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Hybrid Choice Models: Progress and Challenges

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Abstract

We discuss the development of predictive choice models that go beyond the random utility model in its narrowest formulation. Such approaches incorporate several elements of cognitive process that have been identified as important to the choice process, including strong dependence on history and context, perception formation, and latent constraints. A flexible and practical hybrid choice model is presented that integrates many types of discrete choice modeling methods, draws on different types of data, and allows for flexible disturbances and explicit modeling of latent psychological explanatory variables, heterogeneity, and latent segmentation. Both progress and challenges related to the development of the hybrid choice model are presented.

Key words: choice modeling, mixed logit, logit kernel, simulation, estimation, latent variables

Introduction

Many disciplines are interested in the choice process, including economists, engineers, psychologists, marketers, and planners. Figure 1 illustrates the domains of choice research. Predictive choice models emphasize the regularities of choice behavior in quantitative models that can be used for prediction in the disciplines listed above. This view emphasizes the systematic, invariant features of choice behavior that can be used for forecasting. On the other hand, behavioral choice analysis deconstructs the choice process by concentrating on revealing irregularities and idiosyncratic features of choice behavior. The views are different, primarily due to the differences in research objectives, prediction (economics, marketing, planning and engineering) versus deconstruction (psychology). However, strong cross-fertilization has occurred among these fields.

Within the domain of predictive choice models is the random utility model (RUM), a powerful tool for approximating the systematic aspects of choice behavior. There is incomplete overlap between behavioral and predictive approaches to analyzing choice, and there are good reasons for pursuing each in parallel without necessarily trying to force integration. However, each endeavor can benefit from the other's results, and the end goal

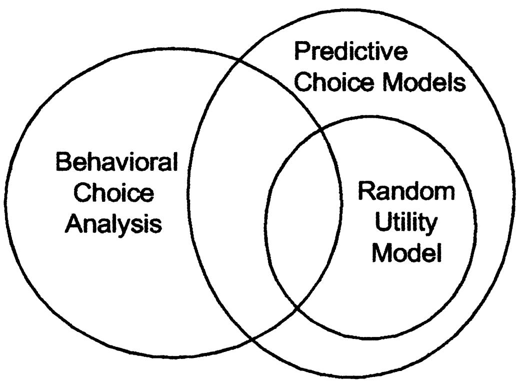

Figure 1. Domains of Choice Research.

is an integrated treatment of choice behavior that incorporates the invariant elements obtained by deconstructionist study of choice processes and is predictive for practical applications.

An important objective of the research efforts presented in this paper is to develop practical models that go beyond the RUM model in its narrowest formulation and incorporate elements of cognitive process that have been identified as important. This view based on a behavioral approach is summarized in Figure 2. In the past, the solid arrows in this figure were mostly examined; now the dashed arrows have growing importance. The expanded behavioral framework emphasizes that:

- Choice is governed by perceptions, information processing and cognitive processes.
- History matters, for example, influencing context (motivation, affect, etc.), status quo, and state dependence.
- Heterogeneity across decision-makers due to different attitudes and perceptions is important.
- There are important latent constructs (such as perceptions and latent classes) that are influenced by psychological factors and external constraints.

Hybrid Choice Model

The objective is to explicitly model the choice behavior relationships depicted in Figure 2. To do this, we propose an expanded discrete choice framework, termed the hybrid choice model (HCM), which integrates many types of discrete choice modeling methods. The framework for the HCM is shown in Figure 3.

The traditional RUM is shown on the vertical axis of this figure: observable explanatory variables (including characteristics of the decision-maker and attributes of the alternatives)

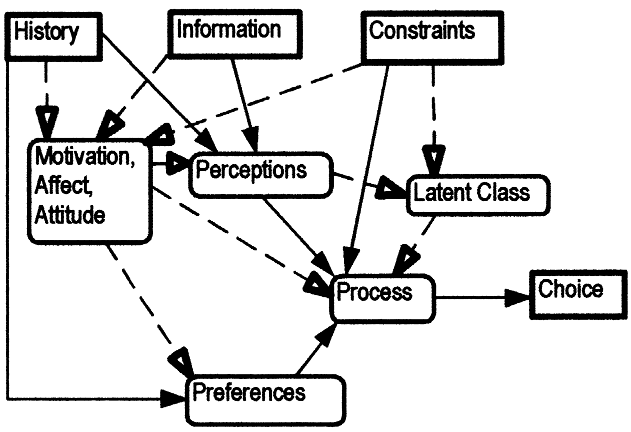

Figure 2. Choice Behavior (McFadden 2001).

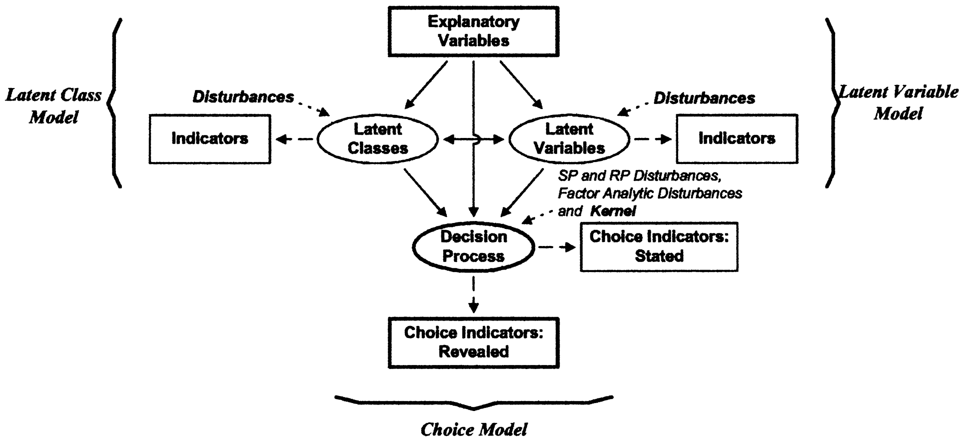

Figure 3. Hybrid Choice Model (Walker and Ben-Akiva 2001).

are shown to influence the decision process (utility maximizing) leading to the revealed choices. In such traditional quantitative choice models, the emphasis has been on the systematic, invariant aspects of choice behavior, which are assumed to dominate the choice process.

Researchers have long been focused on improving the specification of the traditional RUM, and numerous enhancements have been developed that aid in predicting realizations of the choice behavior depicted in Figure 2. These methods are integrated in the general HCM by relaxing the basic RUM core, such as incorporating non-RUM decision protocols, in an effort to relax simplifying assumptions and enrich underlying behavioral characterizations. The extensions include:

- The addition of flexible disturbances (e.g., *factor analytic*) to mimic any desirable error structures (such as relaxing the IIA structure of logit or specifying random parameters).
- The explicit modeling of latent psychological factors such as attitudes and perceptions (*latent variables*). That is, combining "hard information" (such as reasonably well-measured socio-economic characteristics) with "soft information" on population heterogeneity (such as indicators for psychological characteristics that are difficult to measure, for example, risk attitudes, impatience, and self control) in discrete choice models. The aim is to "explain" seemingly irrational behavior, that is, model structurally using economic and psychological data, a substantial part of the unobserved heterogeneity.
- The inclusion of latent segmentation of the population (*latent classes*), which allows for different decision protocols including non-RUM, market segmentation, and choice set formulation.

It is often difficult to estimate such a model with revealed preferences (that is, market behavior) alone, and therefore other indicators are incorporated into the framework to aid in estimation of the behavioral relationships. These additional indicators include stated preferences and psychometric measurements (e.g., survey questions regarding attitudes and perceptions).

Note that the HCM does not require the assumption of RUM since HCM can allow for the incorporation of non-RUM decision protocols, and RUM-based HCM is a restricted form of a general HCM. If the model of interest is not RUM, then the utility model can be used as a paramorphic representation of non-RUM behavior, with the advantage of inference through the use of a statistical model.

Progress

A more limited version of the above HCM framework was considered at the previous Choice Symposium, see Ben-Akiva et al. (1999). More recently, Walker and Ben-Akiva (2001) demonstrated the extended HCM framework. They estimated travel mode choice models using revealed and stated preference data, latent perceptions of comfort and convenience, and taste heterogeneity in the form of random parameters and latent class segmentation. Estimation of the HCM was shown to be practical, and the resulting model incorporated the benefits of each individual method. The addition of stated preferences made a key policy variable significant that was insignificant in the revealed preference model. It also allowed estimation of coefficients for variables not included in the revealed preference data (in this case, ‘amenities’). The latent factors provided for a richer behavioral representation of the choice process (although not a significant improvement in overall model fit); and the inclusion of taste heterogeneity improved the explanatory power of the model. Given that the HCM framework is constructed by integrating modular components such as latent variable models, flexible disturbances, general RUM and non-RUM model cores, disparate sources of preference data, etc., the development of HCM has been catalyzed by technical developments and growing practical experience with each of the modular components. Also, developments in simulation-based estimation techniques and Bayesian procedures for complex model systems such as HCM make HCM more feasible and practical. We highlight such developments below.

Flexible Choice Models

Significant progress has been made in terms of the flexibility of choice models (McFadden, 2001). New GEV and mixed logit models have been developed and extensively applied that avoid the restrictions of standard logit and nested logit. The Generalized Extreme Value (GEV) family of models is a rich set of models. The choice probabilities have closed form, which aids in their estimation compared to models like probit and mixed logit that require simulation. Applications have tended to use only a few of the numerous possible GEV models (Bierlaire 2001). However, new specifications have been developed recently that further exploit their flexibility. As a unifying concept, Daly (2001) presents a nested GEV formulation that includes all the GEV models previously reported in the literature.

As an example of the expanding development of GEV models, Karlstrom (2001) uncovered new forms of the GEV model that perform better than previously known GEV forms.

McFadden and Train (2000) have shown that mixed logit can approximate *any* RUM and have developed tests for determining the appropriate mixing distribution. Mixed logit encompasses latent class models and neural nets, as well as random parameters and error component specifications. The model is also referred to as “logit kernel” to reflect that the core of the model (as well as the resulting probability simulator) is a logit formulation. It has also been referred to as “logit kernel probit” in the special case when the formulation is used to approximate or extend the probit model. The functional form is also applicable for many non-RUM decision processes such as habitual behavior or rule-based decision-making.

Latent Variables

Researchers are investigating the benefits of incorporating latent psychological constructs, such as attitudes and perceptions, into choice models. Eymann, Boersch-Supan and Euwals (2001) modeled the effect of risk attitudes on portfolio choice. Standard portfolio models assume that the household’s choice among assets is guided by the assets’ risk and return and by individual risk aversion. These models usually predict that every individual holds a positive percentage of risky financial assets. This prediction, however, is in stark contrast to actual asset choice: most households do not hold risky assets such as stock. Clearly, asset choice is also influenced by credit constraints, income uncertainty, information costs, risk aversion, impatience, and self-control—all concepts that are difficult to measure and probably quite heterogeneous in the population. The key contribution of this research is the development of a joint choice and latent variable model (using the framework presented in Ben-Akiva et al. 1999), in which a non-linear MIMC-type latent variable model is used to relate latent individual traits to observable determinants. They found that the structural equations model is superior to the more conventional “proxy variable specification” in terms of its predictive accuracy. Estimation results also show that individuals differ in their attitudes towards risk and time; and such taste variations are *systematic* rather than *stochastic.* Hence, there is considerable gain in explicitly modeling the heterogeneity.

Combining Revealed and Stated Preferences

Procedures for combining revealed and stated preference data have been developed in the late 1980’s and are being widely applied (as discussed in a previous Choice Symposium paper: Ben-Akiva et al. 1994). More recently, Bhat and Castelar (2002), Brownstone, Bunch, and Train (2000) and Walker and Ben-Akiva (2001) have integrated revealed and stated preference data while also utilizing the flexibility of mixed logits to obtain more realistic representations of differences across various sources of data.

Estimation by Simulation

A key factor promoting the application of flexible model forms is the advance of simulation techniques. Much recent work has focused on the generation of simulation draws. Bhat (2001a) describes the use of Halton draws, which is a type of quasi-random Monte Carlo (QMC) method. He found that Halton draws improve the estimation of mixed logits by making it faster and more stable with fewer draws compared to pseudo-Monte Carlo draws. Train (1999) and Munizaga and Alvarez-Daziano (2001) have confirmed these results. Subsequently, Bhat (2001b) addressed two aspects of standard Halton draws that can limit their applicability. First, coverage of the integration domain deteriorates quite rapidly in higher dimensions. Second, the standard Halton draws are deterministic, which prevents the computation of statistical error bounds associated with simulation noise. To address these limitations, Bhat proposed the use of scrambled Halton draws to ensure good coverage in higher dimensions and a randomization technique to allow measurement of simulation variance.

Bayesian Procedures

The existing literature predominantly uses the maximum simulated likelihood estimator (MSLE). As described by Train (2001), the simulated mean of the Bayesian posterior (SMPE) provides an alternative estimator to MSLE. SMPE is consistent and efficient under less stringent conditions than MSLE and avoids the numerical difficulties associated with locating the maximum (including the issue of whether a local maximum is global.) Countering these advantages, SMPE usually relies on iterative sampling procedures, namely MCMC methods, under which it can be difficult to determine whether convergence has been achieved. The researcher's choice between MSLE and SMPE often hinges on their relative convenience in terms of programming and computer time. Bolduc, Fortin and Gordon (1997) compared the two procedures for probit models and found run times to be considerably lower for SMPE. Train (2001) compared run times and programming issues for mixed logit under various specifications. When all coefficients are independently normal, the two procedures are similar in terms of run time and programming difficulty. Correlated coefficients and lognormal distributions are more easily handled by SMPE. If the model is specified to include fixed and random coefficients and/or uses distributions with bounded support, such as triangular, MSLE is faster and easier. These comparisons were conducted using random draws in both procedures.

Identification

Discrete choice models always require normalization or identifying restrictions, and there are numerous identification issues that arise. The difficulty is compounded by the fact that unidentified models can often be estimated by available software, due to numerical round-off and other issues, with the software not determining whether the model is identified. Ben-Akiva, Bolduc, and Walker (2001) investigated identification issues in the context of mixed logit (or logit kernel) models. They showed how seemingly straightforward ways of normalizing a mixed logit could actually impose restrictions on the model beyond normalization. They developed specific rules for the identification of mixed logit, including models with heteroscedasticity, nesting, and random parameter specifications.

Behavioral Dynamics and Interactions

Discrete choice models have mainly been used in a static context. However, repetitive choices are best modeled by transition probabilities. De Palma and Kilani (2001) developed such dynamic discrete choice models. They considered a two-period model, and found that for additive random utility models, analytical expressions of the transition probabilities can be derived for changes in the systematic utility (for example price or quality change). In the simplest case, the random disturbances remain the same before and after the change. For this case, the transition probabilities can be computed up to a one-dimensional integral for the GEV model, while they have a closed form expression for a logit model. Monte-Carlo simulations for a logit model show that using information on the transitions rather than on the static choices improves the accuracy of parameter estimates.

Data Imputation

Discrete choice models are frequently estimated using proxy or imputed variables. In these situations, the inevitable imputation errors cause biased inference. Brownstone et al. (2001) developed and demonstrated a technique to perform consistent inference without fitting a complicated joint imputation and choice model. The procedure builds upon Rubin's (1987) multiple imputation method, which is a Bayesian technique with good classical inference properties. Multiple imputed values are generated for each observation, and separate choice model estimates are made for each set of imputations. The results from these separate choice model estimates are easily combined to yield final estimates. This multiple imputation technique is particularly valuable when validation data are available to allow direct observation of the imputation error. Brownstone et al. (2001) use data from driving cars down a corridor to assess the error in travel time computations from loop detectors imbedded in the roadway surface. They find that the loop detector data are downward biased by 50%. Multiple imputation of the correct travel times is used to estimate value of time-savings from taking a toll road. Even though the multiple imputation procedure increases the standard errors of the parameter estimates, the correction significantly lowered value of time estimates.

Discrete/Continuous Models

When a continuous variable depends on the outcome of an endogenous discrete choice, the regression model for the continuous variable can be estimated by instrumental variable

(IV) instead of ordinary least squares, to avoid endogeneity bias in the estimated coefficients. However, adjustments are also needed for hypothesis testing. Bolduc, Khalaf and Moyneur (2001) demonstrated theoretically and through Monte Carlo experiments that standard asymptotic tests over-reject the null hypotheses in a regression models with discrete endogenous regressors. They extended the exact split-sample IV-based tests proposed by Dufour and Jasiak (2000) to discrete/continuous models and showed that this procedure achieves perfect type I error control. These split sample tests are straightforward to implement.

Software

The availability of estimation procedures is critical for the development of hybrid choice models. Many advanced discrete choice techniques are now available in commercial statistical packages. Techniques have been developed so that joint estimation with revealed and stated choice data can be performed with nested logit software. Several commercial discrete choice software vendors have recently introduced routines for mixed logit and probit; and researchers are making their code available on the web. For example, Kenneth Train provides Gauss-based code for mixed logit (http://emlab.berkeley.edu/users/train/index.html). A new freeware package for the estimation of GEV models (http://rosowww.epfl.ch/mbi/biogeme), developed by Michel Bierlaire, is designed for maximum-likelihood estimation of the GEV model family. Based on an efficient optimization algorithm, this software allows for several modeling options, such as scaled utility functions, Box-Cox and Box-Tukey transforms, and performs constrained estimation. The availability of such a tool will help investigate various members of the GEV family (Bierlaire, Axhausen and Abay 2001). The software is now being extended to mixed GEV models.

Challenges

The availability of the new computational tools provides a wide array of opportunities for estimating hybrid choice models. However, complex issues arise with respect to a number of facets. This section highlights particular challenges, and presents examples of related research.

Model Formulation and Estimation

Determining what is Deterministic and what is Random. It is often difficult to distinguish where to build the complexity in the model, that is, by using a complex error structure or by building up the systematic portion of the utility. The consensus is that a good error is a zero error; that is, it is desirable to expand on the systematic term thereby reducing the disturbance term. However, various limitations (for example, data) place restrictions on this objective. Furthermore, a complex but incorrect specification of the systematic portion can be worse than implementing a simple systematic term and complex disturbance term.

Specification of Mixed Models. The choice probability in mixed models is the average of a function over a mixing distribution. In specifying these models, there is often a dilemma as to whether to represent heterogeneity and other issues through the mixing distribution or the function that is being mixed. A mixed logit formulation can quickly lead to high dimensional simulation, since all heteroscedasticity and covariance is captured through the mixing distribution. A mixed GEV (or a GEV kernel) formulation allows some of the heteroscedasticity/covariance to be captured by the function being mixed, thereby reducing the dimension of simulation. For example, a mixed Nested Logit induces heteroscedasticity and correlations between alternatives, thereby reducing dimensionality of the mixing distribution.

Comparisons across Alternative Model Forms. With numerous statistically different yet behaviorally analogous model structures, there is a need to compare alternative model structures (and estimation algorithms) across computational requirements, ability to capture underlying behavior, severity of biases due to misspecifications, and predictive capability. Munizaga and Alvarez-Daziano (2001) compared the prediction performance of mixed logit and traditional models using both simulated and real data. They show that probit models consistently underestimated the presence of correlation between alternatives, logit and nested logit models predict poorly when the error terms are non-IID, and the predictions of probit models are similar to those of mixed logit with a normal mixing distribution. More research is needed to determine the relative performance of models in various contexts.

Identification. Necessary and sufficient conditions for the identification of the general HCM are yet to be developed. For HCM with latent classes, it is often difficult to distinguish between theoretical identification and empirical identification issues when the analyst attempts to capture latent classes that the data do not support. It is conjectured that incorporating indicators of latent classes may lend character to the latent classes since indicators may be viewed as attributes of latent classes, and thus provide adequate information for the empirical identification. (See a previous Choice Symposium paper, Ben-Akiva et al. (1997), for more discussion.)

Simulation. Bhat (2001b) raises several research issues regarding simulation techniques that need careful investigation in the future. First, there is a need to evaluate alternative QMC sequences. Second, there are several ways to randomize a QMC sequence and a comparison of these alternative randomization schemes would be helpful. Third, the application of variance reduction techniques to randomized QMC estimators can provide substantial estimation benefits. Fourth, there is a need to extend the scope of the QMC analysis to higher dimensions. Fifth, it would be useful to examine the effectiveness of

QMC sequences within a Bayesian estimation framework for complex discrete choice models.

Data

There are numerous data issues. Often the sample sizes are too small. This is critical as Monte Carlo results in Bunch (2001), while exploring practical issues related to data requirements, show that sample size requirements increase substantially for estimating non-IID models. In a stated preference context, the study controlled attribute configurations for choice alternatives, choice sets, and sample sizes. The estimation of complex choice models increasingly relies on such data collection. Initial parameter estimates obtained from field experiment data provided the basis for further explorations via Monte Carlo experiments. The analysis of asymptotic covariance matrix (ACM) of the maximum likelihood estimator showed substantial differences in statistical efficiency from alternative experimental designs and important interactions between designs and alternative model forms. ACM analysis is recommended to enhance the understanding of a range of practical issues related to data and sample size requirements.

There also can be too much data, for example, scanner and click-stream data. The quality of the data is also an issue. For example, in the portfolio choice model of Eymann, Boersch-Supan and Euwals (2001) the estimated extent of heterogeneity in risk attitude and impatience is not sufficiently large to explain why households hold so few risky assets. While the model of asset choice improves on the existing literature by using not one but two indicators for risk attitude and by modeling risk as well as impatience, this is obviously not yet sufficient. A crucial task for future research is therefore to sufficiently enrich data to capture population heterogeneity and include richer stated preference data in conventional surveys.

Inherent Limitations

Despite the increasingly flexible model forms and fast estimation techniques, there are inherent limitations to the discrete choice framework. One issue is that the models are centered on an individual, in which it is difficult to capture interdependent (joint) decisions arising from social interaction and the influence between decision-making units. There are complex behavioral influences such as social norms, learning by doing and by observation, choice as a strategy for search and learning, and choice as strategic behavior in games. While aspects of these factors could be built into the model structure, the increased complexity quickly reaches a dimensionality (and therefore computation) barrier.

Interpretation of hybrid choice models is also an issue. For example, while the new GEV forms explored by Karlstrom (2001) provide improved fit to the data, there is no clear behavioral interpretation of the additional parameters. Often there are several different model structures that are indistinguishable in their predictive accuracy. It is not always clear how to distinguish the behavior that is being captured by the model, and it is often left up to interpretation. For example, is the observed behavior a result of changing perceptions or changing preferences? A specification may appear to be capturing one form of behavior, when it is really capturing something else.

Conclusion

We presented the HCM—an integration of models and methods that extend traditional discrete choice analysis and the RUM. We highlighted recent advances that indicate the potential of the HCM to enhance the capabilities of predictive choice models. The added complexity raises a number of challenges and issues for further research.

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