A TWO-STAGE EQUILIBRIUM TRAVEL DEMAND MODEL FOR SKETCH PLANNING

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ABSTRACT

This paper describes a two-stage equilibrium travel demand model. The unique feature of this model is that it takes time-of-day traffic counts instead of land use and demographic data as inputs to derive spatial and temporal travel demand patterns. The first stage of the model is a traffic count-based trip matrix estimator; the second stage is an elastic-demand network flow estimator, which recognizes latent demand shifts while performing mode split, time-of-day split, and traffic assignment in a multi-level equilibration. The main purpose of this paper is to share with readers our model development, algorithm design, and software implementation experiences. An example application illustrates how the model is used to evaluate multi-class, multi-mode and multi-period network flow patterns for sketch planning purposes.

Keywords: Travel demand forecasting, network equilibrium, origin-destination trip matrix, traffic assignment, convex optimization, nonlinear complementarity, sketch planning

BACKGROUND AND INTRODUCTION

From its beginning in the mid 1950s, travel demand forecasting has been a central process and played a prominent role in decision making of urban and regional transportation planning. The last decades observed a large number of creations and improvements of travel demand forecasting methods for various demographic and institutional contexts and technical requirements. Across these extensive research and development efforts in travel demand forecasting, two major modeling paradigms have been formalized and used to characterize different types of travel demand models: trip-based and activity-based. Trip-based models, due to their relatively simple structure and appealing computational tractability as well as the availability of mature commercial software packages, have been intensively studied and dominantly used in the past and current transportation planning practice. Under the trip-based modeling framework, travel demand forecasting is typically described as a sequential process of four modeling steps: trip generation, trip distribution, mode split, and traffic assignment, in its most common formation. Each of these steps represents an aggregation of the demand-supply interactive results of individual travel choices of a certain type.

Since its first conceptualization, travel demand modelers have realized that an iterative procedure must be introduced into the sequential process so as to find a consistent agreement between the inputs and outputs of all modeling steps (*1*). More specifically, the estimated travel demand patterns must be a demand-supply interaction result that reflects the basic flow-cost consistency in a synthetic equilibrium manner, by which every individual makes all his/her decisions in terms of travel benefits/costs of all available alternatives provided by the entire travel environment. The earliest effort of implementing such an iterative procedure relied on heuristic methods (*2*), which, for the well-known reason, may not guarantee the desirable flow-cost consistency. The flow-cost convergence requirement called for a need of the so-called equilibrium demand model, which was formally characterized and interpreted by Sheffi and Daganzo (*3*) and Safwat and Magnanti (*4*). From the supply perspective, such a model is often included in the broad category of network equilibrium models, since the resulting travel demand patterns are a demand equilibrium result over the network.

The origin of equilibrium demand models or network equilibrium models is generally attributed to the seminar work of Beckmann et al. (*5*), though, due to the algorithmic and computational reasons, their work did not lead to an immediate application for the transportation planning practice at that time. They formulated for the first time the traffic assignment problem with variable demand as a convex optimization (CO) problem, which integrates the origin-destination, route and link flows on a congested traffic network, as a function of flow-dependent link costs. The CO formulation warrants a unique, consistent flow-cost solution of travel demand patterns. Though it does not explicitly incorporate all trip-based modeling elements and recognize location and destination choices, this model can be conceptually regarded as an equilibrium device of combining trip generation, trip distribution and traffic assignment (*6*).

A number of equilibrium demand models and solution methods were developed in subsequent decades. Florian et al. (*7*), Evans (*8*), Erlander (*9*), and Lundgren and Patriksson (*10*) proposed CO models for the combined trip distribution and traffic assignment problem and demonstrated the application of linearization and partial linearization algorithms for network equilibrium solutions. Florian (*9*), Abdulaal and LeBlanc (*10*), Fisk and Nguyen (*11*), Aashtiani and Magnanti (*12*), Dafermos (*13*), Florian and Spiess

(*14*), Fernandez et al. (*15*), Cantarella (*16*), Boyce and Bar-Gera (*17*), and Wu and Lam (*20*) suggested various nonlinear complementarity (NC), variational inequality (VI), or fixed-point problem (FP) formulations for the combined mode split and traffic assignment problem and intensively investigated the formulation and solution properties.

Due to the existence of the asymmetric Jacobian matrix that is caused by different impacts on traffic delays from alternative transportation modes, a travel demand model that includes the mode split component in general cannot be written into a mathematical program of the convex form. By using separate traffic and transit subnetworks, asymmetric Jacobian elements may be removed; following this approach, Florian and Nguyen (*21*), for example, presented a CO model for the combined trip distribution, mode split and traffic assignment problem. With a similar subnetwork-hypernetwork idea, Sheffi and Daganzo (*3*) embedded the probit model into the same problem to specify all travel choices. Though the model is initially represented by an NC system, it can be potentially represented by an equivalent convex program (see *19*). By combining the mode split and traffic assignment into a single step, Safwat and Magnanti (*4*) eliminated the asymmetric Jacobian issue in constructing their CO model for the combined trip generation, trip distribution, mode split and traffic assignment problem. In another attempt, by a cost-to-time transformation for the link cost function, Lam and Huang (*20*, *21*) reported an alternative CO model for the same problem. As a synthetic review, Oppenheim (*22*) summarized a set of CO models and provided detailed solution approaches and applications for equilibrium demand models under different behavior assumptions and modeling dimensions. It should, however, be noted that relaxing the asymmetric restriction inevitably leads to a certain level of unrealistic modeling results. To avoid this deficiency, Friesz (*23*), by using a set of VIs to represent the user-equilibrium relationship, formulated the combined trip distribution, mode split and traffic assignment problem that contains the asymmetric restriction into a novel mathematical program. The disadvantage of this formulation, however, is that it is of a nonconvex functional form, where the nonconvexity is caused by the existence of the VI constraints.

Recent research advances have been more focused on the implementation and computational issues in equilibrium travel demand forecasting, including the work of, for example, Boyce et al. (*24*), Safwat et al. (*25*), Slavin (*29*), Abrahamsson and Lundqvist (*26*), Boile and Spasovic (*27*), Florian et al. (*28*), Bar-Gera and Boyce (*29*, *30*), Siegel et al. (*31*), and Zhang et al. (*32*). Among these studies, a unique feature distinguishing Abrahamsson and Lundqvist (*26*), Florian et al. (*28*), and Zhang et al. (*32*) from others is that a hierarchical travel choice structure (in the form of the nested logit model) is explicitly incorporated into these travel demand models, in either CO, VI, or FP forms. This nested setting better reflects the conditional relationship between different levels of travel choices in an integrated system. A detailed treatment of the nested travel choice structure in demand models of mathematical programming without inter-mode interactions is contained in Oppenheim (*22*). For a comprehensive analysis of equilibrium demand models and methods, interested readers are referred to Boyce et al. (*33*), Boyce and Bar-Gera (*34*), and Boyce (*2*).

The fundamental advantage of applying these network equilibrium models for travel demand forecasting is rooted from their rigorous mathematical formulations and properties, which allow for the development of computer algorithms that can rapidly approach the flow-cost convergence point and mathematical tools that can perform various network analysis and system evaluation tasks in an analytical way. In a few example real-world applications, for instance, Boyce et al. (*35*) and Siegel et al. (*31*) showed that those equilibrium models powered by optimization algorithms find near-convergence solutions much faster and more precisely than the sequential procedure with feedbacks.

Despite continuous and fruitful research contributions made to the development of equilibrium demand models in past decades, some common deficiencies still pertain to most of these existing equilibrium models and may restrain their further applications in practice. First, it is noted that most of these models at least exclude the trip generation phase from the iterative procedure (i.e., trip productions and attractions are estimated exogenously and not the functions of travel costs). This reduced form limits the capability of the models in capturing all travel choice behaviors in highly interactive urban travel circumstances and the use of these models for supporting all-dimensional travel demand forecasting tasks. It is well known that in most applications, trip generation profiles are either estimated from the regional demographic and socio-economic data, or alternatively, based on the traffic count data. The latter represents an inexpensive, efficient method for inferring trip tables in place of the conventional trip generation and distribution phases, which is therefore gaining increasing popularity in the transportation planning community, especially for subarea travel demand analyses. On the other hand, as originally established for estimating steady-state travel demand patterns, all these existing models do not include the time dimension in their model structures. Lack of a departure time choice function and a reaction mechanism to time-dependent network conditions makes these models incapable of reflecting travelers' time-related travel behaviors and assessing the network's time-dependent congestion levels. Resorting to activity-based, time-varying models is a promising approach. But most of such dynamic tools are highly computationally demanding and data hungry and hence are still very difficult to be applied to networks of realistic size and settings, at least at present. Moreover, calibrating a dynamic travel demand model poses a very challenging task.

To overcome these deficiencies and create a practically operationable travel demand forecasting process, this paper presents an extended equilibrium travel demand model, by synthesizing many advanced demand modeling and solution techniques recently developed. The model takes traffic counts as inputs for inferring trip tables, captures induced demands in the elasticity form, and is capable of performing mode split, time-of-day split, and route split in a multi-class equilibrium manner. In our case, traveler classes are defined in terms of value of time; use of multiple traveler classes implies a heterogeneous travel population by travel cost perception and produces distinguishing travel choice behaviors across different traveler classes. The model is proposed for the special purpose of sketch planning, which requires an inexpensive, fast-response, yet comprehensively functional tool that works on readily available data sets and is capable of properly evaluating and distinguishing a large number of network improvement projects of various types prior to the implementation of a full-scale regional demand model. By its name, the main operations of the model are conducted in two stages: trip matrix estimation and network flow estimation. The trip matrix estimation stage estimates the base origin-destination flow matrix from traffic count data for each time-of-day period, while the network flow estimation stage produces a set of equilibrium network flow patterns across origin-destination pairs, transportation modes, time-of-day periods, and network links.

Though the term of equilibrium implies different modeling principles or behavioral assumptions for different levels of travel choices, the equilibrium problem in each stage of the model is formulated as a NC system, in which the set of equations and inequalities provide us with a direct description of the equilibrium conditions of the problem. In the following, we first present and interpret the mathematical details of the model formulations and solution methods. A slightly different version of the two-stage model has been implemented in a sketch planning toolkit developed for Texas Department of Transportation (TxDOT). For readers' interests, we share our software design and implementation experiences in the next section. For the purpose of demonstration, we then apply this two-stage model to evaluate the travel demand patterns of an example sketch network and examine the network performance under a number of proposed network expansion and management strategies. The last section finally concludes the paper and suggests extra modeling elements that can be added into the two-stage model and improved solution algorithms that may accelerate the computational processes.

MODEL FORMULATIONS AND SOLUTION METHODS

Notation and problem definition

Our modeling discussion starts with defining the supply and demand data sets of the travel demand forecasting problem. On the supply side, consider a traffic network $G = (N, A, D)$ in an urban area, where $N = \{n\}$ is the set of nodes, $A = \{a\}$ is the set of links, and $D = \{d\}$ is the set of time periods. The combination of all time periods covers a typical weekday in the area. The sets of origin and destination nodes, $R = \{r\}$ and $S = \{s\}$, are subsets of N, i.e., $R \subseteq N$ and $S \subseteq N$. In addition, let us assume that there exist a number of different transportation modes in the network with mode-specific vehicle occupancy rates and passenger car equivalents and use $M = \{m\}$ represent the set of these transportation modes. On the demand side, it is assumed that traffic count data are available on all or part of links of the network, collected by traffic sensors in each time period or estimated from the diurnal curves applied to the daily traffic counts. The set of links covered by traffic sensors in time period d, A_d, is a subset of A, i.e., $A_d \subseteq A$. Note that two covered link sets from two different time periods d_1 and d_2 may not be necessarily the same, i.e., $A_{d_1} \neq A_{d_2}$. The traveler population is categorized into a number of classes in terms of values of time. We use $K = \{k\}$ to represent the set of traveler classes.

Following the definition of supply and demand data sets, for discussion convenience, we further define the parameters and variables of the model, which are all contained in Table 1.

Given the problem settings above, we can distinguish traffic flows in the network by different criteria, such as traveler classes, transportation modes, and time-of-day periods. On the link, path, and origin-destination levels, traffic flows with different classes and modes are mixed together and share the limited network capacity; on the other hand, traffic flows belonging to different periods do not spatially interact with each other (except travelers switch between periods by acting on their time-of-day choices) and we evaluate traffic congestions for each period separately.

The generalized travel cost we employ for the model is defined as the sum of two parts: travel time and monetary cost. Specifically, the link cost function for a specific combination of traveler class k, transportation mode m, and time-of-day period d has the following functional form:

$$\begin{aligned} g_{a,m,d}^{k} &= t_a\left(f_{a,d}\right) + \frac{c_{a,m,d}}{\gamma^k} \\ &= t_a^0\left(1 + \alpha_a\left(\frac{f_{a,d}}{u_a}\right)^{\beta_a}\right) + \frac{c_{a,m,d}}{\gamma^k} \qquad \forall a, m, d, k \end{aligned} \tag{1}$$

where it should be noted that we represent the generalized travel cost in travel time instead of monetary cost. The underlying reason for this representation can be found in Lam and Huang (*20*, *21*) and Yang and Huang (*36*). The first term of the link cost function represents the link travel time, quantified by the well-known Bureau of Public Roads function, while the second term is the link monetary cost, including operating cost, toll, fare, and any other fixed costs, which typically are not flow-dependent. In our case, travel time, as a function of traffic flow rate, is period-specific; in contrast, monetary cost may differ across transportation modes and time-of-day periods and will be converted to different travel time values for different traveler classes.

The two-stage equilibrium demand model employs elastic-demand functions to mimic the aggregate trip generation and distribution behaviors of individual travelers. No matter what functional form the elastic-demand function uses, a basic/initial trip rate is typically needed for each origin-destination pair, which, along with an elasticity parameter, jointly specifies the relationship between the equilibrium demand rate and travel cost. Therefore, as a complete demand modeling scheme, it is necessary to include a procedure of estimating base demand rates for origin-destination pairs in our model. This task can be either accomplished by utilizing the conventional trip generation and distribution techniques based on the regional socio-demographic data or developing a direct origin-destination trip matrix estimation procedure using traffic counts collected in the network. It is well known that the latter represents a much more economic and time-saving approach and avoids the modeling and computational complexity of distinguishing trips by purpose in trip generation as well as its subsequent steps. The first stage of our model contains such a traffic count-based trip matrix estimation procedure for each time-of-day period, which we name *trip matrix estimator*. The second stage takes the period-specific trip matrices estimated by the first stage as the input data sets to estimate an equilibrium network flow pattern across origin-destination pairs, transportation modes, time-of-day periods, and network links, performing as a *network flow estimator*.

Trip matrix estimation

It is well known that in most cases there exist multiple feasible origin-destination trip matrices corresponding to any given set of link counts. The proposed trip matrix estimator is an optimization-based procedure that infers the most likely origin-destination flow pattern in the entropy maximization framework subject to the traffic count match and equilibrium routing principle. The approach of using traffic counts as the major input data source (with or without other data sources, including trip production and attraction rates, target trip matrices, and probe vehicle, cellular phone, or any other traffic data from tracking individual vehicle trajectories) and enforcing the network flow pattern into the equilibrium state (either explicitly or implicitly) receives a large number of attentions in past decades. In principle, such a model can be viewed as a reduced version of combined trip distribution and traffic assignment models, since it in fact estimates origin-destination flows and link flows simultaneously! In particular, the network equilibrium implied in those existing models of this type is specified by two types of mathematical forms: 1) nonlinear forms of link flows, typically written as a CO or VI (*37*-*43*), and 2) linear forms of origin-destination costs (*44*-*47*), where origin-destination costs can be conveniently calculated from the given link counts in concert with link cost functions.

What makes this trip matrix estimator differs from all those previous competitors is that it explicitly models traffic flows in multiple traveler classes and transportation modes and takes both travel time and monetary cost as the individual travel impedance. This more complex flow and cost structures represent a more realistic modeling setting and ensure an inter-stage modeling consistency between the trip matrix estimator and the network flow estimator. Because the model is established in terms of a couple of optimization principles (i.e., maximum entropy and least squares), which can be conveniently accommodated by an objective function of an optimization model, we first present the CO form of the model. The CO form favors a direct application of a number of solution algorithms for problem solutions. By checking the optimality conditions of its Lagrangian relaxation problem, we then derive its NC form. The NC form defines the equilibrium conditions of the problem in a direct manner.

The nonlinear program of the multi-class trip matrix estimation problem is written as follows,

$$\min \quad z(\mathbf{f}_d, \mathbf{y}_d) = \omega_x \sum_{rs} \left(f_{rs,d} \ln f_{rs,d} - f_{rs,d} \right) + \omega_y \sum_{a \in A_d} \left(\left(y^+_{a,d} \right)^2 + \left(y^-_{a,d} \right)^2 \right)$$
$$+ \omega_v \sum_a \left(\int_0^{f_{a,d}} t_a(\omega) d\omega + \sum_k \sum_m \frac{f^k_{a,m,d} c_{a,m,d}}{\gamma^k} \right) \qquad (2.1)$$

$$\text{subject to} \quad f_{a,d} + y^+_{a,d} - y^-_{a,d} = \bar{f}_{a,d} \qquad \forall a \in A_d \qquad (2.2)$$

$$y^+_{a,d}, y^-_{a,d} \geq 0 \qquad \forall a \in A_d \qquad (2.3)$$

$$f^k_{rs,p,m,d} \geq 0 \qquad \forall r, s, p, m, k \qquad (2.4)$$

$$\text{where} \quad f_{rs,d} = \sum_k \sum_m f^k_{rs,m,d} \qquad \forall r, s \qquad (2.5)$$

$$f^k_{rs,m,d} = f_{rs,d} p^k p^k_m \qquad \forall r, s, m, d, k \qquad (2.6)$$

$$f_{a,d} = \sum_{rs} \sum_p \sum_k \sum_m f^k_{rs,p,m,d} \, \delta^{rs}_{a,k} \qquad \forall a \qquad (2.6)$$

$$f^k_{a,m,d} = \sum_{rs} \sum_p f^k_{rs,p,m,d} \delta^{rs}_{a,k} \qquad \forall a, m, k \qquad (2.7)$$

where ω_x, ω_y, and ω_v are a set of prespecified weights assigned to the three terms in the objective function. Given $0 \leq \omega_x, \omega_y, \omega_v \leq 1$ and $\omega_x + \omega_y + \omega_v = 1$, the objective function is a convex combination of these terms. The first term is the negative of the sum of origin-destination entropies over the network; the second term is the sum of least squares of the deviations between the observed and estimated link flow rates; the third term is an extension of Beckmann's objective function, promoting the network flow pattern to the Wardropian equilibrium state in terms of the generalized travel cost (i.e., travel time and monetary cost). Note that the generalized link cost in the third term distinguishes among different traveler classes and transportation modes, which explicitly accommodates different monetary costs (e.g., operating cost, toll, etc.) associated with different modes and different values of time associated with different classes. Slack variables, $y^+_{a,d}$ and $y^-_{a,d}$, are used to represent the deviation between the observed and estimated link flow rates. It is readily known that at the optimal solution, at least one of the slack variables $y^+_{a,d}$ and $y^-_{a,d}$ equals zero. Specifically, when $y^+_{a,d} > 0$ and $y^-_{a,d} = 0$, it implies that the observed link flow rate is greater than the estimated rate, i.e., $\bar{f}_{a,d} > f_{a,d}$; when $y^+_{a,d} = 0$ and $y^-_{a,d} > 0$, the observed rate is less than the estimated rate, i.e., $\bar{f}_{a,d} < f_{a,d}$.

Given the Lagrangian relaxation form of the above nonlinear program,

$$\min \quad L(\mathbf{f}_d, \mathbf{y}_d, \boldsymbol{\lambda}_d) = z(\mathbf{f}_d, \mathbf{y}_d) + \sum_a \lambda_{a,d}\left(\bar{f}_{a,d} - f_{a,d} - y^+_{a,d} + y^-_{a,d}\right)$$

$$\text{subject to} \quad y^+_{a,d}, y^-_{a,d} \geq 0 \qquad \forall a \in A_d$$

$$f^k_{rs,p,m,d} \geq 0 \qquad \forall r, s, p, m, k$$

where $\lambda_{a,d}$ is the dual variable corresponding to the link flow conservation constraint $f_{a,d} + y^+_{a,d} + y^-_{a,d} = \bar{f}_{a,d}$, $\forall a \in A_d$, we can readily derive the NC form of the trip matrix estimator by checking the first-order derivatives of the objective function in terms of all the primal and dual variables:

$$f^{*k}_{rs,p,m,d}\left[\omega_x \ln f^*_{rs,d} + \omega_v \sum_a \left(t_a\left(f^*_{a,d}\right) + \frac{c_{a,m,d}}{\gamma^k}\right)\delta^{rs}_{a,p} - \sum_a \lambda^*_{a,d}\delta^{rs}_{a,p}\right] = 0 \qquad \forall r, s, p, m, k \qquad (3.1)$$

$$f^{*k}_{rs,p,m,d} \geq 0 \qquad \forall r, s, p, m, k \qquad (3.2)$$

$$\omega_v \sum_a \left(t_a\left(f^*_{a,d}\right) + \frac{c_{a,m,d}}{\gamma^k}\right)\delta^{rs}_{a,p} - \sum_a \lambda^*_{a,d}\delta^{rs}_{a,p} \geq -\omega_x \ln f^*_{rs,d} \qquad \forall r, s, p, m, k \qquad (3.3)$$

$$y^{*+}_{a,d}\left[2\omega_y y^{*+}_{a,d} - \lambda^*_{a,d}\right] = 0 \qquad \forall a \in A_d \qquad (3.4)$$

$$y^{*+}_{a,d} \geq 0 \qquad \forall a \in A_d \qquad (3.5)$$

$$2\omega_y y^{*+}_{a,d} \geq \lambda^*_{a,d} \qquad \forall a \in A_d \qquad (3.6)$$

$$y^{*-}_{a,d}\left[2\omega_y y^{*-}_{a,d} + \lambda^*_{a,d}\right] = 0 \qquad \forall a \in A_d \qquad (3.7)$$

$$y^{*-}_{a,d} \geq 0 \qquad \forall a \in A_d \qquad (3.8)$$

$$2\omega_y y^{*-}_{a,d} \geq -\lambda^*_{a,d} \qquad \forall a \in A_d \qquad (3.9)$$

It is obvious that the first part of the complementarity relationships (i.e., (2.1)-(2.3)) is defined for paths, while the remaining part (i.e., (2.4)-(2.9)) is defined for links that have observed traffic counts. For the path part, the complementarity relationship states that at the optimal solution, any path between an origin-destination pair r-s with a positive flow rate has its travel impedance equal to the minimum impedance value and any path with a travel impedance greater than the minimum value does not receive any flow. Here the path travel impedance is defined as the sum of path travel cost, $\omega_v \sum_a \left(t_a\left(v_{a,d}^*\right) + c_{a,m,d}/\gamma^k\right)\delta_{a,p}^{rs}$, and path entropy impedance, $-\sum_a \lambda_{a,d}^* \delta_{a,p}^{rs}$, while the minimum impedance between this origin-destination pair is $-\omega_x \ln f_{rs}^*$. Now we can see that the equilibrium path flow pattern is a route choice result of minimizing individual travel cost and entropy impedance simultaneously. It is noted that both path travel cost and path entropy impedance are additive along paths; in other words, path travel cost and path entropy impedance are the sum of their respective link parts. For those links not covered by traffic sensors, their link entropy impedances are equal to zero. Interested readers are referred to Xie et al. (*48*, *49*) for more details about the definition of entropy impedance. On the other hand, for the link part, our attention is only focused on those links covered by traffic sensors. The complementarity conditions for the slack variables $y_{a,d}^{*+}$ and $y_{a,d}^{*-}$ and the dual variable $\lambda_{a,d}^*$ associated with a covered link a in period d can be specified by either of the following two relationships: $y_{a,d}^{*+} > 0$, $y_{a,d}^{*-} = 0$, $\lambda_{a,d}^* = 2\omega_y y_{a,d}^{*+} > 0$, and $y_{a,d}^{*-} > 0$, $y_{a,d}^{*+} = 0$, $\lambda_{a,d}^* = -2\omega_y y_{a,d}^{*-} < 0$.

It should be emphasized that the above nonlinear optimization or complementarity problem is constructed for each time-of-day period separately. Thus, the trip matrix estimator solves this problem $|D|$ times in the first stage. The trip matrix estimator can be solved by a number of well-known CO solution algorithms. Below we depict a linearization procedure based on the spirit of the Frank-Wolfe algorithm for quadratic programming (QP) problems (*50*).

Step 0 (Initialization): Find an initial feasible solution $\left(\mathbf{f}_d^{(0)}, \mathbf{y}_d^{(0)}\right)$. If a historical reference trip matrix $\left[f_{rs,d}^{(0)}\right]$ is available, perform a multi-class user-equilibrium traffic assignment by solving the following equivalent nonlinear programming problem:

$$\min \sum_a \left(\int_0^{f_{a,d}} t_a(\omega) d\omega + \sum_k \sum_m \frac{f_{a,m,d}^k c_{a,m,d}}{\gamma^k} \right) \tag{4.1}$$

$$\text{subject to} \quad \sum_p f_{rs,p,m,d}^k = f_{rs,m,d}^{0,k} = f_{rs,d}^0 p^k p_m^k \qquad \forall r, s \tag{4.2}$$

$$f_{rs,p,m,d}^k \geq 0 \qquad \forall r, s, p, m, k \tag{4.3}$$

$$\text{where} \quad f_{a,d} = \sum_{rs} \sum_p \sum_k \sum_m f_{rs,p,m,d}^k \, \delta_{a,k}^{rs} \qquad \forall a \tag{4.4}$$

$$f_{a,m,d}^k = \sum_{rs} \sum_p f_{rs,p,m,d}^k \delta_{a,k}^{rs} \qquad \forall a, m, k \tag{4.5}$$

and then set $y_{a,d}^{+(0)} = \bar{f}_{a,d} - f_{a,d}^{(0)}$, $y_{a,d}^{-(0)} = 0$, if $\bar{f}_{a,d} \geq f_{a,d}^{(0)}$, or $y_{a,d}^{-(0)} = f_{a,d}^{(0)} - \bar{f}_{a,d}$, $y_{a,d}^{+(0)} = 0$, if $\bar{f}_{a,d} < f_{a,d}^{(0)}$, where $f_{a,d}^{(0)}$ is the link flow rate estimated by the above traffic assignment problem; if no reference trip matrix is available, a handy initial solution may be obtained by assigning each $f_{rs,d}^{(0)}$, $\forall r, s$ with a very small positive value, and then perform an all-or-nothing assignment to get the value of $f_{a,d}^{(0)}$, $\forall a$ and use the same equations presented above to get the values of $y_{a,d}^{+(0)}$ and $y_{a,d}^{-(0)}$, $\forall a \in A_d$. Set $n \coloneqq 1$.

Step 1 (Direction finding): Find an auxiliary solution $(\mathbf{f}_d, \mathbf{y}_d)$ by solving the following linearized problem:

$$\min \left(\omega_x \ln f_{rs,d}^{(n)} + \omega_v \sum_a \left(t_a\left(f_{a,d}^{(n)}\right) + \frac{c_{a,m,d}}{\gamma^k}\right)\delta_{a,p}^{rs}\right) f_{rs,p,m,d}^k + 2\omega_y y_{a,d}^{+(n)} y_{a,d}^{+} + 2\omega_y y_{a,d}^{-(n)} y_{a,d}^{-} \tag{5.1}$$

subject to
$$f_{a,d} + y_{a,d}^{+} - y_{a,d}^{-} = \bar{f}_{a,d} \qquad \forall a \in A_d \tag{5.2}$$

$$y_{a,d}^{+}, y_{a,d}^{-} \geq 0 \qquad \forall a \in A_d \tag{5.3}$$

$$f_{rs,p,m,d}^{k} \geq 0 \qquad \forall r, s, p, m, k \tag{5.4}$$

where
$$f_{a,d} = \sum_{rs}\sum_{p}\sum_{k}\sum_{m} f_{rs,p,m,d}^{k}\,\delta_{a,k}^{rs} \qquad \forall a \tag{5.5}$$

where $f_{rs,d}^{(n)}$, $\forall r, s$, $f_{a,d}^{(n)}$, $\forall a$, and $y_{a,d}^{+(n)}$ and $y_{a,d}^{-(n)}$, $\forall a \in A_d$ are the solution from the last iteration. The linearized problem, however, is not trivial to solve. The column generation procedure developed by Xie et al. (*48*) with a slight modification for accommodating the slack variables $y_{a,d}^{+}$ and $y_{a,d}^{-}$, $\forall a \in A_d$ can be used here for its solution.

Step 2 (Line search): Find an optimal α value for $0 \leq \alpha \leq 1$ by solving a line search problem for combining the latest solution from the last iteration and the auxiliary solution from the current iteration.

Step 3 (Solution update): Find an updated solution by setting $\left(\mathbf{f}_d^{(n+1)}, \mathbf{y}_d^{(n+1)}\right) = \alpha(\mathbf{f}_d, \mathbf{y}_d) + (1-\alpha)\left(\mathbf{f}_d^{(n)}, \mathbf{y}_d^{(n)}\right)$, where $(\mathbf{f}, \mathbf{y})$ is the auxiliary solution obtained from step 1.

Step 4 (Convergence test): If the solution difference between two consecutive iterations satisfies the convergence criterion, stop the procedure; otherwise, set $n \coloneqq n + 1$ and go to step 1.

Network flow estimation

The time-of-day trip matrices, including origin-destination trip rates and costs, obtained from the first stage are the input data sets of the network flow estimator in the second stage. We call these trip rates and costs the *base trip rates* and *costs*, respectively. Note that the base trip rates obtained from the trip matrix estimator are in vehicle trips. In accordance with this definition, all the base flow and cost variables presented in the last section are imposed with a superscript b at this moment. In the second stage, prior to being fed into the network flow estimator, these base trip rates and costs need to be transformed appropriately. First, the time-of-day trip rates need to be converted from vehicle trips to person trips with the given vehicle occupancy rate o_m and passenger car equivalent e_m and then combined across all time-of-day periods with the period duration h_d to form the whole-day person-trip rate matrix $\left[x_{rs}^{b,k}\right]$ for each traveler class k:

$$x_{rs,d}^{b,k} = \sum_m f_{rs,m,d}^{b,k} \frac{o_m}{e_m} \qquad \forall r, s, d, k \qquad (6)$$

$$x_{rs}^{b,k} = \sum_d x_{rs,d}^{b,k} h_d \qquad \forall r, s, k \qquad (7)$$

Second, the whole-day trip costs $[g_{rs}^{b,k}]$ for each traveler class k are calculated as flow-weighted, duration-weighted time-of-day trip costs:

$$g_{rs,d}^{b,k} = \sum_m f_{rs,m,d}^{b,k} g_{rs,m,d}^{b,k} / f_{rs,d}^{b,k} \qquad \forall r, s, d, k \qquad (8)$$

$$g_{rs}^{b,k} = \sum_d f_{rs,d}^{b,k} g_{rs,d}^{b,k} h_d / f_{rs}^{b,k} \qquad \forall r, s, k \qquad (9)$$

The demand-supply interactions in the network flow estimator are characterized by a set of demand and supply functions. In addition to the link cost function given earlier, other supply functions include those used for calculating origin-destination travel costs:

$$g_{rs,m}^{k} = \sum_d h_d f_{rs,m,d}^{k} g_{rs,m,d}^{k} / f_{rs,m}^{k} \qquad \forall r, s, m, k \qquad (10)$$

$$g_{rs}^{k} = \sum_m f_{rs,m}^{k} g_{rs,m}^{k} / f_{rs}^{k} \qquad \forall r, s, k \qquad (11)$$

where $g_{rs,m,d}^{k}$ is the origin-destination travel cost for traveler class k and transportation mode m from origin r to destination s during period d. Given the user-equilibrium setting in the traffic assignment step, $g_{rs,m,d}^{k}$ can be retrieved by searching for the lowest path travel cost:

$$g_{rs,m,d}^{k} = \min_{p \in P_{rs}} \sum_a \left(t_a\left(f_{a,d}\right) + \frac{c_{a,m,d}}{\gamma^k} \right) \delta_{a,p}^{rs} \qquad \forall r, s, m, d, k \qquad (12)$$

The demand functions in the network flow estimator are used to quantify travel choice behaviors or demand split mechanisms, including the origin-destination demand induction, mode split, period split, and route split. These travel choice functions are specified below. The elastic origin-destination demand function $x_{rs}^{k}(\cdot)$ is given as:

$$x_{rs}^{k} = x_{rs}^{b,k}\left(\frac{g_{rs}^{k}}{g_{rs}^{b,k}}\right)^{\eta^{k}} \qquad \forall r, s, k \qquad (13)$$

where $\eta^{k} < 0$ is the class-specific elasticity parameter. The mode choice is specified by the multinomial logit model:

$$P_{rs,m}^{k}\left(g_{rs,m'}^{k}\right) = \frac{\exp\left(-\lambda_{m}^{k} g_{rs,m}^{k}\right)}{\sum_{m'} \exp\left(-\lambda_{m}^{k} g_{rs,m'}^{k}\right)} \qquad \forall r, s, m, k \qquad (14)$$

$$f_{rs,m}^{k} = x_{rs}^{k} P_{rs,m}^{k} \frac{e_{m}}{o_{m}} \qquad \forall r, s, m, k \qquad (15)$$

where λ_{m}^{k} is the class-specific scale parameter of the mode choice model, o_{m} is the vehicle occupancy rate of mode m, and e_{m} is the passenger car equivalent of mode m. The period split is specified by the multinomial logit model as well:

$$P_{rs,m,d}^{k}\left(g_{rs,m,d'}^{k}\right) = \frac{\exp\left(-\lambda_{d}^{k} g_{rs,m,d}^{k}\right)}{\sum_{d'} \exp\left(-\lambda_{d}^{k} g_{rs,m,d'}^{k}\right)} \qquad \forall r, s, m, d, k \qquad (16)$$

$$f_{rs,m,d}^{k} = f_{rs,m}^{k} P_{rs,m,d}^{k} \frac{1}{h_{d}} \qquad \forall r, s, m, d, k \qquad (17)$$

where λ_{d}^{k} is the class-specific scale parameter for the period choice model and λ_{d}^{k} is typically greater than or equal to λ_{m}^{k} because the period split is a lower-level travel choice compared to the mode split. Finally, the route choice is determined by the path with the minimum travel cost:

$$f_{rs,p,m,d}^{k} = f_{rs,m,d}^{k}, \text{ if } g_{rs,p,m,d}^{k} < g_{rs,p,m,d'}^{k}, \text{ where } d' \neq d \qquad \forall r, s, p, m, d, k \qquad (18)$$

$$f_{rs,p,m,d}^{k} = 0, \text{ otherwise} \qquad \forall r, s, p, m, d, k \qquad (19)$$

Now all of these supply and demand functions are collected together and encapsulated into the following NC system, which characterizes a set of multi-level equilibrium network flow patterns defined by the network flow estimator:

$$g_{rs}^{*k}\left[x_{rs}^{*k} - x_{rs}^{k}\left(g_{rs}^{*k}\right)\right] = 0 \qquad \forall r, s, k \qquad (20.1)$$

$$g_{rs}^{*k} \geq 0 \qquad \forall r,s,k \qquad (20.2)$$

$$f_{rs,m}^{*k}\left[f_{rs,m}^{*k} - x_{rs}^{*k} P_{rs,m}^{k}\left(g_{rs,m}^{*k}\right)\frac{e_m}{o_m}\right] = 0 \qquad \forall r,s,m,k \qquad (20.3)$$

$$f_{rs,m}^{*k} \geq 0 \qquad \forall r,s,m,k \qquad (20.4)$$

$$f_{rs,m,d}^{*k}\left[f_{rs,m,d}^{*k} - f_{rs,m}^{*k} P_{rs,m,d}^{k}\left(g_{rs,m,d}^{*k}\right)\right] = 0 \qquad \forall r,s,m,d,k \qquad (20.5)$$

$$f_{rs,m,d}^{*k} \geq 0 \qquad \forall r,s,m,d,k \qquad (20.6)$$

$$f_{rs,p,m,d}^{*k}\left[\sum_a g_{a,m,d}^{k}\left(v_{a,d}^{*}\right)\delta_{a,p}^{rs} - g_{rs,m,d}^{*k}\right] = 0 \qquad \forall r,s,p,m,d,k \qquad (20.7)$$

$$f_{rs,p,m,d}^{*k} \geq 0 \qquad \forall r,s,p,m,d,k \qquad (20.8)$$

$$\sum_a g_{a,m,d}^{k}\left(v_{a,d}^{*}\right)\delta_{a,p}^{rs} \geq g_{rs,m,d}^{*k} \qquad \forall r,s,p,m,d,k \qquad (20.9)$$

The set of mixed complementarity equations and inequalities may be briefly interpreted as follows. The whole set can be virtually grouped into four subsets, each of which corresponds to a travel choice component in the combined equilibrium demand model: 1) demand induction (i.e., (20.1)-(20.2)), 2) mode split (i.e., (20.3)-(20.4)), 3) period split (i.e., (20.5)-(20.6)), and 4) route split (i.e., (20.7)-(20.9)). The demand induction part simply states the flow-cost consistency on the origin-destination level: $x_{rs}^{*k} = x_{rs}^{k}\left(g_{rs}^{*k}\right)$; the mode split subset presents the stochastic user-equilibrium state of mode choice: $f_{rs,m}^{*k} = x_{rs}^{*k} P_{rs,m}^{k}\left(g_{rs,m}^{*k}\right)\frac{e_m}{o_m}$; similarly, the period split subset presents the stochastic user-equilibrium state of time-of-day choice: $f_{rs,m,d}^{*k} = f_{rs,m}^{*k} P_{rs,m,d}^{k}\left(g_{rs,m,d}^{*k}\right)$; finally, the route split component generates the deterministic user-equilibrium flows over routes: if $\sum_a g_{a,m,d}^{k}\left(v_{a,d}^{*}\right)\delta_{a,p}^{rs} = g_{rs,m,d}^{*k}$, $f_{rs,p,m,d}^{*k} \geq 0$; if $f_{rs,p,m,d}^{*k} = 0$, $\sum_a g_{a,m,d}^{k}\left(v_{a,d}^{*}\right)\delta_{a,p}^{rs} \geq g_{rs,m,d}^{*k}$.

In view of the given set of demand and supply functions, the solution existence of the above NC problem is guaranteed and its solution uniqueness can be derived by checking its equivalent VI problem (refer to *55*, for example) under some mild conditions (i.e., monotonicity and Lipschitz continuity). A few popular solution techniques, including, for example, the relaxation method, projection method, and method of successive averages (refer to *55*-*57*), among others, can be used to solve the formed NC/VI problem. We chose the relaxation method as the problem solver, due to its relatively simple implementation (i.e., the relaxed problem by fixing the cross-mode and cross-period interaction effects can be readily formulated as a CO problem and solved by a number of existing solution algorithms) and better convergence performance compared to others mentioned above. In particular, the relaxed problem at the nth iteration of the relaxation method is as follows:

$$\min \sum_a \sum_d \int_0^{f_{a,d}} t_a\left(f_{a,1,d}^{(n)}, \cdots, f_{a,m,d}^{(n)}, \cdots, \omega, \cdots, f_{a,|M|,d}^{(n)}\right) \mathrm{d}\omega + \sum_a \sum_k \sum_m \sum_d \frac{f_{a,m,d}^k c_{a,m,d}}{\gamma^k}$$
$$- \sum_k \sum_{rs} \int_0^{x_{rs}^k} g_{rs}^k\left(f_{rs,1,1}^{k(n)}, \cdots, f_{rs,m,d}^{k(n)}, \cdots, \nu, \cdots, f_{rs,|M|,|D|}^{k(n)}\right) \mathrm{d}\nu$$
$$+ \sum_k \left[\frac{1}{\lambda_d^k}\left(\sum_{rs}\sum_m\sum_d f_{rs,m,d}^k \ln f_{rs,m,d}^k\right) + \left(\frac{1}{\lambda_m^k} - \frac{1}{\lambda_d^k}\right)\left(\sum_{rs}\sum_m f_{rs,m}^k \ln f_{rs,m}^k\right)\right] \tag{21.1}$$

subject to $f_{rs,p,m,d}^k \geq 0 \qquad \forall r, s, p, m, d, k \qquad$ (21.2)

where $x_{rs}^k = \sum_m \sum_d \sum_p f_{rs,p,m,d}^k \qquad \forall r, s, k \qquad$ (21.3)

$$f_{rs,m}^k = \sum_d \sum_p f_{rs,p,m,d}^k \qquad \forall r, s, m, k \tag{21.4}$$

$$f_{rs,m,d}^k = \sum_p f_{rs,p,m,d}^k \qquad \forall r, s, m, d, k \tag{21.5}$$

$$f_{a,d} = \sum_{rs} \sum_p \sum_k \sum_m f_{rs,p,m,d}^k \, \delta_{a,k}^{rs} \qquad \forall a, d \tag{21.6}$$

$$f_{a,m,d}^k = \sum_{rs} \sum_p f_{rs,p,m,d}^k \delta_{a,k}^{rs} \qquad \forall a, m, d, k \tag{21.7}$$

Note that $g_{rs}^k(\cdot)$ is the inverse of the demand function of $x_{rs}^k(\cdot)$ in (13):

$$g_{rs}^k = \left(x_{rs}^k\right)^{-1}(\cdot) = g_{rs}^{b,k}\left(\frac{x_{rs}^k}{x_{rs}^{b,k}}\right)^{1/\eta^k} \qquad \forall r, s, k \tag{22}$$

And $f_{a,m,d}^{(n)}$ and $f_{rs,m,d}^{k(n)}$ are the current values of vehicle flow rates in mode m during period d on the link level and vehicle flow rates of class k in mode m during period d on the origin-destination level, respectively, at iteration n. They are fixed in the above relaxed problem.

Interested readers can easily identify themselves the equivalency of the first-order derivative conditions of the above optimization problem to the complementarity relationships in (20) and prove the problem's solution uniqueness. Oppenheim (*25*) provides a good reference for formulating and solving the class of equilibrium demand problems of the mathematical programming type analogous to the above. Furthermore, this relaxed problem can be efficiently solved by the partial linearization algorithm developed by Evans (*8*) with slight modifications. Incorporating the Evans algorithm into the relaxation procedure provides a complete algorithmic scheme for the network flow estimator.

In summary, the relaxation method can be simply implemented by repeatedly executing the Evans algorithm:

Step 0 (Initialization): Find an initial feasible solution $(\mathbf{x}^{(0)}, \mathbf{f}^{(0)})$. An initial solution can be readily obtained by setting $\mathbf{f}^{(0)} = \mathbf{f}^b$, where $\mathbf{f}^b$ is the network flow pattern obtained from the first stage. Set $n \coloneqq 1$.

Step 1 (Relaxation): Find an updated solution $(\mathbf{x}^{(n+1)}, \mathbf{f}^{(n+1)})$ by solving the nth relaxed problem in (21) using the Evans algorithm.

Step 2 (Convergence test): If the solution difference between two consecutive iterations satisfies the convergence criterion, stop the procedure; otherwise, set $n \coloneqq n + 1$ and go to step 1.

It should be noted that for an efficient implementation, the solution of the relaxed problem at step 1 of the relaxation method is not required and not desirable to reach a very high precision. For example, in two numerical experiments of using the Frank-Wolfe algorithm to solve the relaxed problem of an asymmetric traffic assignment problem, Sheffi (*58*) and Mouskos and Mahmassani (*59*) reported that they use only one iteration and no more than four iterations of the Frank-Wolfe algorithm, respectively. In our implementation, we use the solution from executing three iterations of the Evans algorithm.

SOFTWARE IMPLEMENTATION

The two-stage model has been proposed and designed as a travel demand forecasting module used in a sketch planning software toolkit. The toolkit is a spreadsheet-based application (constructed on Microsoft Excel), which is capable of anticipating traffic changes and evaluating the long-term effects of a variety of transportation network improvements in terms of economic, environmental and safety performance measures. The spreadsheet feature of the toolkit provides users with a very user-friendly interface and the advantage of making use of powerful data manipulation and visualization functions embedded in Microsoft Excel. On the other hand, given the fact that travel demand forecasting is the most computation-intensive task in the entire planning process, the travel demand model is coded in C++ and compiled as two executable programs, i.e., the trip matrix estimator and network flow estimator, respectively. The data communication function between the spreadsheet interface and the executable programs is established by a group of spreadsheet-embedded Visual Basic for Applications (VBA) scripts.

As an overview, the toolkit's software structure is illustrated by the diagram in Figure 1, which contains the following three functional components:

- Executable programs: Travel demand forecasting module
- Spreadsheets: Other functional modules and data storage, manipulation and visualization environment
- VBA macros: Data communication module between the spreadsheets and executable programs

In our case, this modular design results in at least three development and application advantages:

- While the interface is fully contained in spreadsheets, which is intuitive and user-friendly, the most computationally intensive functions are coded in C++ and executed as external programs, reducing computational bottlenecks to the maximum extent;
- The external programs for travel demand forecasting can be modified and operated independently without interfering the spreadsheet interface, enabling advanced users to directly manipulate, test and diagnose the computational process of the travel demand forecasting module and analyze its results; and
- In case another program or process for travel demand forecasting is preferred, its outputs can be conveniently fed into the toolkit as inputs via a separate input module without altering the toolkit's existing structure and other modules.

For detailed information about the software implementation and application, interested readers are encouraged to review the research report of the project (*54*). As for advanced users who are interested in using the executable programs of the travel demand model as a separate tool or an integrated component of the toolkit, they are referred to the technical document of the travel demand model (*55*).

EXAMPLE APPLICATION

This section reports the numerical results from an example application of the two-stage demand model for network evaluation. This example is rather synthetic and we present it here mainly for the purpose of illustration; we make no claims on any investment or policy recommendations or behavioral findings implied by the evaluation results.

The example sketch network used here is extracted from the regional network of Austin, Texas. The network is constructed by only selecting freeways and major arterials in the urban area of Austin and its skeleton topology is shown in Figure 2. This network contains 62 nodes and 194 links, which is trivial compared to the size of its regional counterpart: 7,388 nodes and 18,961 links. It is not rare that in a typical urban area like Austin, the socio-demographic data sets from surveys or interviews for such a highly synthetic network are not readily available. On the other hand, a large number of its major roadways are monitored by traffic sensors and the hourly or daily traffic counts are regularly collected and stored. For example, the U.S. Highway Performance Monitoring System (HPMS) contains such a database that has archived daily traffic counts and other highway performance data from major highways and arterials nationwide since 1978. It is more important that automatically collected traffic data are generally with less noises and errors than survey or interview data, the quality of which is subject to various subjective and objective factors and data collection and aggregation mechanisms and which inevitably include more or less human errors. This advantage of data availability and quality is an important reason that we developed this traffic count-based demand model. In this example application, all links in the network are covered with traffic counts in all time-of-day periods.

The problem settings and parameter values for the example problem used on the network level are specified in Table 2. Other local parameters such as link-specific parameters are omitted here; they are part of the network files. The key problem settings include five time-of-day periods: P1: morning peak, P2: midday, P3: afternoon peak, P4: off-peak, and P5: evening; four transportation modes: M1: drivealone, M2: 2-passenger shared-ride, M3: 3+-passenger shared-ride, and M4: truck; four traveler classes: C1: truck-specific class, C2: high-income class, C3: median-income class, and C4: low-income class. It should be noted here that the first traveler class is reserved for the truck mode, which distinguishes its cost perception behavior from all other classes; other transportation modes (from M1 to M3) cross with other traveler classes (from C2 to C4). As a result, we have ten mode-class combinations in total in the system (see Table 2).

An important issue pertaining to any demand model prior to its implementation is parameter calibration; our model is not an exclusive case. The set of estimated behavioral parameter values for the example are included in Table 2. They are either estimated exogenously or calibrated by matching the model outputs to the given traffic counts and other measureable flow quantities. Specifically, the mode- and class-specific system and behavior parameters are directly from the Austin regional travel demand model maintained by TxDOT and the scale parameters of mode and time-of-day split models are suggested to their typical values in reference to some previous studies. Other supply parameters, for example, the link cost function parameters, are specified by the Highway Capacity Manual (HCM). Under such a setting, the traffic counts for the base year are used as both the model inputs and the calibration target. This is a unique feature and advantage of the model development and calibration pertaining to our travel demand modeling experiences. We will elaborate the model calibration process and results in a subsequent paper.

The alternative network scenarios have been developed to accommodate three candidate projects of upgrading a U.S. 290 segment (of 8 links in the sketch network) from an arterial to a freeway, upgrading it from an arterial to a toll freeway, and simply adding lanes to the existing arterial roadway. The capacity and toll settings of these candidate projects are shown in Table 3. For illustration, we only list the basic economic performance measures, such as traveler surplus, vehicle hours traveled (VHT), and vehicle miles traveled (VMT), among others, as directly reported by the network flow estimator. The network flow estimator approximates the traveler surplus change using the rule-of-half method. A preliminary assessment on the computation results can support at least two points of view. First, all the three upgrading projects bring a positive traveler surplus change when their network flow patterns are compared to that of the base scenario, which justifies the possible implementation feasibility and provide a sufficient condition for a further in-depth analysis of these projects. Second, it is noted that compared to the base scenario, all these alternative scenarios induce more demands (in terms of the VMT increases) while reducing the congestion level (in terms of the VHT decreases). This phenomenon confirms the attractiveness of these alternative network scenarios in terms of network performance improvement. However, no single project is manifestly superior to others in terms of the given performance measures. To make a comprehensive benefit-cost evaluation, other types of performance matrices such as vehicle emissions and crash rates need to be incorporated into the evaluation system and the construction, maintenance, and any other costs related to the project implementation should be considered as well. This is beyond the scope of this paper. A detailed project evaluation and analysis of using the travel demand model and the sketch planning toolkit for a set of real-world networks can be found in Fagnant (*56*).

CONCLUSIONS AND FUTURE TASKS

The two-stage equilibrium demand model presented in this paper is a unique product designed for sketch planning as well as general transportation planning, which synthesizes many recent research advances in combined or integrated network equilibrium modeling and solution techniques. It consists of two computational components: trip matrix estimator and network flow estimator. The trip matrix estimator uses traffic counts as inputs to infer origin-destination trip matrices, while the network flow estimator takes the inferred time-of-day trip matrices to estimate integrated network flow patterns across origin-destination pairs, transportation modes, time-of-day periods, and network links. To ensure the modeling consistency, both of the stages model and evaluate traffic flows in multiple traveler classes and transportation modes under the equilibrium settings. By equilibrium, the model achieves the flow-cost consistency in two levels. On the individual level, travelers develop their travel decisions in response to prevailing network congestion conditions in such a way as to minimize their personal travel costs or disutilities (in deterministic or stochastic ways) and gain no further improvement by altering any choices. This disaggregate equilibrium occurs in the mode choice, time-of-day choice, and route choice. On the market level, the model treats the network or network components as a whole and uses the point of intersection of an upward-sloping supply curve and a downward-sloping demand curve in the flow-cost coordinate system to determine an aggregate demand-supply consistency state in the market. The equilibrium associated with elastic demands on the origin-destination level belongs to this aggregate case.

Various improvements can be made to enhance the functionality and applicability of the current version of the equilibrium demand model for future applications. An immediate need is to include the transit mode and a mixed highway-transit assignment procedure into the network flow estimator. Specifically, this added transit component should be at least capable of modeling transit routes and stops, service schedule and frequency, vehicle capacity and discomfort, and transit-specific cost structure. Alternative travel choice structures, for example, the nested logit structure, are an option to improve the travel choice modeling mechanism across multiple levels. This incorporation will inevitably change the existing structure of the model and requires a recast of the demand induction and traffic assignment procedures. In the current version of the model, both computational stages are powered by some well-known, easy-to-implement solution algorithms, i.e., the Frank-Wolfe algorithm is used to solve the trip matrix estimator and the Evans algorithm is iteratively executed in the relaxation solution framework to solve the network flow estimator. The two algorithms belong to the Jacobi/Gauss-Seidel type, which are based on the linear approximation principle and converge to the optimal solution at only a sublinear or linear rate. To improve the computational efficiency, further efforts will be shifted to investigating the feasibility of implementing solution algorithms of the Newton or quasi-Newton type, which use the quadratic approximation strategy and can potentially achieve a superlinear convergence rate.

The work presented in this paper is only one part of a series of research efforts in developing advanced travel demand forecasting techniques and tools. More structurally complex but still computationally tractable demand models and methods need to be further pursued to integrate various travel dimensions and restrictions, accommodate network variations and demand uncertainties, and incorporate more realistic travel behavior mechanisms and alternative travel impedance components.

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Table 1 Notation

Parameters	
α_a, β_a	Parameters of the link cost function of link a
t_a^0	Free-flow travel time of link a (hr)
u_a	Capacity of link a (veh/hr)
$c_{a,m,d}$	Monetary cost of transportation mode m on link a in period d ($)
λ_m^k	Scale parameter of the mode split model of class k
λ_d^k	Scale parameter of the period split model of class k
h_d	Relative duration of period d, where $\sum_d h_d = 1$
η^k	Parameter of the elastic demand function of class k
p^k	Proportion of class k in the vehicle population
p_m^k	Proportion of mode m in class k in the vehicle population
γ^k	Value of time of class k
o_m	Vehicle occupancy rate of mode m (per/veh)
e_m	Passenger car equivalent of mode m
ω_x, ω_y, ω_v	Weighting coefficients of the objective function of the trip matrix estimator, where $0 \leq \omega_x$, ω_y, $\omega_v \leq 1$ and $\omega_x + \omega_y + \omega_v = 1$
$\bar{f}_{a,d}$	Observed vehicle flow rate on link a in period d
Variables	
$f_{a,d}$	Vehicle flow rate on link a in period d (veh/hr)
$f_{a,m,d}^k$	Vehicle flow rate of class k in mode m on link a in period d (veh/hr)
$f_{rs,d}$	Vehicle flow rate between O-D pair r-s in period d (veh/hr)
$f_{rs,m}^k$	Average vehicle flow rate in mode m between O-D pair r-s over the entire analysis period (veh/hr)
$f_{rs,m,d}^k$	Vehicle flow rate of class k in mode m between O-D pair r-s in period d (veh/hr)
$f_{rs,p,m,d}^k$	Vehicle flow rate of class k in mode m along path p between O-D pair r-s in period d (veh/hr)

x_{rs}^{k}	Average person trip rate of class k between O-D pair r-s over the entire analysis period (per/hr)
$x_{rs,d}^{k}$	Person trip rate of class k between O-D pair r-s in period d (per/hr)
$g_{a,m,d}^{k}$	Generalized travel cost of class k in mode m on link a in period d (hr)
g_{rs}^{k}	Generalized travel cost of class k between O-D pair r-s (hr)
$g_{rs,d}^{k}$	Generalized travel cost of class k between O-D pair r-s in period d (hr)
$g_{rs,m}^{k}$	Average generalized travel cost of class k in mode m between O-D pair r-s over the entire analysis period (hr)
$g_{rs,m,d}^{k}$	Generalized travel cost of class k in mode m between O-D pair r-s in period d (hr)
$g_{rs,p,m,d}^{k}$	Generalized travel cost of class k in mode m along path p between O-D pair r-s in period d (hr)
$P_{rs,m}^{k}$	Probability of a traveler of class k between O-D pair r-s choosing mode m
$P_{rs,m,d}^{k}$	Probability of a traveler of class k in mode m between O-D pair r-s choosing period d
$y_{a,d}^{+}$, $y_{a,d}^{-}$	Slack variables of the trip matrix estimator (veh/hr)

Table 2 Problem settings and parameter values of the example network

System parameters					
Time-of-day periods	P1	P2	P3	P4	P5
Duration	0.208	0.125	0.208	0.167	0.292
Transportation modes	M1	M2	M3	M4	
Vehicle occupancy rate (per/veh)	1	2	3.2	1	
Passenger car equivalent (veh)	1	1	1	1.8	
Operating cost ($/mi)	0.25	0.25	0.25	5	
Traveler classes	C1	C2	C3	C4	
Value of time ($/hr)	50	30	10	5	
Population proportion	0.1	0.1	0.2	0.6	
Behavior parameters					
Time-of-day periods					
Scale parameter	0.1				
Transportation modes	M1	M2	M3	M4	
Mode proportions in C1†	-	-	-	1	
Mode proportions in C2	0.863	0.082	0.055	-	
Mode proportions in C3	0.692	0.202	0.106	-	
Mode proportions in C4	0.630	0.250	0.120	-	
Scale parameter	0.1				
Traveler classes	C1	C2	C3	C4	
Elasticity parameter	-0.20	-0.35	-0.45	-0.50	

† Note that the mode proportions presented here are defined in term of number of vehicles, not number of persons, which are only used in the first stage of the model, i.e., the trip matrix estimator.

Table 3 Inputs and outputs of the base and alternative scenarios of the example network

	No build	Scenario 1	Scenario 2	Scenario 3
Input (An I-290 segment)				
Roadway type	Arterial	Freeway	Toll freeway	Lanes added
Capacity (veh/hr)	1,360-1,720	3,820	3,820	2,040
Toll ($)	0	0	2	0
Output (Network performance)				
Traveler welfare change ($)	0	276,678	217,860	126,790
Total travel cost ($)	15,958,800	15,893,500	15,905,700	15,720,770
Total travel time (hr)	715,513	704,858	706,938	703,908
Vehicle Hours Traveled (VHT) (hr)	585,045	574,880	576,125	575,538
Vehicle Miles Traveled (VMT) (mi)	17,881,350	17,887,500	17,886,370	17,886,850

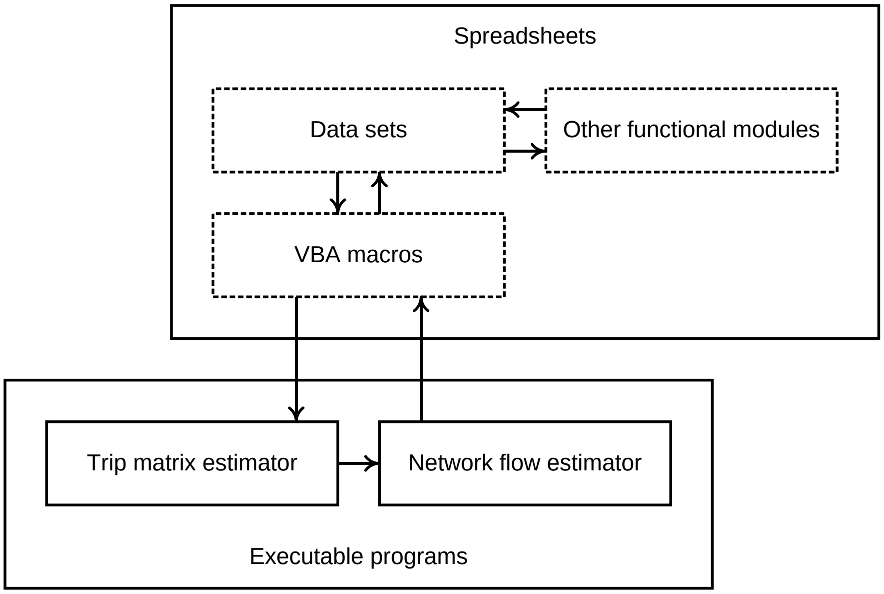

Figure 1 Software structure of the sketch planning toolkit

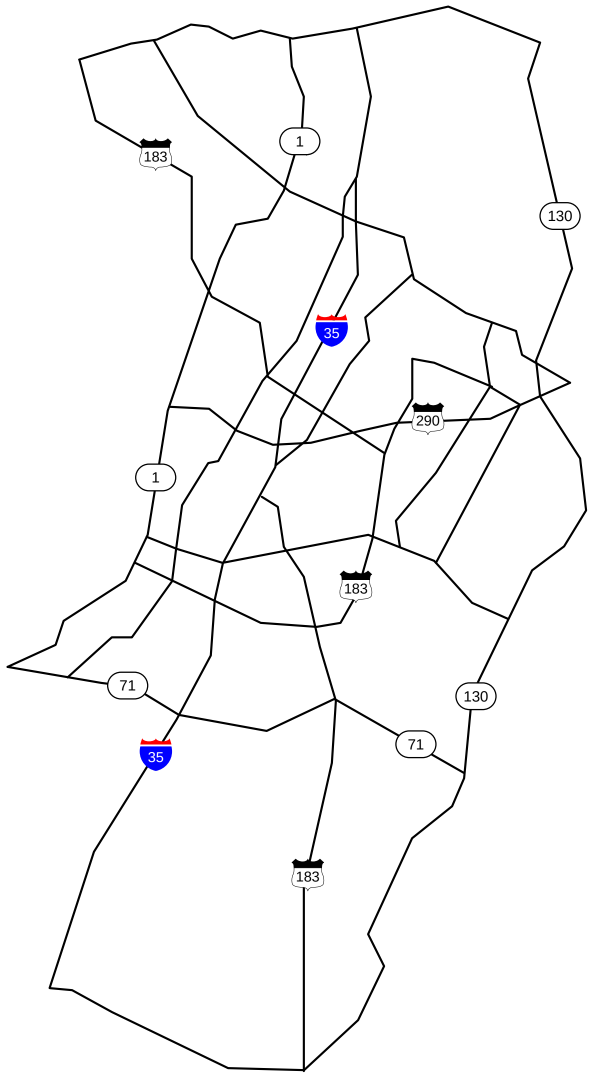

Figure 2 The example network: Austin sketch network