Econometric calibration of the joint time assignment – mode choice model

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Abstract

This paper describes the derivation and the econometric calibration of a joint time assignment – mode choice model with a microeconomic foundation, to be applied to the TASTI (Time ASsignment Travel and Income) database. The econometric procedure is a full information maximum likelihood with three nonlinear continuous equations and one discrete choice. We use Lee's transformation to include correlations between the continuous and discrete equations. This allows us to estimate (a) the value of time as a resource or value of assigning time to a pleasurable activity, (b) the value of assigning time to work, and (c) the value of assigning time to travel. We apply the method and obtain reasonable results. Finally, we identify some econometric challenges for further research.

INTRODUCTION

Traditional transport mode choice models are based on random utility maximization (RUM) theory in which choice is based on a utility function V_j that depends mainly on the travel cost and travel time of mode j. As pointed out by Train and McFadden (1978), V_j can be viewed as an indirect utility function that originates from consumer behavior theory in which both goods and leisure are considered as sources of utility (see also Gronau 1973, Becker 1981; 1965, Mincer 1962; 1963, and DeSerpa, 1971). This behavioral framework has been further enhanced in the last decade, incorporating contributions from the field of home production. In this broader microeconomic framework, utility is viewed as depending on all activities participated in by the individual and the goods consumed while pursuing those activities. From this perspective, discrete mode choice models, time assignment models and goods consumption models originate from a common microeconomic framework. However, most of the efforts to date adopting such a perspective have been rather theoretical in nature.

At the same time that theoretically derived microeconomic frameworks have been developed to examine time-use in activities in the economics and home production fields, there have also been parallel empirically-driven research efforts to study time use in the travel behavior field to better understand and forecast travel (see Bhat and Koppelman, 1999; Pendyala and Goulias, 2002; and Arentze and Timmermans (2004). Some of these travel behavior studies are based on frameworks that are not derived from utility theory (for example, see Allaman *et al.*, 1982; Damm and Lerman, 1981; van Wissen, 1989; Lu and Pas, 1999; Golob, 1998; Meka *et al.*, 2002; Fujii *et al.*, 1999; Bhat, 1998), while others use frameworks with utility theory as the fundamental basis for time use (Munshi, 1993; Kitamura, 1983; Kitamura *et al.*, 1996; Yamamoto and Kitamura, 1999; Bhat and Misra, 1999; Meloni *et al.*, 2004; Bhat, 2005; Ettema, 2005; Srinivasan and Bhat, 2006; Chen and Mokhtarian, 2005). An important difference, though, between the latter group of travel behavior studies and the economics studies is that the travel behavior studies have generally considered time as being the only constraint in time allocation, ignored goods consumption in the formulation, and focused on discretionary activities.

In the past few years, the research work of Jara-Diaz and his colleagues has straddled this economic-travel behavior divide. In particular, their line of research has focused on using a complete microeconomic framework and translating this into an empirically estimable model system. In this context, a recent paper by Jara-Díaz and Guevara (2003) developed and estimated a behavioral model that encompasses time assigned to work and mode choice. However, this model has three important limitations. First, the only non-work activity to which time is assigned is to travel. This generates a work duration model that depends only on travel cost and travel time. Second, the assignment of time to other activities is not considered. Third, the authors assume independence between the error terms of the work and travel mode choice equations for simplicity, an assumption that was later relaxed by Munizaga *et al.* (2006). The microeconomic model was theoretically extended by Jara-Díaz and Guerra (2003) to include all activities, goods consumption, and travel choice. However, they did not develop an econometric framework to estimate their microeconomic model. In this paper, we present an econometric approach to calibrate Jara-Díaz and Guerra's (2003) model for activities and travel, and apply the approach using data from a Chilean time-use survey.

In the rest of this section, we provide a brief overview of the model developed by Jara-Díaz and Guerra (2003) and identify the equation system to be calibrated. In section 2, we summarize the econometric tools available to estimate the different components of the proposed equation system. We then formulate an econometric approach to calibrate the new model system and specify the likelihood function to be maximized. In section 3, we provide an overview of the data and sample used in the empirical analysis. In section 4, we present the empirical results. The final section concludes the paper.

Model of Jara-Díaz and Guerra

The microeconomic model developed by Jara-Díaz and Guerra (2003) follows the general approach by DeSerpa (1971), where individual utility comes from the activities people participate in and the goods consumed while pursuing the chosen activities. There are two budget constraints, one that includes income from different sources and all the expenditures, and another that accounts for total available time. Finally, technological constraints are considered regarding goods consumption and time assigned to activities.

Let T_i be the time assigned to activity i and let X_j be the amount of good j consumed during period τ, with minima given by T_i^{Min} and X_j^{Min}, respectively. Define T_w as the time assigned to work, P_j as the price of good j, w as the wage rate, c_f as the total fixed expenditure (does not depend on goods consumption) and I_f as the exogenous fixed income. If utility is given by a Cobb-Douglas form, further define η_j and θ_i as the exponents associated with good j and activity i, respectively, and Ω as a positive constant. Then consumer behavior regarding time assignments and goods consumption can be described by the constrained utility maximization problem in Equations (1) to (5). Note that monotonic transformations of utility do not change the problem, which implies that the sum of all exponents can be used to normalize utility such that the summation of all normalized exponents is equal to one. A slightly different but useful normalization is presented below. The signs of the marginal utilities are the signs of the θ_i and η_j exponents. Second derivatives have the opposite signs.

$$
\begin{aligned}
&Max \quad U = \Omega T_w^{\theta_w} \prod_i T_i^{\theta_i} \prod_j X_j^{\eta_j} && (1)\\
&s.t. \\
&I_f + wT_w - \sum_j P_j X_j - c_f \geq 0 \quad \leftarrow \quad \lambda \text{ (Income-expenditure constraint)} && (2)\\
&\tau - T_w - \sum_i T_i = 0 \quad \leftarrow \quad \mu \text{ (Time constraint)} && (3)\\
&T_i - T_i^{Min} \geq 0 \quad \forall i \quad \leftarrow \quad \kappa_i \text{ (Minimum time investment constraint)} && (4)\\
&X_j - X_j^{Min} \geq 0 \quad \forall j \quad \leftarrow \quad \varphi_j \text{ (Minimum good consumption constraint)} && (5)
\end{aligned}
$$

where the Lagrange multipliers associated with each constraint have been included on the right side of the constraint. Note that, by definition, μ/λ is the value of time as a resource or value of leisure. Let I be the set of freely chosen activities, R the set of activities assigned the minimum required T_r^{Min}, K the set of freely chosen goods, and J the set of goods for which the minimum required X_j^{Min} is consumed. Note that unconstrained activities (those that are freely assigned more time than the minimum) must have equal positive marginal utilities (all equal to μ), otherwise they would not be undertaken. Besides, every unpleasant activity will be assigned the exogenous minimum, because the sign of its marginal utility is the same irrespective of duration under this specification. This does not mean that an activity that is assigned the minimum time is necessarily unpleasant, because the optimal time assignment could be less than the exogenous minimum.

From the first order conditions of this optimization problem, Jara-Díaz and Guerra (2003) obtain an equations system for time assigned to work (6), time assigned to unconstrained activities (7) and goods consumption (8).

$$T_w^{\ *} = \left[\left(\tau - T_f\right)\beta + \frac{G_f}{w}\alpha\right] + \sqrt{\left[\left(\tau - T_f\right)\beta + \frac{G_f}{w}\alpha\right]^2 - \frac{G_f}{w}\left(2\alpha + 2\beta - 1\right)\left(\tau - T_f\right)} \qquad (6)$$

$$T_i^{\ *} = \frac{\widetilde{\theta}_i}{\left(1 - 2\beta\right)}\left(\tau - T_w^{\ *} - T_f\right) \quad \forall i \in I \qquad (7)$$

$$X_k^{\ *} = \frac{\widetilde{\eta}_k}{P_k\left(1 - 2\alpha\right)}\left(wT_w^{\ *} - G_f\right) \quad \forall k \in K \qquad (8)$$

where $\alpha=(A+\theta_w)/2(A+B+\theta_w)$, $\beta=(B+\theta_w)/2(A+B+\theta_w)$, A is the addition of the θ exponents over all unconstrained activities and B is the addition of the η exponents over all unconstrained goods. The other terms are defined in Equation (9). Note that G_f deals with expenses on goods in J, and T_f is the time committed to activities in R.

$$G_f = \left(\sum_{j \in J} P_j X_j^{\ Min} + c_f - I_f\right), T_f = \sum_{r \in R} T_r^{\ Min}\ , \widetilde{\theta}_i = \frac{\theta_i}{\left(A + B + \theta_w\right)}, \widetilde{\eta}_k = \frac{\eta_k}{\left(A + B + \theta_w\right)} \qquad (9)$$

Equation (6) says that if α and β are positive, then work time increases with G_f/w, which is the minimum work period to cover fixed expenses, and with available time. On the other hand, expressions (1-2β) and (1-2α) are equal to $A/(A+B+\theta_w)$ and $B/(A+B+\theta_w)$ respectively, the former associated with aggregated leisure and the latter with aggregated discretionary goods consumption. Then, Equation (7) says that time assigned to leisure activity i increases with θi and with available time, and Equation (8) says that consumption of good k increases with η_k and with available income.

Due to the existence of time and income budget constrains, only n-1 time assignment and goods consumption equations can be calibrated (where n is the number of unconstrained activities or goods). For each restricted variable, a discrete choice model could be specified and calibrated if there are data available, as explained below. In some cases, it may not be clear which activities (or goods) are restricted, but this can be explored empirically.

Once the time assignment and goods consumption equations are derived, Jara-Díaz and Guerra obtain an expression for the indirect utility function by replacing the optimal values from (6), (7) and (8) into (1), which generates an indirect utility function $V(w, G_f, T_f)$. If constrained activity i is characterized by time t_i and cost c_i, the indirect utility can be trivially transformed into a conditional indirect utility function V_i by simply considering t_i and c_i explicitly as part of T_f and G_f , respectively. In other words, one can make $T_f=T_f'+t_i$ and $G_f= G_f'+c_i$. This way, the resulting function $V_i(t_i, c_i, w)$ is, by definition, the maximum utility that can be obtained conditional on alternative i. If time and cost refers to travel, then this is the conditional indirect utility function commanding travel choice.

The model system described above not only allows the efficient calibration of parameters, but also enables the calculation of the different values of time. These are: the value of time as a resource (value of leisure), the value of assigning time to a particular activity, and the value of saving time in a particular restricted activity. Following Jara-Díaz and Guerra (2003), the value of leisure can be calculated as:

$$\frac{\mu}{\lambda} = \frac{(1-2\beta)}{(1-2\alpha)} \frac{\left(wT_w^* - G_f\right)}{\left(\tau - T_w^* - T_f\right)} \ . \tag{10}$$

The value of assigning time to work is given by

$$\frac{\partial U/\partial T_w}{\lambda} = \frac{(2\alpha+2\beta-1)}{(1-2\alpha)} \frac{\left(wT_w^* - G_f\right)}{T_w^*} \ . \tag{11}$$

Finally, the value of assigning time to a restricted activity t, $(\partial U/\partial T_t)/\lambda$ can be calculated from

$$\frac{\kappa_t}{\lambda} = \frac{\mu}{\lambda} - \frac{\partial U/\partial T_t}{\lambda} \ . \tag{12}$$

As shown by Jara-Díaz and Guevara (2003), κ_t/λ can be directly obtained from a discrete choice model (travel) as the ratio between the marginal utility of time and cost in the conditional indirect utility function, which is the value of saving time in that activity. Subtracting this from the value of leisure yields the value of assigning time to that particular restricted activity. Note that this latter is the value of the marginal utility, *i.e.* the pleasure or displeasure of travel, which is different from the commonly known subjective value of travel time savings (κ_t/λ) that includes both the opportunity cost and the (dis)pleasure of assigning time to travel.

Therefore, to be able to calculate values of time (10), (11) and (12), we have to estimate the parameters α, β, and κ_t/λ from the (6)-(7)- $V_i(t_i, c_i, w)$ model system, where the time assignment equations are continuous and nonlinear, while the conditional indirect utility function represents a discrete choice problem. These equations can be calibrated separately as independent equations, or jointly acknowledging the presence of correlation among equations due to common variables and parameters. The next section discusses the econometric methods to estimate the equations.

2. ECONOMETRIC CALIBRATION OF A TIME ASSIGNMENT AND MODE CHOICE MODEL SYSTEM WITH CORRELATION

2.1 Discrete-Continuous Model Systems

The methods developed for discrete-continuous choices (*i.e.*, where one or more continuous variable choices are related in some way to a discrete choice in a way that requires the joint modeling of the continous and discrete choices) typically fall under one of two categories: structural equations model systems (see Golob, 1998; Simma and Axhausen, 2001; and Schwanen *et al.*, 2004 for applications in a travel behavior context) and econometric model systems (see, for example, Kitamura, 1983; Mannering and Hensher, 1987; and Bhat, 2005). The first approach is a powerful statistical multivariate analysis technique based on path diagrams, which represent the researcher's beliefs about causal effects. The second approach has a more fundamental basis in utility theory. Both approaches have been used extensively in the literature. In the current paper, we use the second approach since our model system is based on a theoretical microeconomic framework that determines the nature of the relationship between the discrete and continuous choices.

If the individuals have *n* different activities to assign their time (excluding constrained activities that are assigned the exogenous minimum), then our model system has *n*-1 nonlinear continuous equations and one discrete choice. The calibration of a system of nonlinear simultaneous equations, with correlated errors, can be accomplished using the method proposed by Gallant (1975), as an extension of Zellner (1962)'s seemingly Unrelated Regression method. The general idea of the method proposed by Gallant (1975) is to perform a first stage, where the error covariance matrix is estimated by applying Nonlinear Minimum Squares to each equation separately, and then estimate all the parameters simultaneously applying Aitken Generalized Least Squares to the whole system using the estimated covariance matrix. A more efficient approach is the Full Information Maximum Likelihood method that assumes a normal additive error for each equation. From this assumption, a joint density function can be expressed for the error terms. This function depends on the models parameters and on standard deviations and correlations of the error terms. The method searches the set of parameters that maximizes the likelihood function evaluated for the calibration sample. Both the model parameters and the parameters that describe the error structure are calibrated simultaneously, using all the information available. The simple Multinomial Logit (MNL) model, which assumes iid Gumbel error terms, is assumed for the mode choice model in this study.

Within the context of a system of discrete-continuous equations, there are approaches to deal with basically two types of problem: endogeneity bias and selectivity bias. Endogeneity bias is expected when the continuous equation includes an endogenous variable. Selectivity bias occurs when the observed values of the variable in a continuous equation are related to a particular choice in the discrete process. To deal with selectivity bias, Heckman (1979) proposed a method that has been widely used in labor supply models (where the number of working hours can only be observed for those individuals who actually work). The main idea of the method is to calibrate the discrete choice model first, assuming a normal distribution for the error terms (Probit model), and then introducing a correction term in the continuous equation. The selectivity correction term is calculated from the choice probabilities predicted by the Probit Model. The continuous equation error term is also assumed to be distributed normal.

A problem with the above procedure is that it does not correct for endogeneity, which can be present if there are common parameters in the discrete and continuous equations, and if the error terms of both types of equations are correlated. Lee (1983) proposed the calibration of the discrete-continuous model system by maximizing a joint full information maximum likelihood function. The method involves transforming *a priori* assumed marginal distributions for each error term into the standard normal, and generating a joint multivariate normal distribution of the resulting transformed error terms. Within the context of a time assignment model, Lee's method has been applied by Barnard and Hensher (1992) to examine shopping destination choice and retail expenditure, and by Bhat (1998) to jointly model the decisions of participating in home versus out of home activities and how much time to allocate to each of them. Another application has been that of Munizaga *et al.* (2006), for a continuous equation of time assigned to work and a discrete equation of mode choice. The approach proposed by Lee (1983) is general enough and practical to tackle the calibration of the models proposed by Jara-Díaz and Guerra (2003), as it allows multiple alternatives for the discrete choice, and permits the inclusion of correlation between this and the continuous equations. This is discussed next.

2.2 Specification of Time Assignment and Mode Choice Equations

The equation system we consider includes three time assignment continuous equations, one for work (Equation 6) and one each for personal care and entertainment (Equation 7). We do not consider an equation for consumption of goods (Equation 8) because of lack of data on this dimension. For clarity, the continuous equation system is rewritten by defining D_q as in Equation (15), obtaining Equation (13). The η terms represent normally distributed error terms and the sub index q stands for individual.

$$T_{Wq}{}^{*} = \left(\tau - \sum_{r\in R} T_{rq}{}^{Min}\right)\left(\beta + D_q\alpha + \sqrt{\left[\beta + D_q\alpha\right]^2 - D_q\left(2\alpha + 2\beta - 1\right)}\right) + \eta_{T_{Wq}} \tag{13}$$

$$T_{lq}{}^{*} = \frac{\tilde{\theta}_l}{1-2\beta}\left(\tau - T_{Wq}{}^{*} - \sum_{r\in R} T_{rq}{}^{Min}\right) + \eta_{T_{lq}} \quad l = Personal\ Care,\ Entertainment \tag{14}$$

where

$$D_q = \frac{G_{fq}}{w_q\left(\tau - \sum_{r\in R} T_{rq}{}^{Min}\right)} \tag{15}$$

Equations (13) and (14) could be calibrated separately. Equation (13) allows estimating α and β using information on time assigned to work, time assigned to restricted activities, fixed expenditures and the wage rate. Equation (14) allows estimating $\tilde{\theta}_l/(1-2\beta)$ and α using information on time allocated to the non restricted activity l, total time allocated to restricted activities, fixed expenditures and the wage rate. Note that travel is included in R and the travel time used has to be that of the actually observed choice.

As indicated earlier, we assume additive error terms η with a different variance in each equation. We expect that the presence of common parameters (α and β) and common exogenous variables would cause correlation among equations, so that this should be considered in the error structure. Now we will explain how we derive the likelihood function for the joint activities equations system, using some general properties of the distribution functions. Taking $Y = (y_1, y_2, ..., y_n)$ as a vector of random variables, μ as a vector that contains the means and Σ its covariance matrix, the general expression for the joint density function is:

$$f(Y) = (2\pi)^{-n/2}|\Sigma|^{-1/2}\exp\left[-\frac{1}{2}(Y-\mu)'\Sigma^{-1}(Y-\mu)\right] \tag{16}$$

This density function can also be expressed from the marginal and conditional distributions. Let y_1 be any subset of the variables, including the case of only one variable, and let y_2 be the remaining variables. We can partition μ and Σ in the same way, so

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \qquad \Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \tag{17}$$

According to the marginal and conditional normal distributions, if $[y_1, y_2]$ follow a joint multivariate normal, then the marginal distributions are

$$\begin{aligned} y_1 &\sim N(\mu_1, \Sigma_{11}) \\ y_2 &\sim N(\mu_2, \Sigma_{22}) \end{aligned} \tag{18}$$

The conditional distribution of y_1 given y_2 is also normal:

$$y_1 / y_2 \sim N(\mu_{1,2}, \Sigma_{11,2}) \tag{19}$$

where $\mu_{1,2} = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(y_2 - \mu_2)$ and $\Sigma_{11,2} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$.

Using this theorem, the joint density can be expressed as the product of two terms:

$$f(y) = f_{1,2}(y_1 / y_2) f_2(y_2) \tag{20}$$

where $f_{1,2}$ is the conditional utility function of y_1 in y_2 and f_2 is the marginal function of y_2.

From these general expressions, we have written the equations for a three variables case, applying equation (20) two times. The three normally distributed variables are the error terms of the time assigned to work equation and the equations of two non-restricted activities (personal care and entertainment). The computation of the likelihood requires defining the normalized error term for each equation:

$$\nu_q = \frac{T_{Wq} - \left(\tau - \sum_{r\in R} T_{rq}{}^{Min}\right)\left(\beta + D_q\alpha + \sqrt{\left[\beta + D_q\alpha\right]^2 - D_q\left(2\alpha + 2\beta - 1\right)}\right)}{\sigma_W} \quad W = Work \tag{21}$$

$$\omega_q = \frac{T_{Cq} - \frac{\tilde{\theta}_C}{1-2\beta}\left(\tau - T^*_{Wq} - \sum_{r\in R} T_{rq}{}^{Min}\right)}{\sigma_C} \quad C = Personal\ Care \tag{22}$$

$$a_q = \frac{T_{Eq} - \frac{\tilde{\theta}_E}{1-2\beta}\left(\tau - T^*_{Wq} - \sum_{r\in R} T_{rq}{}^{Min}\right)}{\sigma_E} \quad E = Entertainment \tag{23}$$

We will also define some auxiliary variables that will appear in the final likelihood function that originate from the elements of Σ in equation (19)

$$\rho^* = \sqrt{1 - \frac{1}{1-\rho^2_{W,C}}\left(\rho^2_{W,E} - 2\rho_{W,C}\rho_{W,E}\rho_{C,E} + \rho^2_{C,E}\right)} \tag{24}$$

$$a_q{}^* = a_q - \frac{1}{1-\rho^2_{W,C}}\left[\left(\rho^2_{W,E} - \rho_{W,C}\rho_{C,E}\right)\nu_q + \left(\rho^2_{C,E} - \rho_{W,C}\rho_{W,E}\right)\omega_q\right] \tag{25}$$

where σ_i is the standard deviation of the error terms of equation i, $\rho_{i,j}$ is the correlation between the errors of equation i and equation j, and T_{iq} is the time individual q assigns to activity i. The joint density function for this three-variable case can be expressed as:

$$f\left(\eta_{T_W}, \eta_{T_C}, \eta_{T_E} / \alpha, \beta, \theta_C, \theta_E\right) = \frac{1}{\sigma_W\sigma_C\sigma_E\rho^*\sqrt{1-\rho^2_{W,C}}}\phi\left(\nu_q\right)\phi\left(\frac{\omega_q - \rho_{W,C}\nu_q}{\sqrt{1-\rho^2_{W,C}}}\right)\phi\left(\frac{a_q{}^*}{\rho^*}\right) \tag{26}$$

where we have transformed a variable distributed normal into standard normal using

$$f(y) = \frac{1}{\sigma}\phi\left(\frac{y-\mu}{\sigma}\right) \tag{27}$$

Finally, the log likelihood of the sample can be written as:

$$LL(\alpha, \beta, \theta_C, \theta_E) = \sum_q ln\left[\frac{1}{\sigma_W \sigma_C \sigma_E \rho^* \sqrt{1-\rho_{W,C}^2}}\phi(\nu_q)\phi\left(\frac{\omega_q - \rho_{W,C}\nu_q}{\sqrt{1-\rho_{W,C}^2}}\right)\phi\left(\frac{a_q^*}{\rho^*}\right)\right] \tag{28}$$

So far, we have only derived the log-likelihood function for the three continuous choice time assignment equations. However, we also have a discrete mode choice multinomial logit model (some more general specifications such as Nested Logit and Mixed Logit were also calibrated, but they where not significantly better than the simpler MNL). Mode *i* will be chosen if its utility is larger than the utility of all the other alternatives, as expressed in Equation (29).

$$U_i \geq \underset{\substack{\forall j \\ j \neq i}}{Max}\, U_j \tag{29}$$

U_i can have an observable component V_i and an error term ε_i (iid Gumbel for the case of MNL). So condition (29) can be written as:

$$V_i \geq \left(\underset{\substack{\forall j \\ j \neq i}}{Max}\, U_j - \varepsilon_i\right) \equiv \omega_i \tag{30}$$

Given the properties of the Gumbel distribution, this new error term ω_i distributes Logistic, so the distribution function is:

$$F(V_i) = \frac{exp(V_i)}{exp(V_i) + \sum_{\substack{\forall j \\ j \neq i}} exp(V_j)} \tag{31}$$

As mentioned in Section 1, there is an analytical expression for V_i derived from the direct utility function, which is a very complex nonlinear equation. For simplicity, we will only use here a linear approximation of that expression, which depends on time, cost, and a mode constant, as shown in Equation (32).

$$V_{iq} = \gamma_i + \gamma_t t_{iq} + \gamma_c c_{iq} \tag{32}$$

This model is calibrated by Maximum Likelihood. The logarithm of the likelihood of a calibration sample with independent observations is given by:

$$lnL(\theta) = \sum_q \sum_{A_i \in A(q)} \delta_{iq} ln(F_{iq}) \tag{33}$$

where δ_{iq} is equal to one if individual q chooses alternative i and zero otherwise and F_{iq} is the choice probability given by (31).

To incorporate correlation between the continuous equations and the discrete choice, using the method proposed by Lee (1983), ω_i defined in Equation (22) can be transformed into a standard normal term by applying the inverse normal function. Let D_{iq} be a dummy variable which is equal to one if individual q chooses mode i and zero otherwise. Then Equation (30) can be written as:

$$D_{iq} = 1 \quad \Leftrightarrow \quad V_i \geq \omega_i \tag{34}$$

This is a monotonical transformation, so the inequality that rules mode choice (Equation 34) can be written as:

$$D_{iq} = 1 \quad \Leftrightarrow \quad J_i(V_i) = \Phi^{-1}(F_i(V_i)) \geq \Phi^{-1}(F_i(\omega_i)) = \omega_i^* \tag{35}$$

Next, we can express the likelihood function using Equation (35) and the fact that T_{Wq}, T_{Cq} and T_{Eq} are observed when $D_{iq} = 1$. Given the trivariate normal distribution of $\eta_{T_{Wq}}$, $\eta_{T_{Cq}}$ and $\eta_{T_{Eq}}$, and the standard normal distribution of ω_i^*, we now have a multivariate normal distribution of $\eta_{T_{Wq}}$, $\eta_{T_{Cq}}$, $\eta_{T_{Eq}}$ and ω_i^*.[1]

To write the likelihood function of this system, we use Equations (21) to (25), and add:

$$y_{iq} = \Phi^{-1} F_i(\gamma z_{iq}) \tag{36}$$

$$\hat{\rho}_i = \frac{1}{1 + 2\rho_{W,C}\rho_{W,E}\rho_{C,E} - \rho_{W,C}^2 - \rho_{W,E}^2 - \rho_{C,E}^2} \begin{bmatrix} \rho_{W,i}^2 + \rho_{C,i}^2 + \rho_{E,i}^2 - \rho_{W,i}^2\rho_{C,E}^2 - \rho_{C,i}^2\rho_{W,E}^2 - \rho_{E,i}^2\rho_{W,C}^2 \\ +2\rho_{W,i}\rho_{C,i}\rho_{W,E}\rho_{C,E} + 2\rho_{W,i}\rho_{E,i}\rho_{W,C}\rho_{C,E} + 2\rho_{C,i}\rho_{E,i}\rho_{W,E}\rho_{W,C} \\ -2\rho_{W,i}\rho_{C,i}\rho_{W,C} - 2\rho_{W,i}\rho_{E,i}\rho_{W,E} - 2\rho_{C,i}\rho_{E,i}\rho_{C,E} \end{bmatrix} \tag{37}$$

$$y_{iq}^* = \frac{1}{1 + 2\rho_{W,C}\rho_{W,E}\rho_{C,E} - \rho_{W,C}^2 - \rho_{W,E}^2 - \rho_{C,E}^2} \begin{bmatrix} \left(\rho_{W,i} - \rho_{W,i}\rho_{C,E}^2 - \rho_{W,C}\rho_{C,i} - \rho_{W,E}\rho_{E,i} + \rho_{W,C}\rho_{C,E}\rho_{E,i} + \rho_{W,E}\rho_{C,E}\rho_{C,i}\right) v_q + \\ \left(\rho_{C,i} - \rho_{C,i}\rho_{W,E}^2 - \rho_{W,C}\rho_{W,i} - \rho_{C,E}\rho_{E,i} + \rho_{W,C}\rho_{W,E}\rho_{E,i} + \rho_{W,E}\rho_{C,E}\rho_{W,i}\right) \omega_q + \\ \left(\rho_{E,i} - \rho_{E,i}\rho_{W,C}^2 - \rho_{W,E}\rho_{W,i} - \rho_{C,E}\rho_{C,i} + \rho_{W,C}\rho_{W,E}\rho_{C,i} + \rho_{W,C}\rho_{C,E}\rho_{W,i}\right) a_q \end{bmatrix} \tag{38}$$

where y_{iq} is Lee's transformation for travel mode choice, F_i is the MNL probability

[1] There is a strong normal distribution assumption being made about the error terms in the discrete and continuous equations. Alternatively, one can use semi-parametric or non-parametric assumptions to generate the correlation between the discrete and continuous equations, but such methods are not easy to apply for a system with more than one continuous equation (see Lewbel and Linton, 2002 and the references therein for recent developments in the area of semi-parametric and non-parametric specifications in the context of limited-dependent variable models)

function, γ is the vector of parameters of the discrete mode choice model and z_{iq} are the level of service variables that individual q observes for alternative i. The correlation between the continuous equations and the chosen transport mode is $\rho_{l,i}$, where l is the continuous equation (time assigned to work, personal care or entertainment) and i is the chosen mode.

With these definitions, the log likelihood of the whole sample can be written as:

$$LL = \sum_q \sum_i D_{iq} ln \left[\frac{1}{\sigma_W \sigma_C \sigma_E \rho^* \sqrt{1-\rho_{W,C}^2}} \phi(v_q) \phi\left(\frac{\omega_q - \rho_{W,C} v_q}{\sqrt{1-\rho_{W,C}^2}} \right) \phi\left(\frac{a_q^*}{\rho^*} \right) \Phi\left(\frac{1}{\sqrt{1-\hat{\rho}_i}} \left(y_{iq} - y_{iq}^* \right) \right) \right] \tag{39}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the density and distribution functions of a standard normal. The maximization of this function can have local solutions, which makes difficult to find the global maximum of the function. In our empirical analysis, we obtained multiple convergent values based on the initial starting values, and we chose the one that did not provide degenerate solution of unrealistic values. We then selected the model results that provided the best log-likelihood value at convergence from among the realistic results.

There are more parameters that could have been included such as additional mode specific variables, standard deviations, and correlations for different chosen modes. We decided not to use such flexibility, as the number of parameters to be calibrated would get large, making more difficult and less reliable the calibration process.

3. EMPIRICAL APPLICATION

3.1 Sample Description

The database used to calibrate the model described above is drawn from a time use survey of 290 individuals who work in the Santiago CBD, and who live in the Southern part of the city along an important corridor. The sample also has mode choice information, including the level of service variables for a sub-sample of the reported trips. Of the 290 workers surveyed, 42.4% are women and 67.6 are married. The most frequent age range is 35 to 49 years (47.9%). Most people in the sample have university or technical studies (67.2%). The average monthly income in the sample is US $868. Within a national context, we could say that this is a typical middle income worker sample.

The time use survey includes the activities pursued by an individual for three days (a working day, Saturday and Sunday). The activities were classified into 38 categories, plus the travel activity, coded separately for 21 modes. From these 38+21 detailed activities, we can obtain time use information for aggregate categories. The most useful aggregation we have identified is based on six categories: **work** (in and out of the workplace); **personal care** (eating at home, resting, washing up, dressing, etc.); **sleep**; **entertainment** (in and out of home activities, such as watching TV, visiting friends, eating at a restaurant, sports, religious or political activities); **shopping and errands** (shopping for food, clothes or durables, looking after children or elderly, domestic work and personal business such as paying bills, going to the hairdresser, doctor or dentist); and **travel**. Weekly observations of time assignment can be generated by repeating the week day observation five times (assuming is the survey weekday a representative weekday) and adding the Saturday and Sunday information.

Other variables that were collected in the survey included individual income, socio-demographic variables and travel cost[2]. As explained below, fixed expenditures by income strata were calculated from other sources.

The mode choice database includes access, egress, waiting, and in vehicle time, and travel cost. This information is available for the morning trip to work (from the house to the work place), and also for the return work-to-home trip in the cases it was a direct trip (without

[2] The data collection process is reported in detail in Munizaga *et al.* (2004).

intermediate stops). In this application we only use the morning commute trip for each individual.

Using this information, the activities and mode choice models were calibrated both separately and jointly.

3.2 Modeling and Results

To perform the analysis, the activities were aggregated into six categories: work, personal care, sleep, entertainment, shopping and errands, and travel, discussed above. The last two were assumed to be restricted activities (those that will be assigned the minimum possible time). The activities included in the model as free activities (those that will be assigned an optimum amount of time) are work, personal care and entertainment. Sleep is also considered as a free activity, but it is not included in the model system as it is determined by the other two (due to the total time restriction). In the fixed expenses G_f, we include the weekly expenditure in transport and an approximation of the basic weekly expenses in other items such as housing, education, health care. This information was obtained from the Fifth Family Budget Survey, conducted by the Instituto Nacional de Estadísticas (INE) during 1996 – 1997. The values were income segment specific and were determined as a percentage of the individual income. Income from other non-work sources was also included in G_f.

Regarding mode choice, it was modeled with a linear utility function using level of service variables collected during the morning trip from home to work. The modes available were: (1) car driver, (2) car driver – metro, (3) car companion, (4) car companion - metro, (5) bus, (6) bus - metro, (7) shared taxi, (8) shared taxi - metro, and (9) metro[3].

[3] These are the usual transport modes in Santiago. Metro is the underground system. Car companion refers to individuals who travel with someone else who drives to work, usually a friend or a relative who lives and works nearby. Shared taxi is a formal transport system where a professional driver carries up to five people in a regular service.

Some observations had to be excluded due to missing variables or because one of the dependent variables was not observed. The modeling sample has 174 observations with complete information on mode choice, positive times assigned to all modeled activities, and complete information on expenses, income and wage rate.

In Table 1 we report the parameters calibrated with both the independent and joint calibration processes. We include only the best specifications according to the traditional statistical indicators. All the estimators of the parameters of the continuous and discrete equations have reasonable values and are statistically significant. The first set of parameters reported is that of the mode choice model. There are eight mode constants that have the role of reproducing the sample market shares, and time and cost marginal utilities that represent the negative effect of having to pay and having to dedicate time to travel. Parameters α and β do not have a direct interpretation, but are very important because the different components of the value of time are calculated from them. The personal care and entertainment θ parameters are directly related to the exponents of those activities in the direct utility function, so their positive values confirm the hypothesis that they are pleasurable activities.

The rest of the parameters are related to the error structure. We report the standard deviation of the continuous equations error terms, the correlation parameters for the continuous equations, and the discrete-continuous correlations. Six discrete-continuous correlation parameters turned out to be statistically significant. The interpretation of these correlations has to be made with the opposite sign because, when applying Lee's transformation, the error term of the discrete choice is included with a negative sign. Therefore, the positive value of the parameter $\rho_{\text{work \& cardriver\_metro}}$ indicates that there are unobserved factors that make some people dedicate more time to work and have a lower propensity to use the car driver-metro transport mode. These correlations come from unobserved effects which are difficult to identify. For example, the correlation of car companion mode choice with work and personal care, may be due to the fact that people who use the car companion mode have to adjust to someone else's schedule, and cannot

Table 1. Calibration results

	Mode choice	Activities	Simultaneous
	Par (t-st)	Par (t-st)	Par (t-st)
Mode constants			
Car driver	2.4 (1.7)	-	2.0 (1.5)
Car driver–Metro	1.0 (1.4)	-	0.8 (1.1)
Car companion	-2.1 (-2.0)	-	-2.3 (-2.2)
Car companion–Metro	-1.2 (-1.4)	-	1.4 (-1.7)
Bus	0.4 (0.5)	-	0.2 (0.3)
Bus–Metro	-0.6 (-0.9)	-	-0.7 (-1,1)
Shared taxi–Metro	0.3 (0.5)	-	0.3 (0.4)
Metro	0.9 (1.1)	-	0.8 (0.9)
Mode choice taste parameters			
Total time	-0.0741 (-3.5)	-	-0.0845 (-4.0)
Cost	-0.0023 (-2.5)	-	-0.0023 (-2.4)
Activities models parameters			
α	-	0.2915 (16.3)	0.2868 (16.5)
β	-	0.0958 (17.6)	0.0977 (18.3)
θ Personal care	-	0.1803 (36.3)	0.1841 (36.3)
θ Entertainment	-	0.1587 (23.8)	0.1627 (22.3)
Standard deviations			
σ_{Work}	-	380.2 (18.6)	365.6 (19.5)
$\sigma_{\text{Personal care}}$	-	419.7 (18.7)	415.5 (18.9)
$\sigma_{\text{Entertainment}}$	-	604.3 (18.7)	599.2 (19.0)
Correlations (activities)			
$\rho_{\text{Work \& Personal care}}$	-	-0.2527 (-3.6)	-0.2717 (-4.1)
$\rho_{\text{Work \& Entertainment}}$	-	-0.2576 (-3.6)	-0.2397 (-3.6)
$\rho_{\text{Personal care \& Entertainment}}$	-	-0.5282 (-9.7)	-0.5276 (-9.9)
Correlations (discrete-continuous)			
$\rho_{\text{Work \& Car driver-Metro}}$	-	-	0.6761 (4.5)
$\rho_{\text{Entertainment \& Car driver-Metro}}$	-	-	-0.3341 (-2.7)
$\rho_{\text{Work \& Car companion}}$	-	-	-0.6155 (-4.3)
$\rho_{\text{Personal care \& Car companion}}$	-	-	0.5591 (3.7)
$\rho_{\text{Entertainment \& Bus}}$	-	-	0.2816 (2.6)
$\rho_{\text{Work \& Shared taxi–Metro}}$	-	-	0.5356 (4.1)
Statistical indicators			
LR test value of correlated equation system relative to independent equation system	-	112.8	44.2
LR value for comparing final specification to model with all correlated discrete-continuous elements	-	-	10.3
Average log-likelihood	-1.2565	-22.3161	-23.4456
Subjective values of time [US$/hour]			
Leisure (μ/λ)	-	2.77 (13.4)	2.75 (-14.1)
Assigning time to work ($(\partial U/\partial T_w)/\lambda$)	-	-1.68 (-8.6)	-1.70 (-9.1)
Wage rate (w)	4.45	4.45	4.45
Saving travel time(K_t/λ)	3.07 (2.0)	-	3.49 (2.0)
Assigning time to travel($(\partial U/\partial T_t)/\lambda$)	-	-	-0.74 (-0.4)

freely choose how much time to allocate to activities. This could explain why people who travel to work as car companions dedicate less time to personal care and more time to work (arrive earlier) than other people with the same explanatory variables. Other possible correlations could be due to hidden segmentation effects. Significant correlations were found for car driver-metro mode with work and entertainment. This may be because people who use the car driver-metro combination to work, may be young or wealthy individuals who dedicate less time to work and more time to entertainment compared to their observationally equivalent non-car driver-metro mode users. Note that car ownership and income are highly correlated in Santiago.

The likelihood ratio test (Ortúzar and Willumsen, 1994) indicates that the activities model system reported is statistically superior to an independent version (critical χ^2 table value (5%, 3) = 7.82). The likelihood ratio tests also indicate that the joint model is better than the independent model (44.2 > χ^2 (5%, 6) table value of 12.59), and as good as the model with all discrete-continuous correlation terms (10.3 < χ^2 (5%, 21) table value of 32.67).

In the last section of Table 1, we present the subjective values of time. It can be observed that the subjective values of leisure and work are not very different if the estimation is separate or simultaneous. The value of leisure is positive as expected; but, it is not equal to the wage rate as predicted in the labor economics literature. We also find that the marginal utility of work is negative, showing that, at the margin, people dislike working. This confirms the importance of including work in utility instead of imposing *a priori* that its marginal utility is zero, as implicitly assumed in the goods-leisure trade off approach. In this application the absolute value is nearly 40% of the wage rate.

In the simultaneous estimation, the subjective value of travel time savings is 14% larger than the value obtained from the independent mode choice model. The value of saving travel time is positive, showing that people are willing to pay close to 80% of their wage rate to reduce travel time. Also, it is larger than the value of time as a resource (value of leisure), showing that the time assigned to travel generates disutility (it is an unpleasant activity at the margin). According to Equation (12), if we separate the value of travel time savings into a component related to the alternative use of time (value of time as a resource) and the disutility of assigning time to travel (the difference), we find that the first term is much larger, showing that the possibility of re-assigning time to more pleasant or more profitable activities is more important for the individuals than the displeasure caused by the trip. If we calculate the value of assigning time to travel from the two independent models using Equation (12), we get –0.29 [US$/hour], which is only 40% the value obtained with the joint estimation.

3.3 Applying the model

The parameters estimated in the joint model system can be used to predict changes in time assignment and/or mode choice in response to different scenarios. For instance, policy measures implying changes in the explanatory variables of the continuous model (such as changes in the wage rate, weekly travel time and level of service variables of the morning trip to work) can be evaluated with this model. An important change in the travel time of a particular mode, for example, will affect the choice probabilities of the different modes, but will also have an effect on the time assigned to the modelled activities, as the weekly travel time is one of the explanatory variables of the continuous model.

In Table 2, we present the variations in time assignment and mode choice due to changes in the level of service variables. The "Base case" column shows the values observed in the database. The other columns show the model predictions for each variable under each policy scenario. It can be seen that in a scenario where the travel time of bus reduces to one half, the market share of bus increases 90%, making both the average travel time and travel cost to decrease. This last effect is due to the fact that new users came from more expensive modes. The effect on time assignment is similar to the combined effect of more slack in the time and income constraints (less time and money on travel), free activities are assigned more time, and a slightly smaller increase is observed for work time. This last effect is due to the fact that travel time is part of the time assigned to constrained (unpleasent) activities, and therefore, there is an elasticity $E_{T_f}^{T_w}$ of the work time with respect to travel time. It is a rather moderate effect ($E_{T_f}^{T_w} = -0,13$), and it is further reduced as it only affects those individuals who use public transport in the base case or those who will change to public transport in the new scenario. Similarly, an increase in car cost reduces the choice of the car driver travel mode, as expected. It also has the effect of decreasing the total transport expenditure, because some users change to less expensive modes. As a consequence of this, people would work less and assign more time to free activities. An increase in the bus fare causes the opposite effect.

Table 2. Time variation and mode choice due to changes in level of service variables.

	Scenarios						
	Base case	Bus travel time reduces ↓50%		Car cost increases ↑50%		Bus fare increases ↑50%	
	Value	Value	% var.	Value	% var.	Value	% var.
Choice of Mode [%]							
Car Driver	6	3	-50.0	1	-83.3	7	+16.7
Car Driver-Metro	12	5	-58.3	5	-58.3	13	+8.3
Car Companion	13	6	-53.8	15	+15.4	15	+15.4
Car Companion-Metro	15	6	-60.0	17	+13.3	17	+13.3
Bus	65	124	+90.8	68	+4.6	57	-12.3
Bus-Metro	18	11	-38.9	19	+5.6	16	-11.1
Shared Taxi	4	2	-50.0	5	+25.0	5	+25.0
Shared Taxi-Metro	21	8	-61.9	23	+9.5	23	+9.5
Metro	20	9	-55.0	21	+5.0	21	+5.0
Expected travel time [min]	45.5	29.6	-34.8	46.0	+1.1	45.2	-0.7
Expected travel cost [US$]	0.77	0.60	-22.1	0.66	-14.3	0.90	+16.9
Work [h]	45.97	46.08	+0.2	45.88	-0.2	46.27	+0.7
Personal care [h]	21.91	22.19	+1.3	21.92	+0.1	21.85	-0.3
Entertainment [h]	19.29	19.53	+1.3	19.30	+0.1	19.23	-0.3
Sleep [h]	54.52	55.22	+1.3	54.56	+0.1	54.37	-0.3

As an example of the sensibility of the activities model to the explanatory variables, we would like to mention that, according to the model, an increase on the wage rate of 50% will have the effect of reducing by 11.4 % the time assigned to work, and increase all the times assigned to discretionary (pleasant) activities by 5.5% .

4. CONCLUSIONS

We have successfully developed a methodology to calibrate a novel time assignment – mode choice model system. This methodology is based on Lee's transformation and expands on the work by Bhat (1998) and Munizaga *et al.* (2006). We used a database collected expressly to estimate the proposed model system. The methodology worked well and generated reasonable results. It permits the calculation of the values of work, leisure and time assigned to travel.

The results show that there is significant correlation between time assignment and mode choice. When the mode choice model is calibrated independently of the time assignment model system, the subjective value of assigning time to travel is underestimated by 60%. The continuous equations are more stable, and do not change much between independent and joint estimations. Even though some elasticities of variables in the continuous models with respect to those of the discrete models are small, the joint estimation is justified because it includes correlation parameters that are significant, and the joint model is econometrically superior than the independent version. The estimated model system can be applied to predict time assignments due to changes in the transport system such as travel time, travel cost or other explanatory variables.

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