A NEW APPROACH FOR TRAVEL DEMAND MODELING: LINKING ROY'S IDENTITY TO DISCRETE CHOICE

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ABSTRACT

The variety of choice alternatives in travel contexts has led to significant simplifications of behavior in models of these complex decisions. Typically, several demand submodels are run independently, producing relatively disconnected estimates of trip generation, destination choice, mode, and time of day. This work relies on nested behavioral models for cost minimization and applications of Roy's Identity to the ensuing comprehensive cost values. The end result is a behaviorally grounded model of travel demand across any number of choice dimensions. These are subject to a general budget constraint based on time and money limitations. Unlike existing models, the model produces rigorous welfare measures recognizing all aspects of travel choice.

For purposes of illustration, the model was calibrated using Austin, Texas, travel-diary data and a modified-translog indirect utility specification. Results indicate that Austinites are less flexible about mode choice than destination choice for non-work trips and that the elasticity of trip generation with respect to travel times and costs is very low. In addition, welfare analyses using equivalent variation measures were performed under various network and policy scenarios, including congestion pricing. These strictly accommodate the welfare impacts of network and land use changes on trip-generation and other travel choices; the resulting estimates suggest flexibility in trip-making.

Keywords

Simultaneous travel demand models, Roy's Identity, logit models, multivariate negative binomial, effective time budget

INTRODUCTION & BACKGROUND

Human transport is a complex commodity. Key dimensions of interest, for purposes of systems modeling, are the number, timing, and spatial distribution of trips by mode. Given the continuous nature of time and space, the observed behaviors are infinitely faceted. Aggregation of such properties can still result in a multitude of alternatives. To model such choices comprehensively is a challenging task.

The nested logit model (McFadden 1974, 1981) has proven very successful in distilling information on attributes across a broad, though discrete, set of choice alternatives into a single "index of desirability" or "inclusive price of travel." (McFadden and Domencich 1975, McFadden 1981) McFadden and Domencich's nested design of household and work location choice, vehicle ownership, trip versus no trip, destination, time of day, and mode choices reduced statistical complexity, thus greatly facilitating model estimation and application.

Multinomial and nested logit models can be developed from a rigorous behavioral theory of utility maximization. However, standard multinomial logit models require discretization of choices (e.g., peak vs. off-peak travel times, trip vs. no trip); this causes a loss of cardinality and continuity, which define many travel choices, such as time of day and number of trips made. Microeconomists have long applied Roy's Identity (Roy 1943) to derive systems of demand equations for continuous consumption of goods, subject to budget constraints. (See, e.g., Varian 1992, Stone 1954, Pollack and Wales 1978 & 1980, Deaton and Muellbauer 1980a & 1980b.) This approach can be highly useful in predicting the number of times individuals choose certain activities or trip types.

Kockelman (1998, 2001) extended Roy's Identity to the time domain for use in travel demand prediction, where time expenditures are fundamental. She modeled households' consumption of various discretionary "activities" across zones as a function of access travel times and both income and time budgets. Using a flexible translog formulation of indirect utility (Christensen et al. 1975) and a multivariate negative binomial specification for activity-participation rates, she estimated household members' activity participation across trip purposes and across highly aggregated equal-opportunity zones. For limited choice sets (e.g., choice of retail centers for shopping excursions), this approach is very valuable. It offers non-linear values of time and flexible expenditure functions (for estimates of both time- and money-compensated and -equivalent variations, for use in welfare analyses); it also permits a variety of interesting hypothesis tests (e.g., existence of constant travel-time budgets). However, in applying this model across more than a few types of activities and zones, the flexibility of the functional specifications can dramatically limit model estimation. For example, in the case of four activity alternatives, Kockelman estimated 24 parameters; for 10 distinct alternatives, one would need to estimate 87 parameters, in order to achieve the same level of functional flexibility. Less flexible and less complex functions can be used, however, particularly if the requirements of symmetry, homogeneity, and quasiconvexity are not required, as Kockelman (1998, 2001) discussed for the time domain.[1]

There is a clear need for a model which adequately represents generated trip counts (which are cardinal in nature) while simultaneously recognizing the multiple attributes of any trip taken (e.g., time of day, mode, and destination). For practical applications across large choice sets, the model must be relatively parsimonious in parameters[2] – yet behaviorally rigorous and flexible. This work defines such a model.

MODEL DESCRIPTION

The dual problem of profit maximization is cost minimization (see, e.g., Varian 1992). Similarly, the dual of utility maximization is expenditure minimization[3] – subject to a minimum-utility constraint. Of course, utility maximization is equivalent to disutility minimization. If units of utility (or disutility) for choice alternatives can be converted into equivalent prices (through, for example, a constant marginal utility of money), one may consider applying Roy's Identity to such equivalent prices. The standard form of Roy's Identity is the following:

$$X_i^* = -\frac{\frac{dv}{dP_i}}{\frac{dv}{dY}}, \forall i \tag{1}$$

where X_i^* is the optimal, long-run rate of consumption per period of good (or activity) type i, v is indirect utility (i.e., the maximum level of utility one can attain facing a set of prices and one's own constrained income), P_i is the (exogenous) unit price of good i, and Y is the (exogenous) income constraint. Kockelman (1998, 2001) made the following extension:

$$X_i^* = -\frac{\frac{dv}{dt_i}}{\frac{dv}{dT}}, \forall i \tag{2}$$

where t_i is the travel time to access activity i and T is the time constraint (per period, on the decision-maker). Kockelman applied this model in the time domain, though the indirect utility function was also a function of monetary income. As she noted, two systems of demand equations can be simultaneously estimated based on Eqs. 1 and 2. Such an approach is relatively complex but avoids one having to convert time to money (or vice versa).

As discussed above, there is a tradeoff in model flexibility and parameter parsimony. Yet, routinely, there are thousands of possible choice alternatives available in most applied models. For example, travel demand models of 1000-zone systems, across 3 trip purposes, 2 times of day, and 3 modes generate 18,000 functionally different trip-type alternatives. The parsimony and rigor of random-utility discrete-choice models (e.g., McFadden's logit [1974] and the probit model, which was first proposed by Thurstone in 1927 [Maddala 1983]) make good sense for defining most if not all of these trip dimensions. For modeling total trip generation and/or trip generation by trip type, however, one needs a model that respects the cardinality inherent in the choice (e.g., zero, one, two, three, and more trips observed per day or week). If trip generation is, fundamentally, a continuous choice process, negative binomial or other statistical specifications, based on continuous optimal demand levels (e.g., Eqs. 1 and 2) and linked to other choice dimensions (such as mode and destination), are attractive.

Given a logit specification of travel choices across modes (and given a particular destination i), one has the following "logsum" expression for the expected maximum utility offered by choice of mode (see, e.g., Ben-Akiva and Lerman [1985]):

$$E(\max U_1) = \Gamma_1 = \frac{1}{\mu_1} \ln\left[\sum_{m=1}^{M} \exp(\mu_1 V_m)\right] \tag{3}$$

where μ_1 is the distribution's scale parameter[4] and V_m is the systematic-utility expression for mode *m* (e.g., $V_m = \beta_{time}\, t_{dm} + \beta_{cost}\, c_{dm}$, where t_{dm} and c_{dm} represent the travel time and cost required by mode *m* to reach a given destination *d*, respectively).

When two choice attributes are modeled in a single nested-logit formulation, the expected maximum utility of the combination requires a nested logsum. For example, if mode is modeled as the lower-level nest, and destination is the higher-level nest, one has the following expression:

$$E(\max U_2) = \Gamma_2 = \frac{1}{\mu_2} \ln\left\{\sum_{d=1}^{D} \left[\exp(\mu_2 V_d + \mu_2 \Gamma_1)\right]\right\} \tag{4}$$

where μ_2 characterizes the dispersion within the destination-choice portion of the model, V_d is the systematic utility expression for destination *d* (e.g., $V_d = \beta_3 A_d$, where A_d is the attractiveness value of destination *d*), and other variables are defined as in Eq. 3.

In this way, one can collapse all attributes of a choice subset into a single logsum measure, in units of utility (per unit of the good, activity, or trip). One may choose to do this across subsets defined by trip purpose. And, following transformation of these utility values to effective prices (or any other form wherein the price units are consistent with budget units), one can apply Roy's Identity for trip generation rates (by purpose, in this example).

Application of Roy's Identity in the Presence of Logsum Effective Prices

Transformation of utility units to monetary units is not necessarily trivial. Utility is only an ordinal construct, whereas prices and budgets are cardinal. Differences in utilities can be directly transformed if one makes an assumption of constant marginal utility of money/income (which is what almost all modelers do, consciously or unconsciously). This is the case when the systematic-utility parameter associated with travel cost (β_{cost} in the example definition for Eq. 3) is a constant – and income does not enter the lower-level specifications. This assumption permits computation of *changes* in effective costs by multiplication of changes in expected maximum utility by the negative of the inverse of the marginal utility of money (i.e., $1/\beta_{cost}$). However, for *standard* application of Roy's Identity, one requires the absolute price levels. Moreover, the property of summability in demand (so that total expenditures equal total income) is highly desirable, not to mention the properties of homogeneity, symmetry and convexity (see, e.g., Kockelman [1998, 2001], Deaton and Muellbauer [1980], or Varian [1992]). These issues are fundamental and require earnest consideration.

In standard form, indirect utility is a function of prices and income: $v(\vec{P}, Y)$. When time use is important, as in travel and activity participation, Kockelman's (1998, 2001) expanded form is useful: $v(\vec{P}, \vec{t}, Y, T)$. If "time is money" and these can be continuously traded (e.g., by working more or less and paying others to undertake certain activities, such as housecleaning) at a constant value of time, *VOT*, one can merge time and money into single indices of effective prices; for example, $v(\vec{P}+VOT\cdot\vec{t},Y+VOT\cdot T)=v(\vec{P}^{e},Y+VOT\cdot T)$.

Making these same assumptions while leaving effective, logsum prices (for different trip purposes, for example) in units of utility (per trip, in this example), one can transform income and time budgets into a single effective budget, in units of utility:

$$Y^e = \beta_{cost}Y + \beta_{cost}\,VOT \times T = \beta_{cost}Y + \beta_{time}T \tag{5}$$

where Y^e is effective budget and other parameters are defined as above.

Roy's Identity (Eqs. 1 and 2) will then continue to hold, when applied with respect to effective (or generalized) prices and effective budget, both in units of utility:

$$X_i^{\ *} = -\frac{\dfrac{dv}{dP_i^e}}{\dfrac{dv}{dY_i^e}}, \forall i \tag{6}$$

Trip Generation Model Specification

Christensen's et al.'s (1975) and Kockelman's (1998, 2001) translog specification of indirect utility, *v*, is flexible to the second order. It was examined here first, for application of Roy's Identity and estimation of travel demand functions; and it is as follows:

$$v = \alpha_o + \sum_i \alpha_i \ln P_i + \sum_{ij} (1/2)\beta_{ij} \ln P_i^e \ln P_j^e + \sum_i \gamma_i \ln Y^e \ln P_i^e + \gamma_Y \ln Y^e \tag{7}$$

Given this functional assumption, the demand equations to be estimated as a system are as follows:

$$X_i^{\ *} = \frac{-\dfrac{1}{P_i^e}\left(\alpha_i + \sum_j \beta_{ij} \ln P_j^e + \gamma_i \ln Y^e\right)}{\dfrac{1}{Y^e}\left(\sum_j \gamma_j \ln P_j^e + \gamma_Y\right)} \tag{8}$$

Three trip purposes were modeled in this application, so indices *i* and *j* take values from 1 to 3, which correspond to home-based non-work (HBNW), home-based work (HBW) and non-home-based (NHB) trips, respectively.

Levels of trip-making across these three trip classes were modeled as a system, using logsums of prices and the multivariate negative binomial specification described below.

STATISTICAL SPECIFICATION

As suggested, one can estimate money and time parameters (β_{cost} & β_{time}) using a linear-in-parameters mode-choice model (or any other model where money and time costs characterize the alternatives). And, after calibrating a logit model (nested or non-nested), one can estimate effective prices for choice subsets via the negatives of logsum expressions (e.g., Eqs. 3 & 4).

When one requires a trip (or activity-participation) count, however, one must turn to more continuous models of behavior. For count processes, a Poisson distribution is a common assumption. This assumes there is a constant, underlying rate generating the observed counts.

Given a Poisson process assumption for trip generation by any single household, a multivariate negative binomial expression involving a single gamma parameter permits both (a specific form of) heterogeneity across observational households and correlation in travel patterns (at the household level). This distributional assumption also results in a closed-form likelihood and, across observational units (households, in this instance), a form of overdispersion, which can be quite common in behavioral data (Cameron and Trivedi, 1998; Hausman et al. 1995; Kockelman, 1998 and 2001). Apparent overdispersion could also result from a dual-regime model, such as the zero-inflated Poisson (ZIP) and negative binomial (ZINB) distributions (see, e.g., Shankar et al. 1997, Washington et al. 2003). For example, a preponderance of zeros may be expected for work trips, due to the existence of households without any workers. Zero-inflated applications presently exist for univariate, uncorrelated responses.

The following specification produces a series of correlated integer responses and was used for this work's application to Austin's 1996 travel survey data:

$$X_i \sim Poisson(\lambda_i = X_i^{*}\varepsilon)\ \forall i \tag{11}$$

where ε is gamma distributed (with a mean of one) across individual observational units (but is constant for any single individual's set of demands). Thus, total demand by any observational unit is negative-binomially distributed:

$$\sum_{i=1}^{I} X_i = X_T \sim \text{Poisson} \underset{\lambda_T}{\wedge} \text{Gamma}(m,m) = \text{Neg.Bin.}(m, p^{*}),$$
$$\text{where } \frac{m(1-p^{*})}{p^{*}} = \sum_{i=1}^{I} X_i^{*}. \tag{12}$$

Moreover, knowing the total demand, X_T (e.g., total trip-making), one can estimate the individual demand levels (X_i) using a multinomial model, where each demand's probability (p_i) is the ratio of its optimal, underlying, continuous level to the total:

$$\Pr(\vec{X} \mid p_1, ..., p_I) = \text{Multinomial } \Pr(\vec{X} \mid \vec{p}, X_T)\text{Neg.Bin. }\Pr\left(X_T \mid \sum_{i=1}^{I} X_i^{*}\right)$$
$$= \left(\frac{X_T!}{\prod_{i=1}^{I} X_i!}\prod_{I=1}^{I} p_i^{X_i}\right)\left(\frac{\Gamma(X_T + m)}{X_T!\Gamma(m)}\right)(1-p^{*})^{X_T}(p^{*})^{m}, \tag{13}$$
$$\text{where } p_i = \frac{X_i^{*}\varepsilon}{\sum_{j=1}^{I}\left(X_j^{*}\varepsilon\right)} = \frac{X_i^{*}}{\sum_{j=1}^{I} X_j^{*}}.$$

Eqs. 11 through 13 are used in the application provided here, with $I = 3$ trip types. However, more complex and flexible stochastic specifications certainly are viable, though their calibration most likely will require likelihood simulation (see, e.g., Train and McFadden 2000, Bhat 2001). For example, one may hypothesize that the error terms ε covary across trip types i, according to a multivariate lognormal distribution; for maximum likelihood estimation, the resulting probabilities of demand vectors will require approximation of the multivariate integrals (through simulation or other methods). A similar situation will arise for multivariate zero-inflated distributions.

DATA DESCRIPTION AND MODEL DETAILED SPECIFICATION

To illustrate the proposed approach, the 1996 Austin Travel Survey data were assembled for model calibration. Total trip-making per household on a single weekday was defined across three trip types: home-based work (HBW), home-based non-work (HBNW), and non-home-based (NHB) trips. For purposes of trip-cost computations, home was the origin for the first two trip types. For NHB trips, if work were the purpose at one end, its location was the assumed trip origin; otherwise, the true trip origin was used.

1074 traffic serial zones (TSZs) are defined by Austin's Metropolitan Planning Organization (the Capital Area MPO, or CAMPO), encompassing the counties of Hays, Williamson, and Travis. Interzonal travel times, for actual trips and trip alternatives, were taken as given by CAMPO. These varied by time of day (peak and off-peak) and mode (automobile and non-automobile). Automobile trips were assumed to cost 30¢ per network mile, and other mode costs were taken as given by CAMPO. Among the 15 different mode categories available from the survey, 10 corresponded to automobile trip types and 5 corresponded to other modes. Trip-weighted averages of the automobile modes and non-automobile modes were used for their respective (average) travel time and cost values, with respect to home location (and with respect to work location, in the case of work-based NHB trips, and with respect to trip origin, in the case of NHB trips). Such aggregation creates loss of information for model estimation; but the focus here is on method illustration, rather than parameter estimation.

Nested logit (NL) models were investigated, for destination (1074 zones) and mode choice (automobile vs. non-automobile). These models were developed separately for each of the three trip purposes. Two standard NL specifications are possible: Specification 1, with mode at the lower nest level, and Specification 2, with mode at the upper level. Both were estimated, and both share the following general random utility specification:

Specification 1 (Destination at the higher level and mode at the lower level):

$$U_{dm} = \tilde{V}_m + \tilde{V}_d + \tilde{V}_{dm} + \varepsilon_d + \varepsilon_{dm} \tag{14}$$

where ε_{dm} and $\varepsilon_d + \varepsilon_{dm}$ are Gumbel distributed with scale parameters μ_m and μ_d, respectively. Specification 1's nested logit choice probability is given by the following expression.

$$P(md) = P(m \mid d)P(d) \tag{15}$$

where

$$P(m \mid d) = \frac{e^{(\tilde{V}_m + \tilde{V}_{dm})\mu_m}}{\sum_{m'} e^{(\tilde{V}_{m'} + \tilde{V}_{dm'})\mu_m}} \tag{16}$$

$$P(d) = \frac{e^{(\tilde{V}_d + \Gamma_{md})\mu_d}}{\sum_{d'} e^{(\tilde{V}_{d'} + \Gamma_{md'})\mu_d}} \tag{17}$$

$$\Gamma_{md} = \frac{1}{\mu_m} \ln \sum_{m'} e^{\left(\tilde{V}_{m'} + \tilde{V}_{dm'}\right)\mu_m} \tag{18}$$

$$\tilde{V}_{dm} = \beta_{time} time + \beta_{cost}\, cost \tag{19}$$

$$\tilde{V}_d = \beta_{emp} emp \tag{20}$$

$$\tilde{V}_m = \beta_{auto}\delta_{auto} \quad \text{(where } \delta_{auto} = \text{ 1 if mode = auto, else 0)} \tag{21}$$

For consistency with utility maximization (McFadden 1981), Specification 1's scale parameters should obey the following: $0 \le \mu_d \le \mu_m$.

Specification's 2 choice probability is given by the following expression.

$$P(md) = P(d \mid m) P(m) \tag{22}$$

where

$$P(d \mid m) = \frac{e^{\left(\tilde{V}_d + \tilde{V}_{dm}\right)\mu_d}}{\sum_{d'} e^{\left(\tilde{V}_{d'} + \tilde{V}_{d'm}\right)\mu_{d'}}} \tag{23}$$

$$P(m) = \frac{e^{\left(\tilde{V}_m + \Gamma_{dm}\right)\mu_m}}{\sum_{m'} e^{\left(\tilde{V}_{m'} + \Gamma_{dm'}\right)\mu_m}} \tag{24}$$

$$\Gamma_{dm} = \frac{1}{\mu_d} \ln \sum_{d'} e^{\left(\tilde{V}_{d'} + \tilde{V}_{dd'm}\right)\mu_d} \tag{25}$$

All the other terms are same as above. For consistency with utility maximization, Specification 2's scale parameters should obey the following: $0 \le \mu_m \le \mu_d$.

Application of the trip generation system-of-demand-equations model requires effective budget and price information from the NL results. The effective budget, in utils per day, comes from Eq. 5: $Y^e = \beta_{\text{cost}}\left(\frac{Y}{365}\right) + \beta_{time}(24 \times 60 \times N)$, where income Y is yearly household income (as reported in the Austin Travel Survey) and N is the household size.[5] While income can be controlled, to some extent, by the household, time per member cannot: everyone enjoys exactly 24 hours per day. Fixed budgets are standard for household consumption and expenditure studies (Varian, 1992), but Kockelman (1998) discusses the base model's adaptations for exogenous wage (rather than income) and time constraints.

The effective prices come from the results of the second nested logit model specification and are computed as the following nested logsums:

$$P_i^e = -E(\max U_m) = -\frac{1}{\mu_m}\ln\left(\sum_{m=1,2} e^{\left(\Gamma_{dm}+\tilde{V}_m\right)\mu_m}\right), \tag{26}$$

where Γ_{dm} and $\tilde{V}_m$ are as defined previously, for Eqs. 25 and 20.

RESULTS

The nested logit models were estimated using Limdep (Econometric Software, 2000) and GAUSS software (Aptech, 1999) packages[6]. Specification 1's results were not consistent with utility maximization for either HBW or HBNW trips, and its likelihood maximization routine would not converge for NHB trips. Specification 2's results were consistent with utility maximization for both HBNW and NHB trips, though not for HBW trips. The preferred specification for the HBW trips was simply a multinomial logit (rather than a nested logit) model. The resulting nesting structures suggest that people are more willing to change their destinations, as compared to their mode, following a change in the transportation system (e.g., congestion pricing). This makes sense in Austin, because no strong competitors/alternatives to the automobile mode presently exist in most locations. Moreover, the destinations for HBNW and NHB trips are not fixed, like work destinations. The fixity of work locations along with the lack of modal alternatives is believed to create inflexibilities along both choice dimensions, for the HBW trips. Therefore, a joint logit model seems the most appropriate model for capturing that behavior.

Parameter estimates for the simultaneously estimated constant-VOTT model specification are shown in Table 1. For this suite of models, the scale parameter of the HBW MNL model was set to one, so both scale parameters for HBNW and NHB trips were statistically identifiable. The results suggest that HBW trips are more attracted by employment, when compared to HBNW and NHB trips. In addition, the bias/innate preference for the automobile is lowest for HBNW trips, which makes sense given the large number of walk and transit trips to school and for local recreation.

The coefficients in all these models are consistent with expectations. For example, travel time and travel cost coefficients are negative, while those for total employment (at the destination) and the automobile-specific-constant are positive. However, the values of travel time (VOTT) obtained from these models – whether estimated separately or in tandem (as shown in Table 1) – are very low, for all trip types. The VOTT obtained from the simultaneously estimated set of models again is \$3.39/hour. For purposes of model illustration, this value is used. However, models which recognize more than two mode alternatives and more than just travel time, cost and destination employment estimates are likely to result in more reasonable VOTT predictions.

Inclusion of presently omitted variables, such as retail space and recreational opportunities at destinations, in-vehicle as well as out-of-vehicle travel time, weather conditions, and parking costs across the many mode alternatives in the ATS data set may yield different results. The simplified nested logit models used here yield basic price estimates, thus permitting application of this paper's proposed analytical methods. These upstream models are for illustration, and are not the focus of this work.

The trip generation model was estimated using full-information maximum likelihood code programmed in GAUSS software (Aptech 1999). The maximum-likelihood parameter estimates for this model appear in Table 2. The inverse of the estimated gamma parameter (1/*m*) is 0.5784. This term is the "overdispersion" parameter of the associated negative binomial model, and its practical and statistical significance suggest that significant unobserved heterogeneity may be present in trip generation across households (assuming that trip rates are Poisson distributed, when conditioned on the multiplicative gamma error term).

Comparisons of aggregate predictions with observed trip-making are shown in Table 3. Households were classified according to annual household income: Group 1 has incomes under $25,000; Group 2 lies between $25,000 and $40,000; Group 3 between $40,000 and $65,000; and Group 4 lies above $65,000[7]. It is clear from the above table that the model does well in predicting aggregate trip generation behavior.

Model estimates permit one to estimate a variety of behavioral measures, such as elasticity of trip generation/demand with respect to income, travel times, and travel costs. Elasticities were calculated by changing the appropriate variables by 10% and then recomputing and normalizing demands. These elasticities are at the disaggregate (i.e., household) level, and quartiles of the vector of ATS household elasticities are provided, with respect to income (Table 4), travel times (Table 5), and travel costs (Table 6). All elasticity estimates support *a priori* expectations that income has a positive impact on trip generation, even while holding time budget (essentially household size) constant. The income elasticity estimates clearly suggest that HBNW trips are more responsive to income changes, when compared to HBW and NHB trip types. An increase in household income provides more purchasing power, and that may manifest itself as more recreational and shopping trips. The low values of trip demand elasticities with respect to travel time and cost suggest that total trip generation rates are not significantly affected by travel times and costs. (This is consistent with Kockelman's results for San Francisco Bay Area households [1998, 2001] and with Golob et al.'s [1981] hypotheses.)

WELFARE ANALYSIS

The microeconomic rigor of this model allows one to study welfare implications of various transportation policies. The affects of various transportation policies such as congestion pricing can be calculated in terms of equivalent variation in dollars. Equivalent variation (Varian, 1992; Small and Rosen, 1981) is the change in income required at the current prices that will be equivalent to the impact on utility due to the proposed policy. The difference between the expenditure functions at the same utility level is the equivalent variation. The expenditure function can be obtained by inverting the indirect utility function with respect to income budget. The expenditure function obtained by inverting the modified translog utility function is shown below:

$$e(u, \vec{p}) = \exp\left(\frac{u - \sum_i \alpha_i \ln P_i^e - \sum_{ij} \left(\frac{1}{2}\right) \beta_{ij} \ln\left(P_i^e\right) \ln\left(P_j^e\right)}{1 + \sum_i \gamma_i \ln\left(P_i^e\right)} \right) \tag{27}$$

where $e(u, \vec{p})$ is the expenditure function and u is the utility level. All other terms are as defined previously.

Equivalent variation (EV) is given by the following equation:
$EV = e(u^1, \vec{p}^{\,0}) - e(u^1, \vec{p}^{\,1})$, where $\vec{p}^{\,0}$ and $\vec{p}^{\,1}$ are the initial and final price vectors. Various scenarios for which policy analysis was performed are described in Table 7. The quartiles of the EV welfare measure are given in Table 8.

The EV values for the first scenarios indicate that the welfare reductions are more severe for 10% increases in auto travel costs than for 10% increases in auto travel times. Austin households are estimated to be willing to pay a median amount of $106[8] per year (1996 dollars) to avoid a 10% increase in average daily auto travel costs, but just $35 to avoid a 10% increase in auto travel times. Both amounts intuitively seem low, particularly when compared to estimates of total yearly delay costs (which are on the order of $1,000 per year for each peak-period driver [TTI 2002]); this is in part due to the low VOTT estimate ($3.39/hr) obtained in this study (TTI researchers [2002] used $12 per hour). Also, welfare impacts due to increases in non-auto costs and/or travel times are not nearly as large as the impacts due to changes in auto costs and travel times. This result is as expected, since auto is the predominant mode of travel.

The seventh scenario relates to congestion pricing. Congestion pricing schemes charge road users for the delay that they cause to other users of the road. In standard traffic assignment models, the cost of travel is taken to be the average cost, which often is characterized by the following link performance function:

$$t = t_f \left[1 + \alpha \left(\frac{v}{c} \right)^{\beta} \right] \tag{28}$$

with α and β varying across roadway classes (but often equal to 0.15 and 4.0, respectively).

However, the cost due to each additional road user is given by the marginal cost:

$$MC = t_f \left[1 + \alpha \left(\frac{v}{c} \right)^{\beta} \right] + t_f \beta \alpha \left(\frac{v}{c} \right)^{\beta} = AC + t_f \beta \alpha \left(\frac{v}{c} \right)^{\beta} \tag{29}$$

Therefore, each additional road user should be charged the difference between the *MC* and *AC*. In this seventh policy scenario/application, the two major Austin highways (IH-35 and Loop 1) were priced during the morning peak period (7:15-9:15 AM). In TransCAD (Caliper 2001) the link performance functions for all the links corresponding to these freeways were modified to appear as their *MC* functions. The four-step model was used to estimate the OD matrix, and this OD matrix was assigned to the network under a base scenario (i.e., with no pricing and all performance functions taken to be *AC* curves) and under this pricing scenario.

The resulting flows and travel times from these two scenarios were used to calculate the EV welfare measures which, while small, were generally estimated to be positive. This result suggests that this policy has welfare-improving effects for most users. This is likely due to reduced congestion on these key freeways, enabling people to commute faster (after some persons have opted for reasonable substitute corridors, modes, destinations, and/or trip-making).

Another approach to congestion is the credit-based congestion-pricing scheme proposed by Kockelman and Kalmanje (2003). This policy allows for the return[9] of all the revenue raised by pricing of roads, through uniform "credit allowances" to all travelers. In this application, the total revenue raised per day is the sum of the products of all links' l tolls/prices and volumes (v_l):

$$VOTT\sum_{l} t_{f,l}\beta_l\alpha_l\left(\frac{v_l}{c_l}\right)^{\beta_l} v_l \ .$$

The total revenue raised from pricing these two corridors during the morning peak period was estimated to be $44,819/day. The revenue per Austin household came out to be just $0.11/day. Since, this amount is being given back to the households under a credit-based policy, it can be added to the EV values for analyzing the welfare implications of this policy. The return of the raised revenue makes this policy better than the tolling policy considered; however, several implementation issues have to be tackled before this policy can be used in practice.

LIMITATIONS AND EXTENSIONS

The marginal utility of money is almost certainly not constant across individuals or across prices and budget levels (though it may be close to constant for many applications). This detail impacts both model specification and estimation. Permitting parameter heterogeneity across individuals would enable variations in values of time and money; such models require estimation through likelihood simulation, but this is perfectly feasible given present computing power. More fundamental are the entangling of time and money, for effective prices and effective budgets and subsequent reliance on Roy's Identity. Essentially, any model is an idealization of reality. And, to some extent, if we are comfortable with linear-in-unknown specifications of indirect systematic utility for calibration of *logit* models, we should be comfortable here, knowing that constant marginal-utility-of-time and -money assumptions are not more heroic. However, this facet of this work deserves more attention; its consideration is likely to require some advanced mathematics.

In this application of the proposed modeling framework, the models were run at the household level, but they may better apply at the individual level. Household-level incomes were given in the ATS data set, and these cannot be divided appropriately among all household members without further information. The budgeting process and optimization practices (if they exist) among a group of persons are complex, and no clear answers yet exist on that topic, particularly with regard to travel demand. It also would of interest to calibrate this model for activity participation rates. One issue in this context is that the transport-cost of any activity requires knowing one's origin, so the approach requires further consideration. The current model also lacked a time-of-day choice for trip-making, which means it does not allow for time-of-day shifts under the various policy scenarios. Inclusion of this dimension will explicate more response flexibility/options for travelers; therefore, it should lead to reduced welfare losses under various policies (and enhanced gains).

Finally, more flexible functional forms for indirect utility should be pursued, including those that impose summability. Additionally, the statistical assumption of a single gamma term for each household, while permit a tractable, closed-form likelihood specification, ideally should be relaxed. Likelihood simulation methods can achieve this.

CONCLUSIONS

This study proposes a microeconomically rigorous method to characterize travel demand across a great variety of choice dimensions, including trip generation. It applies a multivariate negative binomial model for trip demand functions derived from an underlying translogarithmic indirect utility function. Both time and money budgets were incorporated in the model structure via an effective or generalized budget constraint. A nested logit model of trip mode and destination was used to calculate the effective prices for each trip purpose via nested logsum expressions. Time-of-day and other choice attributes could be added.

Implicitly, this approach responds to supply-side (i.e., level of service) variables for trip generation modeling; typically, these are neglected in conventional models. The microeconomic rigor of this model allows one to study the equivalent-variation impacts of various policies, like credit-based congestion pricing on trip generation, destination, and mode choices, which have important consequences for VMT, delays, emissions, and social welfare.

Good predictive capability, along with accommodation of unobserved heterogeneity and correlations in the unobserved terms across trip purposes, makes this approach a highly valuable alternative to the independent linear regression equations typically used to model trip generation and unlinked random-utility models used for other facets of travel choice. The rigorous economic and statistical bases appear very sound, relative to existing approaches. More exploration will be helpful in the context of time valuation in order to combine or distinguish time and money budgets.

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ENDNOTES

[1] Symmetry is a property of the Jacobian matrix of *compensated* (constant-utility) demand functions, with respect to prices. Homogeneity relates to human response to pure price inflation. And quasiconvexity relates to combinations of price extremes being preferred to averages. These do not hold when examined in the context of time costs, since time is experienced directly by humans (rather than money being simply a tool for simplifying trades).

[2] In this instance, parsimony in parameters does not mean inclusion of fewer explanatory variables. It means a thoughtfully structured choice set so that parameter sets which provide behavioral flexibility do not overwhelm the problem, undermining estimation.

[3] Strictly speaking, only linear optimization problems can have “dual problems”, so utility maximization and expenditure minimization would not qualify as duals, though this term is often used.

[4] μ is also called the dispersion parameter. It is inversely related to the standard deviation (σ) of the assumed *iid* Gumbel distribution ($\mu = \pi / \sqrt{6}\sigma$).

[5] The effective budget, Y^e, is in units of utils per day, while units on β_{cost} and β_{time} are utils per dollar and utils per minute. Thus, this equation requires conversions of days per year (365), minutes per hour (60), and hours per day (24).

[6] Limdep’s Version 7.0 (2000) estimates distinct nested logit models consistent with random utility maximization (RUM) via its “RU2” option. Hensher and Greene (2002) provide a detailed description of this model structure. To make sure all estimates are proper, Limdep’s estimates were compared to estimates obtained from a NL estimation using GAUSS (Aptech, 1998). GAUSS also was necessary for the simultaneous estimation results, where VOTTs were constrained to be constant across trip types.

[7] These income categories most closely approximate the household quartiles in the ATS data set.

[8] The number of workdays per year is estimated to be 250. The median amount a household is willing to pay each (work) day to avoid a 10% increase in travel cost is estimated to be $0.424. Therefore, the median amount a household is willing to pay every year to avoid such an increase is predicted to be on the order of $106.

[9] Some of the revenue raised will be used to pay for program administration. However, that amount is likely to be somewhat negligible (perhaps 5%, which is typical of credit card company charges to businesses, so such fees have not been quantified in this application.

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LIST OF TABLES

Table 1: Constrained Joint Choice Model Estimation Results

	HBNW		HBW		NHB	
Variables	**Estimates**	**t-stat**	**Estimates**	**t-stat**	**Estimates**	**t-stat**
Constant, Auto (β_{auto})	1.3029	4.48	2.1399	18.2	2.6535	3.20
Travel Time (min.)	-0.0170	-19.7	-0.0170	-19.7	-0.0170	-19.7
Travel Cost ($)	-0.3011	-29.2	-0.3011	-29.2	-0.3011	-29.2
Employment at Destination	1.362E-04	14.4	4.322E-04	25.9	1.17E-04	13.3
Mode Scale Parameter (μ_m)	0.8448	6.18	1.00	--	0.7614	3.82
Destin. Scale Parameter (μ_d)	1.6069	27.1	1.00	--	1.6892	27.0
Log-Lik	-16,614.42					
Log-Lik (Equal Shares)	-31877.91					
Adj R^2	0.478					
N_{obs}	11029 (4571 + 2009 + 4449)					
VOTT ($/hr)	$3.39/hour					

Table 2: Trip Generation Model Results

Variables (Parameters)	**Estimates**	**SE**	**t-stat**	**p-value**
$\ln P^{e}_{HBNW}$ (α_1)	-0.0336	0.1012	-0.33	0.37
$\ln P^{e}_{HBW}$ (α_2)	-0.0619	0.0136	-4.56	0.00
$\ln P^{e}_{NHB}$ (α_3)	-0.0915	0.0607	-1.51	0.07
$\ln Y^{e} \ln P^{e}_{HBNW}$ (γ_1)	0.0047	0.0050	0.93	0.18
$\ln Y^{e} \ln P^{e}_{HBW}$ (γ_2)	0.0118	0.0014	8.40	0.00
$\ln Y^{e} \ln P^{e}_{NHB}$ (γ_3)	0.0135	0.0020	6.63	0.00
$\ln P^{e\;2}_{HBNW}$ (β_{11})	-0.1376	0.1330	-1.03	0.15
$\ln P^{e}_{HBNW} \ln P^{e}_{HBW}$ (β_{12})	-0.0117	0.0154	-0.76	0.22
$\ln P^{e}_{HBNW} \ln P^{e}_{NHB}$ (β_{13})	0.0156	0.0833	0.19	0.43
$\ln P^{e\;2}_{HBW}$ (β_{22})	-0.0234	0.0078	-2.99	0.00
$\ln P^{e}_{HBW} \ln P^{e}_{NHB}$ (β_{23})	-0.0073	0.0088	-0.84	0.20
$\ln P^{e\;2}_{NHB}$ (β_{33})	-0.0797	0.0527	-1.51	0.07
Gamma Parameter (m)	1.7274	0.3154	5.48	0.00
Log-Lik	-14,200			
N_{obs}	1925			

Table 3: Aggregate Trip Generation Rates Prediction (per Household)

	Observed (ATS)			Predicted		
	HBNW	**HBW**	**NHB**	**HBNW**	**HBW**	**NHB**
Group 1	4.58	1.40	2.03	3.43	1.56	2.23
Group 2	4.75	2.18	3.06	4.44	1.92	2.86
Group 3	5.23	2.26	3.35	5.24	2.17	3.31
Group 4	5.66	2.50	4.30	7.38	2.66	4.27
All	5.03	2.05	3.13	5.02	2.05	3.12

Table 4: Quartiles of Elasticity of Trip Generation with respect to Income

	HBNW	HBW	NHB
Minimum	0.011	-0.076	0.007
25%	0.230	0.151	0.186
Median	0.358	0.222	0.275
75%	0.470	0.298	0.368
Maximum	0.831	0.557	0.681

Table 5: Quartiles of Elasticity of Trip Generation with respect to Travel Time

	HBNW	HBW	NHB
Minimum	-0.053	-0.076	-0.134
25%	-0.017	0.002	-0.015
Median	-0.011	0.019	-0.004
75%	-0.007	0.034	0.009
Maximum	0.013	0.099	0.175

Table 6: Quartiles of Elasticity of Trip Generation with respect to Travel Cost

	HBNW	**HBW**	**NHB**
Minimum	-0.141	-0.227	-0.192
25%	0.012	0.222	0.318
Median	-0.039	-0.022	0.009
75%	-0.025	0.036	0.048
Maximum	-0.018	0.063	0.083

Table 7: Policy Scenarios

Scenario	Description
1	10% increase in Auto travel time throughout the network
2	10% increase in Auto travel cost throughout the network (similar to a gas tax or VMT tax)
3	10% increase in Non-Auto travel time throughout the network
4	10% increase in Non-Auto travel cost throughout the network
5	10% increase in both Auto and Non-Auto travel cost throughout the network
6	10% increase in both Auto and Non-Auto travel time throughout the network
7	Congestion pricing of roads (i.e., charging the difference between the marginal cost and average cost of road usage)

Table 8: Quartiles of Equivalent Variation ($/Household/Day)

	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7
Minimum	-$1.05	-4.21	-0.34	-0.05	-4.21	-1.38	-0.15
25%	-0.26	-0.90	-0.09	0.03	-0.92	-0.42	0.00
Median	-0.14	-0.42	-0.05	0.05	-0.46	-0.24	0.01
75%	-0.05	-0.18	-0.02	0.09	-0.21	-0.12	0.02
Maximum	0.20	0.06	0.15	0.27	0.04	0.08	0.09