THE PROPAGATION OF UNCERTAINTY THROUGH TRAVEL DEMAND MODELS: AN EXPLORATORY ANALYSIS

by

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ABSTRACT

The future operations of transportation systems involve a lot of uncertainty – in both inputs and model parameters. This work investigates the stability of contemporary transport demand model outputs by quantifying the variability in model inputs, such as zonal socioeconomic data and trip generation rates, and simulating the propagation of their variation through a series of common demand models over a 25-zone network. The results suggest that uncertainty is likely to compound itself – rather than attenuate – over a series of models. Mispredictions at early stages (e.g., trip generation) in multi-stage models appear to amplify across later stages. While this effect may be counteracted by equilibrium assignment of traffic flows across a network, predicted traffic flows are highly and positively correlated.

KEYWORDS

sequential models, estimate uncertainty, error propagation, travel demand models

INTRODUCTION

The future operations of transportation systems involve a lot of uncertainty. Modeling these complicated systems requires many variables and behavioral components whose variability may be poorly identified or simply ignored. Without explicit and rigorous statistical recognition of uncertainty in transportation demand forecasts, transportation planning of towns, cities, and metropolitan areas takes on unnecessary risk. Transportation plans and polices based on these forecasts may be inaccurate and even misleading. As a result, transport facility investments may be poorly directed.

Generally, large-scale transport demand models are estimated sequentially, with the results or estimates of one model acting as inputs to subsequent models. In almost all cases, only point estimates are passed forward, rather than estimates of variation and covariation. Such modeling processes limit the final results to point estimates, so comparisons of plans or scenarios based on the results may be incorrect. In reality, outcomes of alternative plans or scenarios may overlap, and the difference between alternatives may not be statistically significant.

This work investigates the nature of uncertainty propagation in contemporary transport demand models, by quantifying variability in model outputs and tracking the sources of this variability – in the form of variable model inputs and parameters. The work's objective is a comparison of point estimates under input variation. Monte Carlo simulation and sensitivity analysis are used to investigate error propagation over an 818-link network covering a 25-zone area of the Dallas-Fort Worth metro region.

The following sections of this paper include a background and literature review, model specification and assumptions, simulation results, and a sensitivity analysis. The paper concludes with a summary of the research findings and identification of possible extensions to this work.

BACKGROUND

There are many sources of forecast errors. Modelers can do relatively little about errors due to mis-measurement, poor sampling, mis-computation, model mis-specification, and data aggregation (e.g., spatial aggregation). (Barton-Aschman *et al*,1997). In contrast, purely stochastic errors can be accommodated statistically and explicitly. Components of these stochastic errors arise from three sources, which here are termed "inherent uncertainty", "input uncertainty", and "propagated uncertainty". Since travel demand model parameters are random variables, estimated from samples of the population, model estimates are associated with variations and covariations. These variations constitute inherent uncertainty. Also, the use of predictions of future demographic data (e.g., employment and land use) as inputs to traffic demand forecasting models contributes input uncertainty. Moreover, since transport demand models are generally estimated and applied sequentially, the results or estimates of one model act as input to subsequent models. Their uncertainty is passed forward, producing propagated uncertainty. The cumulative impact of these three forms of uncertainty is the focus of this research.

Unfortunately, current travel-demand-modeling practice does not acknowledge all these sources of uncertainty, especially input uncertainty. For example, rigorous statistical models produce estimates of variance and covariance along with their point (or mean) estimates. However, only point estimates (of variables' mean values) are carried forward through travel demand models. The covariance information is generally lost. Many variables used as inputs to transport demand models come from other models, whose associated uncertainty is not known or incorporated. If point estimates of these future variables (such as population, housing, and automobile ownership) are to be used in travel demand models, an appreciation of variability in all results requires distributional information on the inputs.

Modeling methods based on point estimates dramatically constrain all final results into point estimates, and the point estimates may be highly biased. Alonso(1968) raised this question in land use and transportation prediction. For example, the expected value of a linear function of independent variables requires only mean values of the input variables. However, non-linear functions and any functions involving correlated variables require distributional information in order to avoid bias when estimating the function's mean value (see, e.g., Rice, 1995). Comparisons of alternative transportation plans or scenarios based on these do not convey information regarding uncertainty in estimates – or the statistical significance of differences. Neglect of data and parameter uncertainties and their correlation ultimately weaken the reliability of transportation planning, policy-making, and infrastructure decisions. For example, Rodier and Johnston (2001) suggested that the plausible uncertainty in population and employment projections may result in a region's transportation plan not meeting the air quality conformity tests in a five- or ten-year time horizon.

To assess some forms of uncertainty in model predictions, most transport modeling processes employ "model validation" to test a model's forecast ability. Although validation compares model predictions with the observed data that are not used in model estimation, this procedure can only assess the model's predictive strength for contemporary situation. Variations in future forecasts due to input and inherent uncertainty, however, change over time. Thus, there is no guarantee that future predictions will be bounded by an acceptable range.

Barton-Aschman *et al.* (1997) have provided a set of specific guidelines for model validation and have recognized that input error and inherent uncertainty add to overall uncertainty. There is the concern that each step in the Urban Transportation Planning System (UTPS) models could possibly increase the overall error. They write that "while there is a potential for the errors to offset each other, there is no guarantee that they will." (1997, p. 12) but make no attempt to quantify the propagated uncertainty.

A "before and after" study is another method used to assess a model's predictive accuracy. But it is difficult to draw useful conclusions from an individual study (Aitken and White, 1972); examples include Horowitz and Emsile (1978), ITE (1980), and Mackinder and Evans (1981). Comparisons of predicted and observed volumes via percent root mean square error (%RMSE) provide validation tools for traffic assignment models. Practical results suggest that average hourly or daily flow forecasts come with %RMSE of 30 to 50 percent (Barton-Aschman *et al* 1997, Martin 1998), and links with low flows tend to have higher %RMSE than those with high flows. However, without sensitivity analysis, one does not know which inputs contribute most to final uncertainty. Mackinder and Evans (1981) have suggested that the errors in socioeconomic variables might dominate highway volume forecast errors, but their work did not explicitly investigate this hypothesis.

There is a fair amount of transportation research focused on modeling uncertainties. For example, Robbins (1978) estimated the possible error in each of the four-step models. However, several of his assumptions were simplistic. For example, he used a fixed-proportion mode split model. Bonsall (1977) proposed a more systematic approach with sensitivity analysis, but no particular input distributions were specified; instead, an *ad hoc* set of values was used.

More sophisticated approaches have adopted simulation to capture random input patterns. Ashley (1980) studied the probability distribution of various outputs from an interurban highway forecasting model due to various input uncertainties. His correlated inputs were drawn from multivariate probability distributions; but he neglected many forms of uncertainty (e.g., destination choice), did not detail the specifics of his simulations, and did not investigate which error sources contributed most to overall uncertainty. In contrast, Pell's (1984) work examined forecast variability by identifying those sources of input uncertainty and error that make the largest contributions to forecast uncertainty. Pell proposed two criteria for selecting the most important error sources: the sensitivity of forecasts to input errors, as measured by elasticity; and the magnitude of forecast errors, as measured by coefficients of variation (also called "percentage error" or "relative error"). His 73 simulations suggested link-flow coefficients of variation of 0.30 to 2.0, but they did not employ correlated inputs. For practical applications, Pell recommended fewer simulation runs after one has identified the influence of a small number of uncertain sources.

There are several other, less relevant studies in uncertainty analysis. For example, Rose's network study (1986) focused on flow predictions but did not permit correlated inputs. And Leurent (1998) developed a sensitivity and uncertainty analysis method for the equilibrium solution to a dual-criteria model on a small-scale network.

In summary, many researchers have examined the propagation of uncertainty through travel demand models. Simulation techniques are suggested as one of the most useful methods in this field because one can simulate uncertainty from a variety of sources simultaneously and impose correlation across inputs. Sensitivity analysis is another effective tool for studying uncertainty. It traces output uncertainty back to inputs, revealing both linear and non-linear relationships. However, due to cost, computational, and other limitations, prior studies exhibit common weaknesses. Few large-scale data applications have been undertaken, and few firm conclusions have been reached.

MODEL APPLICATION

This work investigates the stability of transportation demand model outputs by using traditional, four-step urban transportation planning process (UTPP) models on a Dallas-Ft. Worth (DFW) subregion. Inputs and parameters are varied randomly, to approximate errors and uncertainties; and Monte Carlo simulation and sensitivity analysis are the primary tools used (see, e.g., Hahn and Shapiro [1967] and Cullen and Frey [1999]). A multivariate regression analysis of results (as a function of input levels) along with linear and rank correlation coefficients suggests dependencies and sensitivities between input and output uncertainties

This work considers the traditional UTPP model paradigm via its primary components: trip generation, trip distribution, mode choice, and route selection. There are a number of alternative model formulations one might use, along with a variety of specifications one might choose for each model component. As the first reasonably comprehensive investigation of its type, this study only adopts general, simplified specifications. Such focus permits a clearer picture of output dependencies and general model behaviors – while reducing the computational effort involved in simulating and solving the full UTPP model 100 times. Table 1 provides average parameter values for each of the model components, and the component specifications are described here now.

Trip Generation

Trip generation models have two basic structures: (1) regression equations at an aggregate (zonal) or disaggregate (household/person) level, and (2) cross-classification of trip rates at an aggregate level. This study uses the following simplified cross-classification models to calculate the home-based work trips (HBW).

Trip Production:

$$T_i = \alpha HH_i \qquad (1)$$

where T_i is the number of HBW trips produced in zone i¸ HH_i is the total number of households in zone i, and α is the trip production rate.

Trip Attraction:

$$A_i = \sum_{k,l} \beta_{kj} EMP_{ik} x_{il} \qquad (2)$$

where A_i is the number of HBW trips attracted to zone i, EMP_{ik} is the total number of jobs of type k in zone i, x_{il} is an indicator variable for zone type (*i.e.*, 1 if this zone is of type l, 0 otherwise), and β_{kj} is the trip attraction rate of employment of type k in zone type l.

In this study, three types of employment are used: basic employment, retail employment, and service employment.[1] Four zone types are specified based on the population and employment density: these are business district, urban residential, suburban residential, and rural. To balance total trip productions and attractions, this study constrains HBW trips to equal the estimated trip attractions.

The mean values of demographic inputs (*i.e.*, number of household and different employment types) come from the Dallas-Ft. Worth (DFW) travel model's data set. These are shown in Table 1, along with other model component parameter values. For clarity and focus, the coefficients of variation are all set to 0.30 (and later to 0.10 and 0.50). Thus, the standard deviations (SDs) are determined by multiplying mean values by 0.30. As previously discussed, the actual values may be higher or lower; it depends on the model specifications and data sets used for calibration. Coefficients of variation of 0.30 suggest t-statistics of 3.33, signifying values that differ from zero in highly statistically significant ways; such statistical significance in parameter estimates is common to many behavioral models of travel demand.

The distribution of demographic inputs (i.e., household and employment numbers across zones) is assumed to be multivariate normal with a correlation coefficient of +0.30 across all variables. One would expect a positive correlation in these forecasts, given a general increase or reduction in population and jobs for the region. But, depending on the predictive models used (for households and employment), actual correlations may be weaker or stronger[2]; they also may fall with distance between zones. Estimates from long-term population and economic/jobs forecasting models would be needed for determination of actual correlations.

Trip Distribution

The most common model form used for trip distribution is the gravity model, and this is the model used here. This model form, subject to a production constraint, is defined as follows:

$$T_{ij} = T_i \left(\frac{A_j F(t_{ij})}{\sum_i A_k F(t_{ik})} \right) \qquad (3)$$

where T_{ij} is the number of trips from zone *i* to zone *j*, T_i is the number of trip productions in zone *i*, A_j is the number of trip attractions in zone *j*, t_{ij} is the impedance (time or generalized cost) from *i* to *j*, and $F(t_{ij})$ is the impedance function recognizing travel cost between zones *i* and *j*.

The impedance function should be inversely related to zonal separation. Gamma, power, or exponential functions usually are used. Here a simple exponential function is used, as follows:

$$F(t_{ij}) = t_{ij}^{\gamma} \tag{4}$$

where γ is the impedance parameter.

Equation (3) yields a trip matrix consistent with the number of productions in each zone but not with the number of attractions. Thus, this form of the gravity model is "singly constrained". This study applies three iterations of proportional fitting, switching between the attraction- and production-constrained calculations to meet the margins totals of the trip matrix. To some extent, this fitting of the trip table dampens the effects of the trip distribution parameters.

Murchland (1978) has suggested, via extensive calculation, that for small errors in both trips generated and impedance matrix values, the relative variance (i.e., the coefficient of variation squared) of the resultant cell values is approximately the sum of the relative variances of the input.

Mode Split

Multinomial and nested logit models are very common models of mode choice. A multinomial logit (MNL) specification essentially assumes equal competition across alternatives. Using this model, the proportion of trips made by mode *m* between zones *i* and *j* is the following:

$$\Pr_{m|ij} = \frac{e^{V_{m|ij}}}{\sum_{l} e^{V_{l|ij}}} \tag{5}$$

where $V_{m|ij}$ is the utility of mode *m* given origin *i* and destination *j*. $V_{m|ij}$ is specified to be a linear function of trip time, cost, and other variables. Here, a simple linear function is used:

$$V_m = \theta_m + TT_m \delta + \varepsilon_m \tag{6}$$

where TT_m is total travel time by mode *m*, ε_m represents unobserved heterogeneity (assumed to be *iid* GEV), and θ_m and δ are model parameters.

So the total number of trips by mode *m* from zone *i* to zone *j*, T_{ijm}, is the following:

$$T_{ijm} = T_{ij} \Pr_{m|ij} \tag{7}$$

This study simplifies the travel mode choice by allowing only two options: drive alone and all other modes (based on public transit travel times).

Route Choice

Network assignment of trips can include several common features. For example, an all-or-nothing method assigns all traffic flows between an origin-destination (O-D) pair to the shortest path. Capacity-restrained assignments attempt to approximate an equilibrium solution by iterating between all-or-nothing traffic loading and recalculating link travel times based on link capacity functions. User equilibrium (UE) methods utilize an iterative process to achieve a convergent solution ("equilibrium") in which no traveler can improve his/her travel time by shifting routes.

The uncertainty in assignment model results appears to be small if equilibrium techniques are used. Leurent (1998) suggested that an equilibrium network assignment is very stable, given well-defined criteria and constraints. Indeed, in congested networks the equilibration process may reduce the magnitude of uncertainties from the distribution models, in reproducing of link flows.

This study employs a user equilibrium method in its trip assignment model. UE algorithms incorporate link capacity functions in their search for convergence to an equilibrium state. A common link performance function, developed by the Bureau of Public Roads, is the following:

$$t = t_f \left[1 + \alpha_0 \left(\frac{q}{q_{\max}} \right)^{\beta_0} \right] \qquad (8)$$

where t is the impedance of a given link at flow q, t_f is free flow impedance of the link, $q_{\max}$ is link "capacity", and α_0 and β_0 are volume/delay coefficients.

The traditional BPR values for α_0 and β_0 are 0.15 and 4.0, respectively, but these are based on using a $q_{\max}$ for level of service C. For a q_{max} corresponding to true capacity (*i.e.*, maximum flow under level of service E), *NCHRP Report 365* (Martin 1998) suggests larger values, of 0.84 and 5.5, respectively. These larger values are applied here.

All together, this sequence of four sub-models produces a set of link-flow estimates. These are the model outputs of greatest interest in this work, and their variability is due solely to input and parameter uncertainties. These uncertainties are simulated by first specifying their distributions and then randomly generating values from these distributions. To impose sign constraints on many of these variables (for example, trip generation rate cannot be less than zero), lognormal distributions are used. To accommodate covariation across input and parameter values, multivariate distributions were specified, including the multivariate lognormal distribution.

The four-step model approach is applied into a road network (see Figure 1 in Appendix) with 25 zones and 818 links, which is separated from the Dallas-Fort Worth highway system. The area contains about 18,000 households and represents about 2.5% of DFW region. It is located around Irving, Texas (to the northwest of Dallas). For outside inputs, this study uses the demographic data associated with the network data. For model parameters, it uses mean values from the DFW area travel model description report (NCTCOG 1999). Necessary simplifications and modifications have been made based on *NCHRP Reports 187* (Sosslau *et al*. 1978) and *365* (Martin 1998). However, there are several variation and covariation assumptions; these include a single coefficient of variation for all inputs and parameters and a single correlation coefficient (of +0.30) relating all demographic data inputs. More reliable estimates of variation and covariation are likely to require model estimation using actual travel data, since estimates of variation and covariation are rarely reported in the literature. The rather simplistic assumptions used here provide a general example of variations; certainly, some models will offer stronger parameter estimates than others, and covariances can be both positive and negative. The simplicity of the approach used here permits a clarity in focus on the problem of primary interest: the sensitivity of model outputs to the various inputs and parameter values.

The modeling software used here for the first three sub-model steps (i.e., trip generation, trip distribution, and mode choice) is @Risk (Palisade 1998), which loads through Microsoft Excel software. This is a very flexible and user-friendly software for Monte Carlo simulation and risk analysis; however, many standard programming languages and other software packages are viable for such techniques. TransCAD(Caliper Co., 1996) is used here for the final, trip assignment sub-model in order to apply its commercialized UE algorithm. The convergence of a UE assignment is assumed when the maximum absolute change in all link flows between consecutive iterations is less than 5 vehicles per hour.

The results of greatest interest are variations of link flows and their matrices of covariation, across model simulations. These are discussed in the following section.

SIMULATION RESULTS

The sequence of four-step sub-models produces a set of link-flow estimates. The study simulates the forecasting approach by running the four-step models 100 times, using 100 different sets of input and parameter values. In general, the number of simulations run needs to be large enough to obtain robust and accurate results. When simulation samples of size 10 and 20 were used here, average coefficients of variation in total VMT and VHT (two primary output indicators) were found to range from 0.22 to 0.26. In this study, 100 simulations/replications were used, so a substantially more stable estimate of output variation is expected (e.g., on the order of 2.2 to 3.2 times narrower a band than for the N=20 and N=10 scenarios). Adopting an even larger number of runs would further improve accuracy in estimates of final uncertainties, but it would require substantially more computational effort and time. Based on the smaller-sample estimates obtained, the 100-run simulation appears sufficient to provide robust and accurate results for this 25-zone sub-network.[3]

Final link flows were obtained from the converged UE assignment results. Most of the ratios of volume versus capacity were relatively low (e.g., 85% of them were less than 0.76 and the mean was 0.39), indicating that the assignment equilibrium was not heavily congested. In fact, the result is a portion of a general assignment; it only includes morning peak hour home-based work auto trip assignment. The flow volumes from one assignment are shown in Figure 2. Two example arcs are chosen for explicit consideration. Link one (Rochelle Blvd. between Northgate and Rochelle) represents the general pattern of congested links, while link two (SH183 eastbound passed Story Road ramp) represents other, uncongested links. The flow distributions of 100 simulation results for these two links are shown in Figure 4. Not surprisingly, given the lognormal distribution assumptions of input and parameters, the resulting distributions appear approximately lognormal.

The overall uncertainty results are shown in Table 2. As evident in these results, the variability of the selected link flows is sizable. Both coefficients of variation of the two link flows are larger than 0.30, which suggests the final uncertainty may be compounded and end higher than any input or parameter uncertainty. The flow uncertainty appears not to have a strong relation with congestion, as suggested by Figure 4, which plots the uncertainty of all loaded links versus their volume-to-capacity (v/c) ratios. As can be seen, most link flow uncertainties are larger than 0.30, no matter what their v/c ratios are. Some points in the lower-left area provide a possibility that under very low v/c levels, overall uncertainty may be reduced to some degree. However, the average link travel times exhibit a relatively strong relation to congestion. The travel time uncertainty of the example congested link, 1.899, is much higher than that of the uncongested link, 0.127.

The coefficient of variation estimated for VMT is just 0.236, which is relatively low. This also is true for uncertainty in total vehicle hours traveled (VHT) across the network. As shown in Table 3, the link flows show great correlation between one another. For probabilistic simulations, correlations greater than 0.5 between inputs and outputs suggest substantial dependence. Since total VMT is the weighted sum of all link flow volumes, there is a strong correlation between total VMT and individual link flows.

Overall, the uncertainty propagation process through the four-step travel demand forecast model is shown in Figure 5. In each model step, there is a finite amount of inputs and outputs. Given the distribution assumption of the input and parameters of the model, the simulation yields 100 observations of each output. Although the amount of outputs of each step is different, the average COV, as a scaleless measurement, can be collected to track the changes in uncertainty through model stages. The five percentile and ninety-five percentile of the uncertainty among each step are also shown in Figure 5 to indicate the variability of the uncertainty. Even though all the input uncertainties are set to be the same value, 0.30, the actual simulation data drawn from certain distributions may contain uncertainties slightly different from this value. Thus, the 5% and 95% of demographic input uncertainty are 0.2592 and 0.3397, respectively.

As can be seen, the increasing average uncertainty in the first three step models suggests significant uncertainty propagation through those models. Nevertheless, the final step assignment model somehow reduces the previous compounded uncertainty, but generally not lower than the input uncertainty. The expanding 5% and 95% bound suggests that through the four-step model, the variability of final uncertainty extends. Thus, some link flows' uncertainty may be reduced substantially while others may increase considerably, which indicates the possibility of wide swings in the system. However, one still can improve UTPP model forecasting by providing information on the associated uncertainty of final results. In this way, policymakers will be aware of the uncertainty when comparing scenarios.

Similar results are found in Figure 6, where all input and parameter COVs are assumed to be 0.1 or 0.5, rather than 0.3. The first three model steps compound the uncertainty, while the final step appears to reduce the propagated uncertainty.

The simulation results suggest the trip assignment equilibrium technique may reduce the overall uncertainty, which is partially consistent with Leurent's (1998) study. Leurent suggested that in congested networks the equilibration process may reduce the magnitude of uncertainties in the reproduction of link flows. One possible explanation is the capacity constraint restricts the variability of link flows. However, in this study, relatively few of the links (6%) are congested; the average volume-to-capacity ratio is just 0.39. The coefficient of variation (COV) of a sum of independent random variables is less than the average COV of such variables. Notationally:

$$COV\left(\sum_i a_i X_i\right) = \frac{\sqrt{\sum_i a_i^2 \sigma_i^2}}{\sum_i a_i \mu_i} \le Avg.COV_i = \frac{\sum_i \frac{a_i \sigma_i}{\mu_i}}{\sum_i a_i},$$

wheere X_i's are independent random variables and a_i's are constants

Since link flows essentially are the sum of variable flows between various O-D trip pairs, one might expect a reduction in the coefficient of variation *a priori*. Strong positive correlation dilutes this effect to a certain degree, but it is still evident here.

For better understanding and interpretation of the four-step model results, sensitivity analysis was used to identify model inputs that are key contributors to uncertainty in model output. First, the sample correlation coefficients (Table 4) indicate the linear correlation between inputs and outputs. Since there are many demographic input variables (i.e., the number of households and jobs in each zone), only the sums of these variables across zones are presented. One can compare the output's sensitivity to parameters in each model step. Not surprisingly, the parameter which has the strongest correlation with link flows is the trip generation rate. This is partially consistent with Smith and Cleveland's results (1976). Also, the overall outputs are sensitive to the demographic inputs. Most zonal demographic inputs contribute substantially to the overall uncertainty in link flows. Given the linear function pattern of the trip generation model, it is not surprising that the demographic inputs and the trip generation parameters show strong linear correlation with the overall outputs. Moreover, the rank correlation coefficients (Table 5) show the non-linear correlation between inputs and outputs. The results are somewhat similar to the linear correlation analysis.

To further identify the most important contributors to overall uncertainty, a regression analysis was conducted. Figure 7 shows the final model results (following a series of stepwise deletions of statistically insignificant (at 0.10 level) variables). Before the computation of regression coefficients, the variables are standardized by dividing each observation on a variable by its standard deviation. In Figure 7, the lengths of these bars stand for the standardized coefficient, or beta weight coefficient values. They measure the effect of a one-standard-deviation change in an independent variable on the dependent variable (also measured in standard deviation units). For a selected link, the major contributors to variation in flow estimates are the parameters from trip generation step and total employment input levels[4]. Similar results for total VMT can be seen in Figure 7. Thus, the demographic inputs and parameters to trip generation are primary contributors to the total VMT output. It is not surprising that the trip attraction rates of basic and service employments for land use type 3 (suburban residential) show stronger correlation to final results than other parameters in trip generation, because most zones in this study area belongs to suburban residential and basic and service employments are the main employment types in these zones. In addition, the parameters in mode split are found to play important roles in result variation. In contrast, results exhibit relatively little sensitivity to the parameters of the trip distribution and trip assignment models; this result may be due to the less-than-straightforward application of those models – due to iterative trip-balancing for trip distribution and user-equilibrium feedbacks used in trip assignment. Furthermore, the constraint that trip productions equal attractions negates the effects of the single, multiplicative trip generation rate, permitting trip attraction rates to play the important role in final estimates.

CONCLUSIONS

This work investigated the stability of contemporary transport demand model outputs by simulating a four-step travel demand model over a 25-zone network. Point estimates of outputs were compared following a series of input variations. Sensitivity analyses also were undertaken, to suggest ways for more effective direction of modeling and planning resources.

The results of this work suggest that uncertainty is somewhat compounded over the four stages of the travel demand model and is highly correlated across outputs. Mispredictions at early stages of the multi-stage model (e.g., trip generation) appear to be amplified across later stages. In particular, traffic flow uncertainty appears to vary substantially across links: some link flows are much more variable than others. However, network-predicted flows across various links were relatively stable across simulations, probably as a result of equilibrium assignment (which acknowledges congestion feedbacks). Trip assignment, the final step of the traditional, four-step model, was found to reduce uncertainties developed in the first three steps; however, in general, it could not reduce final flow uncertainties below the levels of input uncertainty. Overall, the results indicate that predictions from many travel demand models may be highly uncertain, due to input and parameter uncertainties. The sequence of models and equilibrium assignment do not attenuate the underlying uncertainties.

To clarify the outcomes and emphasize the model components having greatest impact, this study applied simple model specifications on a sub-network. The results are focused yet general – providing greater applicability to a variety of contexts than more complex or constrained specifications. It is hoped that this work provides a clear starting point and valuable tools for additional analysis of variation in travel demand model outputs.

Further work on this issue and related topics is still needed. For example, applications on more realistic networks may be examined with more simulation runs. And a variety of common model specifications (e.g., a stochastic user equilibrium trip assignment) may be estimated and then tested. In addition, feedbacks of travel-time estimates to destination, mode, and route choices would be valuable. Also, factorized "experiments" rather than random simulations may be more efficient at sampling the set of possible environments and distinguishing the contributions and interactions of different random inputs and parameters. Such work will help identify which aspects of modeling practice are the biggest contributors to result uncertainty – and where modeling improvements are likely to be most effective for added precision.

In general, since inputs and parameter estimates are uncertain, transportation modelers would do better to recognize, estimate, and specify result uncertainties. In addition, policymakers should appreciate these uncertainties and incorporate such information in their decision-making. This work represents a step in this direction.

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ENDNOTES:

[1] Basic employment consists of agriculture, mining, construction, manufacturing, transportation, communications, utilities, wholesale, and non-retail employment. Retail employment consists of jobs in businesses primarily engaged in selling goods to the public. Service employment includes financial, insurance, real estate, and other non-governmental service jobs.

[2] For example, it may be that a single target population or jobs number is forecast for the region, and these are then distributed in simple proportion to current numbers – producing perfect positive correlation.

[3] A more complicated set of travel demand models, using more parameters and inputs, would likely require more simulations, to achieve stable estimates. However, a factorial design of the experiment (introducing orthogonality across experimental values) and more efficient sampling methods across input spaces (e.g., Latin Hypercube [Fishman 1996]) may enhance simulation and stability of estimates.

[4] 100 simulation observations are not sufficient to estimate more than 100 unknown parameters; so only the *total* number of households and employment (of the three different types) across the 25 zones are used in the regression.

TABLE 1. SIMULATION SET-UP: Model Parameters*

Model	Parameter	Mean	SD	Coef. of Variation	Distribution	Covar.
Trip Generation	α	2.303	0.691	0.30	Lognormal	-
	$\beta_{1,2}$	1.389	0.417	0.30	Lognormal	-
	$\beta_{1,3}$	1.328	0.398	0.30	Lognormal	-
	$\beta_{1,4}$	1.309	0.393	0.30	Lognormal	-
	$\beta_{1,5}$	1.476	0.443	0.30	Lognormal	-
	$\beta_{2,2}$	1.396	0.419	0.30	Lognormal	-
	$\beta_{2,3}$	1.530	0.459	0.30	Lognormal	-
	$\beta_{2,4}$	1.448	0.434	0.30	Lognormal	-
	$\beta_{2,5}$	1.386	0.416	0.30	Lognormal	-
	$\beta_{3,2}$	1.304	0.391	0.30	Lognormal	-
	$\beta_{3,3}$	1.371	0.411	0.30	Lognormal	-
	$\beta_{3,4}$	1.369	0.411	0.30	Lognormal	-
	$\beta_{3,5}$	1.392	0.418	0.30	Lognormal	-
Trip Distribution	γ	1.16E-3	3.48E-4	0.30	Lognormal	-
Model Split	$\theta_{transit}$	-0.549**	0.165	0.30	MVLognormal*	ρ =0.67
	δ	-0.0297	0.0089	0.30	MVLognormal*	
Traffic Assignment	α_0	0.84	0.252	0.30	Lognormal	-
	β_0	5.50	1.65	0.30	Lognormal	-

* The mean parameter values come from the DFW area travel model report. (NCTCOG 1999).
**To impose negativity, these parameters are drawn from a multivariate lognormal distribution and then given negative signs.

TABLE 2. NETWORK FLOW SIMULATION RESULTS*

Variable	Description	Mean	SD	Coef. of Variation	Avg. V/C Ratio
f_1	Main direction flow on link 1	1172	363	0.310	1.116
f_2	Main direction flow on link 2	1522	489	0.322	0.235
T_1	Average travel time on link 1 (hour)	0.1058	0.201	1.899	-
T2	Average travel time on link 2 (hour)	0.0137	0.0017	0.127	-
Total *VMT*	Total vehicle-miles traveled on the network	129518	30579	0.236	-
Total *VHT*	Total vehicle-hours traveled on the network	3347	777	0.232	-

* All the results are based on converged UE assignments for 100 runs. The total demand (morning peak hour HBW auto trips) has a mean of 23856 and an SD of 5503.

TABLE 3. CORRELATION COEFFICIENTS BETWEEN LINK FLOWS

	f_1	f_2	**Total *VMT***	**Total *VHT***
f_1	1.000	0.601	0.849	0.862
f_2	0.601	1.000	0.724	0.725
Total *VMT*	0.849	0.724	1.000	0.983
Total *VHT*	0.862	0.725	0.983	1.000

TABLE 4. SAMPLE CORRELATIONS BETWEEN INPUTS AND OUTPUTS

Model	Parameter	f_1	f_2	Total *VMT*	Total *VHT*
Trip Generation	α	0.0589	0.1280	0.1024	0.0990
	$\beta_{1,2}$	0.0345	0.0133	-0.0399	-0.0283
	$\beta_{1,3}$	0.2150*	0.3182*	0.3396*	0.3204*
	$\beta_{1,4}$	-0.0274	-0.0594	-0.0262	-0.0269
	$\beta_{1,5}$	0.0467	0.0343	-0.0008	0.0035
	$\beta_{2,2}$	0.0869	-0.0248	0.0549	0.0562
	$\beta_{2,3}$	-0.1094	0.0394	-0.0086	-0.0004
	$\beta_{2,4}$	0.0091	-0.0123	-0.0023	-0.0076
	$\beta_{2,5}$	0.1270	0.2089	0.1500	0.1483
	$\beta_{3,2}$	0.1013	0.1582	0.0326	0.0488
	$\beta_{3,3}$	0.6052*	0.3646*	0.5944*	0.5987*
	$\beta_{3,4}$	-0.0356	-0.0226	-0.0636	-0.0555
	$\beta_{3,5}$	-0.1701	-0.1753	-0.1259	-0.1297
Trip Distrib.	γ	0.0244	0.0099	0.0084	0.0049
Mode Split	$\theta_{transit}$	0.0711	0.1558	0.1121	0.1075
	δ	0.0457	0.1651	0.1327	0.1271
Traffic Assign.	α_0	-0.0431	-0.0427	-0.0793	-0.0628
	β_0	-0.0409	0.0305	0.0223	0.0080
Inputs	Total Households	0.4419*	0.3354*	0.4719*	0.4791*
	Total Basic Employment	0.4511*	0.3230*	0.5639*	0.5706*
	Total Retail Employment	0.5212*	0.3244*	0.5347*	0.5427*
	Total Service Employment	0.6055*	0.3872*	0.6427*	0.6517*

Note: An "*" indicates the correlation is significant at the 0.05 level (2-tailed).

TABLE 5. RANK CORRELATIONS BETWEEN INPUTS AND OUTPUTS

Model	Parameter	f_1	f_2	Total *VMT*	Total *VHT*
Trip Generation	α	0.0698	0.0959	0.1558	0.1596
	$\beta_{1,2}$	0.0191	0.0220	-0.0433	-0.0291
	$\beta_{1,3}$	0.1471*	0.2296*	0.3019*	0.2827*
	$\beta_{1,4}$	0.0594	-0.0509	0.0585	0.0602
	$\beta_{1,5}$	0.0713	0.0387	0.0211	0.0248
	$\beta_{2,2}$	0.1254	-0.0109	0.0930	0.1001
	$\beta_{2,3}$	-0.1326	-0.0495	-0.0485	-0.0474
	$\beta_{2,4}$	-0.0254	-0.0053	0.0178	0.0050
	$\beta_{2,5}$	0.1982	0.2266*	0.1897	0.1909
	$\beta_{3,2}$	0.0291	0.1156	0.0031	0.0155
	$\beta_{3,3}$	0.5879*	0.3360*	0.5517*	0.5531*
	$\beta_{3,4}$	-0.0836	-0.0899	-0.1048	-0.1050
	$\beta_{3,5}$	-0.1582	-0.1437	-0.1548	-0.1625
Trip Distrib.	γ	0.0057	-0.0184	-0.0327	-0.0399
Mode Split	$\theta_{transit}$	0.0963	0.1187	0.1227	0.1139
	δ	0.0815	0.1530	0.1303	0.1282
Traffic Assign.	α_0	-0.0068	-0.0534	-0.0469	-0.0308
	β_0	-0.0430	0.0641	0.0045	-0.0053
Inputs	Total Households	0.4408*	0.3548*	0.4679*	0.4727*
	Total Basic Employment	0.4276*	0.3172*	0.5327*	0.5391*
	Total Retail Employment	0.4950*	0.3334*	0.4924*	0.5010*
	Total Service Employment	0.5680*	0.3867*	0.6093*	0.6141*

Note: An "*" indicates the correlation is significant at the 0.05 level (2-tailed).

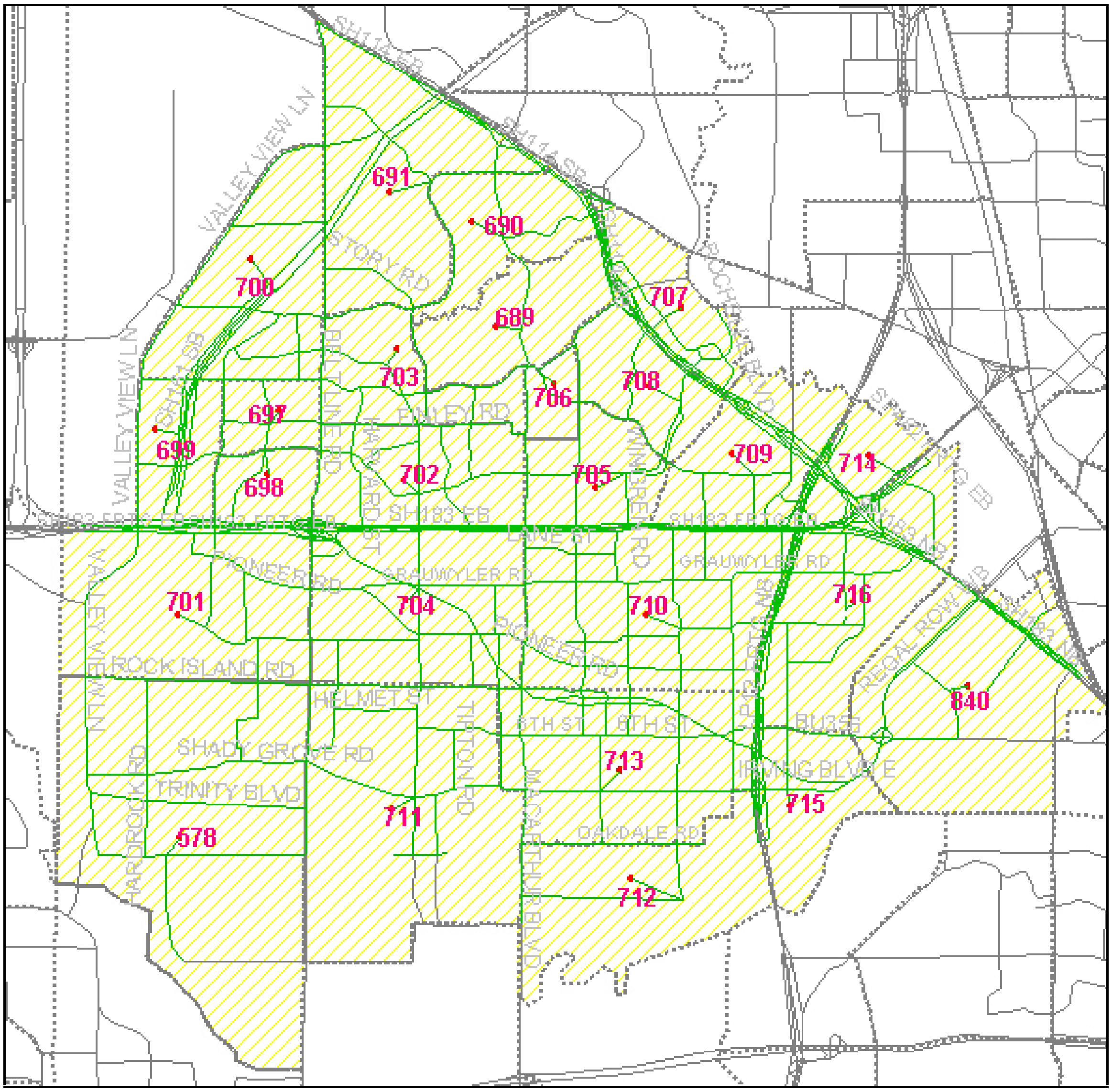


Figure 1. 25-zone subnet from the Dallas-Fort Worth highway network

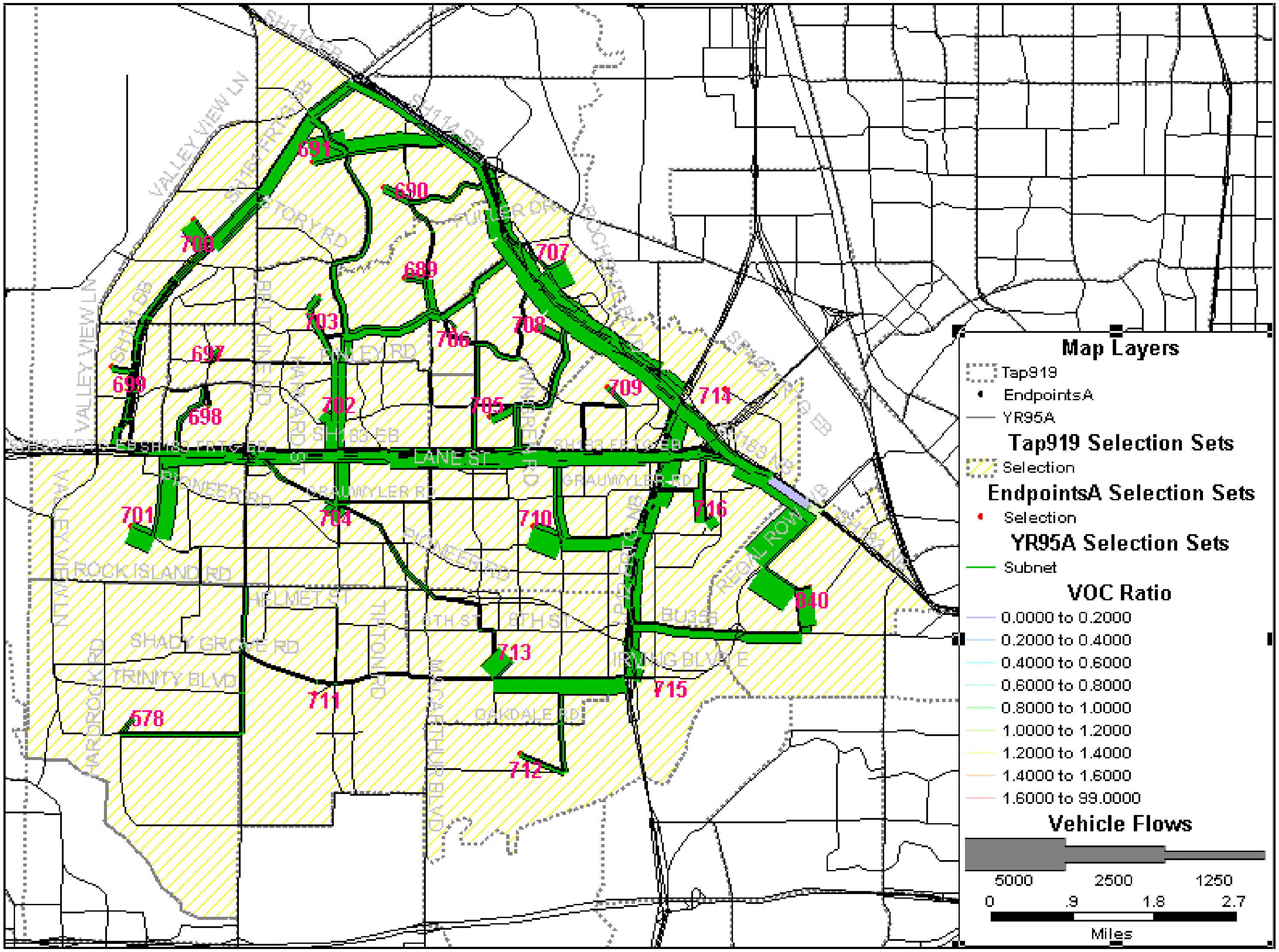
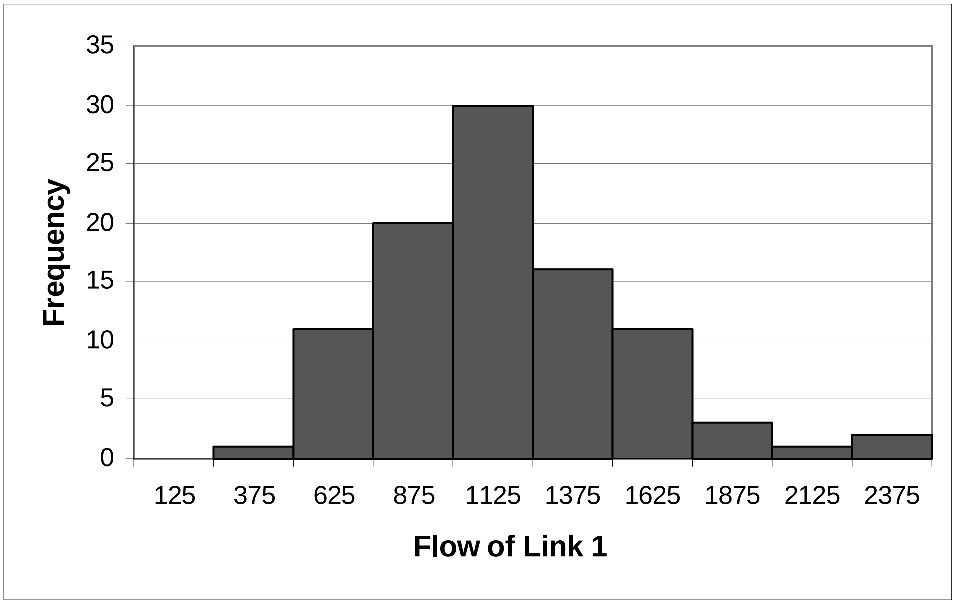


Figure 2. One UE assignment result for the 25-zone subnet

Link 1's Flow Distribution



Link 2's Flow Distribution

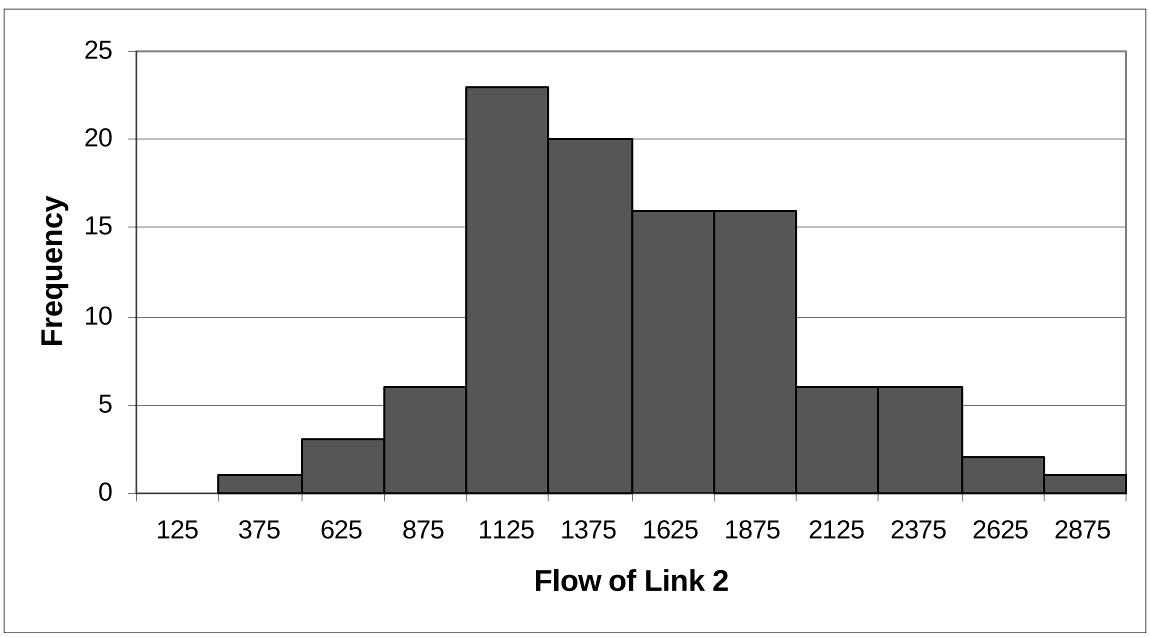


Figure 3. Distribution of 100 assignment results for selected links

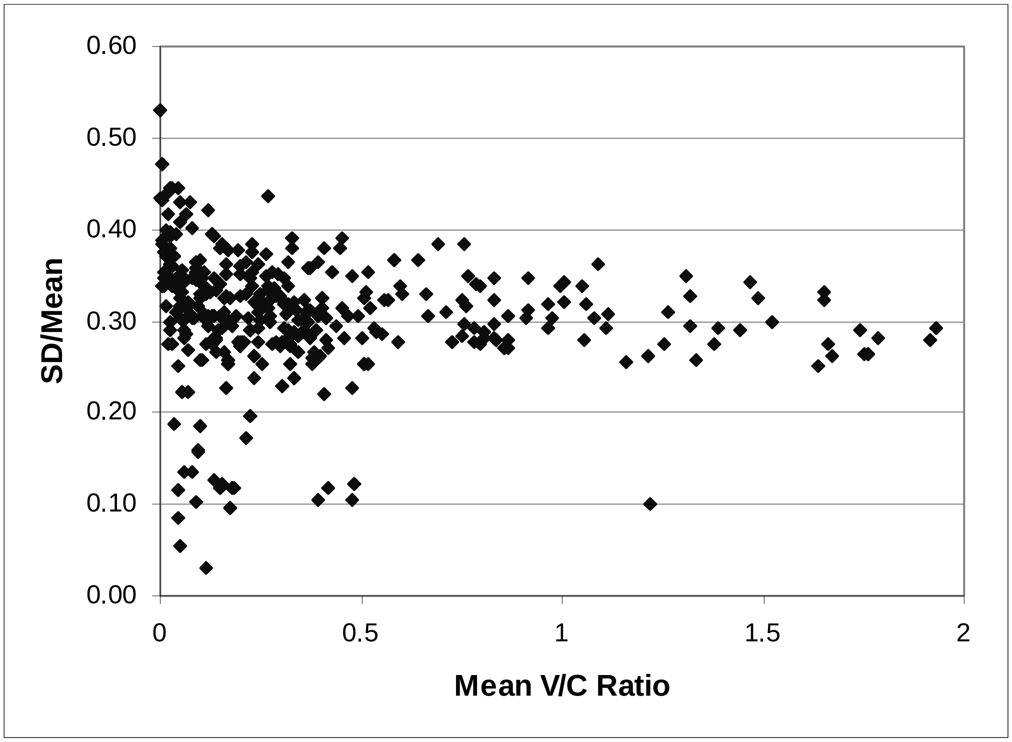


Figure 4. Scatter plot of uncertainty and volume/capacity ratios

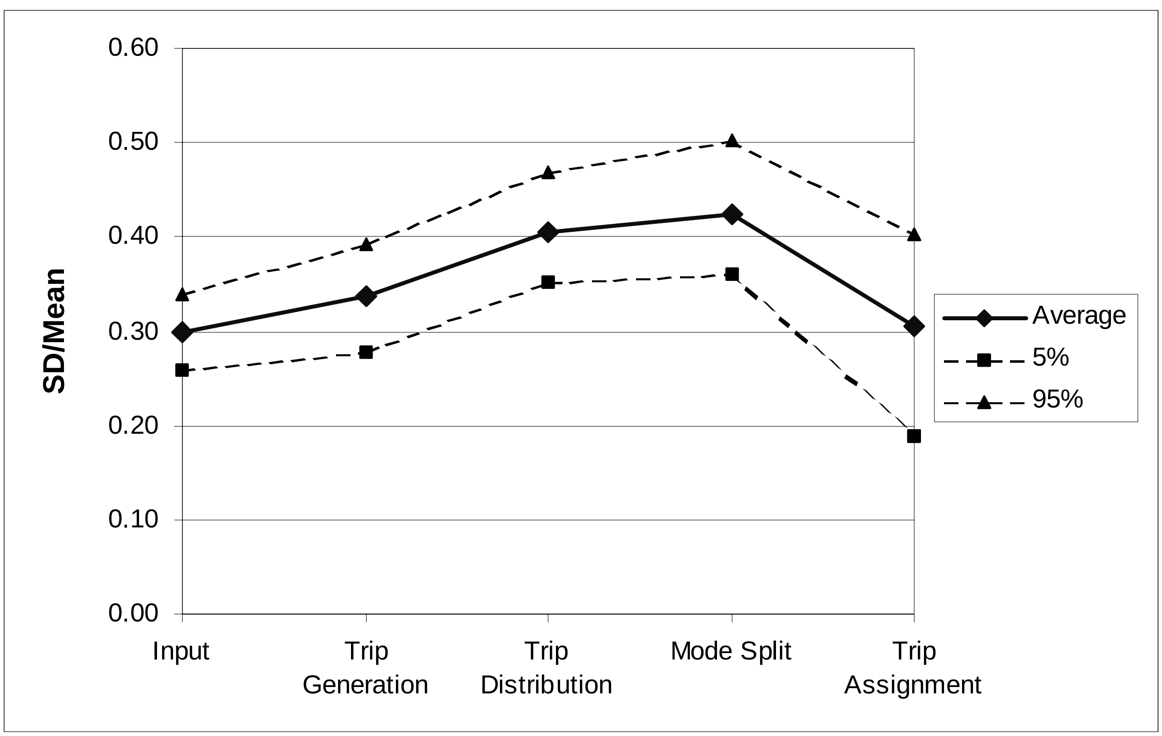


Figure 5. Uncertainty propagation through 4-step models

Note: There are 117 random input variables, 50 random trip generation outputs, 625 trip distribution outputs, 625 mode split (DA) outputs, and 818 trip assignment outputs.

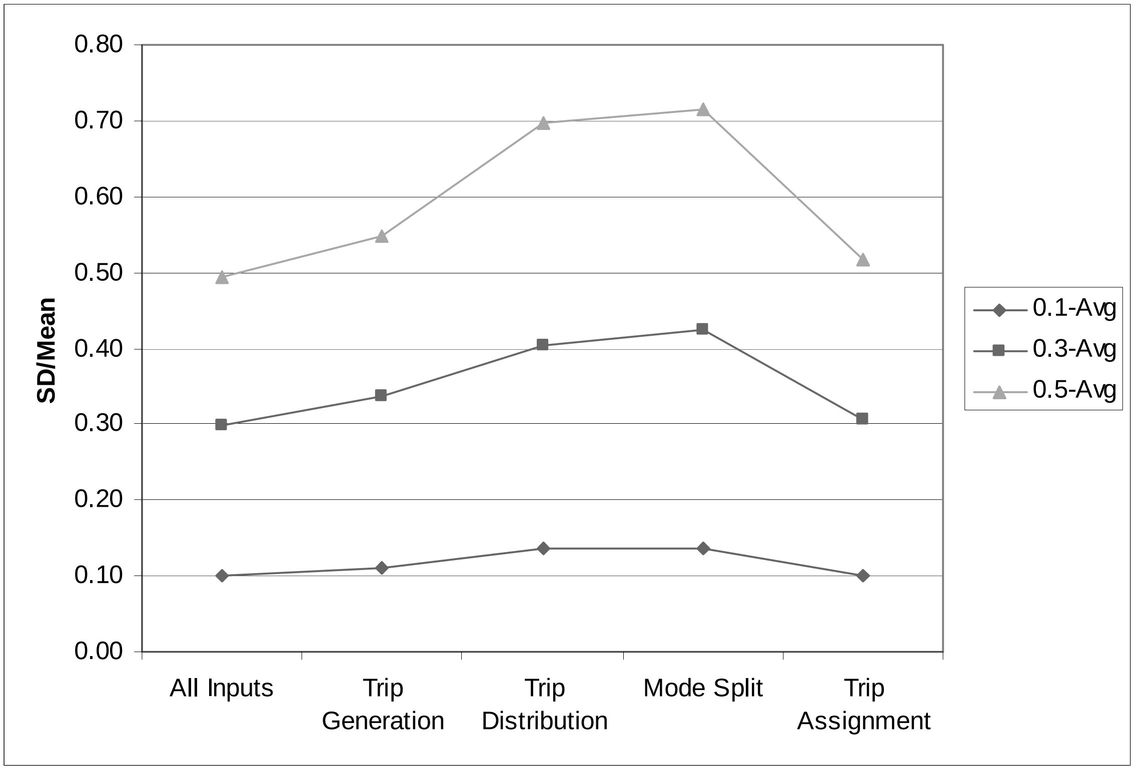


Figure 6. Uncertainty propagation through 4-step models with different input/parameter uncertainty levels

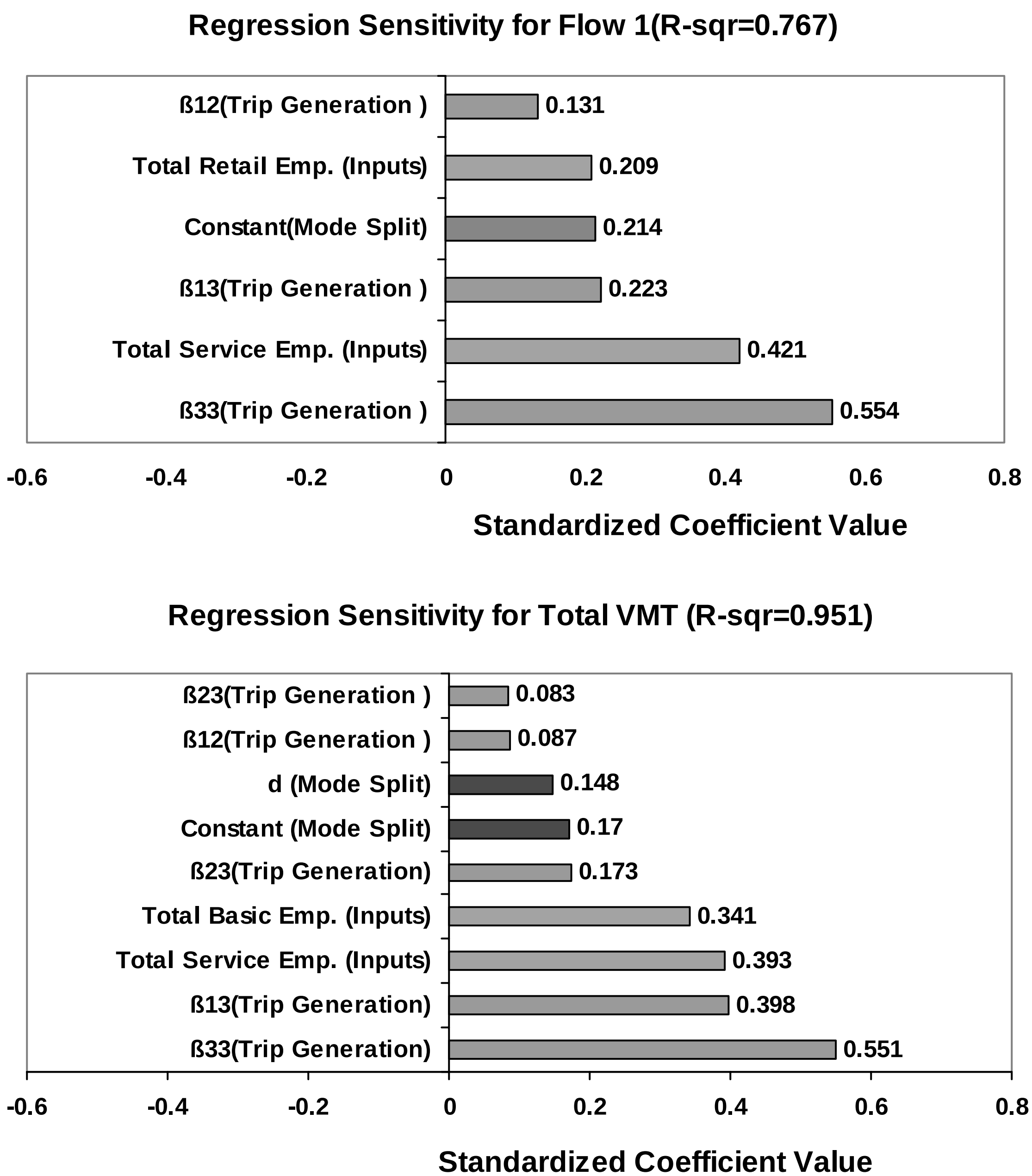


Figure 7. Regression-based sensitivity analysis for final outputs