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Technical Brief

Symmetry Considerations When Using Proper Orthogonal Decomposition for Predicting Wind Turbine Yaw Loads

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In an earlier study, the authors discussed the efficiency of low-dimensional representations of inflow turbulence random fields in predicting statistics of wind turbine loads that included blade and tower bending moments. Both root-mean-square and 10-min *extreme statistics for these loads were approximated very well when time-domain simulations were carried out on a* 600 kW *two-bladed turbine and only a limited number of inflow "modes" were employed using proper orthogonal decomposition (POD). Here, turbine yaw loads are considered and the conventional ordering of POD modes is seen to be not as efficient in predicting full-field load statistics for the same turbine. Based on symmetry arguments, reasons for a different treatment of yaw loads are presented and reasons for observed deviation from the expected monotonic convergence to full-field load statistics with increasing POD mode number are illustrated.* [DOI: 10.1115/1.2349541]

Introduction

In an earlier study [1], the authors reported on the application of proper orthogonal decomposition [2,3] (POD) techniques to construct efficient low-dimensional representations of the along-wind (also referred to as longitudinal or streamwise) inflow turbulence field and demonstrated its use for deriving wind turbine load statistics. In that study, numerical results from simulations on the two-bladed teetered-rotor 600 kW Advanced Research Turbine [4] (ART) with a blade-pitch control system showed that, with a few inflow turbulence POD patterns/modes, the variance and 10-min extremes of flapwise bending moment (FBM) at blade root, edgewise bending moment (EBM) at blade root, and fore-aft tower bending (TBM) at base were close to levels based on full-field inflow simulation. These results support use of the reduced-order inflow turbulence representations based on the POD technique to predict wind turbine load statistics of FBM, EBM, and TBM. A detailed discussion on the inflow turbulence and turbine response simulations as well as on the POD analysis may be found in the authors' earlier study [1].

Following discussions with Hansen [5], a question was raised regarding the efficiency of low-dimensional inflow representations (such as POD) in predicting statistics of turbine loads that are sensitive to asymmetric spatial sampling of the inflow turbulence field, such as is the case for yaw loads. The issue of interest is whether convergence of load statistics for such asymmetric loads may be different than was the case for FBM, EBM, and TBM. To investigate this, we employed the same POD procedure [1] to decompose an along-wind turbulence field into several proper orthogonal modes. It is worth pointing out here that POD techniques are not best suited for modeling general atmospheric turbulence flow fields. Other data processing techniques and modeling procedures may be more appropriate, especially when one encounters nonstationary flows under non-neutral atmospheric conditions. The limitations of the POD procedures discussed here are that they apply to stationary flows in near-neutral atmospheric conditions.

Numerical Studies

The three-dimensional stationary inflow turbulence field is simulated using SNwind [6] on a 6×6 square grid over the 42 m $\times$ 42 m rotor plane of the ART machine (see Fig. 1). The Kaimal spectral model and the exponential coherence model recommended in the International Electrotechnical Commission (IEC) guidelines [7] for wind turbine design are used for this inflow stochastic simulation. The mean wind velocity profile is assumed to follow a power law variation, with an exponent of 0.2, as specified in the IEC guidelines [7]. The mean wind velocity at the 36-m hub height of the turbine under consideration is chosen to be 15 m/s. A POD analysis is carried out using the simulated along-wind turbulence data and, therefore, a total of 36 inflow turbulence POD modes are derived to represent the field. (Again, detailed discussions on the POD analysis may be found in the previous work by the authors [1].) The eigenvalues λ_i (associated with the kinetic energy of the inflow modes) and the first nine eigenmodes ϕ_i of the sample covariance matrix of the simulated along-wind turbulence field are shown in Figs. 2 and 3, respectively. The first 18 inflow mode shapes based on the POD analysis are also plotted over the rotor plane in Figs. 4(*a*) and 4(*b*) emphasizing, by shades of grey, the relative modal amplitudes at various locations on the plane. The less energetic higher mode shapes not shown here are far more complex.

In the standard approach using POD [1], reduced-order representations for the along-wind inflow turbulence field are constructed by simply truncating higher inflow modes corresponding to low levels of energy. Here, ten 10-min wind turbine FAST [8] simulations on the ART machine are performed by employing such reduced-order POD representations of the inflow turbulence field with different numbers of POD modes, along with complete representations of the across-wind (also referred to as lateral or cross-wind) and vertical turbulence components, to derive statis-

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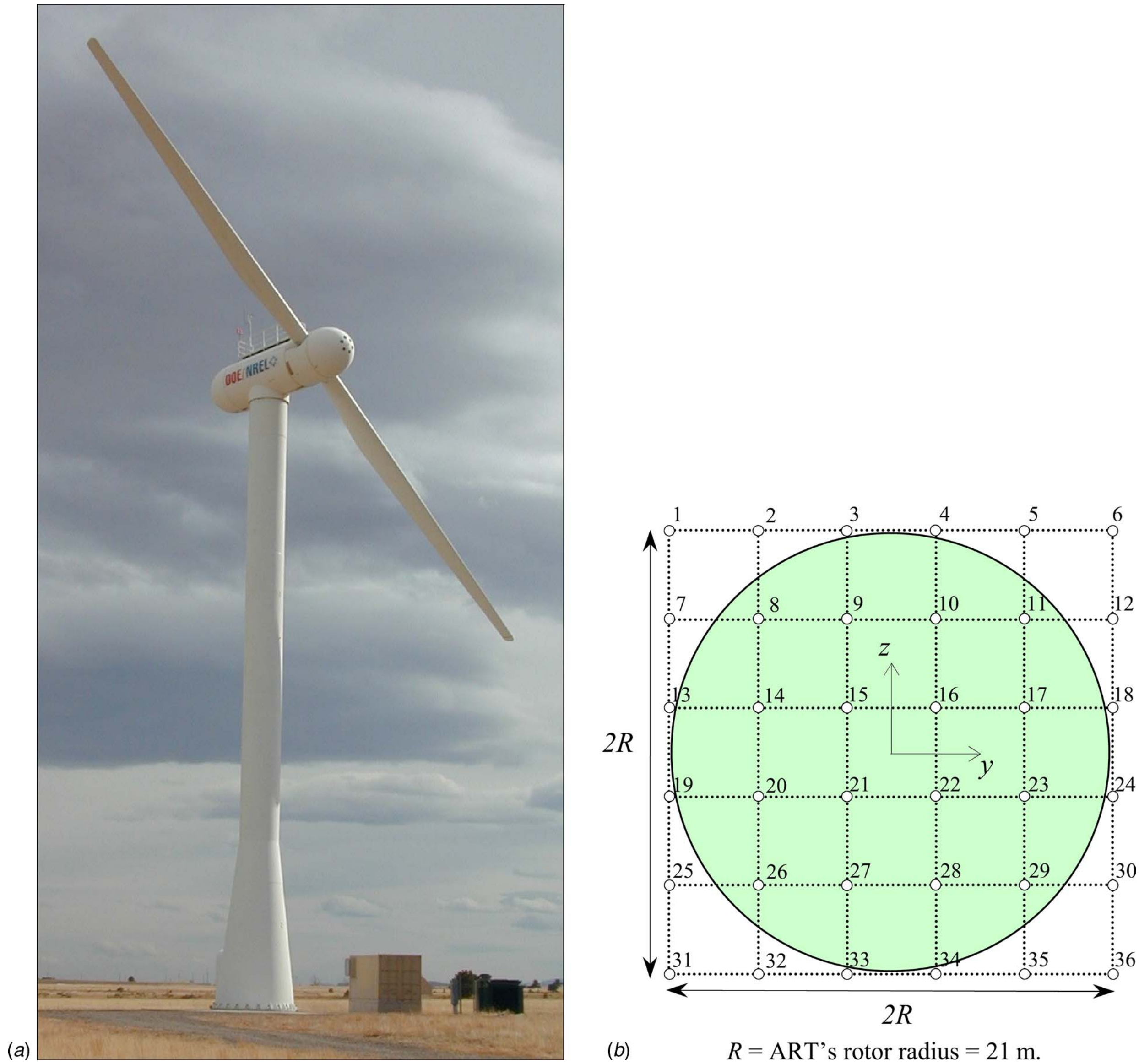


Fig. 1 **(*a*) ART machine, (*b*) spatial grid for simulations of turbulence on the 42 m×42 m rotor plane**

tics of the FBM, EBM, TBM, and the yaw bending moment (YBM). (Note that these yaw bending loads were not considered in the earlier study [1].) Results, as shown in Figs. 5 and 6, reveal that convergence rates for the variance and 10-min extremes of the yaw load are very slow compared with those for the blade and tower bending loads. Even though the ten most energetic inflow POD (out of 36) modes carry more than 85% of the total energy of the along-wind turbulence field, they provide only roughly 65% and 80%, respectively, of the target variance and 10-min extreme yaw load levels.

A possible physical explanation for these results (i.e., slower convergence for yaw loads) is as follows. For a wind turbine with a rigid rotor system, the dominant contribution to yaw bending moment is from the difference in flapwise bending loads on each of the blades that can bring about a moment in the appropriate direction of yaw [9]. This imbalance will, in general, be largest on a two-bladed turbine when the blades are aligned almost horizontally. On the other hand, for a turbine with a teetered-rotor system, as is the case here, the flapwise moments cannot be transmitted through the teetering pin and, hence, yaw loads here are likely brought about by the imbalance of other internal moments and shear forces on each of the blades. For a teetered-rotor system, if the rotor axis is tilted, yaw loads can be caused by the imbalance of edgewise bending loads, particularly when the blades are aligned almost vertically. In either case, i.e., for a rigid or teetering rotor, the yaw load is not influenced very much by POD modes of the inflow turbulence field that exhibit symmetry such that modal amplitudes at two diametrically opposite points are at similar levels (such as is the case for modes 4, 5, and 6 in Fig. 4(*a*)), even when these modes carry a significant portion of the energy in the inflow turbulence field. In contrast, a slight imbalance in internal forces and moments on the two blades caused by asymmetry of even relatively less energetic inflow patterns may result in large yaw loads. Truncating modes only on the basis of energy of the inflow turbulence as is done in traditional low-dimensional POD representations, may therefore lead to inaccurate predictions of statistics of yaw loads. The significance of such asymmetric inflow modes on yaw loads has been studied in great detail by Hansen [9] and later verified in field measurements by Tangler et al. [10]. In other words, for asymmetric loads such as yaw moment, retaining inflow modes based on decreasing eigenvalues (energy) without consideration for symmetry may not be

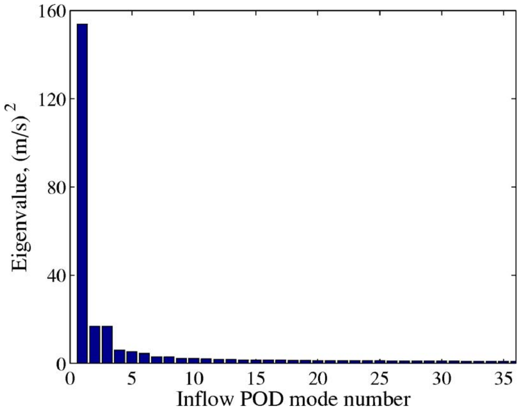


Fig. 2 Eigenvalues λ_i of the sample covariance matrix of the simulated along-wind turbulence field

optimal or efficient. As a consequence, the application of POD's compact inflow representations to estimate asymmetric turbine loads must be used with care.

Symmetry Considerations for Yaw Loads

This hypothesis that symmetry considerations are important for yaw loads may be verified numerically here by calculating the contribution to the variance of the yaw load process from each along-wind inflow mode (additional to that caused by the mean wind profile). Results are shown in Fig. 7 where when compared to Fig. 2, one can see that the variance of the yaw load process does not decrease monotonically with mode number. The four most important modes for yaw load variance are, in decreasing order, mode numbers 2, 3, 8, and 1, respectively. (Also, some fairly insignificant higher modes based on Fig. 2 such as mode numbers 10, 18, 19, and 23 contribute significant energy to the yaw load.) These four most important modes for yaw loads are studied in more detail in Fig. 8 where the four shapes are shown overlaid by snapshots of blade positions that can result in significant yaw loads due to asymmetry. Several other modes (e.g., mode 4 which exhibits much symmetry) are relatively less important even though they account for considerable energy in the turbulence field because they have no way of bringing about large yaw loads (see Fig. 4(*a*)). Clearly both energy as well as asymmetry of the mode shape are important. Mode 1, for example, appears among the top four modes though it is almost uniform in shape only because it accounts for 62.3% of the energy in the inflow field. The results in Figs. 2 and 7 confirm that the relative importance of each inflow mode to the yaw load cannot be evaluated merely by looking at the kinetic energy of that mode. Similar results regarding the importance of asymmetric inflow turbulence modes were also obtained when other load types such as the side-to-side bending moment at the tower base were studied. As a consequence, POD procedures for the analysis of any turbine load sensitive to asymmetric flow modes (such as yaw bending moments, side-to-side tower bending moments, etc.) must be used with caution, taking care not to omit some modes with potentially greater influence though associated with lower inflow energy.

Figures 9 and 10 show estimates of the variance and mean 10-min extremes, respectively, of the yaw loads that result from using two different orderings for truncation of inflow modes: (i) based on inflow energy (Fig. 2); and (ii) based on the actual contribution to yaw load variance (Fig. 7) as determined from turbine load simulations. When the basis of ordering is changed from (i) to (ii), increases in convergence rates (albeit, quite small) for both variance and 10-min extremes of the yaw loads are seen with each additional mode.

It should be noted that despite reordering modes in low-order

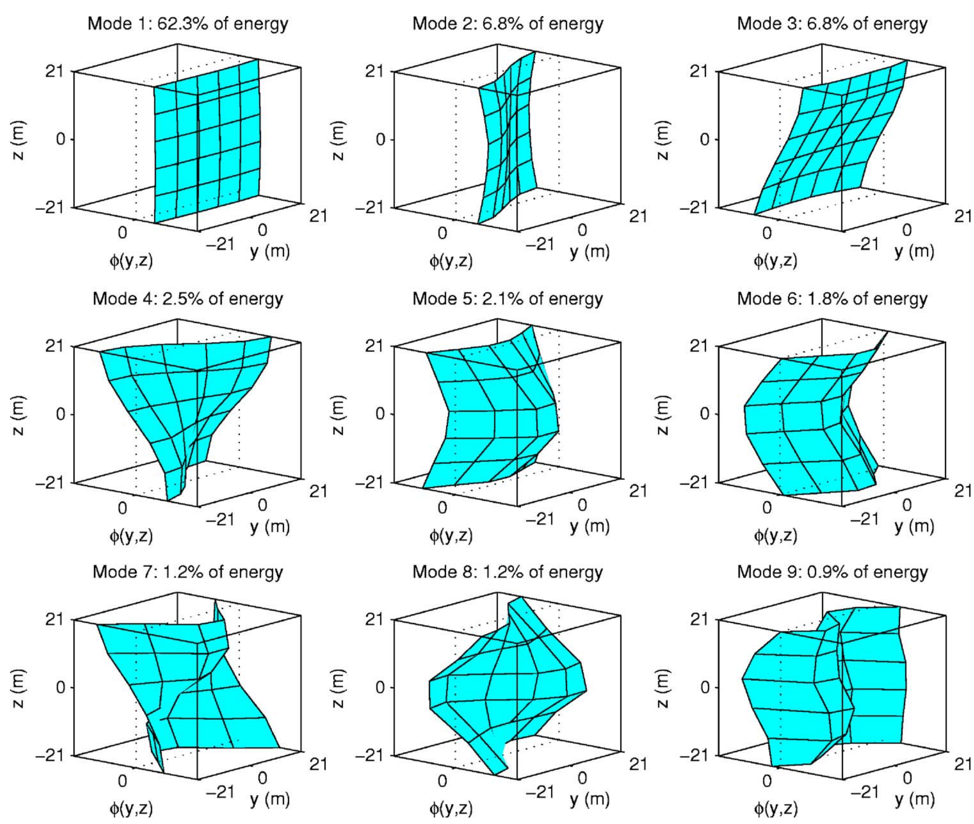
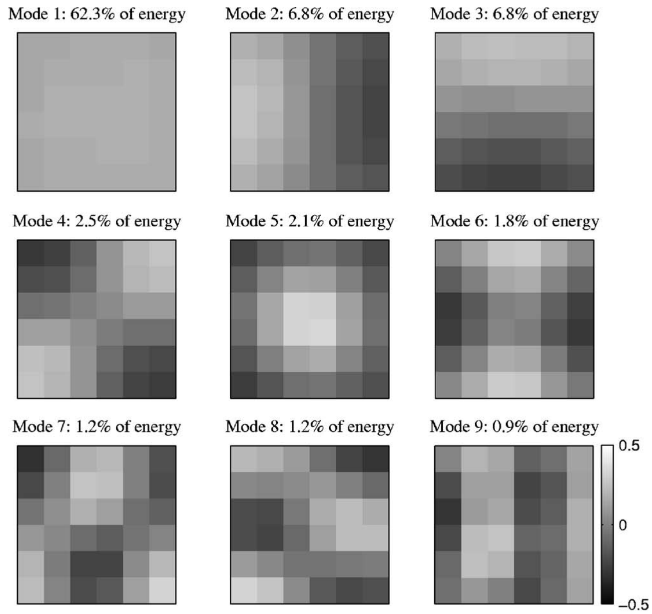
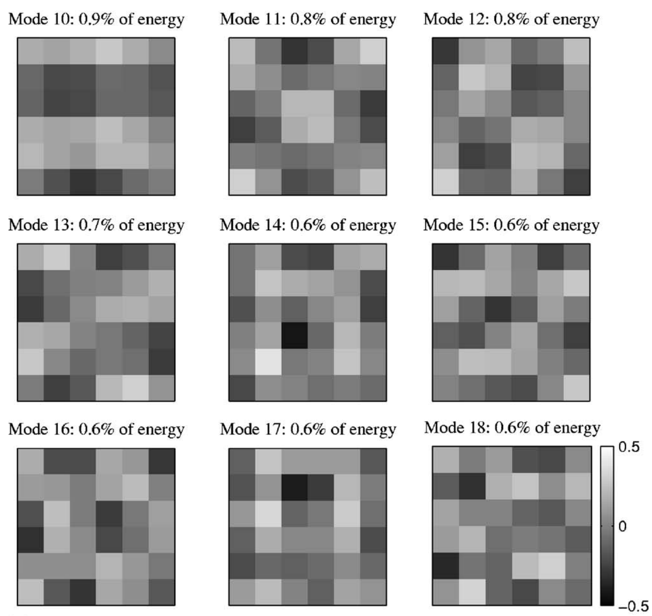


Fig. 3 First 9 (out of 36) eigenmodes of the simulated along-wind turbulence field over the 42 m×42 m rotor plane of the ART machine with the corresponding fraction of total energy



(*a*)



(*b*)

Fig. 4 (*a*). The first through ninth eigenmodes of the simulated along-wind turbulence field over the 42 m×42 m rotor plane of the ART machine with the corresponding fraction of total energy; (*b*) the 10th–18th eigenmodes of the simulated along-wind turbulence field over the 42 m×42 m rotor plane of the ART machine with the corresponding fraction of total energy

representations, based on Fig. 7, convergence rates for variance and extremes of yaw loads as seen in Figs. 9 and 10 are not nearly as good as for FBM, EBM, and TBM (Figs. 5 and 6). For instance, with 20 POD modes, variance convergence for YBM is still about 10% from the target full-field level whereas for FBM, EBM, and TBM, this deficit in variance is only 2–3% from the target. Extreme convergence rates are somewhat better with an increase in the number of modes with greatest benefit from the first few modes but lower rates of convergence than all the other three loads (FBM, EBM, and TBM) after say the first ten modes are included. The nature of yaw loads is such that they often result from small disturbances in the turbulence field that cannot be

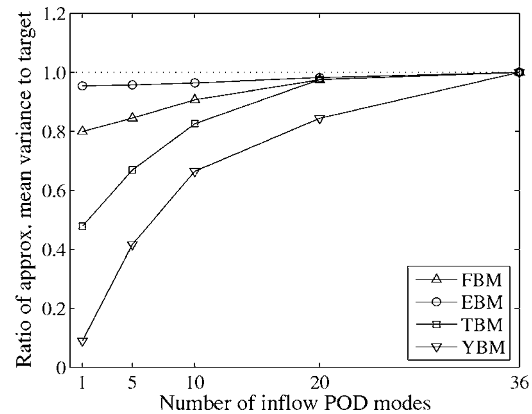


Fig. 5 Ratio of variance of turbine load measures based on 1, 5, 10, and 20 POD modes to that based on full-field inflow simulation

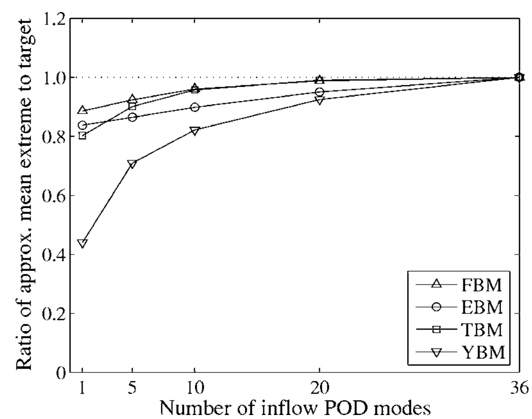


Fig. 6 Ratio of 10-min extreme of turbine load measures based on 1, 5, 10, and 20 POD modes to that based on full-field inflow simulation

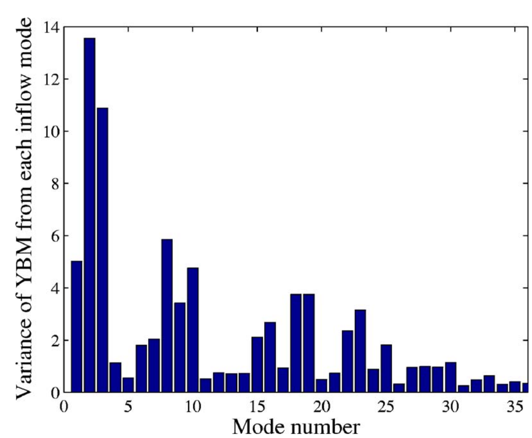


Fig. 7 The variance of the yaw bending moment (YBM) process contributed by each inflow POD mode (additional to that caused by the mean wind profile)

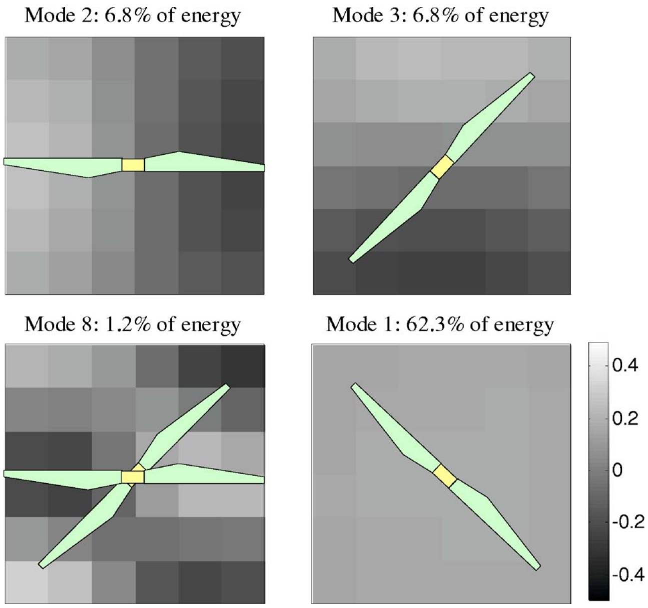


Fig. 8 The first four most important inflow mode shapes for yaw load based on the results from Fig. 7 together with the positions of the blades where imbalance of internal forces on the two blades brings about large yaw loads. The amount of field inflow energy from each mode is also shown.

captured by using only a small number of inflow POD modes. Such small disturbances rely on more complex higher modes, often of low energy, that can contribute significantly to yaw load statistics. Hence, a slower convergence rate for YBM results and, as can be confirmed by studying Fig. 7, a large number of POD modes show significant contribution to yaw loads; for FBM, EBM, and TBM, the relative importance of the modes looks more like that of the inflow energy itself (Fig. 2) suggesting faster convergence should result. In summary, then, even with appropriate ordering of the inflow POD modes, a greater number of modes are generally required to obtain the same level of accuracy in yaw load statistics compared to the blade and tower loads we had studied before [1].

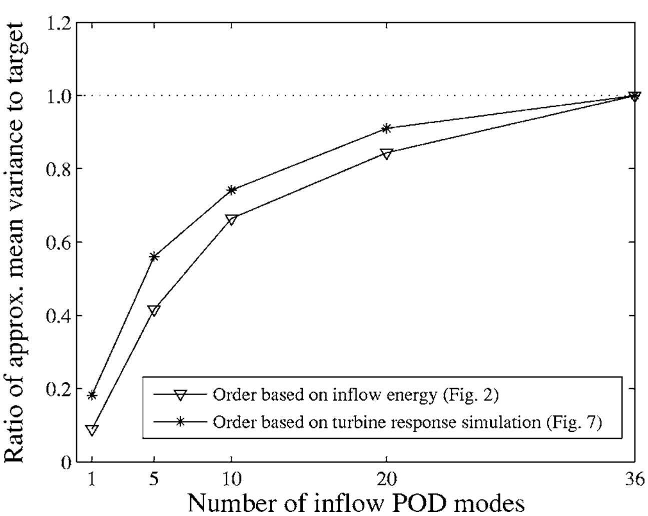


Fig. 9 Ratio of variance of yaw bending moment for two different POD mode orderings to that based on full-field inflow simulation

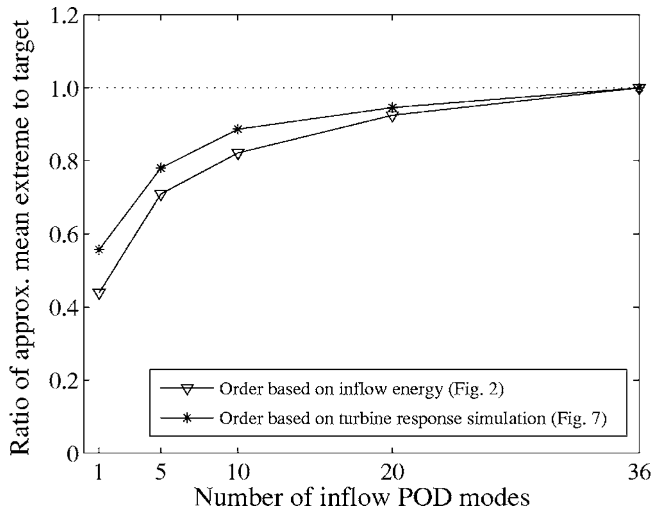


Fig. 10 Ratio of the mean 10-min extreme of yaw bending moment for two different POD mode orderings to that based on full-field inflow simulation

Conclusion

An earlier study [1] by the authors had reported on the efficiency of proper orthogonal decomposition for establishing an efficient low-dimensional representation of the inflow turbulence field that could be used for deriving wind turbine load statistics, including the variance and 10-min extremes of flap and edge bending moments at a blade root and tower bending moment at the base. With a relatively small number of inflow POD modes, approximate statistics were close to target (or full-field) levels. The present study reveals that the convergence rate for yaw load statistics with increase in the number of inflow POD modes employed is comparatively slower. This is because, apart from the kinetic energy of each mode, the yaw load is also sensitive to asymmetric inflow spatial patterns evident in these modes. To obtain a better prediction of yaw loads, such asymmetric inflow modes need to be incorporated in low-dimensional representations even when they, in some cases, account for relatively low levels of inflow kinetic energy. Using a mode reordering based on the actual contributions to the yaw load variance as determined from turbine load simulations, some improvement in the convergence of yaw load statistics was found to result. However, compared to blade and tower bending moments, yaw load statistics converge more slowly with increase in number of POD modes included, regardless of the basis for the ordering of importance. This is at least partly due to the nature of yaw loads that can result from small disturbances in the turbulence field that in turn require inclusion of higher POD modes often of low energy. Therefore, even with different orderings of the inflow POD modes, a larger number of inflow POD modes are generally required to obtain the same level of accuracy in load statistics compared to that for the other turbine loads studied before.

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Nomenclature

FBM = flapwise blade bending moment
EBM = edgewise blade bending moment

TBM = fore-aft tower bending moment
YBM = yaw moment

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