked conditions, we have:

$$
\begin{aligned}
P_f &= P[L > L_{des}] = P[\text{fail in operating or parked conditions}] \\
&= P[\text{fail in operating conditions}] \\
&\quad + P[\text{fail in parked conditions}] \\
&= P_{f,op} + P_{f,pk} \\
&= \iint_{op} I[g_{op} < 0] f_{V,H}(v,h)\, dv\, dh \\
&\quad + \iint_{pk} I[g_{pk} < 0] f_{V,H}(v,h)\, dv\, dh
\end{aligned}
\tag{4}
$$

where $P_{f,op}$ and $P_{f,pk}$ are the failure probabilities for the operating and the parked states, respectively, and g_{op} and g_{pk} are the corresponding limit state functions. According to Eq. 4, we can use a full integration approach to determine the failure probability for each state separately. Obviously, however, integrating over the entire range of environmental variables (to infinity in many cases) is practically impossible. Fortunately, as wind speed and wave height increase to very large values, the joint probability density function becomes negligibly small, as can be seen in Figs. 1 and 2. Hence, at both sites, the integration in Eq. 4 was limited to the range of 0 to 45 m/s for the 10-min mean wind speed, V, and 0 to 6 m for the significant wave height, H.

A Gauss quadrature scheme was selected for the numerical integration in order to achieve reasonable accuracy. In this scheme, 300 × 300 integration cells and 3 × 3 Gauss points for each cell were employed. Eq. 4 can then be evaluated numerically as follows:

$$
\begin{aligned}
P_f &= P[L > L_{des}] \\
&\approx \sum_{\substack{k=1\\op}}^{300\times300} \sum_{i=1}^{3}\sum_{j=1}^{3} I[g_{op} < 0] f_{V,H}(v(i,k), h(j,k))\, w_i w_j \\
&\quad + \sum_{\substack{k=1\\pk}}^{300\times300} \sum_{i=1}^{3}\sum_{j=1}^{3} I[g_{pk} < 0] f_{V,H}(v(i,k), h(j,k))\, w_i w_j
\end{aligned}
\tag{5}
$$

where $w_i w_j$ is the product of weights associated with integration point (i, j), while $v = v(i,k)$ and $h = h(j,k)$ at the integration point in cell k. To assess the accuracy of the Gauss quadrature scheme, it was employed to determine the volume under the joint probability density function whose exact value is unity. Results suggested an error on the order of 10^{-7}. Below, we determine the true failure probability associated with the derived design loads based on the environmental contour method presented above. The true failure probability, P_f, is expected to be close to the target probability, P_T (corresponding to a 50-year return period), if the environmental contour method is an accurate approach. Numerical values of the true failure probability for each site are discussed next.

Rødsand. The environmental contour method predicted that design loads for both shear (1.02 MN) and overturning moment (31.40 MN-m) are controlled by the turbine's operating state with the target probability, P_T, from this state alone, of 1.05×10^{-4}. The failure probability for the parked state was ignored in the calculations. However, these results appear to be inconsistent with the true failure probability computed by full integration. For the shear capacity of 1.02 MN, the true failure probabilities are 2.30×10^{-6} and 7.11×10^{-5}, respectively, for the operating and parked states. The total failure probability is then equal to 7.34×10^{-5} (compared with the target failure probability of 1.05×10^{-4}). It is seen that the operating state contributes very slightly (approximately 3%) to the total failure probability, clearly contradicting what the environmental contour method suggests. When considering the operating state alone, the true failure probability of 2.30×10^{-6} is about 46 times smaller than the target probability of 1.05×10^{-4}. Such a large error would, in general, be unacceptable. However, the overall error is not great, as the target probability (1.05×10^{-4}) implied by the environmental contour method is only about 1.4 times larger than the true total failure probability (7.34×10^{-5}). Similar calculations are performed for the overturning moment design load. The true operating state failure probability is found to be very small (1.3×10^{-8}), about 8000 times smaller than the target probability. The target probability of 1.05×10^{-4} is about 12 times larger than the true total failure probability of 8.61×10^{-6}. Again, these findings are inconsistent with results from the environmental contour method, and we can begin to see at this point how the method can provide inaccurate results as well as wrong insights. Table 2 summarizes the various results for the Rødsand site.

Horns Rev. The environmental contour method predicted that the design load of 1.39 MN for shear is controlled by the turbine's parked state with target probability, P_T, due to this state alone, of 9.13×10^{-5}. The failure probability for the operating state was implicitly assumed to be very small and was ignored in the calculations. These results appear to be consistent with the true failure probability computed by full integration, where the total failure

Site	Loads	P_T	V (m/s)	H (m)	L_{des} (MN, MN-m)	$P_{f,op}$	$P_T/P_{f,op}$	$P_{f,pk}$	$P_T/P_{f,pk}$	P_f
Rødsand	Shear	1.05×10^{-4}	25.0	2.45	$1.02(op)$	2.30×10^{-6}	45.6	7.11×10^{-5}	1.4	7.34×10^{-5}
	Moment	1.05×10^{-4}	25.0	2.45	$31.40(op)$	1.30×10^{-8}	8081.9	8.60×10^{-6}	12.2	8.61×10^{-6}
Horns Rev	Shear	9.13×10^{-5}	37.8	5.01	$1.39(pk)$	≈ 0	–	8.92×10^{-5}	1.0	8.92×10^{-5}
	Moment	9.13×10^{-5}	25.0	3.97	$31.71(op)$	3.59×10^{-8}	2546.3	6.79×10^{-5}	1.3	6.79×10^{-5}

Table 2 Fifty-year design loads and environmental conditions at Rødsand and Horns Rev sites based on environmental contour method and corresponding true failure probability values computed for design loads by using full integration

probability of 8.92×10^{-5} is attributed largely to the parked state. The true failure probability in the operating state is negligibly small and thus consistent with the results from the environmental contour method. The true total failure probability is almost equal to the target probability. For the overturning moment case, the environmental contour method predicted that the design load of 31.71 MN-m was controlled by the operating state with a target probability of 9.13×10^{-5}. In this case, the failure probability associated with the parked state was neglected. Based on full integration, however, the true failure probabilities are found to be 3.59×10^{-8} and 6.79×10^{-5} for the operating and parked turbine states, respectively. The total failure probability is then equal to 6.79×10^{-5}. It is seen that operating state failures contribute only very slightly (approximately 0.5%) to the total failure probability. Clearly, this conclusion is contrary to what is implied by the environmental contour method. When considering the operating state alone, the true failure probability of 3.59×10^{-8} is more than 2500 times smaller than the target probability of 9.13×10^{-5}. However, the overall error in total probability is not that large, as the target probability is only about 1.3 times larger than the true total failure probability, but this is because of the offsetting large failure probability associated with the parked state that was ignored in the environmental contour method. Table 2 summarizes the various results for the Horns Rev site.

It is clear that, for both sites, the environmental contour method predicts quite different probabilities for a given load than are found using full integration, except for the design shear at the Horns Rev site.

True Design Loads

Finally, to assess the accuracy in the design loads obtained by the environmental contour method, we use full integration to establish the true design load, L_{des}. For the exact choice of design load, L_{des}, the integration in Eq. 3 should lead to the desired target probability of failure, i.e., we should find that $P_f = P_T$.

Rødsand. Figs. 6a and b show the long-term distributions of the shear force and overturning moment at the Rødsand site. The total failure probability as well as the separate failure probability for each turbine state is plotted against the load. The contribution of each turbine state to the failure probability at any load level can be easily determined. It can be concluded from the figures that the turbine is more likely to fail under operating conditions (governing state) than under parked conditions (secondary state) for lower return periods associated with load levels that are small (less than 0.97 MN for shear and 31.4 MN-m for overturning moment). For rarer and higher loads levels, however, the turbine will more likely fail under parked conditions than under operating conditions.

The exact design loads for shear and overturning moment associated with a 50-year return period are 1.00 MN and 31.28 MN-m, respectively. For the 50-year shear force, the failure probability is approximately 1.50×10^{-5} when operating and 9.01×10^{-5} when parked, suggesting that when considering shear failure, the turbine

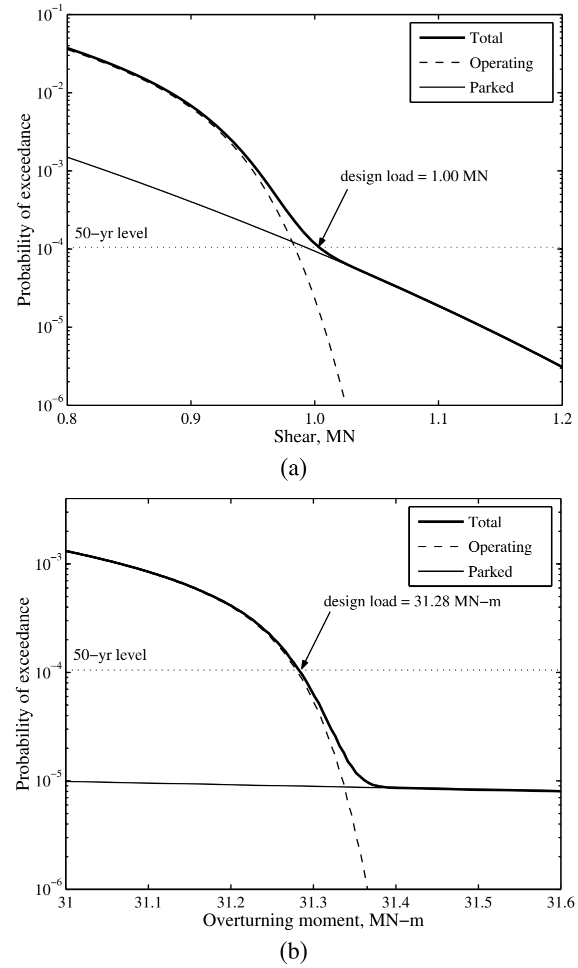

Fig. 6 Fifty-year return period values of (a) shear force and (b) overturning moment for Rødsand site based on full integration

is more likely to fail under parked conditions. It is observed from the response surface in Fig. 3a and the joint probability density function in Fig. 1b that the probability density in the area where the shear exceeds 1.00 MN is on the same order (approximately 1×10^{-4} and smaller) for the operating and parked domains. However, the unsafe area in H-V space, where shear values greater than 1.00 MN can occur, is larger in the parked case; hence, the probability that the turbine fails in the parked state is larger than it is in the operating state (by a factor of about 6). The reverse is the case for overturning moment.

In Fig. 7, A_{op} represents the area on the unsafe side of the limit state function, $g_{op}=0$, in the operating state, and A_{pk} denotes the area on the unsafe side of the limit state function, $g_{pk}=0$, in the parked state. For the 50-year overturning moment, the failure probability is approximately 9.58×10^{-5} under operating conditions and 8.73×10^{-6} under parked condition, suggesting that when considering failure due to overturning, the turbine is more

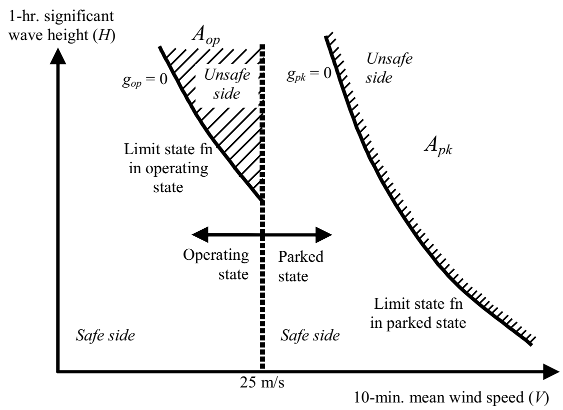

Fig. 7 Limit state functions in operating and parked turbine states in physical $\boldsymbol{X}$ space

likely to fail under operating conditions. Even though the area in the unsafe region in the operating state, A_{op}, is smaller than that in the parked region, A_{pk}, the probability mass in the parked region is much smaller (being farther away from median/central values for H and V), as can be confirmed by studying Fig. 1b, so that the probability of failure in the parked state is smaller than that in the operating state.

Horns Rev. Figs. 8a and b show the long-term distributions of the shear force and overturning moment at the Horns Rev site. The exact design loads for shear and overturning moment associated with a 50-year return period are 1.39 MN and 31.62 MN-m, respectively. For the 50-year shear force, the failure probability is negligibly small under operating conditions and approximately 9.13×10^{-5} under parked conditions, suggesting that when considering shear failure, the turbine is more likely to fail under parked conditions. For the 50-year overturning moment, the failure probability is approximately 2.22×10^{-5} under operating conditions and 6.91×10^{-5} under parked conditions, suggesting that when considering failure due to overturning as well, the turbine is more likely to fail under parked conditions.

When comparing the design shear forces at the 2 sites, it is observed that, at the Horns Rev site, this design shear force is almost 40% larger than that at the Rødsand site. This can be explained by studying the joint probability density function of the environmental random variables in Figs. 1 and 2, where the Horns Rev site appears to experience larger significant wave heights that in turn lead to the larger shear forces at the seabed. For overturning moment failures, however, there is only a very small difference in the design levels at the 2 sites. This is because overturning moment is driven primarily by wind loads, not by wave loads as can be seen in Fig. 3b. The wind conditions at the 2 sites are not very different (see Figs. 1 and 2), and as a result, the design overturning moments are only very slightly different, about 1%.

Table 3 summarizes 50-year shear and overturning moment design loads at the 2 sites as derived from both the environmental contour method and full integration. It also includes the failure probability and the governing turbine state for failure as predicted by each approach. In addition, the table summarizes differences between the derived design loads from the 2 approaches. It is interesting to note that, although the failure probabilities and the turbine states thought to control failure as identified by the 2 methods are often different, the derived design loads (L_{des}) from the 2 methods are comparable with acceptably small differences. This might not be true in general, as discussed below.

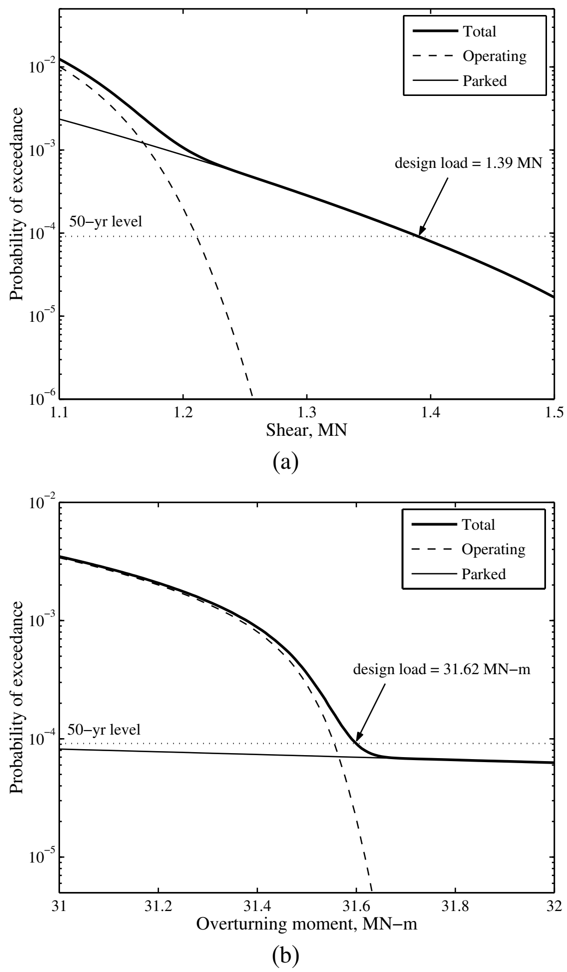

Fig. 8 Fifty-year return period values of (a) shear force and (b) overturning moment for Horns Rev site based on full integration

DISCUSSION

On the basis of the numerical studies presented, it was noted that the 50-year shear force and overturning moment design loads derived by the environmental contour method appear to be only very slightly different from the *exact* design loads as obtained by full integration (and summarized in Table 3). However, there are some apparent contradictions when one studies the results from the 2 approaches a little more closely. For example, when considering the 50-year design shear force at the Rødsand site, even though the computed design loads from both methods are fairly close (1.02 MN by the environmental contour method and 1.00 MN by full integration), the environmental contour method predicts that the machine is more likely to fail under operating conditions, while full integration suggests that the failure probability contribution during parked conditions ($P_{f,pk} \sim 9.01 \times 10^{-5}$) is greater than that during operating conditions ($P_{f,op} \sim 1.50 \times 10^{-5}$). The different conclusions from the 2 approaches probably arise due to violations of the assumptions inherent in the environmental contour method that can, in general, lead to incorrect design loads and/or incorrect identification of the turbine state that affects failures to a greater extent. A key assumption made in the environmental contour method is that the true limit state function may be replaced by a tangent hyperplane at the design point, and that the failure probability may be simply derived using the area on the (unsafe) side of this hyperplane away from the origin in standard normal, $\boldsymbol{U}$, space. Errors in estimating design loads may be significant if the true limit state function is highly nonlinear and/or has no well-defined shape. In addition, the environmental contour method also assumes that there is

Site	Loads	Environmental Contour Method		Full Integration					% error in L_{des}
		P_T	L_{des} (MN, MN-m)	$P_{f,total}$	$P_{f,op}$	$P_{f,pk}$	$P_{f,pk}/P_{f,op}$	L_{des} (MN, MN-m)	
Rødsand	Shear	1.05×10^{-4}	1.02 (op)	1.05×10^{-4}	1.50×10^{-5}	9.01×10^{-5}	6.0	1.00 (pk)	2.0
	Moment	1.05×10^{-4}	31.40 (op)	1.05×10^{-4}	9.58×10^{-5}	8.73×10^{-6}	0.09	31.28 (op)	0.4
Horns Rev	Shear	9.13×10^{-5}	1.39 (pk)	9.13×10^{-5}	≈ 0	9.13×10^{-5}	Inf	1.39 (pk)	0.0
	Moment	9.13×10^{-5}	31.71 (op)	9.13×10^{-5}	2.22×10^{-5}	6.91×10^{-5}	3.1	31.62 (pk)	0.3

Table 3 Fifty-year design loads and implied failure probabilities for parked and operating turbine states at Rødsand and Horns Rev sites based on environmental contour method and full integration

only one mode of failure or, equivalently, one governing turbine state here, and that the failure probability may be computed using that one governing state alone. The probability associated with a given load level due to failures in any secondary turbine state(s) is assumed to be negligibly small and is ignored in the calculations. Hence, implied in the environmental contour approach is the following:

$$\begin{aligned} P[L > L_{des}] &= P[\text{fails in one governing state}] \\ &\approx \Phi(-\beta) = Vol_L \end{aligned} \tag{6}$$

where Vol_L is the volume under the bivariate normal distribution function (in $\boldsymbol{U}$ space) on the unsafe side of the assumed linear limit state surface. In Fig. 9, a schematic representation of a situation is presented where 2 turbine states are considered: State 1 is the governing state, and State 2 a secondary state. A linearization of the limit state function for State 1 is also shown. Errors in the environmental contour approach will be great if any one of the following is true: The linearized limit state surface g_L is a poor approximation of g_1; ignoring State 2 is unjustified because either this secondary state is at a comparable distance to the origin (as State 1), or the unsafe region associated with it is larger than the unsafe region associated with State 1.

For wind turbines, these 2 errors are indeed possible because, as in the illustrations here, there is usually more than one turbine state to consider. For example, here, the turbine can fail either when operating or when parked. In addition, the limit state surfaces are highly irregular. For example, here, State 1's limit state surface is not accurately substituted by a tangent hyperplane. To illustrate how significant error can result in the computed failure probability, in Fig. 10, we show a plot of the environmental contour in $\boldsymbol{U}$ space used to predict the 50-year design shear force at the Rødsand site. It is seen that the true limit state function under operating conditions associated with a design shear force of 1.02 MN cannot be accurately approximated by a tangent hyperplane. It has a notched-like shape due to discontinuity at the cut-out wind speed (25 m/s) and would be very different from a linearized limit state function (or tangent hyperplane) there. Thus, the estimated failure probability for this operating state, using the environmental contour method, is significantly larger than the exact value (by a factor of 46 as seen in Table 2). It is clear that large errors in the computed failure probability can result with the environmental contour method.

It is clear that another source of error in the environmental contour method can arise from ignoring failure in any secondary turbine state. For instance, this could lead to design loads being underestimated because the true failure probability will be greater than the target probability (if the failure probability associated with the secondary state is accounted for). Instead of Eq. 6, the true failure probability, when 2 possible turbine states are taken into consideration, may be expressed as:

$$\begin{aligned} P[L > L_{des}] &= P[\text{fails in State 1 OR fails in State 2}] \\ &= P[\text{fails in State 1}] + P[\text{fails in State 2}] \\ &= Vol_1 + Vol_2 \end{aligned} \tag{7}$$

where Vol_1 and Vol_2 are the volumes under the bivariate normal distribution function associated with the unsafe regions for State 1 and State 2, respectively. No linearization of limit state functions is necessary. Again, considering the case in Fig. 10 (illustrated schematically in Fig. 11), it is seen from Table 3 that there is a 2% reduction in the design shear when both states are accounted for,

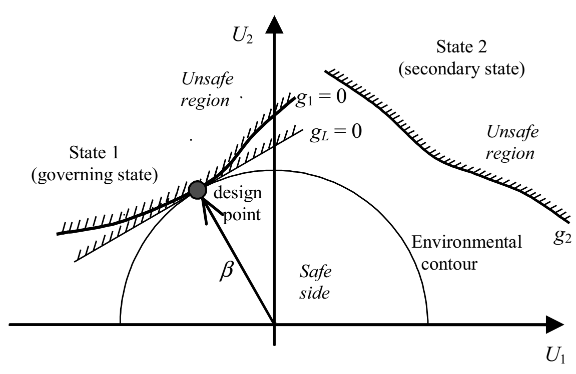

Fig. 9 Schematic representation of environmental contour, limit state functions and design point in $\boldsymbol{U}$ space

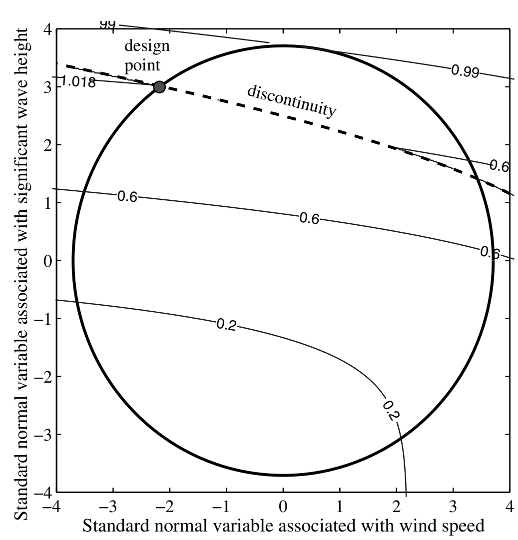

Fig. 10 Environmental contour in standard normal $\boldsymbol{U}$ space associated with 50-year return period at Rødsand site and iso-response curves for shear force at seabed

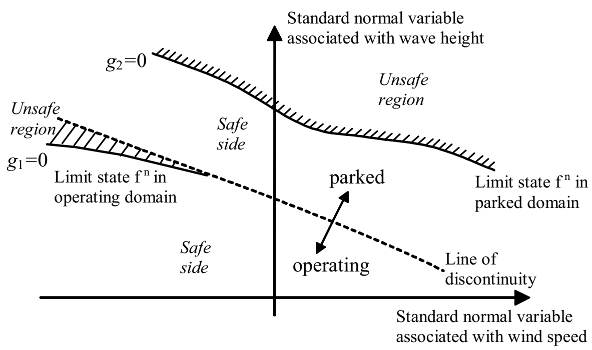

Fig. 11 True limit state functions in operating and parked turbine states plotted in standard normal ***U*** space

as compared to the environmental contour method. This seems like an insignificant difference, but it is small only due to an offsetting effect. The inappropriate linearization of the extremely notched shape of the true limit state function should have led to a gross overestimation of the design shear by the environmental contour method (or, equivalently, the method yields a design level that is associated with a smaller probability than the method suggests). The offsetting effect arises because the overestimation of the failure probability associated with State 1 (operating state) at the design shear is compensated for by the incorrect omission of the failure probability associated with the design shear arising from State 2. The net effect is that, in this case, the design shear from the environmental contour method is only slightly different from the exact value that accounts for both states and makes no linearization simplifications. Note that these offsetting effects may not occur in all situations. Note also that often the environmental contour method did not correctly identify the dominant turbine state for failures (Table 3). In fact, it is only for shear at the Horns Rev site that the environmental contour approach yields negligible error, because for that case the parked state is the governing turbine state, and a tangent hyperplane is then a reasonable approximation of its limit state function, while the secondary (operating) state can be justifiably neglected due to the small probability of failure associated with it.

CONCLUSIONS

We have discussed the use of the environmental contour method to derive design shear forces and overturning moments at the seabed of an active stall-regulated offshore wind turbine. Two different Danish offshore environments, Rødsand and Horns Rev, were considered for the locations of the turbine. The accuracy of the derived design loads was assessed by comparisons based on full integration over the random variables and by accurate descriptions of the failure domain. Even though the environmental contour method is extremely convenient for design purposes, as it uncouples the environment from the response, in some situations it may not yield accurate results. Errors in estimating the design loads are introduced by the fact that the 2 key assumptions used in the environmental contour method are violated:

- The limit state surface, especially for an operating turbine state, may not be well approximated by a tangent hyperplane at the design point.
- Failure associated with secondary turbine states may need to be considered in computing accurate failure probability associated with the design load.

It is recommended that environmental contours and iso-response curves be plotted before establishing design load levels to ensure that the interpretation of results based on this method are meaningful.

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