

**ACCOMODATING MULTIPLE CONSTRAINTS IN THE  
MULTIPLE DISCRETE-CONTINUOUS EXTREME VALUE (MDCEV) CHOICE  
MODEL**

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## **ABSTRACT**

Multiple-discrete continuous choice models formulated and applied in recent years consider a single linear resource constraint, which, when combined with consumer preferences, determines the optimal consumption point. However, in reality, consumers face multiple resource constraints such as those associated with time, money, and capacity. Ignoring such multiple constraints and instead using a single constraint can, and in general will, lead to poor data fit and inconsistent preference estimation, which can then have a serious negative downstream effect on forecasting and welfare/policy analysis.

In this paper, we extend the multiple-discrete continuous extreme value (MDCEV) model to accommodate multiple constraints. The formulation uses a flexible and general utility function form, and is applicable to the case of complete demand systems as well as incomplete demand systems. The proposed MC-MDCEV model is applied to time-use decisions, where individuals are assumed to maximize their utility from time-use in one or more activities subject to monetary and time availability constraints. The sample for the empirical exercise is generated by combining time-use information from the 2008 American Time Use Survey and expenditure records from the 2008 U.S. Consumer Expenditure Survey. The estimation results show that preferences can get severely mis-estimated, and the data fit can degrade substantially, when only a subset of active resource constraints is used.

*Keywords:* Travel demand, multiple discrete-continuous extreme value model, multiple constraints, time use, consumer theory.

## 1. INTRODUCTION

Traditional discrete choice models have been widely used to study consumer preferences for the choice of a single discrete alternative from among a set of available and mutually exclusive alternatives. However, in many choice situations, consumers face the situation where they can choose more than one alternative at the same time, though they are by no means bound to choose all available alternatives. These situations have come to be labeled by the term “multiple discreteness” in the literature (see Hendel, 1999). In addition, in such situations, the consumer usually also decides on a continuous dimension (or quantity) of consumption, which has prompted the label “multiple discrete-continuous” (MDC) choice (Bhat, 2005). Examples of MDC situations abound in consumer decision-making, and include (a) the participation decision of individuals in different types of activities over the course of a day and the duration in the chosen activity types, (b) household holdings of multiple vehicle body/fuel types and the annual vehicle miles of travel on each vehicle, and (c) consumer purchase of multiple brands within a product category and the quantity of purchase. In the recent literature, there is increasing attention on modeling these MDC situations based on a rigorous underlying micro-economic utility maximization framework for multiple discreteness.<sup>1</sup>

The essential ingredient of a utility maximization framework for multiple discreteness is the use of a non-linear (but increasing and continuously differentiable) utility structure with decreasing marginal utility (or satiation), which immediately introduces imperfect substitution in the mix and allows the choice of multiple alternatives. While several non-linear utility specifications originating in the linear expenditure system (LES) structure or the constant elasticity of substitution (CES) structure have been proposed in the literature (see Hanemann, 1978, Kim *et al.*, 2002, von Haefen and Phaneuf, 2005, and Phaneuf and Smith, 2005), Bhat (2008) proposed a form that is quite general and subsumes the earlier specifications as special cases. His utility specification also allows a clear interpretation of model parameters and explicitly imposes the intuitive condition of weak complementarity (see Mäler, 1974), which implies that the consumer receives no utility from a non-essential good’s attributes if she/he does not consume it (see Hanemann, 1984, von Haefen, 2004, and Herriges *et al.*, 2004 for a detailed discussion of weak complementarity). In terms of stochasticity, Bhat (2005; 2008) used a multiplicative log-extreme value error term in the baseline preference for each alternative, leading to the multiple discrete-continuous extreme value (MDCEV) model. The MDCEV model has a closed-form probability expression, is practical even for situations with a large number of discrete alternatives, is the exact generalization of the multinomial logit (MNL) for MDC situations, collapses to the MNL in the case that each (and every) decision-maker chooses only one alternative, and is equally applicable to cases with complete or incomplete demand systems (that is, the modeling of demand for all commodities that enter preferences or the modeling of demand for a subset of commodities that enter preferences).<sup>2</sup> Indeed, the MDCEV and its

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<sup>1</sup> This is in contrast to using “quick-fix” and cumbersome explosion-based single discrete choice models (that is, identifying all bundles of the “elemental” alternatives and treating each bundle as a “composite” alternative in a single discrete choice model), or statistical stitching models that handle multiple discreteness through methods that generate correlation between univariate utility maximizing models for single discreteness (see Manchanda *et al.* 1999, Baltas, 2004, Edwards and Allenby, 2003, and Bhat and Srinivasan, 2005).

<sup>2</sup> In a complete demand system, the demands of all consumption goods are modeled. For instance, one may model expenditures in each of many appropriately defined commodity/service categories that exhaust the consumption space of consumers. However, complete demand systems require data on prices and consumptions of all commodity/service items, and can be impractical when studying consumptions in finely defined commodity/service categories. In such situations, it is common to use an incomplete demand system, typically in the form of a two stage

variants have been used in several fields, including time-use (Kapur and Bhat, 2007, Chikaraishi *et al.*, 2010, Wang and Li, 2011), transportation (Rajagopalan and Srinivasan, 2008, Ahn *et al.*, 2008, Pinjari, 2011), residential energy type choice and consumption (Jeong *et al.*, 2011), land use change (Kaza *et al.*, 2009), and use of information and communication technologies (Shin *et al.*, 2009).

An important assumption, however, in the MDCEV model (as it stands currently) is that consumers maximize utility subject to a single linear binding constraint (the constraint is binding because the alternatives being considered are goods and more of a good will always be preferred to less of a good; thus, consumers will consume at the point where all budget is exhausted). But in most choice situations, consumers usually face multiple resource constraints.<sup>3</sup> Some common examples of resource constraints relate to income (or expenditure), time availability, and space availability, though other constraints such as rationing (for example, coupon rationing), energy constraints, technological constraints, and pollution concentration limits may also be active in other consumption choice situations. For instance, consumers' decisions regarding how they use their time in different activity purposes will naturally be dependent on both an income constraint (the expenditure incurred through participation in the different chosen activity purposes cannot exceed the money available for expenditure) and a time availability constraint (the time allocated to the various activities cannot exceed the available time). Another example relates to households' decisions regarding the quantity of purchase of grocery items. Here, in addition to the income constraint, there is likely to be a space constraint based on the household's refrigerating space or pantry storage space. In such multi-constraint situations, ignoring the multiple constraints and considering only a single constraint can lead to utility preference estimations that are not representative of "true" consumer preferences. For example, consider the time-use of individuals with limited time and limited income. Also, assume that a water park in the area where the individuals live reduces service times (to get on water rides) as a promotion strategy to attract more patrons. This may relax the time constraints of the individuals as they make their participation choices. However, many of the individuals may still decide not go to the

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budgeting approach or in the form of the use of a Hicksian composite commodity assumption. In the two stage budgeting approach, separability of preferences is invoked, and the allocation is pursued in two independent stages. The first stage entails allocation between a limited number of broad groups of consumption items, followed by the incomplete demand system allocation of the group expenditure to elementary commodities/services within the broad consumption group of primary interest to the analyst (the elementary commodities/services in the broad group of primary interest are referred to as "inside" goods). The plausibility of such a two stage budgeting approach requires strong homothetic preferences within each broad group and strong separability of preferences, or the less restrictive conditions of weak separability of preferences and the price index for each broad group not being too sensitive to changes in the utility function (see Menezes *et al.*, 2005). In the Hicksian composite commodity approach, one needs to assume that the prices of elementary goods within each broad group of consumption items vary proportionally. Then, one can replace all the elementary alternatives within each broad group (that is not of primary interest) by a single composite alternative representing the broad group. The analysis proceeds then by considering the composite goods as "outside" goods and considering consumption in these outside goods as well as in the finely categorized "inside" goods representing the consumption group of main interest to the analyst. It is common in practice in this Hicksian approach to include a single outside good with the inside goods. If this composite outside good is not essential, then the consumption formulation is similar to that of a complete demand system. If this composite outside good is essential, then the formulation needs minor revision to accommodate the essential nature of the outside good. The reader is referred to von Haefen (2010) for a discussion of the Hicksian approach and other incomplete demand system approaches such as the one proposed by Epstein (1982) that we do not consider here.

<sup>3</sup> The constraints included in our framework are structural constraints associated with limited resources. Psychological or personal barriers that limit consumption (such as personal tastes or beliefs) are included in the definition of the utility function, and are not modeled as constraints.

water park because of the income constraint they face. The net result would be that a model estimated only with a time constraint would not consider this income constraint effect and would underestimate the time-sensitivity of the individuals. Similarly, consider that the water park decides to reduce its admission fee. But individuals who are time constrained may still not be able to respond. In this case, the net result of ignoring the time constraint and using a single income constraint is an underestimation of the price sensitivity of the individuals. Further, the use of a single constraint in both these situations will likely lead to a poor data fit. The fundamental problem here is that there is a co-mingling of preference and constraint effects, leading to inconsistent preference estimation. Thus ignoring constraints will, in general, have serious negative repercussions for both model forecasting performance and policy evaluation.

To be sure, there has been earlier research in the literature considering multiple constraints (say  $R$  constraints), especially in the context of single discrete choice models. The basic approach of these studies, as proposed by Becker (1965) and sometimes referred to as a “full price” approach, essentially involves solving for  $(R-1)$  of the decision quantities (as a function of the remaining decision quantities) from  $(R-1)$  constraints, and substituting these expressions into the utility function and the one remaining constraint to reduce the utility maximization problem with multiple constraints to the case of utility maximization with a single constraint. Carpio *et al.* (2008) apply this “full price” approach in their model that includes the choice of an outside good and a single discrete choice from among all inside goods. Unfortunately, this single discrete choice-based approach is not easily extendable to the multiple discrete choice case because of the non-linearity of the utility expressions in the decision quantities. Even so, there is another problem with this approach. Specifically, there is an implicit assumption of the free exchangeability of constraints, which may not be valid because of the fundamentally different nature of the constraints. Thus, considering each constraint in its own right is a more direct and appealing way to proceed. Following Larson and Shaikh (2001), Hanemann (2006) provides a theoretical analysis for such a multi-constraint utility maximization problem for two and three constraints, and develops an algorithm to construct the demand functions for such multi-constraint problems by starting off with a system of demand functions that are known to solve the utility maximization problem with a single constraint. While an important contribution, the approach is rather circuitous and does not constitute a direct way of solving utility maximization problems with multiple constraints.

While there has been some research, even if limited, in the area of multiple constraints for single discrete choice models, the consideration of multiple constraints within the context of multiple discrete continuous (MDC) econometric models has received scant attention (though there have been theoretical expositions of such a framework in the microeconomics and home production fields; see Hanemann, 2006 and Jara-Díaz, 2007). The objective of this paper is to contribute to this area by developing a practical multiple constraint extension of the MDCEV model. In doing so, a brief overview of two precursor studies of relevance is in order. The first study by Parizat and Shachar (2010) applied an MDC model with two constraints, based on a constant elasticity of substitution (CES) function with nonlinear pricing. Because Kuhn-Tucker conditions are not sufficient for optimality with non-linear pricing, the estimation procedure is based on numerically locating the constrained optimal point, while taking all constraints into consideration. This is a substantial challenge, as acknowledged by Parizat and Shachar. They undertake the optimization using a simulated annealing algorithm after partitioning the solution space into regions. Of course, the approach obviates the need for a continuous, differentiable, and well-behaved utility function. But the approach loses the behavioral insights usually

obtained from the Kuhn-Tucker first-order conditions, and has to resort to a relatively “brute” force optimization approach rather than use analytic expressions during estimation. The second relevant study by Satomura *et al.* (2011) adopted a Bayesian approach to estimate an MDC model with multiple linear constraints. However, our effort (1) generalizes the restrictive Linear Expenditure System (LES) utility form used by Satomura *et al.*, (2) accommodates a random utility specification on all goods - inside and outside, (3) is applicable to the case of complete demand systems and incomplete demand systems (with outside goods that may be essential or non-essential), (4) allows for the presence of any number of outside goods, (5) shows how the Jacobian structure (and the overall consumption probability structure) has a nice closed-form structure for many MDC situations, which aids in estimation, and (6) is applicable also to the case where each constraint has an outside good whose consumption contributes only to that constraint and not to other constraints.

To summarize, the purpose of this paper is to develop a random utility-based model formulation that extends the MDCEV model to include multiple linear constraints. The model is applied to time-use decisions, where individuals are assumed to derive their utility from participation in one or more activities, subject to a monetary constraint and a fixed amount of time available. The data source used in our empirical exercise is generated by merging time-use data records from the 2008 American Time Use Survey with expenditure records from the 2008 U.S. Consumer Expenditure Survey.

The rest of the paper is structured as follows. Section 2 presents the model structure and estimation procedure. Section 3 illustrates an application of the proposed model for analyzing time use subject to budget and time constraints. The fourth and final section offers concluding thoughts and directions for further research.

## **2. MODEL FORMULATION**

In this section, we motivate and present the multiple constraint-MDCEV (or MC-MDCEV) model structure in the context of the empirical analysis in the current paper. We begin by considering two constraints – one being a money budget (or simply a “budget”) constraint and the other being a time constraint. However, while the alternatives in the empirical analysis refer to activity purposes for participation over a fixed time period, for presentation ease, we will refer to the alternatives in this section generally as goods. Also, the decision variables in our model correspond to the amount of each of several goods consumed over a certain fixed time interval, subject to multiple constraints operating on the consumption amounts. While quite general in many ways, the formulation does not consider multiple dimensions that characterize consumer choice situations in specific choice situations. For example, in a time allocation empirical context, it is not uncommon to consider both time allocations and goods consumption (required for activity participation) separately as decision variables in the utility function, and accommodate technological relationships between goods consumption and time allocations (see DeSerpa, 1971, Evans, 1972, Jara-Díaz, 2007, and Munizaga *et al.*, 2008). Accommodating such multiple dimensions and technological relationships is left for future research.

To streamline the presentation, we first consider the case of complete demand systems or the case of incomplete demand systems in the sense of the second stage of a two stage budgeting approach. Extension to the case of incomplete demand systems in the sense of the Hicksian approach is straightforward, and indeed makes the model simpler (see Section 2.3). In Section 2.4, we formulate a related model in which each constraint has an outside good whose

consumption contributes only to that constraint and not to others. Finally, in Section 2.5, we extend the analysis to include multiple (more than two) constraints.

## 2.1 Model Structure for Complete Demand Systems or the Second Stage of a Two Stage Incomplete Demand System

Consider Bhat's (2008) general and flexible functional form for the utility function that is maximized by a consumer subject to budget and time constraints:

$$\begin{aligned} \max U(\mathbf{x}) &= \sum_{k=1}^K \frac{\gamma_k}{\alpha_k} \psi_k \left( \left( \frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right) \\ \text{s.t.} \quad &\sum_{k=1}^K p_k x_k = E \\ &\sum_{k=1}^K g_k x_k = T \end{aligned} \tag{1}$$

where the utility function  $U(\mathbf{x})$  is quasi-concave, increasing and continuously differentiable,  $\mathbf{x} \geq 0$  is the consumption quantity ( $\mathbf{x}$  is a vector of dimension  $(K \times 1)$  with elements  $x_k$ ), and  $\psi_k$ ,  $\alpha_k$ , and  $\gamma_k$  are parameters associated with good  $k$ . The function  $U(\mathbf{x})$  in Equation (1) is a valid utility function if  $\psi_k > 0$ ,  $\gamma_k > 0$ , and  $\alpha_k \leq 1$  for all  $k$ . The reader will note that there is an assumption of additive separability of preferences in the utility form of Equation (1), as in literally all earlier MDC studies (the reader is referred to Vasquez Lavin and Hanemann (2008) and Bhat and Pinjari (2010) for modifications of the utility function in Equation (1) to accommodate non-additiveness, but we will confine attention to the additive separability case in this paper).<sup>4</sup>

The utility function form in Equation (1) clarifies the role of each of the  $\psi_k$ ,  $\alpha_k$ , and  $\gamma_k$  parameters. In particular,  $\psi_k$  represents the baseline marginal utility, or the marginal utility at the point of zero consumption.  $\gamma_k$  is the vehicle to introduce corner solutions for good  $k$  (that is, zero consumption for good  $k$ ), but also serves the role of a satiation parameter (higher values of  $\gamma_k$  imply less satiation). Finally, the express role of  $\alpha_k$  is to capture satiation effects. When  $\alpha_k = 1$  for all  $k$ , this represents the case of absence of satiation effects or, equivalently, the case of constant marginal utility (that is, the case of single discrete choice). As  $\alpha_k$  moves downward from the value of 1, the satiation effect for good  $k$  increases. When  $\alpha_k \rightarrow 0 \forall k$ , the utility function collapses to the following linear expenditure system (LES) form:

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<sup>4</sup> Additive separability implies that the marginal utility with respect to any good is independent of the levels of all other goods.

$$U(\mathbf{x}) = \sum_{k=1}^K \gamma_k \psi_k \ln\left(\frac{x_k}{\gamma_k} + 1\right).^5 \quad (2)$$

The first constraint in Equation (1) is the linear budget constraint, where  $E$  is the total expenditure across all goods  $k$  ( $k=1,2,\dots,K$ ) and  $p_k > 0$  is the unit price of good  $k$  (if modeling a complete demand system). The second constraint is the time constraint, where  $T$  is the time expenditure across all goods  $k$  ( $k=1,2,\dots,K$ ) and  $g_k > 0$  is the unit time of good  $k$ . Note that the model formulated here is not applicable to settings where  $p_k < 0$  or  $g_k < 0$ . Such a situation can arise, for example, in a time allocation setting in which participation in work activity generates money (since the associated unit price of partaking in work activity takes a negative value equal to the wage per unit of activity time). This setting leads to discontinuities in the money resource constraint with respect to consumption amounts, rendering the regular KT conditions insufficient for optimality.<sup>6</sup> But one way to view our model formulation in the time allocation context is that it is the second stage of a two-stage budgeting approach. In the first step, the individual chooses between work time (that generates money), sleep time, and non-work non-sleep time, given his/her wage. In the second step (at which the model formulation in this paper may be applied), the individual chooses among different non-work non-sleep activities, conditional on the first step budgeting.

To find the optimal allocation of goods, we construct the Lagrangian and derive the Kuhn-Tucker (KT) conditions. The Lagrangian function for the model of Equation (1) is:

$$L = U(\mathbf{x}) + \lambda \left( E - \sum_{k=1}^K p_k x_k \right) + \mu \left( T - \sum_{k=1}^K g_k x_k \right) \quad (3)$$

where  $\lambda$  and  $\mu$  are Lagrangian multipliers for the budget and time constraints, respectively. These values represent the marginal utility of expenditure and time. The KT first order conditions for optimal consumption allocations ( $x_k^*$ ) are:

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<sup>5</sup> Empirically speaking, it is difficult to disentangle the two effects of the  $\gamma_k$  and  $\alpha_k$  parameters separately, which leads to serious empirical identification problems and estimation breakdowns when one attempts to estimate both  $\gamma_k$  and  $\alpha_k$  parameters for each good. Thus earlier studies have either constrained  $\alpha_k$  to zero for all goods (technically, assumed  $\alpha_k \rightarrow 0 \forall k$ ) and estimated the  $\gamma_k$  parameters (as in Equation (2)), or constrained  $\gamma_k$  to 1 for all goods and estimated the  $\alpha_k$  parameters. This is discussed in detail by Bhat (2008), who suggests testing both these normalizations and selecting the model with the best fit.

<sup>6</sup> In traditional time allocation theory (see Jara-Díaz and Guerra, 2003 and Munizaga *et al.*, 2008), this is not an issue because the money resource constraint is expressed in terms of work time and the amount of each of several goods consumed per unit leisure time (in addition to fixed income and fixed expenditures). The utility function is expressed in terms of work times, leisure times, as well as consumption quantities of goods. Essentially, the multidimensional nature of the utility function, combined with the way the constraints are expressed, allows the use of KT conditions for optimality. The authors are currently working on extending the formulation in the current paper to multi-dimensional variables in the utility function.

$$\begin{aligned} \psi_k \left( \frac{x_k^*}{\gamma_k} + 1 \right)^{\alpha_k - 1} - \lambda p_k - \mu g_k &= 0 \quad \text{if } x_k^* > 0, \quad k = 1, 2, \dots, K \\ \psi_k \left( \frac{x_k^*}{\gamma_k} + 1 \right)^{\alpha_k - 1} - \lambda p_k - \mu g_k &< 0 \quad \text{if } x_k^* = 0, \quad k = 1, 2, \dots, K \end{aligned} \quad (4)$$

The optimal demand satisfies the conditions in Equation (1) and both constraints above. The budget and time constraints imply that only  $K - 2$  of the optimal consumptions  $x_k^*$  need to be estimated because, given  $E$  and  $T$ , the quantity consumed of two goods is automatically determined from the quantity consumed for all other goods. Denote goods 1 and 2 as the goods to which the individual allocates non-zero consumption (the individual has to participate in at least 2 of the  $K$  purposes). The KT conditions for these goods are:

$$\lambda + \mu h_1 = \frac{\psi_1}{p_1} \left( \frac{x_1^*}{\gamma_1} + 1 \right)^{\alpha_1 - 1}, \quad \lambda + \mu h_2 = \frac{\psi_2}{p_2} \left( \frac{x_2^*}{\gamma_2} + 1 \right)^{\alpha_2 - 1} \quad (5)$$

where  $h_k = g_k / p_k$ ,  $p_k \neq 0$ ,  $k = 1, 2, \dots, K$ . Solving the above equation system, the values of  $\lambda$  and  $\mu$  are given by:

$$\lambda = \frac{h_1 \tilde{V}_2 \psi_2 - h_2 \tilde{V}_1 \psi_1}{h_1 - h_2}, \quad \mu = \frac{\tilde{V}_1 \psi_1 - \tilde{V}_2 \psi_2}{h_1 - h_2} \quad (6)$$

where  $\tilde{V}_k = \frac{1}{p_k} \left( \frac{x_k^*}{\gamma_k} + 1 \right)^{\alpha_k - 1}$  ( $k = 1, 2, \dots, K$ ). Substituting  $\lambda$  and  $\mu$  into Equation (4), the KT conditions can be rewritten as:

$$\begin{aligned} \tilde{V}_k \psi_k &= (1 - \omega_k) \tilde{V}_1 \psi_1 + \omega_k \tilde{V}_2 \psi_2 \quad \text{if } x_k^* > 0, \quad k = 3, \dots, K \\ \tilde{V}_k \psi_k &< (1 - \omega_k) \tilde{V}_1 \psi_1 + \omega_k \tilde{V}_2 \psi_2 \quad \text{if } x_k^* = 0, \quad k = 3, \dots, K \end{aligned} \quad (7)$$

$$\text{where } \omega_k = \frac{h_1 - h_k}{h_1 - h_2}.$$

The KT conditions above have an intuitive interpretation. Note that, for any good ( $k = 1, 2, \dots, K$ ),  $\tilde{V}_k \psi_k$  represents the price-normalized marginal utility at the optimal consumption point  $x_k^*$ . The term  $\omega_k$  ( $k = 3, 4, \dots, K$ ) serves as a unique adjustment that applies to the marginal utilities of the chosen goods 1 and 2 in the  $k^{\text{th}}$  good's KT conditions. Specifically,  $\omega_k$  takes account of the fact that it is not only the marginal utilities of goods (based on the preferences of the consumer) that play into the optimal consumptions, but also the unit prices  $p_k$  and unit times  $g_k$  of the goods. That is,  $\omega_k$  serves the role of a price-time normalization involving the marginal utilities of the first two goods and good  $k$  ( $k = 3, 4, \dots, K$ ). To illustrate, consider the case when  $h_k = h_2$ , which in the context of our time-use application corresponds to  $p_k = p_2$  (since

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<sup>7</sup> To compute  $\omega_k$ , we need  $h_1 \neq h_2$ , or equivalently  $g_1/p_1 \neq g_2/p_2$ .

$g_k = 1 \forall k$ ). Then,  $\omega_k$  takes the value of one. The KT conditions for this good  $k$  then state that good  $k$ 's optimal consumption will either be (a) positive such that the price-normalized marginal utility at this optimal point is exactly equal to the price-normalized marginal utility of good 2 at good 2's optimal consumption point, or (b) zero if the price-normalized marginal utility at zero consumption for good  $k$  is less than the price-normalized marginal utility of good 2 at good 2's optimal consumption point. On the other hand, when  $h_k = h_1$  (or  $p_k = p_1$ ), the KT conditions for good  $k$  state that the optimal consumption for good  $k$  will either be (a) positive such that the price-normalized marginal utility at this optimal point is exactly equal to the price-normalized marginal utility of good 1 at good 1's optimal consumption point, or (b) zero if the price-normalized marginal utility at zero consumption for good  $k$  is less than the price-normalized marginal utility of good 1 at good 1's optimal consumption point. For other values of  $h_k$  not equal to  $h_1$  or  $h_2$ ,  $\omega_k$  serves to normalize the marginal utilities of goods 1,2, and  $k$  ( $k = 3,4,\dots,K$ ) to enforce the general notion that, for consumed goods, the price-time normalized marginal utilities are the same at the optimal allocations, while, for the non-consumed goods, the price-time normalized marginal utilities at zero consumption are lower than the price-time normalized marginal utilities at the optimal consumptions of the consumed goods.

Of course, as mentioned before, although our empirical setting is time allocation, the proposed model structure is derived in the general context of consumption goods, and is applicable to a wide variety of multiple choice consumer contexts.

## 2.2 Model Estimation

The baseline random marginal utility for each good is defined as:

$$\psi_k = \exp(\boldsymbol{\beta}'\mathbf{z}_k + \varepsilon_k), \quad k = 1,2,\dots,K \quad (8)$$

where  $\mathbf{z}_k$  is a set of attributes that characterize alternative  $k$  and the decision maker (including a constant), and  $\varepsilon_k$  captures the idiosyncratic (unobserved) characteristics that impact the baseline utility of good  $k$ . This parameterization guarantees the positivity of the baseline utility. Substituting this baseline utility form in Equation (7), the KT conditions, after some algebraic manipulations, are equivalent to:

$$\begin{aligned} \ln \tilde{V}_k + \boldsymbol{\beta}'\mathbf{z}_k + \varepsilon_k &= \ln\left((1 - \omega_k)\tilde{V}_1 e^{\boldsymbol{\beta}'\mathbf{z}_1 + \varepsilon_1} + \omega_k \tilde{V}_2 e^{\boldsymbol{\beta}'\mathbf{z}_2 + \varepsilon_2}\right) \quad \text{if } x_k^* > 0, \quad k = 3,\dots,K \\ \ln \tilde{V}_k + \boldsymbol{\beta}'\mathbf{z}_k + \varepsilon_k &< \ln\left((1 - \omega_k)\tilde{V}_1 e^{\boldsymbol{\beta}'\mathbf{z}_1 + \varepsilon_1} + \omega_k \tilde{V}_2 e^{\boldsymbol{\beta}'\mathbf{z}_2 + \varepsilon_2}\right) \quad \text{if } x_k^* = 0, \quad k = 3,\dots,K \end{aligned} \quad (9)$$

Let  $W_k | (\varepsilon_1, \varepsilon_2) = \ln\left((1 - \omega_k)\tilde{V}_1 e^{\boldsymbol{\beta}'\mathbf{z}_1 + \varepsilon_1} + \omega_k \tilde{V}_2 e^{\boldsymbol{\beta}'\mathbf{z}_2 + \varepsilon_2}\right) - \ln \tilde{V}_k - \boldsymbol{\beta}'\mathbf{z}_k$ ,  $k = 3,4,\dots,K$ . Under the assumptions that the unobserved terms  $\varepsilon_k$  are independently distributed across all alternatives ( $k = 1,2,\dots,K$ ) and independent of  $\mathbf{z}_k$ , and follow a standard extreme value distribution with scale parameter  $\sigma$ , the probability that the individual chooses the first  $M$  of the  $K$  goods ( $M \geq 3$ ), given  $\varepsilon_1$  and  $\varepsilon_2$ , is:

$$P(x_1^*, x_2^*, \dots, x_M^*, 0, \dots, 0 | (\varepsilon_1, \varepsilon_2)) = \left\{ \prod_{m=3}^M \frac{1}{\sigma} g\left(\frac{W_m | (\varepsilon_1, \varepsilon_2)}{\sigma}\right) \det(\mathbf{J}) | (\varepsilon_1, \varepsilon_2) \right\} \times \left\{ \prod_{l=M+1}^K G\left(\frac{W_l | (\varepsilon_1, \varepsilon_2)}{\sigma}\right) \right\} \quad (10)$$

where  $g$  is the standard extreme value density function,  $G$  is the standard extreme value cumulative distribution function, and  $\det(\mathbf{J}) | (\varepsilon_1, \varepsilon_2)$  is the determinant of the Jacobian  $\mathbf{J}$  with elements  $J_{in} = \frac{\partial \varepsilon_{i+2}}{\partial x_{n+2}}$  ( $i, n = 1, 2, \dots, M-2$ ) conditional on the error terms of the first two alternatives. The first component on the right side of Equation (10) involves the density of the  $(M-2)$  chosen alternatives based on a change-of-variable calculus (the transformation from the random utility errors  $(\varepsilon_m, m = 3, 4, \dots, M)$  to the consumptions  $(x_m, m = 3, 4, \dots, M)$  generates the Jacobian  $\mathbf{J}$ ; the first and second alternatives do not appear in this term because they can be derived from the consumption of the other goods). The determinant of the Jacobian, conditional on  $\varepsilon_1$  and  $\varepsilon_2$  (see Appendix A for the derivation), has the following closed form:

$$\det(\mathbf{J}) | (\varepsilon_1, \varepsilon_2) = \left[ \prod_{m=3}^M c_m \right] \left[ 1 + \sum_{m=3}^M \frac{p_m b_m | (\varepsilon_1, \varepsilon_2)}{c_m} \right] \quad (11)$$

$$\text{where } c_m = \frac{1 - \alpha_m}{x_m^* + \gamma_m} \text{ and } b_m | (\varepsilon_1, \varepsilon_2) = \frac{(1 - \omega_m) \tilde{V}_1 e^{\beta' z_1 + \varepsilon_1} c_1 + \omega_m \tilde{V}_2 e^{\beta' z_2 + \varepsilon_2} c_2}{(1 - \omega_m) \tilde{V}_1 e^{\beta' z_1 + \varepsilon_1} + \omega_m \tilde{V}_2 e^{\beta' z_2 + \varepsilon_2}}.$$

The second component on the right side of Equation (10) involves the probability of the goods that are not consumed  $(M+1, M+2, \dots, K)$ . This is obtained by integrating  $(\varepsilon_{M+1}^*, \varepsilon_{M+2}^*, \dots, \varepsilon_K^*)$  over the region consistent with no-consumption, based on the KT inequalities in Equation (9). Integrating out the error terms  $\varepsilon_1$  and  $\varepsilon_2$  from Equation (10), the unconditional probability can be computed as:

$$P(x_1^*, x_2^*, \dots, x_M^*, 0, \dots, 0) = \int_{\varepsilon_1 = -\infty}^{\infty} \int_{\varepsilon_2 = -\infty}^{\infty} \left\{ \prod_{m=3}^M \frac{1}{\sigma} g \left( \frac{W_m | (\varepsilon_1, \varepsilon_2)}{\sigma} \right) \det(\mathbf{J}) | (\varepsilon_1, \varepsilon_2) \right\} \times \left\{ \prod_{l=M+1}^K G \left( \frac{W_l | (\varepsilon_1, \varepsilon_2)}{\sigma} \right) \right\} \times f(\varepsilon_1) f(\varepsilon_2) d\varepsilon_1 d\varepsilon_2 \quad (12)$$

where  $f(\varepsilon_1)$  and  $f(\varepsilon_2)$  refer to the extreme value density function with scale parameter  $\sigma$ . Finally, substituting the expression for the Jacobian from Equation (11) into the above equation, we obtain the expression below:

$$P(x_1^*, x_2^*, \dots, x_M^*, 0, \dots, 0) = \frac{1}{\sigma^{M-2}} \left[ \prod_{m=3}^M c_m \right] \int_{\varepsilon_1 = -\infty}^{\infty} \int_{\varepsilon_2 = -\infty}^{\infty} \left[ 1 + \sum_{m=3}^M \frac{p_m b_m | (\varepsilon_1, \varepsilon_2)}{c_m} \right] \times \prod_{m=3}^M g \left( \frac{W_m | (\varepsilon_1, \varepsilon_2)}{\sigma} \right) \times \left\{ \prod_{l=M+1}^K G \left( \frac{W_l | (\varepsilon_1, \varepsilon_2)}{\sigma} \right) \right\} \times f(\varepsilon_1) f(\varepsilon_2) d\varepsilon_1 d\varepsilon_2. \quad (13)$$

In the case when there is only one constraint (*i.e.*,  $t_k = 0 \forall k$ ), the term  $\omega_k$  is equal to zero for all goods. As a result, the KT conditions from Equation (9) are equivalent to the traditional MDCEV's KT conditions, and the term  $b_m$  from the Jacobian is reduced to  $c_1$ . Then, the model collapses to the MDCEV with only one constraint. Thus, the multiple constraint MDCEV (MC-MDCEV) model in Equation (13) is the extension of the single constraint MDCEV model of Bhat (2008).

A couple of remarks about identification in the MC-MDCEV model are appropriate here. First, the scale parameter of the error terms  $\sigma$  is always estimable (at least from a theoretical standpoint) in the case of the MC-MDCEV, since  $h_k$  cannot all be equal to 1 (if this was the case, the model would collapse to a single constraint MDCEV model). That is, when  $h_k$  of at least two of the  $K$  goods are different, Equation (9) does not collapse in a way that can lead to non-identification of  $\sigma$  (see Bhat, 2008, who discusses the fact that, even in a single discrete MDCEV,  $\sigma$  is identified if the unit values of goods characterizing the single constraint are different). Second, as can be observed from the KT conditions in Equation (9), it is not the case in the MC-MDCEV model that only differences in the  $\beta'z_k$  terms matter. This is because the logarithm functional form operates on a function of the sum of quantities associated with the first two goods. However, note that the KT conditions in Equation (9), as well as the probability expression in Equation (13), are essentially derived based on the consumption pattern of only  $K - 2$  goods, since the consumption of the first and second goods may be obtained by solving the two constraints once the consumption pattern of other goods is known. Thus, while the KT conditions themselves (because of their functional form) do not impose any theoretical need for the normalization of constants and consumer-specific variables, it may be desirable to set the component of  $\beta'z_k$  corresponding to these terms to zero for at least one of the first two goods.

### 2.3 Model Structure for a Hicksian Approach-Based Incomplete Demand System

In this section, we consider the case when there are Hicksian composite outside goods and inside goods. This is easily handled with minor revisions to the framework discussed in Section 2.1. For ease in exposition, assume that there are two outside goods, good 1 and good 2 (however, the method proposed can handle as many outside goods as there are in a choice situation). If both of these outside goods are non-essential, the formulation is identical to that in Section 2.1. If both of these are essential, the formulation needs modification and actually simplifies compared to that in Section 2.1. If one of these is non-essential, and the other is essential, the formulation entails a simple modification from the case when both are essential. In this section, we present the case when both the goods are essential. Modifications to the case of more than two outside goods and combinations of essential and non-essential outside goods are also discussed.

As discussed previously, at least two goods have to be chosen when individuals face two constraints. Assume also that there is a minimum consumption for outside good 1, given by  $\gamma_1$  (the case of no minimum consumption becomes a special case with  $\gamma_1=0$ ). Similarly, assume that there is a minimum consumption of good 2, given by  $\gamma_2$ . Following the notation used in Section 2.1, the utility maximization problem is:

$$\begin{aligned} \max U(\mathbf{x}) &= \frac{\psi_1}{\alpha_1} \left[ (x_1 - \gamma_1)^{\alpha_1} - 1 \right] + \frac{\psi_2}{\alpha_2} \left[ (x_2 - \gamma_2)^{\alpha_2} - 1 \right] + \sum_{k=3}^K \frac{\gamma_k}{\alpha_k} \psi_k \left( \left( \frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right) \\ \text{s.t.} \quad & \sum_{k=1}^K p_k x_k = E \\ & \sum_{k=1}^K g_k x_k = T \end{aligned} \tag{14}$$

In the above formula, we need  $\gamma_k > 0$  for all  $k$ . Also, we need  $x_1 - \gamma_1 > 0$  and  $x_2 - \gamma_2 > 0$ . The result of the utility specification above is that an amount equal to  $\gamma_1$  for the first good, and

$\gamma_2$  for the second good, is first allocated to the two outside goods. Satiation effects for these first two goods start to “kick-in” only beyond these minimum consumption levels, at which point the usual satiation-based allocation mechanism sets in to determine consumption levels beyond the minimum quantities for the outside good, and the consumption levels of other inside goods. Since the  $\gamma_k$  and  $\alpha_k$  parameters serve very different roles for the outside goods, they are both theoretically estimable. However, because of the highly non-linear nature of the optimization problem, it is not uncommon to normalize some or all of these parameters to gain stability. A common normalization used in earlier multiple discrete choice studies is to set  $\alpha_k = 0$  (i.e.,  $\alpha_k \rightarrow 0$ ) as well  $\gamma_k = 0$  for the outside goods.

The constraints in Equation (14) are the same as earlier, with  $h_k = g_k/p_k$  ( $p_k \neq 0 \forall k$ ). Using the above formulation, one can go through the same procedure as in the previous section. All expressions provided in the previous section remain valid, with the following substitutions:

$$\tilde{V}_1 = \frac{1}{p_1}(x_1^* - \gamma_1)^{\alpha_1 - 1}, \quad \tilde{V}_2 = \frac{1}{p_2}(x_2^* - \gamma_2)^{\alpha_2 - 1} \text{ and } \tilde{V}_k = \frac{1}{p_k} \left( \frac{x_k^*}{\gamma_k} + 1 \right)^{\alpha_k - 1} \quad (k=3,4,\dots,K).$$

In the case of say three essential outside goods (say the first, second, and third goods), the expressions in the previous section again remain unchanged except that in addition to the substitutions for  $\tilde{V}_1$  and

$$\tilde{V}_2, \text{ we now also have } \tilde{V}_3 = \frac{1}{p_3}(x_3^* - \gamma_3)^{\alpha_3 - 1}.$$

In the case that the first outside good is an essential good, but not the second and third, the expressions in the previous section hold except that

$$\tilde{V}_1 = \frac{1}{p_1}(x_1^* - \gamma_1)^{\alpha_1 - 1} \text{ and } \tilde{V}_k = \frac{1}{p_k} \left( \frac{x_k^*}{\gamma_k} + 1 \right)^{\alpha_k - 1} \quad (k=2,3,4,\dots,K).$$

In this way, any number of outside goods (and any combination of essential and non-essential outside goods) can be accommodated.

#### 2.4 Model Structure for a Hicksian Approach-Based Incomplete Demand System with Constraint-Specific Numeraire Essential Outside Goods

In this section, we consider the case with two outside goods, denoted as the first and second goods. Let the first good be the numeraire good with respect to the budget constraint, so that  $p_1 = 1$  and it does not appear in the time constraint ( $g_1 = 0$ ). Let the consumption of the first good be denoted by  $x_1$  in money units. Similarly, let the second good be the numeraire good with respect to the second constraint, so that  $g_2 = 1$  and it does not appear in the budget constraint ( $p_2 = 0$ ). Let the consumption of the second good be denoted by  $x_2$  in time units. For instance, in the case of time-use, one may use savings as the first good (this has no time investment) and in-home leisure as the second good (this has no expenditure). Assume also that there is a minimum consumption for good 1, given by  $\gamma_1$  (the case of no minimum consumption becomes a special case with  $\gamma_1 = 0$ ). Similarly, assume that there is a minimum consumption of good 2, given by  $\gamma_2$ . Such a situation cannot immediately be handled by the framework in Section 2.3, because  $h_2 = g_2/p_2$  becomes undefined for the second alternative (and formulating the constraints in a form that uses the unit price in the numerator and the unit time in the denominator will not work either because the corresponding value is undefined for the first alternative).

Following the notation used in Section 2.3, the utility maximization problem is:

$$\begin{aligned} \max U(\mathbf{x}) &= \frac{\psi_1}{\alpha_1} [(x_1 - \gamma_1)^{\alpha_1} - 1] + \frac{\psi_2}{\alpha_2} [(x_2 - \gamma_2)^{\alpha_2} - 1] + \sum_{k=3}^K \frac{\gamma_k}{\alpha_k} \psi_k \left( \left( \frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right) \\ \text{s.t.} \quad & \sum_{k=3}^K p_k x_k + x_1 = E \\ & \sum_{k=3}^K g_k x_k + x_2 = T \end{aligned} \quad (15)$$

The Lagrangian function for the model of Equation (15) is:

$$L = U(\mathbf{x}) + \lambda \left( E - \left( \sum_{k=3}^K p_k x_k + x_1 \right) \right) + \mu \left( T - \left( \sum_{k=3}^K g_k x_k + x_2 \right) \right). \quad (16)$$

Following the same procedure as for inside goods, and using the baseline preference structure  $\psi_k = \exp(\boldsymbol{\beta}' \mathbf{z}_k + \varepsilon_k)$  for all alternatives, the KT first order conditions for optimal consumption allocations  $(x_k^*)$ , conditional on  $\varepsilon_1$  and  $\varepsilon_2$ , are:

$$\begin{aligned} \varepsilon_k &= W_k | (\varepsilon_1, \varepsilon_2) \quad \text{if } x_k^* > 0, \quad k = 3, \dots, K \\ \varepsilon_k &< W_k | (\varepsilon_1, \varepsilon_2) \quad \text{if } x_k^* = 0, \quad k = 3, \dots, K \end{aligned} \quad (17)$$

where  $W_k | (\varepsilon_1, \varepsilon_2) = \ln(\tilde{V}_1 e^{\boldsymbol{\beta}' \mathbf{z}_1 + \varepsilon_1} + \tilde{V}_2 h_k e^{\boldsymbol{\beta}' \mathbf{z}_2 + \varepsilon_2}) - \ln \tilde{V}_k - \boldsymbol{\beta}' \mathbf{z}_k$ ,

$$\tilde{V}_k = \frac{1}{p_k} \left( \frac{x_k^*}{\gamma_k} + 1 \right)^{\alpha_k - 1} \quad (k = 3, 4, \dots, K), \quad \tilde{V}_1 = (x_1^* - \gamma_1)^{\alpha_1 - 1}, \quad \text{and} \quad \tilde{V}_2 = (x_2^* - \gamma_2)^{\alpha_2 - 1}.$$

Using the same assumptions on the error terms as earlier, the unconditional probability that the individual chooses the first  $M$  of the  $K$  goods ( $M \geq 3$ ) is:

$$\begin{aligned} P(x_1^*, x_2^*, \dots, x_M^*, 0, \dots, 0) &= \int_{\varepsilon_1 = -\infty}^{\infty} \int_{\varepsilon_2 = -\infty}^{\infty} \left\{ \prod_{m=3}^M \frac{1}{\sigma} g \left( \frac{W_m | (\varepsilon_1, \varepsilon_2)}{\sigma} \right) \det(\mathbf{J}) | (\varepsilon_1, \varepsilon_2) \right\} \\ &\quad \times \left\{ \prod_{l=M+1}^K G \left( \frac{W_l | (\varepsilon_1, \varepsilon_2)}{\sigma} \right) \right\} \times f(\varepsilon_1) f(\varepsilon_2) d\varepsilon_1 d\varepsilon_2, \end{aligned} \quad (18)$$

The elements of the Jacobian are given by:

$$J_{in} | (\varepsilon_1, \varepsilon_2) = \frac{\partial \varepsilon_{i+2}}{\partial x_{n+2}} = p_{n+2} (a_{i+2} + b_{i+2} h_{n+2}) + \delta_{in} c_{i+2}, \quad i, n = 1, 2, \dots, M-2, \quad \text{where} \quad (19)$$

$$\begin{aligned} a_{i+2} &= \frac{\tilde{V}_1 e^{\boldsymbol{\beta}' \mathbf{z}_1 + \varepsilon_1} c_1}{\tilde{V}_1 e^{\boldsymbol{\beta}' \mathbf{z}_1 + \varepsilon_1} + \tilde{V}_2 h_{i+2} e^{\boldsymbol{\beta}' \mathbf{z}_2 + \varepsilon_2}}, \quad b_{i+2} = \frac{\tilde{V}_2 h_{i+2} e^{\boldsymbol{\beta}' \mathbf{z}_2 + \varepsilon_2} c_2}{\tilde{V}_1 e^{\boldsymbol{\beta}' \mathbf{z}_1 + \varepsilon_1} + \tilde{V}_2 h_{i+2} e^{\boldsymbol{\beta}' \mathbf{z}_2 + \varepsilon_2}}, \quad c_1 = \frac{1 - \alpha_1}{x_1 - \gamma_1}, \quad c_2 = \frac{1 - \alpha_2}{x_2 - \gamma_2}, \\ c_k &= \frac{1 - \alpha_k}{x_k + \gamma_k} \quad \text{for } k = 3, 4, \dots, K, \quad \text{and } \delta_{in} = 1 \text{ if } i = n \text{ and } \delta_{in} = 0 \text{ if } i \neq n. \end{aligned}$$

In this case, there is no closed-form structure for the determinant of the Jacobian, because of the presence of the  $h_{n+2}$  term in the  $in^{\text{th}}$  Jacobian element. But each element of the Jacobian

may be constructed in a straightforward fashion based on the expressions above and then its determinant can be taken. If in the development above,  $\alpha_k = 0$  for all  $k$ ,  $\gamma_1 = \gamma_2 = 0$ ,  $\gamma_k = 1$  for  $k = 3, 4, \dots, K$ ,  $\psi_1 = \psi_2 = 1$ , and the error terms  $\varepsilon_1$  and  $\varepsilon_2$  (on the outside goods) are assumed not to exist (that is, their distributions collapse on zero), the result is Satomura *et al.*'s (2011) model.

## 2.5 More Than Two Constraints

Now consider the case with  $R$  constraints and complete demand systems or the second stage of a two stage incomplete demand systems. Each constraint is associated with a limited resource (money, time, space, *etc.*). To estimate the MDCEV model with  $R$  constraints, individuals should consume at least  $R$  goods from the choice set, and the maximization problem is given by:

$$\begin{aligned} \max U(\mathbf{x}) &= \sum_{k=1}^K \frac{\gamma_k}{\alpha_k} \psi_k \left( \left( \frac{x_k}{\gamma_k} + 1 \right)^{\alpha_k} - 1 \right) \\ \text{s.t.} \quad &\sum_{k=1}^K a_k^1 x_k = A^1 \\ &\sum_{k=1}^K a_k^2 x_k = A^2 \\ &\vdots \\ &\sum_{k=1}^K a_k^R x_k = A^R \end{aligned} \tag{20}$$

where  $a_k^r$  is the unitary contribution of good  $k$  to constraint  $r$  ( $a_k^r \geq 0 \forall k = 1, 2, \dots, K$ ,  $\forall r = 1, 2, \dots, R$ ) and  $A^r$  is the total availability of resource  $r$  ( $\forall r = 1, 2, \dots, R$ ). This problem can be solved in the same way as for the case with two constraints, except that the probability expression for the consumption pattern will now involve  $R$  integrals, one for each constraint. Modifications to cases with incomplete demand systems with Hicksian composite outside goods are similar to the two-constraint case.

## 3. APPLICATION

In the past decade and more, the activity-based approach to travel demand analysis has received much attention and seen considerable progress (see Pinjari and Bhat, 2011 and Ronald *et al.* (2008) for recent reviews). A fundamental difference between the commonly-used trip-based approach and the activity-based approach is the way time is conceptualized and represented in the two approaches. In the trip-based approach, time is reduced to being simply a “cost” of making a trip. The activity-based approach, on the other hand, treats time as an all-encompassing continuous entity within which individuals make activity/travel participation decisions. Thus, the central basis of the activity-based approach is that individuals’ travel patterns are a result of their time-use decisions, which determine the generation and scheduling of trips. In this context, the empirical application in the current paper contributes to the now growing number of utility-based micro-economic models of time-use (see Jiang and Morikawa, 2004; Bhat, 2005; Jara-Díaz, 2007; Munizaga *et al.*, 2011).

### 3.1 Data

The data source used for this analysis is obtained by combining two different disaggregate national survey data sets -- the 2008 American Time Use Survey (ATUS) and the 2008 Consumer Expenditure Survey (CES). The ATUS survey provides information on the amount of time individuals spend undertaking various in-home as well as out-of-home activities (such as work, study and recreational activities) on a pre-assigned day of the week (see U.S. BLS, 2011a for details on the ATUS survey). The data was collected through telephone interviews, and only individuals aged 15 years or older were eligible. The survey also obtained socioeconomic and demographic characteristics, the location of activities, and information on accompanying individual(s). The CES survey provides data on the consuming and buying habits of households, both on a weekly basis (information is gathered based on two consecutive one-week survey periods) and over a longer period of time (information is gathered based on a quarterly period of expenditure). The survey (see U.S. BLS, 2011b for additional information) includes information on small and frequent expenditures (such as grocery shopping, personal care, *etc.*) as well as larger and longer- term expenditures (household appliances, vehicles, *etc.*). Dollar amounts of the purchases (both goods and services) made during the survey period are recorded by the respondents irrespective of whether or not payment is made at the time of purchase.

For the current demonstration exercise to show the applicability of our proposed MC-MDCEV model, we used a combined and synthesized weekly time-use and expenditure data that Konduri *et al.* (2011) put together from the ATUS and the CES surveys. Since the ATUS collected time-use data at the individual level, while the CES survey obtained information at the household level, the analysis is confined to single individual households. The final sample used in the current empirical exercise includes the weekly time-use and expenditure patterns of 332 single individual households.<sup>8</sup>

The decision variables used in this application are the weekly times allocated to different activities, measured in minutes. In the ATUS-CES sample developed by Konduri *et al.* (2011), 19 time use categories (by activity purpose) are defined, including work, study, personal business and care, shopping, social, entertainment and travel, separated by in-home and out-of-home activities. The weekly expenditures are categorized into 14 activity purposes, but they are not

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<sup>8</sup> A weekly analysis period is considered here because there is likely to be a weekly rhythm in time use and expenditure patterns (see Habib *et al.*, 2008). For full details of the synthesizing procedure and the scaling approach to a week's period from the ATUS daily time-use data and the CES weekly/quarterly data, the reader is referred to Konduri *et al.* (2011). Essentially, for the ATUS data, individuals who were surveyed on Sunday were chosen and time use patterns for Monday through Saturday were generated by appending records of individuals who reported time use patterns on other days of the week (based on matching on seven socioeconomic characteristics of interest - gender, age, employment status, race, college status, family income, and employment category). A weekly expenditure data set was constructed by applying a simple deflating factor approach on the CES quarterly data. The matching of the time-use and expenditure data was again undertaken based on a set of socioeconomic characteristics. While one can certainly debate the merits and appropriateness of such a synthetic data generation procedure, suffice it to say that the authors were not able to obtain any data set which collected both time-use and expenditure data. Given the importance of this issue in terms of the substantial benefits to be accrued from including time-use and expenditure constraints, it is hoped that concerted efforts will be undertaken in the future to obtain data on both these important drivers characterizing activity participation and time-use. In the meantime, assembling synthetic data to study the issue is the best and only possible way to proceed. Further, the imputation methods used are consistent with approaches used in a variety of fields for data imputation in which missing fields are filled by borrowing information from another record with similar attributes. Of course, in interpreting model results from any synthetic data generation procedure, an added layer of caution needs to be exercised. Also, the intent of this empirical exercise is primarily to show the applicability of the proposed model.

associated one-to-one with the time use categories. To apply our model, we need the time and expenditures for each alternative. Therefore, the time use activity purposes and the expenditure activity purposes are brought to a common four activity purpose classification taxonomy as follows: (1) personal care (includes personal care, child care, healthcare, religious and spiritual activities and phone calls, considering both in and out home activities), (2) eat out (includes all foods and drinks consumed out-of-home), (3) leisure (in and out-of home social activities, recreation, sports, exercise and entertainment) and (4) shopping (both in and out home shopping activities). The budget constraint represents limited purchasing power, and the unit price  $p_k$  was computed for each alternative as the total expenditures (in US dollars) divided by the total time allocated (in minutes) across all individuals. The time constraint represents time as a limited resource, bounded by the available time after performing mandatory activities, such as work and sleep. As discussed earlier, since the decision variables themselves represent time investments,  $g_k = 1 \forall k$ .

Table 1 provides a summary of the final sample used in estimation. The time use by activity purpose shows that the first three alternatives are always chosen (the minimum time allocated is always greater than zero; that is, these three alternatives are “outside goods”). The final activity purpose, shopping, is selected by 97.3% of the individuals (that is, shopping is an “inside good”). The reason for these high levels of participation is the use of a weekly time frame. However, the presence of several outside goods does not pose problems because, as highlighted in Section 2.3, our proposed model can accommodate as many outside goods as there are in any choice context. The time use patterns in the different activity purposes in Table 1 indicate that individuals spend a substantial amount of time on leisure (about 29 hours per week, or 4 hours per day, on average), followed by personal care (about 8.5 hours per week, or 1.2 hours per day, on average). Shopping and eat out, on the other hand, are activity purposes in which individuals generally expend less time. These results are generally consistent with the associated unitary costs: leisure and personal care are the least expensive activities, while the most expensive ones are shopping and eat out. Even this preliminary data analysis suggests that individuals may not only be constrained by time, but also by income.

Information on the independent variables is provided in the bottom half of Table 1. The sample has a slightly higher proportion of males relative to females, and the expected higher share of individuals of Caucasian origin (this includes individuals with a Hispanic background). Given that all individuals in the sample are employed, the percentage of students (both full and part time) is low. Following the definitions made by the U.S. Census Bureau (2010), information regarding the geographic area where the individuals live is also provided, including Midwest, South, and West and Northeast. The age range in the sample is between 20 and 64 years. The average number of hours worked per week is 41.8, which is a little higher than the standard five eight-hour days (almost 10% of the workers work more than 55 hours per week). Finally, the average weekly income is US\$1,048, which roughly translates to an annual household income of about \$54,500.

### **3.2 Variable and Utility Form Specification**

Individual socio-demographics and work-related characteristics were considered in the analysis. Socio-demographics capture the generic contextual and preference differences across individuals, while work-related characteristics capture the effects of more specific work schedules and time flexibility related attributes. In addition, we also considered interaction effects among the two sets of variables. The final variable specification was based on a

systematic process of removing statistically insignificant variables and combining variables when their effects were not significantly different.

As discussed in Section 2.3, we set the baseline preferences for the first good to zero due to stability considerations. A number of alternative model forms were explored for the  $\alpha_k$  and  $\gamma_k$  parameters, including (1) setting  $\alpha_k$  to zero for all goods, and estimating the  $\gamma_k$  values (the  $\gamma$ -profile), (2) setting  $\alpha_k$  to zero for all goods, setting the  $\gamma_k$  values to zero for the outside goods, and estimating the  $\gamma_k$  values for the inside (shopping) good (the  $\gamma^1$ -profile), (3) setting  $\alpha_k$  to zero for all goods, setting the  $\gamma_k$  values for the outside goods to the minimum consumptions as obtained from the descriptive statistics in Table 1 (the  $\gamma^2$ -profile), and estimate the  $\gamma_k$  value for the inside good, (4) setting the  $\gamma_k$  values to zero for the outside goods and one for the inside good (shopping), and estimating the  $\alpha_k$  values (the  $\alpha$ -profile), (5) setting the  $\gamma_k$  values to the minimum consumptions, constraining  $\gamma_k$  for shopping to 1, and estimating the  $\alpha_k$  values (the  $\alpha^1$ -profile), (6) setting the  $\gamma_k$  values to zero for the outside goods, normalizing the  $\alpha_k$  values for the inside goods to zero, and estimating the  $\alpha_k$  values for the outside goods and the  $\gamma_k$  value for the inside good (the  $\alpha\gamma$ -profile), and (7) setting the  $\gamma_k$  values to the minimum consumptions for the outside goods, normalizing the  $\alpha_k$  values for the inside goods to zero, and estimating the  $\alpha_k$  values for the outside goods and the  $\gamma_k$  value for the inside good (the  $\alpha\gamma^1$ -profile). While all of these profiles were estimable, for some of these profiles, we observed convergence and stability problems as manifested in large estimated standard errors. In any case, at the end, the  $\gamma^1$ -profile consistently emerged as the “winner” among these alternative profiles as well as provided stable estimates, and is the one used in this paper (as indicated in Section 2.3, this  $\gamma^1$ -profile has been assumed *a priori* in most earlier studies).

### 3.3 Model Estimation Results

In addition to the multiple constraint MDCEV (MC-MDCEV) model proposed in this paper, we also estimated two single constraint MDCEV (SC-MDCEV) models in which only the time constraint is active or only the money constraint is active. The results of the time constrained MDCEV, money constrained MDCEV and MC-MDCEV models are presented in Table 2 (note that the scale parameter  $\sigma$  is not estimable in the time-constrained SC-MDCEV model; the reason for this is discussed in detail in Bhat, 2008). The comparison of the results of the three models highlights two primary differences in variable effects. First, some variables are statistically insignificant in the SC-MDCEV models (gender in the money-constrained model and weekly income in both the singly constrained models), while they are statistically significant in the MC-MDCEV model. Second, the effects of variables on the choice process differ substantially across the two models, both in sign and magnitude. From a data fit standpoint, the log-likelihood measures for the SC-MDCEV models are -6,639.1 (time-constrained) and -4,744.5 (money-constrained), while the corresponding value for the MC-MDCEV model is -3,018.7. Although the improvement in log-likelihood measures of the MC-MDCEV model over the SC-MDCEV models is readily apparent, one can evaluate the models using a non-nested likelihood ratio test. For presentation ease, we focus on a comparison of the money-constrained SC-MDCEV (that provides a better fit than the time-constrained SC-MDCEV model) and the

MC-MDCEV model proposed in this paper. For this test, we use the base as the convergent log-likelihood value (say,  $L(C)$ ) of the money constrained MDCEV model with only the baseline constants and the shopping satiation parameter. This value, as shown in Table 2, is -4949.5. With respect to this base model, the adjusted rho-bar squared ( $\bar{\rho}_c^2$ ) for the money-constrained MDCEV model is 0.0402, while that for the MC-MDCEV model is 0.3889.<sup>9</sup> The probability that the difference in the adjusted rho-bar squared ( $\bar{\rho}_c^2$ ) values, which is 0.3889, could have occurred by chance is less than  $\Phi\{-[-2 \times 0.3889 \times L(C)]^{0.5}\}$ . This value is literally zero, indicating that the difference in adjusted rho-bar squared values between the two models is highly statistically significant and that the MC-MDCEV model is the “runaway winner” from a data fit perspective. Indeed, the superiority of the MC-MDCEV model, if anything, is rather shocking.

In general, the MC-MDCEV offers plausible behavioral interpretations in the effects of exogenous variables. The gender effect indicates that men are more likely than women to participate in leisure activities. This result reinforces the stereotype of men being “glued to the tube”, a finding also observed in Habib *et al.* (2008) and Carrasco and Miller (2009). The influence of the geographic region of residence suggests that individuals living in the West region of the United States have a higher baseline preference for shopping. There is no obvious explanation for this finding, though the variable helps control for region-level differences in time-use patterns. Among the individual demographic variables, age, race and student status had no significant effects on time use.

The remaining two variables impacting the baseline preferences relate to work characteristics. Individuals who work less than 35 hours per week are more likely to shop than those who work more than or equal to 35 hours per week, possibly a reflection of time constraints that deter participation in shopping, and a preference to participate in recreation and leisure activities after long workdays (see Goulias and Kim, 2001 for a similar result). Finally, low and middle income individuals (earning less than \$1500 per week) participate less in eat-out activities relative to their high income earning counterparts.

The baseline preference constants reflect the higher overall time investment in leisure and lower time investment in shopping and eat-out compared to personal care activities. The translation parameter for shopping allows corner solutions for that activity type.

Finally, because our model is based on constrained utility maximization, the Lagrangian multipliers may be gainfully employed to investigate the money value of leisure time (VLT). In particular, the multiplier  $\lambda$  in Equation (3) is the marginal utility of income (it provides the increase in utility due to an increase in the expenditure constraint by one unit) and the multiplier  $\mu$  is the marginal utility of time (it provides the increase in utility due to an increase in the available time by one unit). Thus, the implied VLT is  $\mu/\lambda$ , which represents the willingness to pay to increase the available time  $T$  by one hour. For each individual in the sample, this VLT may be formulated as the ratio of the right sides of the two expressions in Equation (6), and then estimated by integrating out the stochasticity embedded in the baseline utilities for the first two goods. Then, the average VLT obtained (across individuals) is 62.18 US\$/hour, a value that is similar to that obtained in Konduri *et al.* (2011). This VLT value may be used for user benefits

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<sup>9</sup> The adjusted rho-bar squared value ( $\bar{\rho}_c^2$ ) for each of the SC-MDCEV and MC-MDCEV models is computed as  $\bar{\rho}_c^2 = 1 - [(L(\hat{\beta}) - H) / L(C)]$ , where  $L(\hat{\beta})$  is the log-likelihood at convergence,  $H$  is the number of model parameters (excluding the baseline constants and the shopping translation parameter), and  $L(C)$  is as already defined earlier. In both the SC-MDCEV and MC-MDCEV models,  $H = 6$ .

computations and social welfare analysis to evaluate the cost-benefits of investing in infrastructure improvements or in policies that have the effect of increasing participation in leisure and other non-work activities.

#### 4. CONCLUSIONS

Multiple-discrete continuous choice models have gained attention in recent years to handle choice situations where consumers choose multiple alternatives simultaneously, along with a quantity dimension associated with the consumed alternatives. However, such models have been dominated by the assumption of a single linear resource constraint, which, when combined with consumer preferences, determines the optimal consumption point. However, in reality, consumers typically face multiple resource constraints such as those associated with time, money, and capacity. Ignoring such multiple constraints and instead using a single constraint can, and in general will, lead to poor data fit and inconsistent preference estimation, because there is a co-mingling of preference and constraint effects. In turn, this can have serious negative repercussions for both model forecasting performance and policy evaluation.

In this paper, we have extended the multiple-discrete continuous extreme value (MDCEV) model to accommodate multiple constraints. Our formulation of the multiple constraints-MDCEV (MC-MDCEV) model uses a flexible and general utility function form, accommodates a random utility specification on all (inside and outside) goods, is applicable to the case of complete demand systems and incomplete demand systems (with outside goods that may be essential or non-essential), allows for the presence of any number of outside goods, shows how the Jacobian structure has a nice closed-form structure for many MDC situations, and is applicable also to the case where each constraint has an outside good whose consumption contributes only to that constraint and not to other constraints. Issues associated with identification are also discussed. The proposed MC-MDCEV model is applied to time-use decisions, where individuals are assumed to derive their utility from participation in one or more activities within a fixed time interval and a monetary constraint. The sample for the empirical exercise was generated by combining time-use information from the 2008 American Time Use Survey with expenditure records from the 2008 U.S. Consumer Expenditure Survey.

The final sample included 332 workers who lived alone, and who could choose from among four aggregate activity purpose alternatives within a week: personal care, eat out, leisure and shopping. The first three alternatives are always allocated some positive amount of time by all individuals in the sample. The estimation results show substantial differences across the MC-MDCEV and the SC-MDCEV models in the estimated effects of variables. While it is difficult to definitively state that the parameter estimates from the MC-MDCEV model represent the “true” effects of variables, there is a clear suggestion that the SC-MDCEV models are mis-estimated, given the vast improvement in data fit of the proposed MC-MDCEV model compared to the SC-MDCEV models. Overall, the results strongly reinforce the notion that ignoring multiple constraints when present can have serious consequences for both forecasting purposes and welfare/policy analysis.

Of course, as with any research exercise, there are several avenues for further research. Two of these that the authors are currently pursuing include a deeper analysis of empirical identification and stability issues during estimation, and the development of efficient algorithms for forecasting for the MC-MDCEV model. On the forecasting issue, one approach would be to use an iterative gradient-based algorithm to solve the constrained non-linear optimization problem, but this would be inefficient. Pinjari and Bhat (2010) have recently devised an

algorithm for the single constraint MDCEV case that solves the problem by building on simple, yet insightful, analytic explorations with the Kuhn-Tucker conditions of optimality. One possibility for forecasting with the MC-MDCEV model is to use Pinjari and Bhat's approach in an iterative fashion by cycling among the multiple constraints, while applying the approach for each constraint. Efficient cycling mechanisms should be possible. Other approaches that exploit special properties of the Kuhn-Tucker conditions for the MC-MDCEV model are also being explored.

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## Appendix A: Computation of the Determinant of the Jacobian

For ease in presentation in this Appendix, we will not explicitly indicate that the Jacobian computation is conditional on the error terms  $\varepsilon_1$  and  $\varepsilon_2$ . The elements of the Jacobian are given by:

$$J_{in} = \frac{\partial \varepsilon_{i+2}}{\partial x_{n+2}}, \quad i, n = 1, 2, \dots, M - 2 \quad (\text{A.1})$$

where the error term of alternative  $i+2$  is:

$$\varepsilon_{i+2} = \ln\left((1 - \omega_{i+2})\tilde{V}_1 e^{\beta'z_1 + \varepsilon_1} + \omega_{i+2}\tilde{V}_2 e^{\beta'z_2 + \varepsilon_2}\right) - \ln\tilde{V}_{i+2} - \beta z_{i+2}, \quad i = 1, 2, \dots, M - 2 \quad (\text{A.2})$$

Then, the  $in^{\text{th}}$  element of the Jacobian is:

$$J_{in} = \frac{1}{(1 - \omega_{i+2})\tilde{V}_1 e^{\beta'z_1 + \varepsilon_1} + \omega_{i+2}\tilde{V}_2 e^{\beta'z_2 + \varepsilon_2}} \left[ (1 - \omega_{i+2})e^{\beta'z_1 + \varepsilon_1} \frac{\partial \tilde{V}_1}{\partial x_{n+2}} + \omega_{i+2}e^{\beta'z_2 + \varepsilon_2} \frac{\partial \tilde{V}_2}{\partial x_{n+2}} \right] - \frac{1}{\tilde{V}_{i+2}} \frac{\partial \tilde{V}_{i+2}}{\partial x_{n+2}} \quad (\text{A.3})$$

Given that  $\tilde{V}_1 = \frac{1}{p_1} \left( \frac{1}{\gamma_1} \left( E - \sum_{r \neq 1}^K p_r x_r^* \right) + 1 \right)^{\alpha_1 - 1}$  and  $\tilde{V}_2 = \frac{1}{p_2} \left( \frac{1}{\gamma_2} \left( E - \sum_{r \neq 2}^K p_r x_r^* \right) + 1 \right)^{\alpha_2 - 1}$ , Equation

(A.3) is equivalent to:

$$J_{in} = p_n \frac{(1 - \omega_{i+2})\tilde{V}_1 e^{\beta'z_1 + \varepsilon_1} c_1 + \omega_{i+2}\tilde{V}_2 e^{\beta'z_2 + \varepsilon_2} c_2}{(1 - \omega_{i+2})\tilde{V}_1 e^{\beta'z_1 + \varepsilon_1} + \omega_{i+2}\tilde{V}_2 e^{\beta'z_2 + \varepsilon_2}} + \frac{1}{\tilde{V}_{i+2}} \left( \tilde{V}_{i+2} \delta_{in} c_{i+2} \right) \quad (\text{A.4})$$

where  $c_m = \frac{1 - \alpha_m}{x_m^* + \gamma_m}$ ,  $m = 1, 2, \dots, M$  (all chosen alternatives),  $\delta_{in} = 1$  if  $i = n$  and  $\delta_{in} = 0$  if

$i \neq n$ . Finally, the elements of the Jacobian are given by:

$$J_{in} = p_{n+2} b_{i+2} + \delta_{in} c_{i+2}, \quad i, n = 1, 2, \dots, M - 2; \quad (\text{A.5})$$

$$b_{i+2} = \frac{(1 - \omega_{i+2})\tilde{V}_1 e^{\beta'z_1 + \varepsilon_1} c_1 + \omega_{i+2}\tilde{V}_2 e^{\beta'z_2 + \varepsilon_2} c_2}{(1 - \omega_{i+2})\tilde{V}_1 e^{\beta'z_1 + \varepsilon_1} + \omega_{i+2}\tilde{V}_2 e^{\beta'z_2 + \varepsilon_2}}$$

To compute the determinant of the Jacobian, consider the case where the individual chooses 5 alternatives. In this case the Jacobian is the  $3 \times 3$  matrix presented below.

$$\mathbf{J} = \begin{bmatrix} b_3 p_3 + c_3 & b_3 p_4 & b_3 p_5 \\ b_4 p_3 & b_4 p_4 + c_4 & b_4 p_5 \\ b_5 p_3 & b_5 p_4 & b_5 p_5 + c_5 \end{bmatrix} \quad (\text{A.6})$$

Because of the special structure of the Jacobian, conditional on  $\varepsilon_1$  and  $\varepsilon_2$ , it is straight forward to see that its determinant is given by:

$$\det(\mathbf{J}) = \left( \prod_{i=1}^3 c_{i+2} \right) \left( 1 + \sum_{i=1}^3 \frac{p_{i+2} b_{i+2}}{c_{i+2}} \right), \quad \text{or equivalently, } \det(\mathbf{J}) = \left( \prod_{m=3}^5 c_m \right) \left( 1 + \sum_{m=3}^5 \frac{p_m b_m}{c_m} \right) \quad (\text{A.7})$$

In the more general case with  $M$  consumed alternatives, the Jacobian, after explicitly recognizing the conditionality on the error terms  $\varepsilon_1$  and  $\varepsilon_2$ , takes the form in Equation (11) of the main paper.

**Table 1. Sample Characteristics (332 observations)**

| <b>Alternatives</b>          |                            |                         |                  |                |                |
|------------------------------|----------------------------|-------------------------|------------------|----------------|----------------|
|                              |                            | <b>Mean</b>             | <b>Std. Dev.</b> | <b>Minimum</b> | <b>Maximum</b> |
| Personal care                | Time Use<br>[min]          | 511.1                   | 269.8            | 95             | 1931           |
| Eat Out                      |                            | 258.3                   | 137.8            | 13             | 752            |
| Leisure                      |                            | 1750.0                  | 488.3            | 561            | 3592           |
| Shopping                     |                            | 141.5                   | 114.2            | 0              | 525            |
| Personal care                | Unitary cost<br>[US\$/min] | 0.055                   | 0.135            | 0.004          | 1.943          |
| Eat Out                      |                            | 1.340                   | 3.093            | 0.012          | 43.393         |
| Leisure                      |                            | 0.117                   | 0.203            | 0.001          | 2.331          |
| Shopping                     |                            | 1.922                   | 5.398            | 0.008          | 64.172         |
| <b>Explanatory Variables</b> |                            |                         |                  |                |                |
| <b>Discrete Variables</b>    |                            | <b>Sample Share [%]</b> |                  |                |                |
| <i>Gender</i>                |                            |                         |                  |                |                |
| Male                         |                            | 53.9                    |                  |                |                |
| Female                       |                            | 46.1                    |                  |                |                |
| <i>Race</i>                  |                            |                         |                  |                |                |
| Caucasian                    |                            | 77.1                    |                  |                |                |
| African American             |                            | 19.6                    |                  |                |                |
| Other                        |                            | 3.3                     |                  |                |                |
| <i>Student status</i>        |                            |                         |                  |                |                |
| Student                      |                            | 6.3                     |                  |                |                |
| Not a student                |                            | 93.7                    |                  |                |                |
| <i>Geographic region</i>     |                            |                         |                  |                |                |
| Midwest                      |                            | 27.4                    |                  |                |                |
| South                        |                            | 34.6                    |                  |                |                |
| West and Northeast           |                            | 38.0                    |                  |                |                |
| <b>Continuous Variables</b>  |                            | <b>Mean</b>             | <b>Std. Dev.</b> | <b>Minimum</b> | <b>Maximum</b> |
| Age [years]                  |                            | 43.7                    | 12.0             | 20.0           | 64.0           |
| Hours worked per week        |                            | 41.8                    | 11.2             | 1.5            | 69.5           |
| Weekly income [US\$]         |                            | 1,048.2                 | 795.0            | 188.6          | 5,393.4        |

**Table 2. Estimation Results**

| Explanatory Variable  | Time-Constrained MDCEV | Money-Constrained MDCEV | Multiple Constraints MC-MDCEV |
|---|------------------------|-------------------------|-------------------------------|
|   | Parameter (t-stat)     | Parameter (t-stat)      | Parameter (t-stat)            |
| <i>Gender</i>   |                        |                         |                               |
| Males   |                        |                         |                               |
| Leisure   | 0.4080 (3.108)         | 0.2008 (0.902)          | 0.5025 (6.353)                |
| <i>Geographic region</i>  |                        |                         |                               |
| West  |                        |                         |                               |
| Shopping  | 0.3156 (1.875)         | 0.5561 (1.893)          | 0.2365 (2.555)                |
| <i>Hours worked per week</i>  |                        |                         |                               |
| Less than 35 hours  |                        |                         |                               |
| Shopping  | 0.3676 (2.319)         | 0.7199 (2.602)          | 0.1951 (2.812)                |
| <i>Weekly income [US\$]</i>   |                        |                         |                               |
| Less than 1,500 US\$  |                        |                         |                               |
| Eat Out   | 0.0737 (0.429)         | 0.3260 (1.149)          | -0.9276 (-124.134)            |
| Baseline preference constants   |                        |                         |                               |
| Eat Out   | -0.8245 (-5.116)       | -3.8588 (-14.495)       | -0.0440 (-5.869)              |
| Leisure   | 1.0906 (10.137)        | -0.8021 (-4.494)        | 1.0228 (17.080)               |
| Shopping  | -3.8492 (-19.836)      | -4.2361 (-14.171)       | -3.8724 (-89.275)             |
| Satiation Parameters ( $\gamma$ )   |                        |                         |                               |
| Shopping  | 8.7343 (4.942)         | 4.2436 (3.747)          | 2.6158 (32.664)               |
| Scale Parameter ( $\sigma$ )  | Not estimable          | 1.5722 (39.313)         | 0.4706 (127.643)              |
| Number of Parameters  | 7                      | 8                       | 8                             |
| Log-likelihood at convergence   | -6,639.1               | -4,744.5                | -3,018.7                      |
| Log-likelihood of Money-constrained SC-MDCEV with only baseline constants and translation parameter | -4949.5                |                         |                               |
| Number of Observations  | 332                    |                         |                               |