Appendices to

"A New Utility-Consistent Econometric Approach to Multivariate Count Data Modeling"

by

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APPENDIX A

The notations used here will be the same as those used in the text. Before providing proofs for the theorems in the main text, we provide the following well established results for the multivariate normal distribution, collected together in a single Lemma (without proof).

Lemma 1

1) The multivariate normal density function and cumulative distribution function of dimension R are respectively given by $f_R(\boldsymbol{z};\boldsymbol{\tau},\boldsymbol{\Gamma}) = \left(\prod_{r=1}^{R} \omega_{\Gamma r}\right)^{-1} \phi_R\left(\boldsymbol{\omega}_{\boldsymbol{\Gamma}}^{-1}[\boldsymbol{z}-\boldsymbol{\tau}];\boldsymbol{\Gamma}^*\right)$ and $F_R(\boldsymbol{z};\boldsymbol{\tau},\boldsymbol{\Gamma}) = \Phi_R\left(\boldsymbol{\omega}_{\boldsymbol{\Gamma}}^{-1}[\boldsymbol{z}-\boldsymbol{\tau}];\boldsymbol{\Gamma}^*\right)$, where $\boldsymbol{\Gamma}^* = \boldsymbol{\omega}_{\boldsymbol{\Gamma}}^{-1}\boldsymbol{\Gamma}\boldsymbol{\omega}_{\boldsymbol{\Gamma}}^{-1}$.

2) Let $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ be normally distributed vectors of dimension I_1 and I_2, respectively. The corresponding mean vector and covariance matrix of $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ are $(\boldsymbol{b}_1,\boldsymbol{\Sigma}_{11})$ and $(\boldsymbol{b}_2,\boldsymbol{\Sigma}_{22})$. Defining $\boldsymbol{X}=(\boldsymbol{X}_1',\boldsymbol{X}_2')'$, $\boldsymbol{b}=(\boldsymbol{b}_1',\boldsymbol{b}_2')$ and $\boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12}' \\ \boldsymbol{\Sigma}_{12} & \boldsymbol{\Sigma}_{22} \end{pmatrix}$, where $\boldsymbol{\Sigma}_{12}$ is the covariance matrix between $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$, the conditional distribution of $\boldsymbol{X}_2$ given $\boldsymbol{X}_1$ is $\boldsymbol{X}_2 \mid (\boldsymbol{X}_1=\boldsymbol{x}_1) = MVN_{I_2}[\boldsymbol{b}_2+\boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{11}^{-1}(\boldsymbol{x}_1-\boldsymbol{b}_1), \boldsymbol{\Sigma}_{22}-\boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12}']$. Then,

$$\frac{\partial F_I(\boldsymbol{X}_1=\boldsymbol{x}_1,\boldsymbol{X}_2=\boldsymbol{x}_2)}{\partial \boldsymbol{x}_1} = f_{I_1}(\boldsymbol{x}_1;\boldsymbol{b}_1,\boldsymbol{\Sigma}_{11}) \times F_{I_2}(\boldsymbol{x}_2;\boldsymbol{b}_2+\boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{11}^{-1}(\boldsymbol{x}_1-\boldsymbol{b}_1),\boldsymbol{\Sigma}_{22}-\boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{11}^{-1}\boldsymbol{\Sigma}_{12}').$$

In what follows, we present and discuss four theorems that are key to the proposal in this paper.

Theorem 1

The stochastic transformation of $\eta_q = \text{Max}(\breve{\boldsymbol{U}}_q)$ as $g_q^* = \vartheta\eta_q + W_q$, where ϑ is a constant scalar parameter and W_q is a univariate normally distributed scalar $(W_q \sim N(\mu_q,\upsilon_q^2))$, has a cumulative distribution function and density function as below:

$$H(z;\boldsymbol{d}_q,\boldsymbol{\Sigma}_q,\vartheta,\mu_q,\upsilon_q^2) = F_I\left[z\boldsymbol{1}_I;\ \left(\vartheta\boldsymbol{d}_q+\mu_q\boldsymbol{1}_I\right),\ \left(\vartheta^2\boldsymbol{\Sigma}_q+\boldsymbol{I}_{II}\upsilon_q^2\right)\right]$$

Proof:

$$
\begin{aligned}
H(z;\boldsymbol{d}_q,\boldsymbol{\Sigma}_q,\vartheta,\mu_q,\upsilon_q^2) &= P[g_q^* < z] = P\left[\vartheta\, Max(\breve{\boldsymbol{U}}_q)+W_q < z\right] = P\left[Max(\vartheta\breve{\boldsymbol{U}}_q + W_q\boldsymbol{1}_I) < z\right] \\
&= P\left[\vartheta\breve{U}_{q1}+W_q < z \text{ and } \vartheta\breve{U}_{q2}+W_q < z \text{ and ,..., } \vartheta\breve{U}_{qI}+W_q < z\right] \\
&= P[\boldsymbol{UW_q} < z], \text{ where } \boldsymbol{UW_q} \sim MVN_I(\vartheta\boldsymbol{d}_q+\mu_q\boldsymbol{1}_I, \vartheta^2\boldsymbol{\Sigma}_q+\boldsymbol{1}_{II}\upsilon_q^2) \\
&= F_I(z\boldsymbol{1}_I;\vartheta\boldsymbol{d}_q+\mu_q\boldsymbol{1}_I,\vartheta^2\boldsymbol{\Sigma}_q+\boldsymbol{1}_{II}\upsilon_q^2)
\end{aligned}
$$

Theorem 2 – Proposition (1)

The probability function of $\eta = \text{Max}\ (\breve{\boldsymbol{U}})$ is given by $g(z;\boldsymbol{d},\boldsymbol{\Sigma})$ as follows:

$$
g(z;\boldsymbol{d},\boldsymbol{\Sigma}) = \sum_{i=1}^{I} f\left(z;d_i,\omega_{\Sigma i}^2\right)\times F_{I\text{-}1}\left(z\boldsymbol{1}_{I-1};\boldsymbol{d}_{\text{-}i}+\overline{\boldsymbol{\Sigma}}_{\text{-}i}(\omega_{\Sigma i}^2)^{\text{-}1}(z-d_i),\widetilde{\boldsymbol{\Sigma}}_i\right),
$$

where $\widetilde{\boldsymbol{\Sigma}}_i = \boldsymbol{\Sigma}_{\text{-}i,\text{-}i} - \overline{\boldsymbol{\Sigma}}_{\text{-}i}(\omega_{\Sigma i}^2)^{\text{-}1}(\overline{\boldsymbol{\Sigma}}_{\text{-}i})'$.

Proof:

The cumulative distribution function of $\eta = \text{Max}\ (\breve{\boldsymbol{U}})$ is given by (see Tellambura, 2008):

$$
\begin{aligned}
G(z;\boldsymbol{d},\boldsymbol{\Sigma}) &= Prob[\text{Max}(\boldsymbol{X})<z] = Prob[X_1 < z \text{ and } X_2 < z \text{ and.... } X_I < z] \\
&= F_I(z\boldsymbol{1}_I;\boldsymbol{d},\boldsymbol{\Sigma}).
\end{aligned}
$$

The proof that the density function takes the form as given above can be shown by differentiating $G(z;\boldsymbol{b},\boldsymbol{\Sigma})$ with respect to z and using the last result from Lemma 1.

Theorem 2 – Proposition (2)

The moment generating function of η is given by:

$$
M_\eta(t) = \int_{z=-\infty}^{\infty} e^{tz} g(z;\boldsymbol{d},\boldsymbol{\Sigma})dz = \sum_{i=1}^{I} e^{td_i+\frac{1}{2}\omega_{\Sigma i}^2 t^2}\times F_{I-1}(\mathrm{s}_{\Sigma i}\, t;\boldsymbol{\gamma}_i,\boldsymbol{\Psi}_i),
$$

where $\mathrm{s}_{\Sigma i} = \lambda_i\,\omega_{\Sigma i}$, $\lambda_i = \omega_{\Sigma i}\boldsymbol{1}_{I-1} - \dfrac{\overline{\boldsymbol{\Sigma}}_{-i}}{\omega_{\Sigma i}}$, $\boldsymbol{\gamma}_i = \boldsymbol{d}_{-i} - d_i\boldsymbol{1}_{I-1}$, and $\boldsymbol{\Psi}_{i\xi} = \widetilde{\boldsymbol{\Sigma}}_i + \lambda_i\lambda_i'$.

Proof:

$$
g(z;\boldsymbol{d},\boldsymbol{\Sigma}) = \sum_{i=1}^{I} f\left(z;d_i,\omega_{\Sigma i}^2\right)\times F_{I\text{-}1}\left(z\boldsymbol{1}_{I-1};\boldsymbol{d}_{\text{-}i}+\overline{\boldsymbol{\Sigma}}_{\text{-}i}(\omega_{\Sigma i}^2)^{\text{-}1}(z-d_i),\widetilde{\boldsymbol{\Sigma}}_i\right)
$$

$$= \sum_{i=1}^{I} f(z; d_i, \omega_{\Sigma i}^2) \times F_{I-1}\left(\left(\frac{z\mathbf{1}_{I-1} - d_i \mathbf{1}_{I-1}}{\omega_{\Sigma i}} \right) \omega_{\Sigma i} - \frac{\overline{\mathbf{\Sigma}}_{-i}}{\omega_{\Sigma i}} \left(\frac{z - d_i)}{\omega_{\Sigma i}} \right); \boldsymbol{\gamma}_i, \widetilde{\mathbf{\Sigma}}_i \right)$$

$$= \sum_{i=1}^{I} \frac{1}{\omega_{\Sigma i}} \phi(y_i) \times F_{I-1}\left(\lambda_i\, y_i; \boldsymbol{\gamma}_i, \widetilde{\mathbf{\Sigma}}_i \right)$$

where $y_i = \dfrac{z - d_i}{\omega_{\Sigma i}}$.

The moment generating function of η is given by:

$$M_\eta(t) = \int_{z=-\infty}^{\infty} e^{tz} g(z; \boldsymbol{d}, \mathbf{\Sigma})\, dz = \int_{z=-\infty}^{\infty} e^{tz} \sum_{i=1}^{I} \frac{1}{\omega_{\Sigma i}} \phi(y_i) \times F_{I-1}\left(\lambda_i\, y_i; \boldsymbol{\gamma}_i, \widetilde{\mathbf{\Sigma}}_i \right) dz$$

$$= \sum_{i=1}^{I} \int_{y_i=-\infty}^{\infty} e^{t(y_i \omega_{\Sigma i} + d_i)} \phi(y_i) \times F_{I-1}\left(\lambda_i\, y_i; \boldsymbol{\gamma}_i, \widetilde{\mathbf{\Sigma}}_i \right) dy_i$$

$$= \sum_{i=1}^{I} \int_{y_i=-\infty}^{\infty} e^{td_i} e^{t(y_i \omega_{\Sigma i})} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} y_i^2} \times F_{I-1}\left(\lambda_i\, y_i; \boldsymbol{\gamma}_i, \widetilde{\mathbf{\Sigma}}_i \right) dy_i$$

$$= \sum_{i=1}^{I} \int_{y_i=-\infty}^{\infty} e^{td_i} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(y_i^2 - 2 y_i \omega_{\Sigma i} t + \omega_{\Sigma i}^2 t^2 \right) + \frac{1}{2} \omega_{\Sigma i}^2 t^2} \times F_{I-1}\left(\lambda_i\, y_i; \boldsymbol{\gamma}_i, \widetilde{\mathbf{\Sigma}}_i \right) dy_i$$

$$= \sum_{i=1}^{I} \int_{y_i=-\infty}^{\infty} e^{td_i + \frac{1}{2} \omega_{\Sigma i}^2 t^2} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(y_i - \omega_{\Sigma i} t)^2} \times F_{I-1}\left(\lambda_i\, y_i; \boldsymbol{\gamma}_i, \widetilde{\mathbf{\Sigma}}_i \right) dy_i$$

$$= \sum_{i=1}^{I} e^{td_i + \frac{1}{2} \omega_{\Sigma i}^2 t^2} \int_{u_i=-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} u_i^2} \times F_{I-1}\left(\lambda_i (u_i + \omega_{\Sigma i} t); \boldsymbol{\gamma}_i, \widetilde{\mathbf{\Sigma}}_i \right) du_i, \text{ where } u_i = y_i - \omega_{\Sigma i} t$$

$$= \sum_{i=1}^{I} e^{td_i + \frac{1}{2} \omega_{\Sigma i}^2 t^2} \int_{u_i=-\infty}^{\infty} \phi(u_i) \times F_{I-1}\left(\lambda_i (u_i + \omega_{\Sigma i} t); \boldsymbol{\gamma}_i, \widetilde{\mathbf{\Sigma}}_i \right) du_i$$

$$= \sum_{i=1}^{I} e^{td_i + \frac{1}{2} \omega_{\Sigma i}^2 t^2} \times E_{u_i}\left[F_{I-1}\left(\lambda_i (u_i + \omega_{\Sigma i} t); \boldsymbol{\gamma}_i, \widetilde{\mathbf{\Sigma}}_i \right) \right]$$

$$= \sum_{i=1}^{I} e^{td_i + \frac{1}{2} \omega_{\Sigma i}^2 t^2} \times F_{I-1}\left(\lambda_i\, \omega_{\Sigma i} t; \boldsymbol{\gamma}_i, \widetilde{\mathbf{\Sigma}}_i + \lambda_i \lambda_i' \right)$$

since $E_{u_i}\left[F_{I-1}\left(a + \boldsymbol{B}\, u_i; \boldsymbol{\gamma}_i, \widetilde{\mathbf{\Sigma}}_i \right) \right] = F_{I-1}\left(a; \boldsymbol{\gamma}_i, \widetilde{\mathbf{\Sigma}}_i + \boldsymbol{BB'} \right)$ for all scalar *a,* vector $\boldsymbol{B}$ and random variable $u_i \sim N(0,1)$ (see Marsaglia, 1963 and Gupta *et al.*, 2004).

Finally, $M_\eta(t) = \sum_{i=1}^{I} e^{td_i + \frac{1}{2} \omega_{\Sigma i}^2 t^2} \times F_{I-1}(\mathrm{s}_{\Sigma i}\, t; \boldsymbol{\gamma}_i, \mathbf{\Psi}_i)$.

Theorem 2 – Proposition (3)

This proposition can be proved through straightforward, but tedious, differentiation, and using the results of Lemma 1.

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APPENDIX B: CONSISTENCY WITH TWO-STAGE BUDGETING

The proposed approach that combines a total count model with a model that allocates the count to different event types is analogous to the two stage budgeting procedure in utility-based consumer theory. The basic idea of two stage budgeting is to determine a budget for a specific group of commodities at a first stage (through the development of a scalar price index for the commodity group) in such a way that the first stage utility maximization can progress without the need to worry about allocations to particular commodities within the group. Once the budget is determined at the first stage, the allocation of the budget to individual commodities is pursued in a second stage. The approach makes use of the notion of weak separability of the direct utility function. Our presentation follows that of Hausman *et al.* (1995), except that there is a difficulty with the Hausman *et al.* formulation that makes it incompatible with two-stage budgeting, while our formulation is.

Consider a direct utility function in which a group of commodities is separable from the rest. The group of commodities corresponds to the one whose count is being modeled. So, it may correspond to recreational or grocery shopping trips (with the event type being alternative destinations), or to vehicle ownership level (with the event type being alternative body types), or, as in our empirical application, the number of out-of-home non-work episodes (with the event type being different time periods of the day). The notion of separability implies that the commodity group can be represented by a group utility function in the first stage of the two-stage budgeting process in which the overall budget allocation to the commodity group is being determined in the presence of other commodity groups. It also implies that the optimal allocation of the budget within the commodity group can be determined solely by the group utility function in a second stage, once the budget to the commodity group is determined in the first stage and the prices of individual commodities in the group are known (the reader is referred to Deaton and Muelbaurer, 1980, for a detailed description of the concepts of separability and two-stage budgeting; a comprehensive discussion is well beyond the scope of this paper).

An important issue in the two-stage budgeting is the question of how to determine the budget allocation to the commodity group in the first stage. While one can consider many different formulations, it would be particularly convenient if there were no need to explicitly consider the detailed vector information of the prices of all the individual commodities in the group in this first stage. The question then is whether one can use a group (scalar) price index for the commodity group at this first stage. Gorman (1959) studied this problem in his seminal research, and concluded that one can use a scalar price index if, in addition to the separability property of the overall utility function, this overall utility function in the first stage is additive in the group utility functions <u>and</u> the group indirect utility functions (corresponding to the group direct utility functions) follow what is now referred to as the Gorman Polar Form (GPF). We start with this group indirect utility function of the GPF form for the commodity group of interest. In the following presentation, we suppress the index q for the individual, and, as in Hausman *et al.* (1995), consider the group utility function to be homothetic. Then, we can write the group indirect utility function for the commodity group D as a function of the budget for the commodity group (y_D) and the vector of prices $(\boldsymbol{p_D})$ of the goods within the commodity group D:

$$V_D(y_D, \boldsymbol{p_D}) = \frac{y_D}{r(\boldsymbol{p_D})} \tag{B.1}$$

In the above GPF equation, $r(\boldsymbol{p_D})$ represents the group scalar price index. The functional form of $r(\boldsymbol{p_D})$ must be homogenous of degree one. If this condition is satisfied, then information about the value of $r(\boldsymbol{p_D})$ is adequate to determine the budget allocation to the commodity group in the first stage. That is, the entire commodity group can be viewed as a single commodity with price $r(\boldsymbol{p_D})$ in the first stage budgeting, which takes the form of maximizing a direct utility function that takes consumption in other goods and consumption in a single composite "good" representing the commodity group of interest as arguments (subject to the usual budget constraint). The number of units (the total count) of consumption in the commodity group becomes $g_D = \frac{y_D}{r(\boldsymbol{p_D})}$.

The second stage budgeting of the group budget y_D to individual commodities in the group can be obtained by applying Roy's identity to the indirect utility function of Equation (B.1). Specifically, the conditional number of units of consumption of commodity i can be written as:

$$g_i = -\frac{\partial V_D / \partial p_i}{\partial V_D / \partial y_D} = \frac{y_D}{r(\boldsymbol{p_D})} \times \frac{\partial r(\boldsymbol{p_D})}{\partial p_i} = g_D \times \frac{\partial r(\boldsymbol{p_D})}{\partial p_i}, \tag{B.2}$$

where p_i is now the price of commodity *i* within group *D*. To view the above equation as the second stage of a two-stage budgeting procedure, there are two conditions that $r(\boldsymbol{p_D})$ must satisfy: (1) it must be homogeneous of degree one (that is the requirement of the GPF), and (2) $\sum_{i=1}^{I} \frac{\partial r(\boldsymbol{p_D})}{\partial p_i} = 1$ (this allows the interpretation of g_D as the total units (or count) of consumption across all commodities in group *D*). Hausman *et al.* (1995) choose the expected consumer surplus (or accessibility) measure resulting from a multinomial logit model for $r(\boldsymbol{p_D})$. That is, they write $r(\boldsymbol{p_D}) = E(\eta) = E(\text{Max}(\breve{\boldsymbol{U}}) = \frac{1}{\gamma}\ln\left[\sum_{i=1}^{I}\exp(\gamma p_i)\right]$. With this specification, we have $\frac{\partial r(\boldsymbol{p_D})}{\partial p_i} = \frac{\exp(\gamma p_i)}{\sum_{i'=1}^{I}\exp(\gamma p_{i'})} = \Pr(i)$, and therefore the second condition above on $r(\boldsymbol{p_D})$ is satisfied. However, the form used by Hausman *et al.* for $r(\boldsymbol{p_D})$ does not satisfy the first condition because of the presence of the log transformation. Specifically, $r(\alpha\boldsymbol{p_D}) = \frac{1}{\gamma}\ln\left[\sum_{i=1}^{I}\exp(\alpha\gamma p_i)\right] \neq \alpha r(\boldsymbol{p_D})$. Thus, as pointed out by Rouwendal and Boter (2009), Hausman *et al.*'s model specification is not consistent with a single utility maximization setting. Further, the use of any generalized extreme value (GEV) model for the second stage commodity choice is also not consistent with utility theory because the resulting expression for $r(\boldsymbol{p_D})$ is not homogeneous. Rouwendal and Boter (2009) comment that they have not been able to find an expression for $r(\boldsymbol{p_D})$ that satisfies both the conditions stated above. That is exactly where our proposed model comes in. To our knowledge, we are the first to propose a specification for $r(\boldsymbol{p_D})$ that satisfies both the required conditions discussed above for compatibility of the joint count-event type model with two-stage budgeting, while also allowing the probability of choice of commodity *i* to be a function of individual commodity prices (as they should be). In particular, as in Hausman *et al.*, we propose $r(\boldsymbol{p_D}) = E(\eta) = E(\text{Max}(\breve{\boldsymbol{U}})$, except that we specify $\breve{\boldsymbol{U}}$ to be multivariate normal (see previous section; $\breve{\boldsymbol{U}} \sim MVN_I(\boldsymbol{d}, \boldsymbol{\Sigma})$ after suppressing the index *q* for individuals, where $\boldsymbol{d}$ plays the role of a generalized price vector for the set of individual commodities and is interchangeable with $\boldsymbol{p_D}$ in the theoretical model). This specification has not been considered in econometrics and utility theory in the past because the exact density function and moment generating functions for the maximum of multivariate normally distributed variables were not established until very recently. Specifically, it was not until the research of Arellano-Valle and Genton (2008) and Jamalizadeh and Balakrishnan (2009, 2010) that an exact density function and moment generating function was obtained for the maximum of arbitrarily dependent normally distributed random variables. These works show that the distribution of $\eta = \text{Max}\,(\breve{\boldsymbol{U}})$, when $\breve{\boldsymbol{U}}$ has a general multivariate normal distribution, is a mixture of unified univariate skew-normal distribution functions, and then use this mixture representation to derive the density and moment generating functions of η (in doing so, they invoke the density and moment generating functions of the unified univariate skew-normal distribution functions). In this paper, we derive, apparently for the first time, expressions for the density and the moment generating functions for η directly from first principles (rather than going through the circuitous route of using a mixture representation) and explicitly write out these expressions for η (these are buried within the expressions for the general distribution of order statistics in Jamalizadeh and Balakrishnan, 2010). Also, we have not seen an expression for the first moment (or expected value) of η in the literature, which is important because that is the expression for $r(\boldsymbol{p_D})$ in our econometric model. We explicitly derive this expression from the moment generating function of η. These results are collected below as Theorem 2.

Theorem 2

Let $\boldsymbol{d}_{-i}$ be the sub-vector of $\boldsymbol{d}$ without the i^{th} element, let d_i be the i^{th} element of $\boldsymbol{d}$, let $\boldsymbol{\Sigma}_{-i,-i}$ be the sub-matrix of $\boldsymbol{\Sigma}$ without the i^{th} row and the i^{th} column, let $\omega^2_{\Sigma i}$ be the diagonal entry at the i^{th} row and i^{th} column of $\boldsymbol{\Sigma}$, and let $\overline{\boldsymbol{\Sigma}}_{-i}$ be the i^{th} column of the matrix $\boldsymbol{\Sigma}$ minus the i^{th} row element.

(1) Denote the probability density function of $\eta = \text{Max}\,(\breve{\boldsymbol{U}})$ by $g(z;\boldsymbol{d},\boldsymbol{\Sigma})$. Then:

$$g(z;\boldsymbol{d},\boldsymbol{\Sigma}) = \sum_{i=1}^{I} f\left(z; d_i, \omega^2_{\Sigma i}\right) \times F_{I-1}\left(z\mathbf{1}_{I-1}; \boldsymbol{d}_{-i} + \overline{\boldsymbol{\Sigma}}_{-i}(\omega^2_{\Sigma i})^{-1}(z-d_i), \widetilde{\boldsymbol{\Sigma}}_i\right), \tag{B.3}$$

where $\widetilde{\boldsymbol{\Sigma}}_i = \boldsymbol{\Sigma}_{-i,-i} - \overline{\boldsymbol{\Sigma}}_{-i}(\omega^2_{\Sigma i})^{-1}(\overline{\boldsymbol{\Sigma}}_{-i})'$.

(2) The moment generating function of η is given by:

$$M_\eta(t) = \int_{z=-\infty}^{\infty} e^{tz} g(z;\boldsymbol{d},\boldsymbol{\Sigma})dz = \sum_{i=1}^{I} e^{td_i + \frac{1}{2}\omega_{\Sigma i}^2 t^2} \times F_{I-1}(\mathrm{s}_{\Sigma i}\, t; \boldsymbol{\gamma}_i, \boldsymbol{\Psi}_i), \tag{B.4}$$

where $\mathrm{s}_{\Sigma i} = \lambda_i\, \omega_{\Sigma i}$, $\lambda_i = \omega_{\Sigma i}\mathbf{1}_{I-1} - \dfrac{\overline{\boldsymbol{\Sigma}}_{-i}}{\omega_{\Sigma i}}$, $\boldsymbol{\gamma}_i = \boldsymbol{d}_{-i} - d_i \mathbf{1}_{I-1}$, and $\boldsymbol{\Psi}_{i\xi} = \widetilde{\boldsymbol{\Sigma}}_i + \lambda_i \lambda_i'$.

(3) Let $\boldsymbol{\gamma}_{i,\text{-}l}$ be the vector $\boldsymbol{\gamma}_i$ minus the l^{th} row element, γ_{il} the l^{th} element of the vector $\boldsymbol{\gamma}_i$, $s_{\Sigma il}$ the l^{th} element of the vector $\boldsymbol{s}_{\Sigma i}$, $\boldsymbol{\Psi}_{i,-l,-l}$ the sub-matrix of $\boldsymbol{\Psi}_i$ without the l^{th} row and the l^{th} column, $\sigma^2_{\boldsymbol{\Psi}il}$ be the diagonal entry at the l^{th} row and l^{th} column of $\boldsymbol{\Psi}_i$, $\overline{\boldsymbol{\Psi}}_{i,-l}$ be the l^{th} column of the matrix $\boldsymbol{\Psi}_{i\xi}$ minus the l^{th} row element, and the matrix $\boldsymbol{\Delta}_{il} = \boldsymbol{\Psi}_{i,-l,-l} - \overline{\boldsymbol{\Psi}}_{i,-l}(\sigma^2_{\boldsymbol{\Psi}il})^{-1}\overline{\boldsymbol{\Psi}}'_{i,-l}$.

$$\begin{aligned} E(\eta) \; = \left.\frac{dM_\eta(t)}{dt}\right|_{t=o} &= \sum_{i=1}^{I} \Bigg\{ \; d_i F_{I-1}(\boldsymbol{0}_{I-1}; \boldsymbol{\gamma}_i, \boldsymbol{\Psi}_i) \\ &\quad + \sum_{l=1}^{I-1} s_{\Sigma il} f(0; \gamma_{il}, \sigma^2_{\boldsymbol{\Psi}il}) \times F_{I-2}\left[0_{I-2}; \left(\boldsymbol{\gamma}_{i,\text{-}l} - \overline{\boldsymbol{\Psi}}_{i,-l}(\sigma^2_{\boldsymbol{\Psi}il})^{-1}\gamma_{il}\right), \boldsymbol{\Delta}_{il}\right] \Bigg\}. \end{aligned} \tag{B.5}$$

With the expected value of η as above, we now present the following theorem that is crucial to the utility-consistent nature of our proposed model.

Theorem 3

$E(\eta)$ as defined in Equation (B.5) is both homogeneous of degree one and satisfies the condition $\sum_{i=1}^{I} \dfrac{\partial E(\eta)}{\partial d_i} = \sum_{i=1}^{I} \dfrac{\partial E(\eta)}{\partial p_i} = 1$.

The fact that $E(\eta)$ is homogeneous of degree one is proved by noting that $E(\tilde{\eta}) = E[Max(\alpha\breve{\boldsymbol{U}})]$ corresponds to the expected value of the maximum over random variables that are distributed $MVN_I(\alpha\boldsymbol{d}, \alpha^2\boldsymbol{\Sigma})$. Then, by the application of Equation (B.5), we get $E(\tilde{\eta}) = \alpha E(\eta)$.

The condition $\sum_{i=1}^{I} \frac{\partial E(\eta)}{\partial p_i} = 1$ can be proved in many ways. The easiest is to first define $\widetilde{M}_i$ as an $(I-1)\times I$ matrix corresponding to an $(I-1)$ identity matrix with an extra column of '-1' values added as the i^{th} column. Then, statistically speaking, we can write:

$$E(\eta) = \sum_{i=1}^{I} \left[\int_{k=-\infty}^{k=p_i} \left\{ F_{I-1}(\boldsymbol{0}_{I-1}; \widetilde{M}_i \breve{\boldsymbol{U}}, \ \widetilde{M}_i \boldsymbol{\Sigma} \widetilde{M}_i' \right\} dk \right] = \sum_{i=1}^{I} \left[\int_{k=-\infty}^{k=p_i} \Pr(i)\, dk \right]. \tag{B.6}$$

Then, by the first fundamental theorem of calculus, $\frac{\partial E(\eta)}{\partial p_i} = \Pr(i)$, and therefore $\sum_{i=1}^{I} \frac{\partial E(\eta)}{\partial p_i} = 1$.

Based on the results from Theorem 3, we have proved that setting $r(\boldsymbol{p_D}) = E(\eta) = E(\text{Max}(\breve{\boldsymbol{U}}))$, with $\breve{\boldsymbol{U}}$ arising from a multinomial probit formulation for event type provides a theoretic underpinning to integrate the discrete choice model and a count data model into a single integrated utility maximizing framework. In particular, we can now write Equation (B.2) as:

$$g_i = g_D \times \frac{\partial r(\boldsymbol{p_D})}{\partial p_i} = g_D \times \Pr(i). \tag{B.7}$$

That is, the demand for commodity i is a product of the total count of the units of the commodity group consumed times the probability that commodity i is chosen. But everything above is predicated on using $r(\boldsymbol{p_D}) = E(\eta)$ from the MNP model in the count model. Without introducing this linkage, there is no way that prices of individual commodities enter into the total count model, and the resulting model is not utility-consistent. This linkage is precisely what we accomplish in Equation (10) in Section 2 of the paper, but with an important difference. In particular, we recognize that η has a distribution because of the presence of choice model errors. Thus, the precursor to the latent structure part of Equation (10), after reintroducing the index q for individuals, is as follows:

$$g_q^* = \left(\theta + \widetilde{\theta}_q\right)' \boldsymbol{w}_q + \vartheta\left[E(\eta_q) + \left\{\eta_q - E(\eta_q)\right\}\right] + \zeta_q, \tag{B.8}$$

Equation (10) is the net result.

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APPENDIX C: SAMPLE FORMATION PROCEDURES

Several steps were involved in developing the sample used for the empirical analysis. First, only individuals over 18 years of age, and who participated in at least one work activity episode during the survey day on a weekday (Monday to Friday), were selected. Second, we eliminated individuals whose trip diary did not start or end at home. Third, records that contained incomplete information on individual, household, employment-related, and activity and travel characteristics of relevance to the current analysis were removed from the sample. Fourth, several consistency checks were performed and records with missing or inconsistent data were eliminated. The final estimation sample contained 2,113 person observations. Fifth the trip diaries of these 2,113 individuals were processed to obtain, for each individual, the total number of out-of-home non-work episodes undertaken during the survey day, along with the number of these episodes pursued during each of the five time-of-day blocks identified in Section 3.1. Finally, the accessibility measures by the fifteen different industry types were appended to each time-of-day block for each individual as follows. For the before-work (BW) block, the accessibility measures (by industry type) are based off the time the individual would have had to leave home if s/he went directly to work (computed as the individual's work start time minus the estimated direct home-to-work commute time assuming auto mode of travel and an average speed of 30 mph). That is, the accessibility measures corresponding to the individual's estimated departure time from home to work (assuming a direct home-to-work trip) and for the residential Census tract of the individual are designated as the home end accessibilities for the BW block. For the home-to-work commute (HWC) block, the accessibility measures are based off the individual's work start time. For this block, we create two sets of accessibility measures, one for the home end (based on the Census tract of residence) and another for the work end (based on the Census tract of the individual's workplace location). For the work-based (WB) block, the accessibility measures are based on the off-peak period for the work location Census tract. For the work-to-home commute (WHC) block, the accessibility measures are based off the individual's work end time. For this block, we once again create both a home end set of accessibilities as well as a work end set of accessibilities. For the after home arrival from work (AH) block, the accessibilities are based off the time the individual would have arrived home if s/he went directly back home from work (computed as the individual's work end time plus the estimated direct work-to-home commute time assuming auto mode of travel and an average speed of 30 mph). That is, the accessibility measures corresponding to the estimated arrival time back home and for the residential Census tract of the individual (assuming a direct work-to-home trip) are designated as the home end accessibilities. It is important to note that the accessibility measures, as discussed above, vary across the different time-of-day blocks for the same individual.

Table C.1 provides an unweighted summary of select individual, household, work-related and activity and travel characteristics of the final sample.

Table C.1 Sample Characteristics

Variable	Share [%]	Variable	Share [%]
Individual characteristics		**Household characteristics**	
Race and ethnicity		*Household income [US$/year]*	
Non-Hispanic Caucasian	71.56	Less than 80,000	46.66
Hispanic	9.99	80,000 or more	53.34
Non-Hispanic Asian	9.37	*Home location*	
Non-Hispanic African-American	4.45	Urban cluster	94.18
Non-Hispanic Other[1]	4.63	Not in urban cluster	5.82
Gender		**Work-related characteristics**	
Male	52.25	*Employment Industry*	
Female	47.75	Professional, managerial or technical	48.62
Driver status		Sales or services	23.32
Has driver's license	98.58	Clerical or administrative support	14.59
Does not have a driver's license	1.42	Other[2]	13.47
Highest education level		Is self-employed	9.51
At least some college education	76.53	Has flexible work start time	44.87
No college education	23.47	Has more than one job	9.13
Past week primary activity		Has the option to work at home	13.06
Work	94.18	**Activity and travel characteristics**	
Other activity	5.82	Survey day is Friday	17.79
Shopped via internet in past month		Used public transportation on survey day	3.98
No	57.31	At least one walk trip in past week	63.98
Yes	42.69	At least one bike trip in past week	6.58

Descriptive statistics

Variable	Mean	Std. Dev.	Min.	Max.
Individual characteristics				
Age [years]	46.67	12.70	18.00	86.00
Household characteristics				
Number of adults	2.40	0.92	1.00	7.00
Number of non-adults	0.74	1.05	0.00	6.00
Number of drivers	2.33	0.92	0.00	7.00
Number of vehicles	2.59	1.30	0.00	12.00
Number of workers	1.84	0.82	1.00	5.00
Work-related characteristics				
Distance to work [miles]	13.52	12.56	0.11	97.00

Dependent variable: Number of out-of-home non-work episodes

Time-of-day block	Mean	Std. Dev.	Min.	Max.
Before-work (BW)	0.12	0.44	0.00	6.00
Home-to-work commute (HWC)	0.20	0.56	0.00	11.00
Work-based (WB)	0.23	0.47	0.00	4.00
Work-to-home commute (WHC)	0.43	0.83	0.00	6.00
After-home (AH)	0.56	1.12	0.00	12.00
Total non-work episodes	1.54	1.67	0.00	13.00

[1] Non-Hispanic Other includes American Indian, Alaskan Native (1.23%), Native Hawaiian, or other Pacific Islander (0.52%), Multiracial (0.70%), and other (that is, specified in the survey capture itself as a catch all "other" category (2.18%)

[2] This other category includes Manufacturing, construction, maintenance or farming (12.62%) and other (that is, specified in the survey capture itself as a catch all "other category"): 0.85%

APPENDIX D: MODEL FIT ASSESSMENT USING PREDICTIVE MEASURES

First, define $\mathbf{R}_i$ $(i = 1,2,...,I)$ as an $(I-1)\times I$ matrix that corresponds to an $(I-1)$ identity matrix with an extra column of -1's added as the i^{th} column. Following the notation in Equation (10) and immediately after, define $\mathbf{G}_{qi} = \mathbf{R}_i \mathbf{\Sigma}_q \mathbf{R}_i'$. We can then write the probability that individual (consumer) q chooses alternative i at any choice occasion as:

$$P_{qi} = P[\boldsymbol{M}_{qi}\breve{\boldsymbol{U}}_q < \boldsymbol{0}_{I-1}] = \Phi_{(I-1)}\left[(\boldsymbol{\omega}_{\mathbf{G}_q})^{-1}(-\boldsymbol{d}_q), (\boldsymbol{\omega}_{\mathbf{G}_q})^{-1}\mathbf{G}_{qi}(\boldsymbol{\omega}_{\mathbf{G}_q})^{-1}\right]. \tag{D.1}$$

Next, since this probability does not change across choice occasions, and the individual-specific preferences are already embedded in $\breve{\boldsymbol{U}}_q$ (through the $\boldsymbol{\beta}_q$ vector), the multivariate probability of counts in each time-of-day block (*i.e.*, event type), conditional on the total count level k_q ($k_q > 0$), takes the usual multinomial distribution form:

$$P[(g_{q1} = k_{q1}),(g_{q2} = k_2),...(g_{qI} = k_{qI}) \,|\, k_q] = \frac{k_q!}{\prod_{i=1}^{I} k_{qi}!}\prod_{i=1}^{I}(P_{qi})^{k_{qi}}. \tag{D.2}$$

In our joint model of multivariate counts, the unconditional multivariate probability then takes the form indicated below ($k_q = \sum_{i=1}^{I} k_{qi}$, $k_{qi} = 0,1,2,...,\infty$, $k_q = 0,1,2,...,\infty$):

$$P[(g_{q1} = k_{q1}),(g_{q2} = k_2),...(g_{qI} = k_{qI})] = P[g_q = k_q]\times\left(\frac{k_q!}{\prod_{i=1}^{I} k_{qi}!}\prod_{i=1}^{I}(P_{qi})^{k_{qi}}\right), \tag{D.3}$$

with $P[g_q = k_q]$ as in Equation (13) after replacing n_q (the actual observed total count for individual q in the estimation sample) with an arbitrary value k_q. Using the properties of the multinomial distribution, the marginal probability of k_{qi} counts for time-of-day block i is:

$$P[g_{qi} = k_{qi}] = \sum_{k_q=0}^{\infty}\left[P[g_q = k_q]\times\left(\frac{k_q!}{k_{qi}!(k_q - k_{qi})!}(P_{qi})^{k_{qi}}(1-P_{qi})^{(k_q - k_{qi})}\right)\right] \tag{D.4}$$

In the above expression, the upper bound of the summation is $k_q = \infty$, though the probability values fade very rapidly beyond a k_q value of 10. For the purposes of this paper, we carry the summation up to $k_q = 50$.

With the above preliminaries, the model predictions can be used to evaluate data fit at both the disaggregate and aggregate levels, as well as for both the multivariate count distribution and the marginal count distribution. At the disaggregate level, we estimate the probability of the observed multivariate count outcome for each individual using Equation (B.3), and compute an average probability of correct prediction. Similarly, we also estimate the probability of the observed marginal count outcome separately for each time-of-day period using Equation (B.4), and compute an average probability of correct prediction. At the aggregate level, we design a heuristic diagnostic check of model fit by computing the predicted aggregate share of individuals for specific multivariate outcome cases (because it would be infeasible to provide this information for each possible multivariate outcome). In particular, we compute the aggregate share of consumers for each of six combinations. The first combination corresponds to no participation in any non-work episodes (which we will refer to as the "no participation" combination). The other five combinations correspond to participation in one or more episodes during a specific time-of-day block and no participation in any other time-of-day period (which we will refer to using such labels as the "BW participation only" combination or the "HWC participation only" combination). In addition to these aggregate shares of multivariate outcomes, we also compute the aggregate shares of the marginal outcomes of count values of 0, 1, 2, 3, and 4+ for each time-of-day period, as well as for the total count. As a yardstick to evaluate the performance of the joint model proposed here, we compare the predictions from the joint model with the independent model using the absolute percentage error (APE) statistic for each count value, and then compute a mean weighted APE value across the count values (of 0 1, 2, 3, and 4+) using the observed number for each count value as the weight for that count value.

The disaggregate-level data fit measures indicate an average probability of correct prediction of 13.9% for the multivariate counts and an average probability of correct prediction of 67.6% for the marginal counts. The corresponding values for the independent model are 13.6% and 65.0%, respectively, which are smaller in magnitude than those from the joint model. The aggregate fit measures are provided in Table D.1. The joint model provides a better (lower)

APE value for all the multivariate outcomes in Table D.1 (see upper panel of the table), except for the WB participation only outcome. The APE values are sizeable for both the joint and independent values, but it should be noted that these predictions are for multivariate outcomes. Overall, the mean weighted APE value is about 12% higher for the independent model relative to the joint model. As expected, the APE values are lower for the marginal outcomes (see lower panel of Table D.1) than for the multivariate outcomes. The total count predictions from the joint model are much better than the total count predictions from the independent model. Also, the predictions for the other marginal counts are better from the joint model relative to the independent model (except for the WB block count). These results clearly show that the joint model proposed here outperforms the traditional independent model in the disaggregate level and aggregate level comparisons.

Table D.1 Aggregate Data Fit Measures

Aggregation Level	Combination Event		Observed	Joint Model Predicted	Joint Model APE	Independent Model Predicted	Independent Model APE
Multivariate	No participation		676	669.7	0.9	656.1	2.9
	BW participation only		67	90.5	35.0	95.2	42.1
	HWC participation only		67	85.2	27.1	87.0	29.9
	WB participation only		168	63.0	62.5	67.4	59.9
	WHC participation only		230	153.6	33.2	137.0	40.4
	AH participation only		279	345.4	23.8	347.7	24.6
	Overall mean weighted APE			**19.9**		**22.2**	
Marginal	Total count	0	676	669.7	0.9	656.1	2.9
		1	593	589.1	0.7	585.7	1.2
		2	388	379.1	2.3	377.2	2.8
		3	208	225.6	8.4	253.5	21.9
		4+	248	249.5	0.6	240.5	3.0
	Weighted APE			1.8		4.3	
	BW block count	0	1926	1756.8	8.8	1745.3	9.4
		1	147	295.8	101.2	305.7	108.0
		2	24	47.8	99.1	50.8	111.5
		3	13	7.6	41.5	8.1	37.7
		4+	3	5.0	66.7	3.1	3.3
	Weighted APE			16.5		17.6	
	HWC block count	0	1792	1739.1	3.0	1729.8	3.5
		1	250	317.8	27.1	326.4	30.6
		2	57	45.8	19.6	48.1	15.7
		3	10	5.9	41.0	6.2	38.3
		4+	4	4.4	10.0	2.5	37.5
	Weighted APE			6.5		7.3	
	WB block count	0	1660	1831.5	10.3	1809.9	9.0
		1	421	244.8	41.9	261.2	38.0
		2	29	29.2	0.7	35.0	20.7
		3	2	3.6	77.5	4.7	133.2
		4+	1	3.9	290.0	2.2	120.0
	Weighted APE			16.7		15.1	
	WHC block count	0	1516	1560.2	2.9	1593.4	5.1
		1	397	423.4	6.6	408.7	3.0
		2	131	97.1	25.8	87.2	33.5
		3	45	22.0	51.0	17.6	60.8
		4+	24	10.3	57.1	6.1	74.6
	Weighted APE			6.7		8.4	
	AH block count	0	1465	1216.0	17.0	1201.7	18.0
		1	394	576.9	46.4	589.8	49.7
		2	90	214.3	138.1	221.7	146.3
		3	103	71.9	30.2	71.6	30.5
		4+	61	33.9	44.4	28.2	53.8
	Weighted APE			29.1		31.0	

APPENDIX E: MODEL APPLICATION

The joint model estimated in the paper can be used to examine the impact of changes in socio-demographic characteristics over time as well as the effects of policy actions that involve a change in the accessibility measures and work-related characteristics. In this paper, we demonstrate the application of this model by studying the effects of changes in three selected variables: distance to workplace, retail trade accessibility at the home location, and entertainment accessibility at the home location. These three variables are increased by 20% across all workers. The impact on the frequency and organization of non-work activities is estimated by determining the percentage change in the expected number of non-work episodes (across all workers) for the entire day (*i.e.*, total count) and for each time-of-day block. To demonstrate the potentially misleading inferences from the independent model, we compute the percentage change as predicted by both the joint model as well as the independent model. The emphasis here is not on substantive empirical inferences as much as it is on demonstrating the differences in the inferences from the two models. Table E.1 provides the results.

Three observations may be made from Table E.1. First, in the independent model, a change in the retail trade and entertainment accessibility variables do not have any impact on the total count of non-work episodes over the entire day. This is, of course, because these variables appear only in the event discrete choice model and not the total count model (and the independent model does not have any link between the discrete choice model and the total count model). As indicated earlier in the paper, it is natural to expect that changes in the attributes impacting the attractiveness of alternatives in the choice model (retail trade and entertainment accessibilities in the specific case under discussion) will result not only in substitution among the counts of each discrete choice alternative, but also an overall change in the total count, as appropriately recognized by the joint model. Second, the positive effect (of an increase in the number of retail trade jobs) on the number of non-work tours during the BW, HWC, WHC, and AH periods is underestimated by the independent model, while the negative effect of the variable on WB non-work tours is overestimated by the independent model. Third, a similar result holds also for the influence of the number of entertainment jobs at the home location. Indeed, for this variable, the directionality of the effect on WHC non-work tours is itself different between the independent and joint models. These differences between the models highlight the potentially misinformed policy analyses that result from ignoring the linkage between the frequency of non-work episodes and their organization across time-of-day blocks.

Table E.1 Aggregate Percentage Change in Expected Number of Non-Work Episodes

Effect of 20% increase in …	Time-of-Day Block	Joint Model	Independent Model
Distance to work	All day	-2.51	-2.30
	BW	-2.64	-2.44
	HWC	-2.52	-2.32
	WB	-2.30	-2.08
	WHC	-2.46	-2.27
	AH	-2.46	-2.27
Number of retail trade jobs at the home location	All day	0.53	0.00
	BW	0.75	0.31
	HWC	0.85	0.40
	WB	-1.30	-2.34
	WHC	0.62	0.06
	AH	0.69	0.31
Number of entertainment jobs at the home location	All day	3.44	0.00
	BW	8.31	5.23
	HWC	5.93	2.32
	WB	-2.41	-6.94
	WHC	1.52	-1.75
	AH	3.27	0.37