CONGESTION PRICING’S LONG-RUN EFFECTS ON FIRM AND HOUSEHOLD LOCATION CHOICES IN POLYCENTRIC CITIES

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Submitted for presentation at the 94th Annual Meeting of the Transportation Research Board, January 2015 in Washington D.C. and for publication in Regional Science and Urban Economics

Abstract: This paper develops a spatial general equilibrium model to explore the land use effects of three roadway congestion pricing strategies in polycentric (multi-ring) city settings, where household and firm locations and wage rates are endogenously determined. Simulation results show how Pigouvian tolling of travel in the polycentric setting can cause many jobs (17% in this example) to leave the central business district (CBD) and relocate to a relatively dense but suburban ring. To achieve city-wide welfare gains, efficient land use regulations should permit such job decentralization. Simulations also illuminate how simple, distance-based tolls generate lower welfare improvements, but stimulate similar land use effects. A cordon toll may re-agglomerate firms in a polycentric sub-center ring of development. Overall, results highlight how an urban economic model enabling endogenous business and household decisions can illuminate various travel, congestion pricing, and land use connections.

Key words: Congestion Pricing, Polycentric Urban Economics, Land Use, Job Decentralization.
INTRODUCTION

The use of congestion tolls around the world is rising, in the form of cordon charges, area-wide pricing, and variable-rate highway tolling. Pioneering examples include Singapore’s Area Licensing Scheme in the early 1970s and its Electronic Road Pricing (congestion pricing) policy in 1998 and London’s 2003 start of an area-wide toll (Santos, 2005). By 2011, ten U.S. metropolitan areas had introduced 12 high-occupancy toll (HOT) facilities on freeways, and 13 new HOT lanes were under construction or extension (GAO, 2012). Congestion pricing (CP) schemes in these regions are expected to reduce congestion, moderate negative congestion externalities (like traffic delays, air pollution, and greenhouse gas emissions), and offer revenue to help fund transport system improvements, including public transit. Much literature has focused on the short-term impacts of CP on traffic conditions and mode choices, and CP’s long-term effects on travel preference and climate change (see, e.g., Olszewski, 2005; Beavers, 2005; and Bhatt, 2011). Less attention has been paid to CP’s effects on land use patterns, urban form, and environmental justice, all of which merit further exploration (Levinson, 2010; ULI, 2013).

CP strategies differ from many other sources of transport funding (e.g., fuel, sales, and property taxes), and can influence land use decisions rather directly, since trip charges affect travel routes, destinations, timing, and ultimately home and business location decisions. Tolls can affect firms’ labor costs, productivity, and customer access. Many experts believe that a tax on vehicle-miles traveled (VMT) may accelerate new development of compact, mixed-use, walkable neighborhoods, and may modestly affect commercial land uses, especially retail (ULI, 2013). Gupta et al.’s (2006) simulations of Austin, Texas suggest that CP may catalyze land development around tolled roads, while London’s area-based charge has had a somewhat negative effect on the city center’s economy, particularly in retail (Santos and Shaffer 2004). Associations between congestion tolls and land use patterns in Singapore and Stockholm remain ambiguous (Bhatt, 2011; Litman, 2011).

This paper develops modeling improvements for analyzing CP’s long-term land use effects. Many studies (e.g., Pines and Sadka, 1985; Wheaton, 1998; Brueckner, 2007; Kono and Joshi, 2012) provide theoretically rigorous frameworks to explore land use patterns under marginal cost pricing (MCP) strategies in monocentric settings, with firms’ location decisions exogenously given (i.e., all jobs are placed in the central business district, or CBD). In a city or region with only congestion externalities, MCP is a first-best policy to reflect the gap between marginal social and marginal private costs of each trip. In a closed-form monocentric model, MCP raises residential densities near the CBD, while slightly lowered edge densities (Pines and Sadka, 1985; Wheaton, 1998; Kono and Joshi, 2012). A well-executed lot-size zoning policy can replace such MCP policies and still reach the first-best optimum, including an
upward adjustment of central densities and downward adjustment of edge densities. However, these findings largely rely on the monocentric assumption and hardly reflect most regions’ polycentric reality, with firm location decisions endogenous and dependent, to some extent, on household choices.

Several studies have explored the effects of first-best CP strategies in polycentric cities and their land use effects on both firm and household location choices. For example, Anas and Xu (1999) developed a spatial general equilibrium model without predetermined firm locations to explore the locational effects of MCP in a linear city with discrete zones. They found that the addition of MCP policies could disperse producers away from the regional center while centralizing households, thus bringing jobs and workers closer together. However, their model did not control for the agglomeration economies that can cause firms to locate close to one another, and thus can somewhat misestimate CP’s effects on job dispersion.

Several other studies have built models for continuous space, allowing more direct comparison of results to those of the traditional monocentric setting. For example, Wheaton (2004) extended a monocentric model to involve both congestion and center-agglomeration externalities, and found that higher congestion levels may cause greater job decentralization. Though his model did not test the toll policy’s efficiency, his results suggest that land use-congestion studies of this sort should not overlook interactions between congestion and agglomeration externalities. Recently, Zhang and Kockelman (2014) developed a spatial general equilibrium model allowing for such interactions, and compared socially optimal land use patterns to those under a free-market equilibrium. They found that the MCP strategy (which they called a Pigouvian congestion toll, or PCT) could lead to job decentralization and residential densification. In general, these non-monocentric studies (Anas and Xu 1999, Wheaton 2004, Zhang and Kockelman 2014) focus less on other, second-best CP policies, such as a VMT tax and a cordon toll, which are much more practical for application than is MCP (which varies by location and time of day, requiring more information)\(^1\).

Several theoretical papers have investigated the land use effects of second-best pricing in a monocentric framework (see, e.g., Mun et al., 2003; Verhoef, 2005; De Lara et al., 2013). Some have sought to extend the monocentric model by involving non-monocentric features, like allowing flexible commute-trip destinations, instead of requiring that all such trips head to the CBD (Mun et al., 2005), or positing two CBDs, instead of one (De Lara et al., 2013). Such improvements still heavily rely on the assumption that

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\(^1\) Other researchers tend to focus on second-best land use policies, instead of second-best pricing schemes. These include urban growth boundaries in monocentric regions (Kanemoto, 1977; Pines and Sadka, 1985; Brueckner, 2007) and polycentric regions (Anas and Rhee, 2006; Zhang and Kockelman, 2014), and building size/floor-area-ratio regulations in monocentric regions (Pines and Kono, 2012; Kono et al., 2012).
firms’ location choices are exogenously given, so they cannot anticipate CP’s effects on job location patterns.

This paper extends Zhang and Kockelman’s (2014) model by exploring the land use effects of three CP policies, including MCP, distance-based VMT taxes, and cordon tolls, after controlling for these policies’ effects on firms’ agglomeration economies in a duo-centric (two-ring) urban form. In this way, the work compares the effectiveness of the three CP policies in polycentric settings. The following sections define the model and its optimization problems under different CP schemes, present the simulation algorithm and its many parameters, discuss key simulation results, and offer several conclusions.

MODEL SPECIFICATION
The model developed and applied here assumes a region with continuous circular space. A city locates in this region with an endogenously determined city boundary, $\bar{x}$. The whole city area is assumed to be symmetrical, implying that people need travel only towards or away from the center, along radial street networks, and any circumferential travel is ignored. Only identical households and homogenous firms exist in the city. For all locations $x$ inside the city ($x \in (0, \bar{x})$), land is used only for firms, households, and/or transport infrastructure. $\theta_f(x)$ and $\theta_h(x)$ represent the fractions of land area used by firms and households, while $\theta_t$ represents an exogenously given fraction of land for transportation use. Many of these and following settings are idealistic, without considering non-worker preferences, non-commute trips, heterogeneity in firms and householdss, real transport networks, and the costs of infrastructure construction. However, mimicking an actual existing city’s form is a complicated process, beyond the scope of this analysis. The model applied here rather enables us to discuss the long-run efficiency of CP policies under an polycentric equilibrium framework, superior to traditional monocentric model².

Firm Behavior
Each firm is a price taker in the output and input markets, and decides how much labor and land to use for production, at each location $x$. Each firm’s production per unit of land $p(x)$ at location $x$ is determined by two functions: One is an ordinary, constant-returns production function (per unit of land), $f(n(x))$, that only relates to the labor per unit of land or employment density, $n(x)$. The other is a measure of external economies at location $x$, $A(F(x))$. Thus, $p(x)$ is defined as follows:

$$p(x) = A(F(x))f(n(x))$$

² Those interested in the limitations and benefits of urban economic equilibrium models, especially the traditional monocentric model, can refer to Arnott (2012).
where \( F(x) \) represents a positive agglomeration externality for firms located at \( x \). A larger market may benefit more from the sharing of facilities and suppliers, a better matching between firms and workers, and the facilitation of social learning through knowledge transmission (Rosenthal and Strange, 2004; Puga, 2009). The setup used here mainly considers the agglomeration effects that come from sharing of facilities and social learning, by assuming that clustered firms benefit more from their workers’ knowledge spillovers. Although the model is designed to deliver in a static, long-term spatial equilibrium, it is based on a dynamic agglomeration economy, which assumes that both current and historical economic activities at a given location affect agglomeration economies in production (Henderson, 2003; Rosenthal and Strange, 2004). Thus, \( F(x) \) consists of two components:

\[
F(x) = F^0(x) + F^1(x)
\]

where \( F^0(x) \) represents a given historical agglomeration economy that reflects the natural advantage and long-term benefits from the sharing of facilities at location \( x \), and \( F^1(x) \) is the current agglomeration effect at location \( x \). When \( F^1(x) = 0 \) for any locations, \( F(x) \) becomes pre-determined/exogenous, and the model collapses to a traditional monocentric model. In this paper, \( F^1(x) \) is defined as the integral of exponentially distance-weighted job counts within a given boundary \( \bar{r} \) (Zhang and Kockelman, 2014):

\[
F^1(x) = \zeta \int_0^{\bar{r}} \int_0^{2\pi} r \varphi_f(r) n(r) e^{-\zeta l(x,r,\psi)} d\psi dr
\]

where \( \zeta \) is exogenously determined to describe the strength/level of production externalities that exist, \( \psi \) is the polar angle around the center (ranging from 0 to \( 2\pi \)), and \( l(x, r, \psi) \) is the straight-line distance between a firm at location \( x \) and any firm lying within \( \bar{r} \) miles of the center (at a counter-clockwise angle of \( \psi \) from the first firm).

Based on Eqs. (2) and (3), one can calculate the marginal agglomeration externality (MAE) of labor, \( s_{mae}(x) \), for firms at location \( x \). An additional worker employed in location \( x \) will affect the productivity of firms not only at location \( x \) but nearby (e.g., \( r \) distance away), through \( F(r) \)'s labor effects. As shown in Zhang and Kockelman (2014), \( s_{mae}(x) \) thus equals:

\[
s_{mae}(x) = \zeta \int_0^{\bar{r}} r \varphi_f(r) p_F(r) \int_0^{2\pi} e^{-\zeta l(x,r,\psi)} d\psi dr
\]

---

3 Fujita and Ogawa (1982) first provided a measure of agglomeration economies for firms based on job densities and distances to other firms or workers in a linear city setting (termed locational potential or communication externalities, in Fujita and Thisse [2002]). In their “LRH” model, Lucas and Rossi-Hansberg (2002) extended this idea to circular space. The only difference in the current formulation (provided here) is that LRH’s model considers production externalities from all firms in the entire city (inversely weighted by distance), and assumes a fixed city boundary. Our model assumes production externalities come only from firms within a pre-set area, and the city’s boundary/limit is endogenously determined. Though both setups are imperfect, either a fixed boundary or a fixed agglomeration-area limit appears necessary to achieve solution convergence (as theoretically proven in the Lucas and Rossi-Hansberg paper and experimentally supported in the ZK model).
where \( p_F(r) \) is the marginal production (per unit of land) \( p(r) \) of \( F(r) \), i.e., \( \partial p(r)/\partial F(r) \). The aggregate agglomeration benefit, \( S \), of firms in the city is thus as follows:

\[
S = \int_0^T 2 \pi x \theta_F(x) n(x) s_{mae}(x) dx
\]

The price of firm output is set to 1.0 (as the numeraire) without loss of generality; thus, a firm’s profit per unit of land at location \( x \), \( \Pi(x) \), can be given by the following:

\[
\Pi(x) = f(n(x))A(F(x)) - w(x)n(x) - r_F(x)
\]

where \( w(x) \) is the wage paid to each laborer and \( r_F(x) \) is the rent firms are willing to pay (per unit of land) at location \( x \).

**Household Choices**

Each household chooses its home location \( x \) to maximize its utility function involving goods \( c(x) \) and residential lot size \( q(x) \). Households are assumed to be identical, maximizing utility function \( u(c(x), q(x)) \), subject to their budget constraint:

\[
c(x) + r_h(x)q(x) = y(x)
\]

where \( r_h(x) \) is land rent at location \( x \) and \( y(x) \) is the net income of any household living at \( x \). This net income is the sum of wage income plus any rent taxes and/or tolling/road pricing revenues returned, \( \bar{y} \), net of the worker’s commute costs and possible congestion tolls. Here, \( \bar{y} \) can recognize any lump-sum redistribution of overall land rents that exceed the edge (agricultural use) rent, \( \bar{y}_{rent} \), and any (typically uniform) return of toll revenues, \( \bar{y}_{toll} \). Such rent redistributions would essentially presume that land is owned collectively by the workers or that landlords are taxed away all their extra rents for non-agricultural use. Such toll revenue returns are a way to improve equity in CP policy applications. As proven in Zhang and Kockelman (2014), when households’ utility levels are maximized, net incomes equals the wage (work-based) income of households working at location \( x \), \( w(x) \), plus \( \bar{y} \), as follows:

\[
y(x) = w(x) + \bar{y} = w(x) + \bar{y}_{rent} + \bar{y}_{toll}
\]

Eq. (8) implies that all households living at location \( x \) achieve the same net income, regardless of where they work; that is, no worker can achieve a higher net income by changing his or her job location. Thus, in the household’s partial equilibrium, a net-income gradient \( y'(x) \) or wage gradient \( w'(x) \) varies with total marginal travel costs.

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4 The redistribution of rent and toll revenues to residents guarantees a “fully closed city” (Pines and Sadka, 1986), which is important for comparing the welfare consequence of different policy interventions. If these revenues are not returned to residents or the land is owned by absentee outside the city, the change in aggregate rents due to a policy implementation is a dead loss to the residents of the city (Solow, 1973).

5 This is a major equilibrium condition used in both monocentric models (Solow, 1973) and non-monocentric models (Wheaton, 2004).
\[
y'(x) = w'(x) = t(x) + \tau(x)
\]
where \(t(x)\) is the marginal (private) travel cost and \(\tau(x)\) is a (possible) congestion toll at location \(x\).

**Transport and Congestion**

In a symmetric city, worker travel occurs just radially: inward (toward the city center) or outward. Here, \(t(x)\) represents marginal travel cost (dollar per mile, for example) at location \(x\), with negative values representing inward travel and positive values representing outward travel. Since only one travel mode or transport technology (e.g., the private car) is reflected in this model, the marginal travel cost in an uncongestible network will equal a constant, \(\varphi\) (in dollars per mile).

Since transportation systems are congestible, the true \(t(x)\) contains an additional component to reflect congestion. Here, this second component is assumed proportional to a power of the ratio of travel demand to supply or transport capacity at location \(x\) (as used, for example, by Solow, 1972; Wheaton, 1998, 2004; and Brueckner, 2007). Here, \(D(x)\) represents total travel demand passing location \(x\). When \(D(x) < 0\), travel flow is inward; when \(D(x) > 0\), commute travel is outward; and, when \(D(x) = 0\), no travel demand crosses location \(x\). Under these three potential travel settings, the marginal travel cost is as follows:

\[
t(x) = \begin{cases} 
-\varphi - \rho \left( \frac{|D(x)|}{2\pi x \theta_i} \right)^\sigma & \text{if } D(x) < 0 \\
\varphi + \rho \left( \frac{|D(x)|}{2\pi x \theta_i} \right)^\sigma & \text{if } D(x) > 0 \\
\varphi \text{ or } -\varphi & \text{if } D(x) = 0
\end{cases}
\]

where \(\rho\) and \(\sigma\) (\(\sigma \geq 1\)) are positive parameters and reflect network congestibility. According to Eq.\((10)\), one can calculate the marginal congestion externality (MCE) at each \(x\) as follows:

\[
\tau_{mce}(x) = \frac{\partial t}{\partial |D(x)|} |D(x)| = \begin{cases} 
-\rho \sigma \left( \frac{|D(x)|}{2\pi x \theta_i} \right)^\sigma & \text{if } D(x) \leq 0 \\
\rho \sigma \left( \frac{|D(x)|}{2\pi x \theta_i} \right)^\sigma & \text{if } D(x) > 0
\end{cases}
\]

Here, the derivative of \(t(x)\) of \(D(x)\) represents differential travel cost on each drivers across \(x\) when adding one additional driver, while \(\tau_{mce}(x)\) represents all travel costs on other drivers caused by a driver. Thus, the region’s aggregate congestion externality, or congestion diseconomy, is as follows:

\[
\Gamma = \int_0^\tilde{x} \tau_{mce}(x) D(x) dx = \int_0^\tilde{x} \rho \sigma |D(x)|^{\sigma+1} / (2\pi x \theta_i)^\sigma dx
\]

**Solving for the General Spatial Equilibria**

Four types of spatial equilibrium are discussed here, including the no-toll (i.e., free-market) city, the MCP equilibrium, and the VMT tax and cordon toll equilibria. The existence of both congestion and
agglomeration externalities increases the difficulty of comparing road pricing policies, since the pricing instruments can affect agglomeration economies (Verhoef and Nijkamp, 2004; Zhang and Kockelman, 2014). This paper controls for the toll policy effects on agglomeration externalities, allowing one to more equitably compare the congestion benefits and land use effects of these pricing policies in a closed region. To do so, the equilibrium population and agglomeration benefits under the three pricing policies are set to equal those in the no-toll (base case) equilibrium, with agglomeration benefits over (or under) the no-toll equilibrium level equally taxed (or credited) across workers.

The No-Toll Equilibrium
The no-toll equilibrium is an efficient market solution if both congestion and production externalities do not exist. Thus, given \( t(x) \) and \( F(x) \), the solution to a no-toll equilibrium is achieved by determining five factors, \( \{n(x), q(x), c(x), \theta_f(x), D(x)\} \), at each location \( x \), so as to maximize household utility levels under the five constraints (13)–(17), as defined in Problem 1.

**Problem 1.** Choose functions \( n(x), q(x), c(x), \theta_f(x), D(x) \) so as to maximize

\[
u(c(x), q(x)) \]

subject to the following conditions:

\[
\begin{align*}
(13) & \quad c(x) + r_h(x)q(x) = y(x) = w(x) + \bar{y} \\
(14) & \quad f(n(x))A(F(x)) - w(x)n(x) - r_f(x) \geq 0 \\
(15) & \quad \theta_h(x) + \theta_f(x) + \theta_t = 1 \\
(16) & \quad D'(x) \leq 2\pi x \left( \frac{\theta_h(x)}{q(x)} - \theta_f(x)n(x) \right) \\
(17) & \quad \int_0^\bar{x} \left\{ 2\pi x \left( \theta_f(x)f(n(x))A(F(x)) - \frac{\theta_h(x)}{q(x)} c(x) - (1 - \theta_t)R_A \right) - t(x)D(x) \right\} dx \geq 0
\end{align*}
\]

for all \( x \in [0, \bar{x}] \), with boundary conditions:

\[
\begin{align*}
(18) & \quad r(\bar{x}) = R_A \\
(19) & \quad D(0) = 0 \text{ and } D(\bar{x}) = 0 \\
(20) & \quad \int_0^\bar{x} 2\pi x \frac{\theta_h(x)}{q(x)} dx = N
\end{align*}
\]

where \( R_A \) is the opportunity cost of land inside a city, which is assumed to equal the exogenous rent of agriculture use outside the city (as done by Pines and Sadka [1986] and Bruckner [2007]). \( r(x) \) is the highest bid-rent at location \( x \), so \( r(x) = \max \{ r_h(x), r_f(x), R_A \} \).

Constraint (13) is the household budget constraint, formed by combining Eqs. (7) and (8). Since no toll revenue is earned, \( \bar{y}_{toll} = 0 \) and \( \bar{y} = \bar{y}_{rent} \), where \( \bar{y}_{rent} \) is set as follows:
\[ \bar{y}_{\text{rent}} = \frac{1}{N} \int_{0}^{\infty} 2\pi x (1 - \theta_c) (r(x) - R_A) dx \]

Constraint (14) guarantees non-negative profits for each firm. Constraint (15) represents land market clearance, so that all available land or properties are assigned to agents, while the city’s edge rent equals the agricultural land rent, as defined in boundary condition (18). Constraint (16) guarantees that an additional number of travelers passing the infinitesimal interval \( dx \) (from \( x+dx \) to \( x \) or from \( x-dx \) to \( x \)), \( D'(x)dx \), will not exceed the maximum travel demand generated in the interval \( dx \): 
\[ 2\pi x dx \left( \frac{\theta_h(x)}{q(x)} - \frac{1}{\nu(x)} \right). \]

This constraint relates to boundary condition (19), in which no travel demand exists at the regional center point or at the city’s edge. This ensures a city-wide jobs-housing balance. Finally, Constraint (17) guarantees a non-negative net social surplus. Given that aggregate land rents (net of the opportunity costs) will be returned uniformly to each household (due to the closed-city formulation, which facilitates welfare comparisons across settings, and as done in Solow (1973), Pines and Sadka (1986), Anas and Xu [1999] and Breukner [2007], for example), the net surplus is equivalent to aggregate production minus consumption of goods produced by the firms, plus land opportunity costs, minus commuting costs. In order to arrive at a closed-form solution, the equilibrium population equals an exogenous value, \( N \), as shown in boundary condition (19). The resulting solution will satisfy the following proposition:

**Proposition 1.** In a closed city with \( \bar{u} \) as the equilibrium utility level, the equilibrium solution set \( \{n^*(x), q^*(x), c^*(x), \theta_f^*(x)\} \) satisfies the following equations:

(a) \( n^*(x) = n^*(w(x)) \), and \( n^*(x) \) satisfies \( f_n(n^*(x)) = w(x)/A(F(x)) \);

(b) \( q^*(x) = q^*(w(x), \bar{u}) \) and \( c^*(x) = c^*(w(x), \bar{u}) \), and \( q^*(x) \) and \( c^*(x) \) satisfy the equation set:

\[
\begin{align*}
    c(x) + q(x)u_q/u_c &= y(x) \\
    u(c(x), q(x)) &= \bar{u}
\end{align*}
\]

(c) \( \theta_f^*(x) = \begin{cases} 
1 - \theta_t & \text{if } r_f(x) > r_h(x) \\
(0,1 - \theta_t) & \text{if } r_f(x) = r_h \\
0 & \text{if } r_f(x) < r_h(x)
\end{cases} \)

(d) \( y'(x) = w'(x) = t(x) \)

**Proof.** Appendix A1 provides this proof.

In equilibrium, households pursue optimal good consumption, \( c^*(x) \), and housing lot sizes, \( q^*(x) \), by minimizing expenditures given the target utility level (Proposition 1[b]). Firms pursue optimal employment densities, \( n^*(x) \), in order to maximize their profits (Proposition 1[a]). At the same time,
available land and property are assigned to agents offering the highest bid rents, while city edge rents
equal the background (agricultural) land rent and jobs and housing are in balance, consistent with
Proposition 1(c). Proposition 1(d)’s differential equation suggests that the net-income gradient and the
wage gradient both equal \( t(x) \) only, since no congestion toll is levied on workers/travelers (see Eq. [9]).
This condition guarantees that all workers are equivalent in the eyes of each firm owner, and all firms are
equivalent in the eyes of each worker.

Propositions 1(a)-(c) show how equilibrium values \( n^*(x), q^*(x), c^*(x), \) and \( \theta^*_f(x) \) are only determined
by \( w(x) \), when given \( \bar{u}, F(x), \) and \( \bar{y} \). If the wage function is derived first, all other solution values for this
no-toll equilibrium can then be generated. Moreover, if one knows \( w(0) \) or \( w(\bar{x}) \), one can derive \( w(x) \) at
any other location \( x \), and so derive all other solution values. This suggests that the urban equilibrium
problem here can be resolved using a recursive algorithm, which searches for a unique \( w(0) \) and \( \bar{u} \) until
the boundary conditions (18)-(20) are entirely satisfied. Following Eqs. (5) and (12) and Proposition 1’s
equilibrium solutions, one can derive the agglomeration economies, \( S_{nt} \), and congestion diseconomies,
\( I_{nt} \), under the no-toll equilibrium.

The MCP Equilibrium

The MCP case represents the spatial equilibrium under a “perfect” road pricing policy. Here, negative
congestion externalities are fully internalized in the MCP equilibrium, while the aggregate agglomeration
benefit is endogenously adjusted to equal that arising in the no-toll equilibrium (i.e., \( S_{nt} \)), in order to
equitably compare each policy’s results. The optimization problem setup of the MCP case thus matches
that of Problem 1 (defined above, for the no-toll case), but with an additional constraint on travel costs, as
defined in Eq. (10). By resolving this optimization problem, one can prove that the equilibrium solutions
in Proposition 1(a)-(c) still hold, while the wage gradient in Proposition 1d becomes the following:

\[
y'(x) = w'(x) = t(x) + \tau_{mce}(x)
\]

This condition shows that the net-income and wage gradients need to cover the marginal social costs of
travel, which reflect both marginal private costs and marginal external (delay) costs imposed on other
travelers, \( \tau_{mce}(x) \). Given that Eq. (9) still holds in this MCP equilibrium (for each household’s partial
equilibrium), a congestion toll, \( \tau_{mcp}(x) \), equaling Eq. (11)’s marginal congestion externalities \( \tau_{mce}(x) \),
needs to be levied on each worker/each traveler passing location \( x \):

\[
\tau_{mcp}(x) = \tau_{mce}(x) = \begin{cases} 
-\rho \sigma \left( \frac{|D(x)|}{2\pi \sigma \theta_i} \right)^\sigma, & \text{if } D(x) \leq 0 \\
\rho \sigma \left( \frac{|D(x)|}{2\pi \sigma \theta_i} \right)^\sigma, & \text{if } D(x) > 0 
\end{cases}
\]
Thus, this MCP instrument is an optimal policy for correcting the system’s negative congestion externalities.

In a closed-form city, a lump-sum amount of congestion toll revenues, $\bar{y}_{toll}$, may be returned to each worker, such that:

$$\bar{y}_{toll} = \frac{1}{N} \Gamma_{mcp} = \frac{1}{N} \int_0^\infty \tau_{mcp}(x)D(x) dx$$

In addition, the agglomeration economies under a MCP equilibrium, $S_{mcp}$, can be computed by Eq. (5). Since agglomeration economies over (or under) the no-toll equilibrium level are taxed (or credited to) across workers, the revenue return to each household, $\bar{y}$, thus becomes:

$$\bar{y} = \bar{y}_{rent} + \bar{y}_{toll} + \frac{1}{N} (S_{nt} - S_{mcp})$$

where the subsidy (or tax) $\frac{1}{N} (S_{nt} - S_{mcp})$, on each worker can compensate for the MCP tolls’ effects on agglomeration externalities and thus rescale the agglomeration economies to the no-toll equilibrium level.

The VMT-Tax and Cordon-Toll Equilibria

In practice, second-best CP policies typically involve a cordon or area-based toll ($\bar{\tau}_{ct}$, levied at location $\bar{x}_{ct}$) or a (flat-rate) distance-based (VMT) tax (of $\bar{\tau}_{vmt}$). If $\tau(x)$ represents the congestion toll levied on each worker crossing ring $x$ (positive for outward travel and negative for inward travel), the magnitudes of these two distinctive tolls can be presented as follows:

$$|\tau(x)| = \begin{cases} \bar{\tau}_{vmt} & \text{VMT tax} \\ \bar{\tau}_{ct}, & \text{if } x = \bar{x}_{ct} \\ 0, & \text{otherwise} \end{cases}$$

Proposition 2. In both the cordon-toll and VMT-tax equilibria (agglomeration externalities are under control or corrected), if the aggregate tolling revenues cover $\frac{1}{1+\sigma}$ of the overall social costs of the congestion externality, $\Gamma$ (as defined in Eq. (12)):

$$\int_0^\infty \tau(x)D(x) dx = \frac{1}{1+\sigma} \Gamma$$

then, the corresponding tolling level, $\tau(x)$, is second-best optimal.

Proof. See A2 in the Appendix. An imperfect cordon toll or VMT tax (with revenues lying below or above $\frac{1}{1+\sigma}$ of the overall congestion diseconomies) will lead to labor market distortions, where workers are overpaid or underpaid by firms, to help cover travel costs and/or tolls. Only when the toll equals the

These revenues can be returned to the population in one of two ways: subsidize each household to increase money budget (as done in Equation [8]) or subsidize firms to reduce each worker’s labor cost. These two approaches are equivalent here, because firms will compete on wage offers to workers, returning the subsidy equitably.
optimal level defined in Proposition 2 will it not distort the labor market. Proposition 2 also illuminates
the “second-best” nature of a second-best CP, which demonstrates that such toll policies cannot (fully)
correct the market failure of negative congestion externalities. The second-best optimum only corrects
$\frac{1}{1+\sigma}$ (less than 1.0) of overall congestion externalities.

Based on Proposition 2, one can calculate the optimal VMT tax as follows:

$$\bar{\tau}_{vmt}^* = \frac{\Gamma}{(1+\sigma) \int_0^\infty d(x) dx}$$

and, the optimal cordon toll at (exogenously given) location $\bar{x}_{ct}$ will be:

$$\bar{\tau}_{ct}^* = \frac{\Gamma}{(1+\sigma) \int_0^\infty d(x) dx}$$

Finding optimal prices is almost any urban economic model is a challenge. In traditional monocentric
models, the basic strategy uses a heuristic search method to identify $\bar{\tau}_{vmt}^*$ or $\bar{x}_{ct}$ and $\bar{\tau}_{ct}^*$, by seeking
maximum utility or social surplus (Mun et al., 2003; Verhoef, 2005; De Lara et al., 2013). Proposition 2
provides an alternative, effective approach for non-monocentric simulations, by increasing $\bar{\tau}_{vmt}^*$ or $\bar{\tau}_{cd}^*$
until Eq. (27) is satisfied.

**SYSTEM SIMULATIONS**

The model system and its parameters are specified so as to yield polycentric structures. The general urban
form is largely determined by parameters that reflect past and present contexts, such as $l^0(x)$ and $\bar{r}$,
while specific land use details (like densities and distribution of firms and households) are determined
mostly by other parameters.

All policy scenarios use Cobb-Douglas specifications for the utility function and the two-part production
functions:

$$u(c(x), q(x)) = c(x)^\alpha q(x)^{1-\alpha}, \ 0 < \alpha < 1$$

$$f(n(x)) = \delta n(x)^\kappa, \delta > 0, 0 < \kappa < 1$$

$$A(F(x)) = F(x)^\gamma, \gamma > 0$$

Based on Proposition 1, the equilibrium residential lot size function and the household’s bid-rent function
can be derived as follows:

$$q^*(x) = \alpha^{-\alpha/(1-\alpha)} y(x)^{-\alpha/(1-\alpha)} \bar{a}^{1/(1-\alpha)}$$
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(34)

\[ r^{m}_{h}(x) = (1 - \alpha)\alpha^{a/(1-\alpha)} \left( \frac{y(x)}{\bar{u}} \right)^{1/(1-\alpha)} \]

The equilibrium employment density and the firm’s bid-rent function are as follows:

3

(35)

\[ n^{*}(x) = \left( \frac{\kappa \delta F(x) y}{w(x)} \right)^{1/(1-\kappa)} \]

(36)

\[ r^{m}_{f}(x) = (1 - \kappa)\delta^{1/(1-\kappa)} F(x) y^{(1-\kappa)} \left( \frac{\kappa}{w(x)} \right)^{\kappa/(1-\kappa)} \]

Table 1 shows the parameter values assumed in all simulations. These were developed/calibrated using data from the Austin, Texas metropolitan area and values found in the literature (see, e.g., Lucas and Rossi-Hansberg, 2002; Wheaton, 1998; Brueckner, 2007), and more details can be found in Zhang and Kockelman (2014). The intercept parameter \( \varphi \) in Eq. (16)’s average travel cost function represents the average cost of free-flow travel, and is set at $200 dollar per daily (one-way commute) mile per year. This figure is generated from the assumption that marginal free-flow travel cost is about $0.40 per vehicle-mile and each worker works about 250 days a year. \( \rho \) and \( \sigma \) reflect congestion levels and are set to 0.00001 and 1.5, respectively. In a highly congested location, for example, if there are 50,000 travelers passing a point \( x = 1 \) mile from the region’s centerpoint, the marginal congestion cost will be $0.17 per vehicle-mile here, and account for about 30% of total marginal costs. In a lightly congested location, like 5,000 travelers per day at a distance \( x = 10 \) miles away, the marginal congestion cost accounts for only 0.4% of total MSC.

Table 1 Parameter value assumptions

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( R_{A} )</th>
<th>( \kappa )</th>
<th>( \gamma )</th>
<th>( \delta )</th>
<th>( \zeta )</th>
<th>( \theta_{\epsilon} )</th>
<th>( \lambda )</th>
<th>( \varphi )</th>
<th>( \rho )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>$4,000,000 per sq. mile per year</td>
<td>0.95</td>
<td>0.04</td>
<td>30,000</td>
<td>5</td>
<td>0.3</td>
<td>0.5</td>
<td>200 $/mi</td>
<td>0.00001</td>
<td>1.5</td>
</tr>
</tbody>
</table>

In order to achieve a polycentric equilibrium, we set \( F^{0}(x) \) to be the equilibrium production externality function \( F(x) \), as solved for in a monocentric no-toll equilibrium \( (F^{0}(x) = 0, \bar{r} = 1) \). The agglomeration limit extends to \( \bar{r} = 6 \), while \( \bar{u} \) increases to 4000 utils. These settings will generate a sub-center ring of development/density in the suburbs. This two-center equilibrium can be understood as an evolution from the initially monocentric city, after population, jobs and utility levels grow.

Using the polycentric settings, four policy scenarios (a no-toll base case, an MCP case, a VMT-tax case, and a cordon-toll case) were simulated. The spatial equilibria were solved using MATLAB, following a
fixed-point algorithm, as described in Zhang and Kockelman (2014). Using Proposition 1, given pre-set values of $F(x)$ and $\bar{y}$, the process of finding an equilibrium corresponds to seeking an equilibrium initial wage $w(0)$ to clear all land and labor markets and to satisfy the boundary conditions defined in Eqs. (18)-(20). New $F(x)$ and $\bar{y}$ can be derived, along with a new equilibrium initial wage at the region’s centerpoint, $w(0)$. The equilibrium solutions process achieves convergence when the iterations find fixed-point $F(x)$ and $\bar{y}$ values.

RESULTS

The no-toll polycentric setting’s urban form was endogenously determined after assuming household levels to be 4000 utils, yielding a population of $N = 1,048,000$ workers. The no-toll equilibrium yields two city “centers” or densely developed rings of clustered firms. The first firm cluster, at the city center, is referred to here as the “traditional CBD”, while the second, in the suburban area (about 4.5 to 6.5 miles away from the center), is called the region’s sub-center. Simulation results suggest that the optimal MCP tolls rise as high as $3 per mile of travel, while the average MCP across locations is $0.71 per mile. In addition, the optimal VMT tax is computed to be $0.40 per mile of travel. The cordon toll’s optimal location is found to be about 2 miles away from the city center, with an optimal cordon fee of $1,210 per year per worker – roughly $5 per workday, or $100 per month. The utility values rise about 0.86% in the MCP equilibrium, 0.59% in the VMT-tax equilibrium, and up to 0.58% in the cordon-toll equilibrium (Table 2), while the corresponding average EV values are estimated to be $231 (ranging from $213 to $252), $157 (ranging from $145 to $185), and $156 (ranging from $143 to $180) per worker per year, which amounts to 0.85%, 0.56%, and 0.55% of the average net income.

Similar to the monocentric setting (Verhoef, 2005; De Lara et al., 2013), the polycentric city solutions becomes more compact after MCP and VMT taxes are imposed (Table 2). The city boundary distance falls from 14.76 miles in the no-toll equilibrium to 14.40 miles in the MCP case (a 4.8% drop in total city area) and 13.88 miles in the VMT-tax case (a 6.7% drop in area). The cordon toll policy appears to slightly expand the city, rather than restrict it, with a 1.8% increase in city area. Such pricing policies also reduce average travel distances in the polycentric region, by 20%, 18%, and 11% under the MCP, VMT tax, and cordon toll cases, respectively, relative to the no-toll base case.

7 Cordon locations between 2 and 2.5 miles generate nearly constant maximized utility levels, based on the solution routine’s simulation accuracy. Thus, without loss of generality, we chose 2 miles for the optimal cordon location.
Table 2 Simulation results under different pricing regimes in a polycentric city

<table>
<thead>
<tr>
<th></th>
<th>No-Toll</th>
<th>MCP</th>
<th>VMT Tax</th>
<th>Cordon Toll</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Utility level</strong>, $\bar{u}$ (utils per household)</td>
<td>4000.00</td>
<td>4034.58</td>
<td>4023.56</td>
<td>4023.28</td>
</tr>
<tr>
<td>Average equivalent variation, $\text{EV}$, relative to No Toll case ($/ per worker per year)</td>
<td>231.07</td>
<td>157.43</td>
<td>155.56</td>
<td></td>
</tr>
<tr>
<td>City boundary, $\bar{x}$ (miles)</td>
<td>14.76</td>
<td>14.4</td>
<td>13.88</td>
<td>14.89</td>
</tr>
<tr>
<td>Central wage, $w(0)$ ($/ per year per worker)</td>
<td>28,504</td>
<td>29,070</td>
<td>28,663</td>
<td>28,844</td>
</tr>
<tr>
<td>Central rent, $r(0)$ ($/sq.mi.)</td>
<td>254</td>
<td>173</td>
<td>228</td>
<td>201</td>
</tr>
<tr>
<td><strong>Jobs density</strong>, $n(0)$ (workers/sq.mi.)</td>
<td>169,510</td>
<td>113,081</td>
<td>151,183</td>
<td>132,685</td>
</tr>
<tr>
<td>Residential density at edge, $1/q(\bar{x})$ (hhs/sq.mi.)</td>
<td>1588</td>
<td>1566</td>
<td>1571</td>
<td>1571</td>
</tr>
<tr>
<td>Rent revenues returned, $y_{rent}$ ($/year/worker)</td>
<td>2,256</td>
<td>2,241</td>
<td>2,487</td>
<td>2,053</td>
</tr>
<tr>
<td>Toll revenues returned, $y_{toll}$ ($/year/worker)</td>
<td>0</td>
<td>883</td>
<td>518</td>
<td>418</td>
</tr>
<tr>
<td>Average travel distance per worker (miles/day)</td>
<td>6.27</td>
<td>4.97</td>
<td>5.16</td>
<td>5.57</td>
</tr>
<tr>
<td>Average travel costs ($/year/worker)</td>
<td>2,446</td>
<td>1,582</td>
<td>1,898</td>
<td>1,801</td>
</tr>
<tr>
<td>Average rent in the CBD ($/sq.mi.)</td>
<td>115.73</td>
<td>98.95</td>
<td>83.58</td>
<td>111.89</td>
</tr>
<tr>
<td>Average rent in the sub-center ($/sq.mi.)</td>
<td>18.99</td>
<td>24.26</td>
<td>22.42</td>
<td>22.94</td>
</tr>
<tr>
<td>Jobs in the sub-center (1,000)</td>
<td>470</td>
<td>648</td>
<td>530</td>
<td>659</td>
</tr>
<tr>
<td>Percentage of jobs in the sub-center (%)</td>
<td>44.83</td>
<td>61.8</td>
<td>50.53</td>
<td>62.89</td>
</tr>
<tr>
<td>Job density in the CBD (workers/sq.mi.)</td>
<td>54,373</td>
<td>45,704</td>
<td>55,791</td>
<td>51,537</td>
</tr>
<tr>
<td>Job density in the sub-center (wks/sq.mi.)</td>
<td>9,888</td>
<td>12,716</td>
<td>11,704</td>
<td>11,926</td>
</tr>
</tbody>
</table>

In this two-center city, tolling policies cause interesting effects on land rent distributions. A major tendency is for central-area/CBD land rents to fall significantly, while sub-center land rise (Figure 1). The average CBD rent falls by 15%, 28%, and 3.3% under the MCP, VMT-tax, and cordon-toll equilibria, respectively (Table 2). Meanwhile, the average rent in the sub-center increases by 28%, 18%, and 21% in the MCP, VMT-tax, and cordon-toll schemes. All available land outside the CBD and the sub-center goes to housing. Under the MCP and VMT-tax schemes, residential land rents rise either in the area between the CBD and the sub-center or in the area near the sub-center, dropping near the city edge. Under the cordon-toll equilibrium, residential land rents inside the cordon area mostly rise, while those outside the cordon line fall.
Figure 1 Land rent distribution under different pricing policies in a polycentric setting (N = 1,048,000, \( \bar{p} = 6 \))

Figure 2 Firm distribution and job densities under different pricing policies in a polycentric setting (N = 1,048,000, \( \bar{p} = 6 \))
Figure 2 shows the distinct tendency toward job decentralization after the implementation of pricing policies. In the no-toll equilibrium, about 55% of jobs locate in the CBD and 45% in the sub-center. The MCP scheme causes about 17% of jobs to move outside the CBD and relocate at the sub-center. Levying a VMT tax is associated with a 5% increase in sub-center jobs, while the cordon toll is associated with an 18% increase in sub-center jobs. Pricing also tends to significantly lower CBD job densities, while raising sub-center job densities (Figure 1): average CBD’s job densities are computed to fall 16% and 5.2% under the MCP and cordon-toll equilibria (versus the no-toll base case), but rise 2.6% in the VMT-tax case (Table 2). This VMT-tax result emerges because, while a number of firms depart the center, those remaining in the CBD become more agglomerated (so the CBD’s area becomes smaller). In addition, the average sub-center job densities rise 29%, 18%, and 21% in the MCP, VMT-tax, and cordon-toll equilibria (versus the base case). Firms leaving the CBD will enhance agglomeration economies in the sub-center areas.

Pricing’s effects on residential densities are similar to the monocentric findings (Verhoef, 2005; De Lara et al., 2013). Policymakers’ and planners’ residential density targets in a polycentric city will presumably need upward adjustment near the city center, and downward adjustment near the city boundary (Figure 3). According to Table 2, the average residential density slightly decreases after an imposition of one of these three pricing policies (around 1%).

Figure 3 Household distribution and residential densities under different policies of CP in a polycentric setting ($N = 1,048,000, \bar{r} = 6$)
CONCLUSIONS

The current urban-economics understanding of the land use effects of road pricing policies comes primarily via traditional monocentric models, with exogenously determined firm locations (all in the CBD). Few researches have developed models to address congestion issues in a continuous, polycentric context (with endogenously determined firm locations). To fill this gap, this work provides new models to reflect both positive agglomeration externalities and negative congestion externalities. Three pricing policies (MCP, VMT tax, and a cordon toll) were examined, alongside a (no-toll) base case; and their land use, travel, and rent impacts were compared under polycentric settings.

The simulation results reveal that all pricing policies deliver more compact city/regional forms. Both the VMT tax and the cordon toll can generate somewhat higher household utility, although their welfare improvements are less than that of the MCP policy, as expected. The VMT tax is predicted to generate a more compact urban form than the MCP policy, by incentivizing firms and households to locate more closely, to reduce commuting distance, while the MCP toll may allow firms and/or households to trade a longer travel distance for less congestion. The compactness effects are also reflected in the findings that all three CP policies can reduce daily travel distance by more than 10% (with results ranging from 10% to 20% varying across settings and policies).

The MCP scheme’s land use patterns are more efficient than those in a free (non-tolled, but congestible) market. In the closed-form polycentric-city setting, efficient land use regulation may promote some job decentralization from the CBD to sub-center locations (since simulations showed more than 17% of the CBD-ring jobs moving to the suburban jobs ring). Regulation recommendations for residential densities in a polycentric city are similar to those for the monocentric setting: raise central-area population densities and reduce edge densities. The VMT tax results are not too far from those of the MCP, and should be much easier to achieve in practice; unfortunately, no pricing policy is trivial to get right, especially in the context of heterogeneous and regions and travel plans that shift regularly (from day to day an year to year). Cordon or area tolls are presently more popular in practice, and a cordon line near the edge of a polycentric city’s central ring may cause significant CBD-area job loss (18% simulated here).

Several limitations still merit further exploration. First, land use for transportation infrastructure is exogenous here; the model could be extended to internalize that infrastructure, as in studies by Wheaton (1998) and De Lara et al. (2013). Second, our model could be extended to consider more than one travel mode (like transit), to reflect differences in congestibility and mode-based pricing impacts. Third, a
dynamic framework should be developed, to enable more complicated urban economic models for
exploring urbanization and suburbanization. Of course, firm and household heterogeneity are also
important to permit. Such imperfections will remain the norm, due to information, technology, and other
limitations never absent from our complex communities.

5 APPENDIX

6 A1: Proof of Proposition 1

7 Problem 1’s Hamiltonian function is as follows:

\[ H_1(n, q, c, \theta_f, \beta_3, \beta_2, \beta_3) = \frac{u(c(x), q(x))}{\lambda(x)} \]

\[ + 2\pi x \left[ \theta_f(x)f(n(x))A(F(x)) - \frac{1 - \theta_t - \theta_f(x)}{q(x)}c(x) - (1 - \theta_t)R_A \right] - t(x)D(x) \]

\[ + \beta_1(x)[c(x) + \eta_h(x)q(x) - w(x) - \bar{y}] \]

\[ + \beta_2(x)[f(n(x))A(F(x)) - w(x)n(x) - r_f(x)] \]

\[ + \beta_3(x)2\pi x \left( \frac{1 - \theta_t - \theta_f(x)}{q(x)} - \theta_f(x)n(x) \right) \]

From the Maximum Principle (Pucci and Serrin, 2007), the first-order conditions are as follows:

(A1) \[ \frac{\partial H_1}{\partial n} = 2\pi x \theta_f(x)[f_n(n(x))A(F(x)) - \beta_3(x)] + \beta_2(x)[f_n(n'(x))A(F(x)) - w(x)] = 0 \]

(A2) \[ \frac{\partial H_1}{\partial c} = \frac{u_c}{\lambda(x)} - 2\pi x \frac{1 - \theta_t - \theta_f(x)}{q(x)} + \beta_1(x) = 0 \]

(A3) \[ \frac{\partial H_1}{\partial q} = \frac{u_q}{\lambda(x)} + 2\pi x \frac{1 - \theta_t - \theta_f(x)}{q^2(x)}c(x) + \beta_1(x)\eta_h(x) - \beta_3(x)2\pi x \frac{1 - \theta_t - \theta_f(x)}{q^2(x)} = 0 \]

(A4) \[ \frac{\partial H_1}{\partial \theta_f} = f(n(x))g(F(x)) + \frac{c(x)}{q(x)}\frac{\beta_3(x)}{q(x)} - \beta_3(x)n(x) = 0 \]

(A5) \[ \frac{\partial H_1}{\partial D} = -\beta_3'(x), \text{ and thus } \beta_3'(x) = t(x). \]

(a) (A1) \[ \rightarrow f(n'(x))F(x)^y - w(x) = 0 \text{ and } f(n'(x))g(F(x)) - \beta_3(x) = 0. \] Then, \( \beta_3(x) = w(x), \) and \( f(n'(x)) = w(x)/g(F(x)), \) so \( n^*(x) = n^*(w(x)) \).

(b) Given \( \eta_h(x) = \frac{\gamma(x) - c(x)}{q(x)}, (A2)/(A3) = c(x) + q(x)u_d/u_c = y(x). \) Thus, given \( u(c(x), q(x)) = \bar{u}, \) one can solve for \( q^*(x) = q^*(w(x), \bar{u}) \) and \( c^*(x) = c^*(w(x), \bar{u}) \).

(c) (A4) \[ \Delta H_1 = r_f(x) - \eta_h(x). \] Thus, if \( r_f^*(x) > \eta_h^*(x) \rightarrow \theta_f^* > 0, \) the larger the \( \theta_f(x), \) the larger the \( H. \) Since \( 0 \leq \theta_f(x) \leq 1 - \theta_t, \theta_f^*(x) = 1 - \theta_t. \) Similarly, if \( r_f^*(x) < \eta_h^*(x), \) then \( \theta_f^*(x) = 0. \) If \( r_f^*(x) = \)}
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\( r_1^* (x), \) then \( 0 < \theta_1^* (x) < 1 - \theta_t, \) and both firms and households will locate at location \( x, \) which is a mixed use area.

(d) From (a) and (A5) we have \( w_3' (x) = t(x). \)

**A2: Proof of Proposition 2**

The search for a second-best optimal congestion toll (e.g., a VMT tax and a cordon toll) is equivalent to solving Problem 1’s optimization by adding constraints on \( t(x) \) and \( w(x) \), as shown in Eqs. (9) and (10), and imposing the following condition:

\[ \int_0^\tilde{x} \tau (x) D (x) \, dx = \epsilon \int_0^\tilde{x} \tau_{mce} (x) D (x) \, dx, \epsilon \neq 1: \]

Eq. (10)’s constraint represents the internalized travel cost, while (9)’s guarantees that the wage gradient equals the marginal private travel cost plus a congestion toll, which is a critical condition for Pareto efficiency. The condition (A6) implies that second-best tolls cannot correct all the aggregate congestion externalities; such tolls cover just an \( \epsilon (\epsilon < 1) \) share of those external costs.

With the condition (A6), the first-order condition of the corresponding Hamiltonian function with respect to \( D (x) \) is as follows:

\[ \beta_3 (x) = t (x) + \rho \sigma \left( \frac{|D (x)|}{2\pi x \theta_t} \right)^\sigma + \tau (x) - \epsilon (1 + \sigma) \rho \sigma \left( \frac{|D (x)|}{2\pi x \theta_t} \right)^\sigma \]

Since \( \beta_3 (x) = w (x) \) still holds here (as noted in Appendix A1), (A7) become the following:

\[ w' (x) = t (x) + \rho \sigma \left( \frac{|D (x)|}{2\pi x \theta_t} \right)^\sigma + \tau (x) - \epsilon (1 + \sigma) \rho \sigma \left( \frac{|D (x)|}{2\pi x \theta_t} \right)^\sigma \]

Comparing (A8) and the Pareto condition on the wage gradient (Eq. 9), one can derive that the optimal toll \( \tau^* (x) \) needs to reflect/correct for \( \frac{1}{1+\sigma} \) of overall congestion externalities.

**ACKNOWLEDGEMENTS**

The authors thank Dr. Alex Anas for his insightful suggestions and Ms. Annette Perrone for all her administrative and editorial support.

**REFERENCES**


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