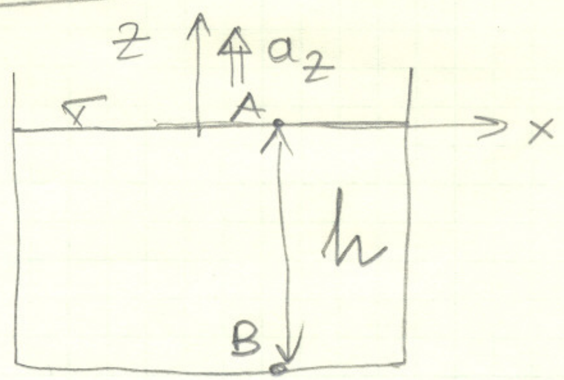
CE319F

$$\boxed{-\frac{\partial(p+\gamma z)}{\partial l} = \rho a_l}$$ 4.17, Euler equ.

A)



$$-\frac{\partial(p+\gamma z)}{\partial z} = \rho a_z \rightsquigarrow$$

$$-\frac{\partial}{\partial z}(p+\gamma z) = \frac{\partial}{\partial z}(\rho a_z z) \rightsquigarrow$$

$$\rightsquigarrow -\frac{\partial}{\partial z}\left[p+\gamma z+\rho a_z z\right] = 0 \rightsquigarrow$$

$$\rightsquigarrow p+\gamma z+\rho a_z z = \text{const.} \rightarrow \boxed{p+\rho(g+a_z)z = \underline{\underline{\text{const}}}}$$

Compare with hydro-static law $p+\gamma z = \underline{p+\rho g z} = \underline{\underline{\text{const}}}$

$$\rightarrow p_B + \rho(g+a_z)z_B = p_A + \rho(g+a_z)z_A$$

$$p_B = \overset{=0 \text{ (gage)}}{\cancel{p_A}} + \rho(g+a_z)\underbrace{(z_A - z_B)}_{h}$$

$$\rightarrow p_B = \rho\underline{(g+a_z)}h$$

Compare with hydrostatic law:

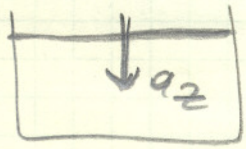
$$p_B = \underline{\rho g h}$$

So, hydro-law can be modified so that $g_{v,z}$ (virtual accel. of gravity) $=$

$$= \underline{\underline{g + a_z}}$$

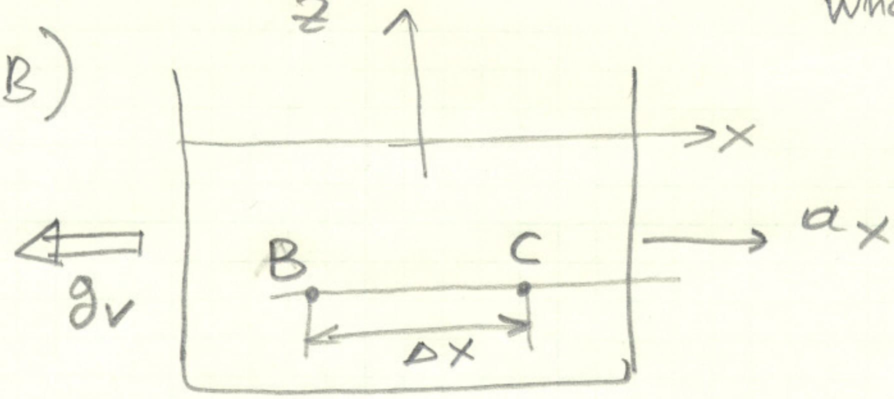
what would happen if a_z point down?

$$P_B = \rho (g - a_z) h$$



what if $\underline{\underline{a_z = g}}$?

B)



$$-\frac{\partial(p + \gamma z)}{\partial x} = \rho a_x \quad \text{or} \quad -\frac{\partial(p)}{\partial x} = \rho a_x \quad \text{or}$$

$$-\frac{\partial p}{\partial x} = \frac{\partial}{\partial x}(\rho a_x x) \rightsquigarrow -\frac{\partial}{\partial x}\left[p + \rho a_x x\right] = 0$$

$\rightsquigarrow$ $p + \rho a_x x = \text{const.}$ (for each z)

<u>Ex.</u> $P_B + \rho a_x x_B = P_C + \rho a_x x_C \rightsquigarrow$

$$\rightsquigarrow P_B = P_C + \rho a_x (x_C - x_B) \rightsquigarrow$$

$$\rightsquigarrow P_B = P_C + \underline{\underline{\rho a_x}} \Delta x$$

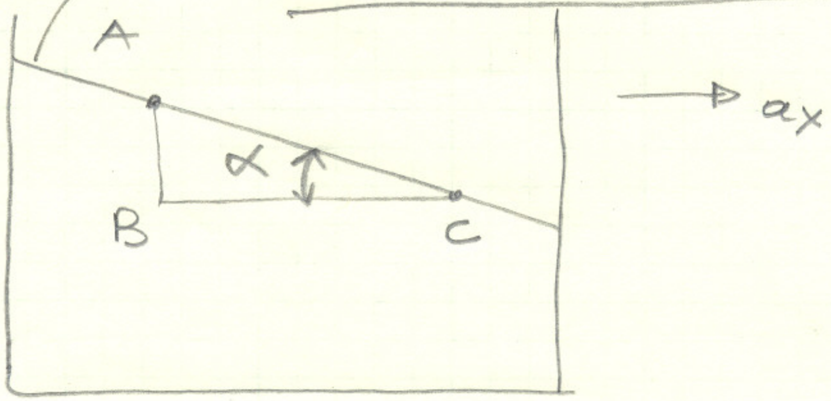
The above looks like hydro-law <u>along</u> $\underline{\underline{x}}$ where $g_{v,x} = a_x$ and in opposite direction.

Along z: $-\dfrac{\partial(p+\gamma z)}{\partial z} = 0$ (if we only have a_x, but $a_z = 0$)

$\rightsquigarrow$ $p + \gamma z = \text{const.}$ for const. $\underline{\underline{x}}$

So, hydrostatic law still applies on the same vertical

How do determine shape of free surface?



$$P_B = P_A + \rho g (AB) \quad (1)$$

$$P_B = P_C + \rho a_x (BC) \quad (2)$$

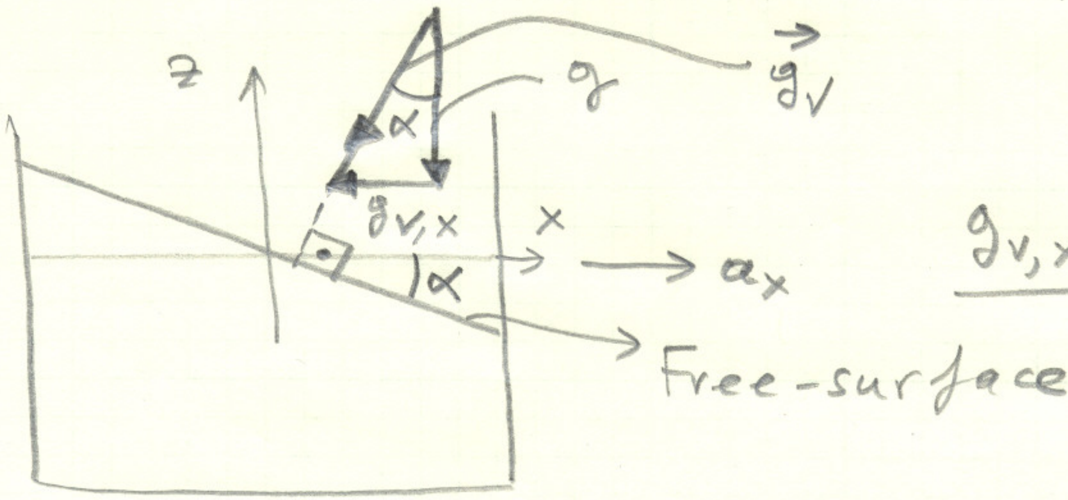
Equating the RHS's of (1) and (2) we get:

$$P_A + \rho g (AB) = P_C + \rho a_x (BC) \quad (3)$$

On the free surface $P_A = P_C = 0$, and thus (3) becomes:

$$\rho g (AB) = \rho a_x (BC) \rightarrow \frac{AB}{BC} = \boxed{\frac{a_x}{g} = \tan\alpha}$$

The above angle α can also be determined by using the virtual acceleration vector $\vec{g}_v$

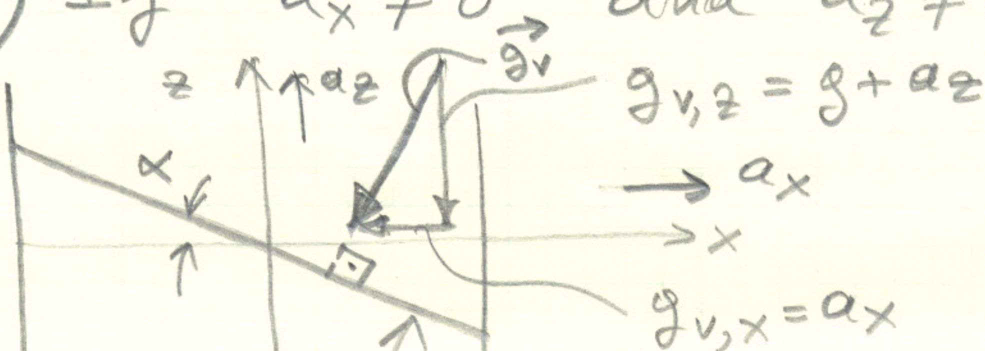


$$\underline{g_{v,x} = a_x}$$

Free-surface must be normal to $\vec{g}_v$ as shown in the figure above.

From geometry $\rightsquigarrow$ $\tan\alpha = \dfrac{g_{v,x}}{g} = \dfrac{a_x}{g}$

c) If $a_x \neq 0$ and $a_z \neq 0$



$$\tan\alpha = \frac{g_{v,x}}{g_{v,z}} = \frac{a_x}{g + a_z}$$