

1 **ELECTRIC VEHICLE CHARGING STATION LOCATIONS:**
2 **RECOGNIZING ELASTIC DEMAND AND USER EQUILIBRIUM**

3
4 **Yantao Huang**

5 Graduate Research Assistant
6 Department of Civil, Architectural and Environmental Engineering
7 The University of Texas at Austin
8 yantao.h@utexas.edu
9

10 **Kara M. Kockelman**

11 (Corresponding Author)
12 Dewitt Greer Centennial Professor in Engineering
13 Department of Civil, Architectural and Environmental Engineering
14 The University of Texas at Austin
15 kkockelm@mail.utexas.edu
16 Phone: 512-471-0210
17

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23 **ABSTRACT**

24 Battery-only electric vehicles (BEVs) generally offer better air quality through lowered emissions, along
25 with energy savings and security. The issue of long-duration battery charging makes charging-station
26 design and placement key for BEV adoption rates. This work uses genetic algorithms to identify profit-
27 maximizing station placement and design details, with applications that reflect costs of installing,
28 operating, and maintaining BEV service equipment, including land acquisition. BEV charging stations
29 (EVCSs) are placed subject to stochastic demand for charging stations under a user-equilibrium traffic
30 assignment. Random utility theory is used to determine BEV users' station choices, considering
31 endogenously determined (congested) travel times and on-site charging queues. The travel assignment
32 with elastic demand problem is formulated as a convex program and is solved using a modified Frank-
33 Wolfe algorithm.

34 Various realistic costs for power delivery and elastic demand patterns (to reflect driver sensitivities to
35 travel times, wait times and charging costs) are used. Results for the Sioux Falls network suggest that
36 EVCSs should locate mostly in the city center and along major highways. If a time horizon of just 3 years
37 is used, and assuming that just 10% of BEV owners seek to charge en route each day, a user fee of \$6 for
38 a 30-minute charging session is not enough for station profitability (assuming land costs of \$10,000 to
39 \$20,000 per year per station). However, a charging fee of \$10 per BEV delivers a profit of about
40 \$130,000 per station, with just 1.3 cords per station on average in this coarse-network application. Based
41 on sensitivity analysis, EVCS owner profits rise with a longer-term view (e.g., 5 to 10 years), shorter
42 charging durations, more EVCS demand, and larger sites with more cords.
43

44 **INTRODUCTION**

45 Battery-only and plug-in electric vehicle (BEV and PEV) popularity is rising, thanks to environmental
46 and energy benefits, falling battery and vehicle prices, and expanding consumer experience and education.
47 PEVs generally offer better air quality through lowered emissions, along with energy savings and security,
48 and lowered carbon footprints. PEVs are not only less expensive to operate and maintain than
49 conventional (internal combustion engine) vehicles (Tuttle and Kockelman 2012; Simpson, 2006; Noori

1 et al., 2015; Sierchula et al., 2014; Reiter and Kockelman, 2017), but also face lower risk of fire and
2 explosionMcDonald, 2016However, long-duration battery charging (vs. gasoline refueling time) makes
3 charging-station placement and design key for many consumers' BEV adoption decisions (He et al., 2018;
4 Chen et al., 2018; Smith and Castellano, 2015; Shabbar et al., 2017). Thanks to Level 3 or direct-current
5 fast chargers (DCFCs) in thousands of high-traffic commercial locations and along major freeways, many
6 U.S., Chinese, European, and other travelers can now deliver significant charge to their PEVs in 30 min
7 or less. For example, Tesla Model S users can add 170 miles to their batteries in 30 minutes at U.S. Tesla
8 supercharging sites, and BMW i3 and Volkswagen e-Golf owners can charge deliver 60 to 80 miles in 30
9 minutes at similar sites (EVTown, 2015; Fleetcarma, 2018). As of August 2018, Tesla operates 10,738
10 superchargers in 1,333 stations worldwide, including 551 stations in the U.S., 53 in Canada, 11 in Mexico,
11 425 in Europe, and 293 in the Asia/Pacific region (Tesla, 2018). But queues are arising at many stations,
12 while BEV and PEV adoption and use rates are rising (Voelcker, 2013). In addition, many travelers do
13 not have decent access to charging stations at their homes (e.g., those in apartment buildings) or at their
14 work places (EverCharge, 2017). Such travelers will not even purchase a PEV if they do not have good
15 access. More stations are needed to support all these types of demand, and thereby encourage greater
16 adoption and use of BEVs and PEVs. A salient question is where they should be placed.

17

18 **LITERATURE REVIEW**

19 Many economic decisions relate to location choices, of firms and firehouses, schools and transshipment
20 warehouses. The goal is to serve demands efficiently while minimizing system costs or maximizing
21 owner profits. Intensive studies of facility location started in the 17th Century when the Fermat-Weber
22 Problem was introduced (Weber, 1909). Rather recently, researchers have investigated EVCS location
23 choice.

24 Most EVCS research is based on optimizing facility location choice. The approaches used vary in
25 terms of their objective functions, decision types, station types, application sizes, and candidate locations.

26 Candidate locations are normally allowed to be a node in the network (He et al., 2018; Capar et al., 2013;
27 Hanabusa and Horiguchi, 2011; Lee et al., 2014; Sageghi et al., 2014; Ghamami et al., 2016;) or special
28 existing infrastructure, like parking lots and gas stations (Chen et al., 2013; Wang et al., 2010; Shahraki et
29 al., 2015; Huang et al., 2016). Chen et al. (2013) formulated a mixed integer programming problem for
30 optimal EV-charging-station location assignments across the Seattle region, minimizing PEV users'
31 station access costs while penalizing unmet demand. Current parking (and thus BEV-charging) demands
32 were estimated via regression equations - as a function of zone accessibility, local jobs and population
33 densities, trip attributes, and other variables available in most regions and travel surveys. When installing
34 just 80 stations, their algorithms were able to serve 78% of parking demand within 1 mile of a CS cord,
35 with a demand-weighted average access distance of 0.69 miles. He et al. (2018) coded a refueling-
36 location model to identify optimal sites for EVCSs, to maximize the share of completed range-constrained
37 long-distance highway travel across the U.S. (after clustering over 4000 National Use Microdata Area
38 Zones into 196 trip-generation and -attraction points). They estimate that 93% and 99% of the nation's
39 long-distance ground-based passenger-vehicle trips can be completed with vehicle ranges of 200 and 300
40 miles, respectively, using just 100 EVCSs, thoughtfully located.

41 Some researchers do not use facility location optimization methods. For example, Shabbar et al.
42 (2017) studied the estimated demand on a charging station by using a birth-and-death Markov-chain
43 network model. They investigated the number of electric sockets needed, PEV waiting times, and average
44 number of PEVs in queue at each station. Profit-maximizing CS locations, under both budget and routing
45 constraints, were selected using a Grey Wolf Optimization algorithm. They conclude that commercial
46 chargers should be used in early stages of infrastructure implementation, with superchargers enabling
47 higher profits once the number of BEVs increases in the transportation network.

48 Some papers also consider queueing models, within their facility-location frameworks. However,
49 they do not consider network congestion. Li and Su (2011) developed an optimal-cost model for EVCS
50 with a minimum total-cost-of-service system, considering a queueing model at EVCS through waiting
51 probability characteristics. Jung et al. (2014) proposed a bi-level simulation-optimization solution method

1 to simulate a fleet of 600 shared-taxis in Seoul, Korea, considering itinerary-interception and queue delay.
2 Hess et al. (2012) presented a model for electric vehicles and their battery depletion, vehicle mobility,
3 charging stations, and give a solution for the optimal placement of charging stations in a smart city. They
4 considered queueing at EVCS and simulate electric vehicles through a genetic programming method.

5 One paper has investigated the EVCS location problem under network congestion. Lee et al.
6 (2014) proposed a bi-level model to minimize the total failure cost under user equilibrium in route choice
7 with a heuristic algorithm of simulated annealing. They applied their work using the Sioux Falls network,
8 but did not consider queueing at stations or the number of chargers to be installed. Yao et al. (2014)
9 developed a model, to minimize the overall annual cost of investment and energy losses while
10 maximizing the annual traffic flow captured by fast charging stations through a user equilibrium-based
11 traffic assignment model. However, it does not have congestion feedback on the station choices of the
12 PEV users, and it does not consider queues at stations while just maximizing the flow at the station.

13 Overall, this work synthesizes the facility location problem with network congestion and queueing
14 at the charging stations, which is not often seen in current research. Further, this work is able to provide
15 suggestion not only on station locations but detailed design in the stations in terms of the chargers so to
16 minimize the cost over a time horizon while ensuring meeting the charging demand during a day. It
17 provides an extension to most of current EVCS research in these aspects.

18 19 **METHODOLOGY**

20 This section describes the simulation framework used to solve this complex problem, including
21 background assumptions and key equations. Assumptions impact travel behaviors (e.g., demand
22 elasticities), EVCS owner costs (for land/space, equipment, operations and maintenance), and EVCS
23 location options. Initial applications are for an entire day's demand (to reflect all possible revenues).
24 Analysis of more specific times of day will allow for greater congestion feedbacks but ignore demand that
25 will impact profitability.

26 27 **Problem Setting**

28 U.S. PEV purchases rose to 1.1% of light-duty vehicle sales in 2016 (Richter, 2017). By March 2018, this
29 share had increased to 1.6% of U.S. sales (EVAAdoption, 2018). For this reason, PEVs are assumed to be
30 1.6% of total passenger vehicles on the Sioux Falls road network in this paper's applications. But not
31 everyone will want to charge while traveling intra-regionally.

32 Among PEVs, battery electric vehicles (BEVs) require charging (since BEVs cannot use
33 gasoline). And BEV owners may choose to charge en route (rather than at one's home or workplace or
34 shopping destination, for example), like when traveling long distances (typically inter-regionally) or when
35 forgetting to charge overnight (and possibly running out of charge that day). In contrast, plug-in *hybrid*
36 electric vehicle (PHEV) owners may not care to wait to charge en route (since gasoline can be added
37 quickly to PHEVs, generally with little to no circuitry in one's routing choice). Of course, hybrid electric
38 vehicle (HEVs) cannot be plugged in and thus would not be charged en route. BEVs were two-thirds of
39 all US PEV sales in 2016 (Statista, 2018), and perhaps one out of every 20 BEVs in use (5% of BEV
40 users en route) will stop for charging within a city network (Hardman et al., 2018). Due to falling battery
41 prices, rising climate change concerns, and other trends, PEV sales and BEV ownership levels are likely
42 to continue rising, around the world. Moreover, shared self-driving or "autonomous" vehicle (SAV) fleets
43 may be largely electric, giving rise to greater EVCS demand and station location solution needs (see, e.g.,
44 Loeb and Kockelman 2018).

45 PEV or BEV owners' en-route station use and station selections will depend on detour distances or
46 travel times involved and queuing or wait times at desired charging stations. This work assumes that
47 travelers are informed of congestion along all routes and at each EVCS, thanks to navigation technologies
48 and charging-station broadcasts of queues. Random utility theory is used here for station choice: BEV
49 users favor shortest total travel+charging time paths (from origin to final destination, recognizing delays
50 to reach and while charging at EVCSs, for those who wish to charge en route that day). Of course,
51 network congestion also affects network demand, for all travelers.

1 Charging station costs vary by cords provided and power rates delivered. Level 3 stations offer
2 power levels from 20 to 50 KW, and thus can deliver 70 to 100 miles of passenger-car BEV range in 30
3 minutes or less. Smith and Castellano (2015) estimate Level 3 charging stations to cost \$10,000 to
4 \$40,000 per charger and \$2,300 to \$6,000 for parts and labor in their installation, so those values are used
5 here. Blink DC fast chargers were installed at an average price \$22,626, and the lowest registered cost
6 was \$8,500 across 22 regions in U.S. (Idaho National Laboratory, 2015).

7 Different station sites carry different land costs, and some carry different energy costs (by rate of
8 power delivery and high-voltage power-grid-access constraints. Here, land in the city center (1/3 of the 24
9 nodes) is assumed to cost \$20,000 per station per year for a station of maximum three charging spot,
10 while land elsewhere is assumed to be \$10,000 per station, based US average land values of \$510,000 per
11 acre (Florida, 2017). Cord installation costs require labor, materials, permits and taxes, and assumed to
12 cost \$21,000 up front, per site. Variable costs include electric power fees, station maintenance, station
13 signage, equipment updates, advertising and credit-card transaction fees. Electricity is assumed to cost 12
14 cents per kWh, or just \$0.1 per minute of Level 3 charge time (assuming 30 minutes to provide over 24
15 kWh of charge, for 70 to 90 miles of driving range). Station owners may charge by the minute, like EVgo
16 does (20¢/minute charge across many states in U.S.) or per visit (like Blink is doing, at \$7 to \$10, and
17 AeroVironment is doing, at \$7.50 per session) (Berman, 2018). Credit card transaction costs are assumed
18 to be 5% of the fee. The average charging time is assumed to be 30 minutes here, but may vary from 10 to
19 60 minutes or more, depending on customer needs and pricing structure used. Other owner costs, as
20 described above, are assumed to be \$10,000 per station per year.

21 Charging stations can be located at any of the network's nodes (just 24 in this initial application).
22 For the Sioux Falls network, these 24 nodes are also origins and destinations of the region's the 3.6
23 million vehicle-trips each day (rather than having separate zone centroids that connect to the network). If
24 an origin-destination pair requires a BEV user to travel further to the EVCS than one would travel to go
25 directly to the destination, the BEV will decide not to charge en route.

26 On-site power supply is an important consideration for EVCS owners. Direct-current fast-
27 charging (DCFC) or super-charging and hyper-charging (like 24 mi/20 minutes at 24kW, 50 mi/20
28 minutes at 50kW and 90mi/20minutes at 90kW with 208/480VAC 3-phase charger) generally require
29 relatively big batteries on site, to avoid overtaxing the grid (and causing brownouts) – and to avoid very
30 high power pricing (by grid managers) (Smith and Castellano, 2015). Putrus et al. (2009) investigated the
31 impacts of PEV charging on power distribution networks in the US that heavy PEV deployment and peak
32 charging can create power-delivery issues for existing power networks, including voltage imbalance and
33 transformer loss. The situation can be ameliorated if fast-charge stations are reasonably distributed across
34 the grid, relative to power generation and transmission stations. Bullis (2013) argues that public fast-
35 charging stations for cars and trucks should not impact the grid much because our commercial grids have
36 transformers and other equipment sized to accommodate large loads, for big businesses, apartment
37 buildings, and so forth. To avoid excessive power demand in any one location, a maximum cord count of
38 3 is assumed at each station.

39 Queues may still be observed at a station. Since static traffic and station assignment algorithms
40 are used here (assuming stationing demand and supply conditions for network links and station cords),
41 BEV users beyond 80% of any EVCS's capacity are assumed to wait some period of time for a charging
42 cord or space to become available. Any EVCS demand levels that exceed station capacity are not able to
43 charge their vehicles, so such revenues are lost.

44 **Problem Formulation**

45 Using the above assumptions and ideas, the profit-maximization problem for charging station provision
46 across a town, city or region can be stated as follows:
47

$$\max_y \left(\sum_{\forall (r,v) \in Z^2} d_c^{rv} (p\varepsilon - p_e) - f_m - \sum_{\forall v \in V} f_v y_v \right) T - \sum_{\forall v \in V} N_v f_v^c y_v \quad (1)$$

$$s.t. \quad y_v = \{0,1\} \quad \forall v \in V \quad (2)$$

$$d^{r's'} = d^{rs} + d_c^{rv} + d_c^{vs} \quad \forall (r,v) \in Z^2 \quad (3)$$

$$N_v = \min \left\{ \frac{\sum_{\forall r \in V} d^{rv}}{S}, M \right\} \quad \forall v \in V \quad (4)$$

$$\sum_{\forall r \in V} d_c^{rv} = N_v S \quad \forall v \in V \quad (5)$$

$$\text{where } d^{rv} = \arg \min_{x,h,d} \sum_{(i,j) \in A} \int_0^{x_{ij}} t_{ij}(x) dx - \sum_{(r,s) \in Z^2} \int_0^{d^{r's'}} D^{-1}(\omega) d\omega \quad \forall (r,v) \in Z^2 \quad (6)$$

$$s.t. \quad x_{ij} = \sum_{\pi \in \Pi} h^\pi \delta_{ij}^\pi \quad \forall (i,j) \in A \quad (7)$$

$$\sum_{\pi \in \Pi} h^\pi = d^{rs} \quad \forall (r,s) \in Z^2 \quad (8)$$

$$1 \quad h^\pi \geq 0 \quad \forall \pi \in \Pi \quad (9)$$

$$d^{r's'} \geq 0 \quad \forall (r,s) \in Z^2 \quad (10)$$

2 In this formulation, Z is the travel demand matrix of all travelers, T is the time horizon, A is the set of
3 directed links in the road network, Π is the set of all used paths (for all OD pairs) in the network, V is the
4 set of nodes in the network (just 24 for the Sioux Falls network), h^π is the flow of travelers choosing
5 path π , D is the demand function for each OD pair (based on shortest-path travel times), x_{ij} is the vehicle
6 flow on link (i,j) , and $t_{ij}(x)$ is the travel time performance function for link (i,j) at flow level x .

7 Equations (1) to (5) are the facility location portion of this EVCS problem. Equation (1) is the profit
8 function of the EVCS owners, and thus the sum of revenues collected from BEV owners that decide to
9 stop and charge their vehicles, minus all other costs (for site rental, equipment provision, operations, and
10 maintenance). t is the analysis time horizon of the investment of building EVCS (e.g., 3 years in the initial
11 Sioux Falls application), ε is the share of revenues that owners keep after credit card fee, p is the price or
12 fee paid by BEV owners that charge at a station en route, p_e is the price of electricity per vehicle charged
13 (10ct/min x 30 min/vehicle charge = \$3/vehicle), f_v^c is the fixed price of each cord (and its installation),
14 f_v is the land or site fees per year per station (much like a mortgage payment), f_m is annual station
15 maintenance cost, d_c^{rv} is the supply of BEVs from node r to node v (i.e., vehicles that seek to charge their
16 batteries after considering a path's EVCS total travel time situation), S is the daily flow a cord can serve,
17 and N_v is the number of cords ultimately provided at station v . The $N_v S$ is the final capacity of a station
18 that can serve daily at node v .

19 The key decision variables are the indicators y_v in Eq. (2): $y_v = 1$ when node v is used/chosen for
20 an EVCS, and 0 otherwise. Another set of decision variables $\{d^{r's'}\}$ are the modified demands from
21 nodes r to nodes s . These are obtained via the network updating process, which returns the OD matrices
22 for both BEVs who would need to charge en route ($d_c^{rv} + d_c^{vs}$) and those do not (d^{rs}), which consist of
23 conventional vehicles, BEVs that do not want to charge and BEVs that would like to charge but decide
24 it's too time-consuming to charge en route. BEVs who would need to charge en route can charge at

origins or destinations when the corresponding origin or destination is opened as a charging station. The number of cords provided at each EVCS (N_v) is also an important decision variable.

Eq. (3) updates and adjusts network trip tables to reflect EVCS use, with the station choice incorporated: d^{rs} is the demand by conventional vehicles from origin r to destination s , d_c^{rv} is the demand of BEVs from origin r to station v , and d_c^{vs} is the demand of BEVs from station v' to destination s . The demand matrix obtained in Eq. (3) is used for the traffic assignment procedure.

Eqs. (6) through (10) solve the user-equilibrium traffic assignment problem with elastic demand and EVCSs present in the network. Their solution delivers link flows and path demands, which are used as inputs to the overall problem's primary objective function: BEVs that stop to charge en route impact Eq. (1) - the profit equation. Based on Eq. (6)'s BEV charging demands, optimal station cord counts are determined. Cord counts are also limited by station sizing (which is assumed fixed here, but can be varied in an expanded formulation), and the maximum (due to size or demand) is shown in constraint Eq. (4). The optimal cord-count decision values (which maximize EVCS owner profits) are shown in Eq. (5), as N_v values.

Travelers are permitted to shift to other modes (or destinations or curtail trip-making altogether) when congestion increases. Thus, demand between each OD pair is elastic, as a function of that OD pair's shortest-path travel time. Sheffi's (1985) an exponential demand function is used here, as follows:

$$D_{rs} = D_{rs}^0 \times e^{-\alpha(\beta_{rs}^t u_{rs} + \beta_{rs}^l l_{rs})}$$

Here, t_{rs} is the shortest path travel time from origin r to destination s , l_{rs} is the shortest-path distance between r and s , β_{rs}^l is vehicle operating cost per mile (40 ct/mile), and $\beta_{rs}^t = \beta_{rs}^l \times VOTT$ is the cost of shortest-path travel time, u_{rs} , where the value of travel time ($VOTT$) is assumed to be \$10/hour. A values of $\alpha = -0.01$ is used here so that demand is not too elastic with respect to travel time and distance; a more accurate calibration of this α value can deliver more realistic results.

EVCS SOLUTION

This section provides the solution methods to the NP-hard facility location problem. The overall problem is solved using a genetic algorithm (GA) approach, and a modified Frank-Wolfe algorithm is used to solve the traffic assignment portion. A station-choice algorithm is also introduced to handle the EVCS choice of the BEV users.

Genetic Algorithm

A GA is a metaheuristic for (approximately) solving complex optimization problems, inspired by the process of natural selection (Mitchell, 1998). GAs rely on bio-inspired operators, including mutation, crossover, and selection across decision variables' values. Here, the GA solution is a combination of the network's node values (0's and 1's), indicating which nodes will be selected to host EVCSs. One updates each generation of solutions by selecting the best solutions (those delivering higher profit levels) from earlier generations, and randomly mutating some values from 0 to 1 or from 1 to 0, or crossing over/exchanging a section of one set of binary values with another, in current-solution vectors. Each iteration's suggestion of EVCS locations results in an updated traffic assignment (reflecting user equilibrium with elastic demand) and an updated cord count recommendation (to maximize profits, given current station locations). The stopping criterion used here is 100 iterations, due to the significant computational load (about 20 minutes per iteration, including 80 traffic assignments), and Figure 1 illustrates the solution process. The design of the stations as well as their locations are determined by the best solution among these 100 iterations.

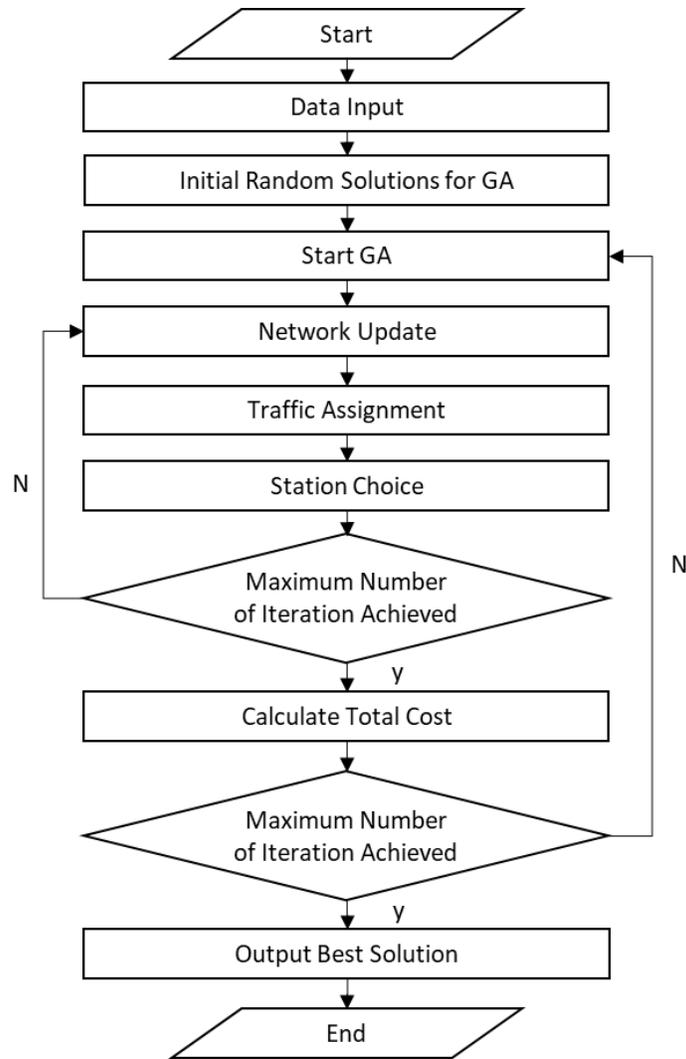


FIGURE 1 Genetic algorithm process used

Here, GA parameters are determined after testing a few sets of parameters to obtain a reasonable computation speed while ensuring the accuracy. The GA population consists of 8 EVCS assignment sets, and thus each GA iteration relies on 8 different traffic assignment solutions. The selection rate assumption used is 0.5, implying that 4 solutions are used to generate 4 solutions after crossover. All binary elements of these 8 solutions have a probability of 0.05 to mutate from 0 to 1 or from 1 to zero.

Traffic Assignment Algorithm

Static traffic assignment is a traditional network problem (Sheffi, 1985). It seeks a user equilibrium (UE) so that minimum travel time paths are achieved for travelers between all OD pairs (Wardrop, 1952). The Frank-Wolfe algorithm is used here, and both those seeking to charge BEVs en route and those not seeking to charge are assigned simultaneously. The relative gap used here, as the stop criteria for convergence, is set to 0.0001. Smaller values will enable clearer convergence, but computing demands on supercomputers will limit this choice.

Station Choice Algorithm

Equilibrium network flows can shift a fair bit, at least for BEV owners seeking to charge en route, due to different EVCS siting decisions. Those who stop to charge en route create two sub-trip tables, from origin to EVCS and from EVCS to final destination. Station choice happens for those trips who share the same

1 OD pair (r, s) . EVCS choice for station v' among all potential station V is determined using a logit choice
 2 model:

$$3 \quad d_c^{rv'} = d_c^{v's} = \frac{\exp(t_{rv'} + t_{v's} + \rho \times w_v)}{\sum_v \exp(t_{rv} + t_{vs} + \rho \times w_v)} \quad \forall (r, s) \in Z^2, \forall v \in V$$

4 where $d_c^{rv'}$ is the demand of BEV users (who need to charge en route) who travel from origin r to station
 5 v' to charge for a trip from origin r to destination s , which equals to the demand $d_c^{v's}$ from station v' to
 6 destination s , $t_{rv'}$ and $t_{v's}$ are the congested travel time from origin r to station v' and from station v' to
 7 destination s , respectively, w_v is expected/average wait time at station v , and ρ is the relative importance
 8 of wait time (in proportion to travel time). Therefore, an EVCS choice depends on the detour time ($t_{rv'} +$
 9 $t_{v's}$) and the waiting time at the station. The total demand arriving at a station v' is

$$10 \quad \sum_{(r,s) \in Z^2} d_c^{rv'}$$

11 For each EVCS siting decision (as given by the GA described above), traffic assignment is first conducted,
 12 and then EVCS wait times are ascertained. Station wait times impact EVCS choices by those wishing to
 13 charge their BEVs en route. Another procedure is conducted to obtain a stable solution of station choice:
 14 under the EVCS patterns, people would no longer shift their station choice, considering the congestion at
 15 the stations. The updated flows to each EVCS are a convex combination of prior BEV assignments to
 16 EVCSs and the newest set of assignments, using the method of successive averages:

$$17 \quad d_c^{rv'} = \frac{1}{\gamma} \times d_{c,new}^{rv'} + \frac{\gamma - 1}{\gamma} \times d_{c,old}^{rv'} \quad \forall (r, v) \in Z^2, \gamma = 2, 3, 4 \dots,$$

$$18 \quad d_c^{v's} = \frac{1}{\gamma} \times d_{c,new}^{v's} + \frac{\gamma - 1}{\gamma} \times d_{c,old}^{v's} \quad \forall (r, v) \in Z^2, \gamma = 2, 3, 4 \dots$$

19 The procedure should iterate until assignments of BEV users to EVCS sites are stable. However, since
 20 there also is a traffic assignment procedure running alongside, this calculation is run just 10 times, to
 21 obtain a relatively stable pattern of station assignments to BEV users, while speeding up the long
 22 computing times.

23

24 SCENARIO TEST

25 Data Input

26 The Sioux Falls' network data were developed by LeBlanc et al. (1975), with 24 nodes and 76 links, as
 27 shown in Figure 2. The OD matrix contains 3.6 million vehicle-trips per day. The Bureau of Public Roads
 28 (TRB, 2000) link-performance function is used here ($t = t_0 * \left(1 + \alpha * \left(\frac{v}{c}\right)^\beta\right)$, where v is the traffic
 29 volume, c is the capacity, t_0 is the free flow travel time, α and β are empirical coefficients), with
 30 parameters $\alpha = 0.84$ and $\beta = 5.5$, to reflect a true capacity values.

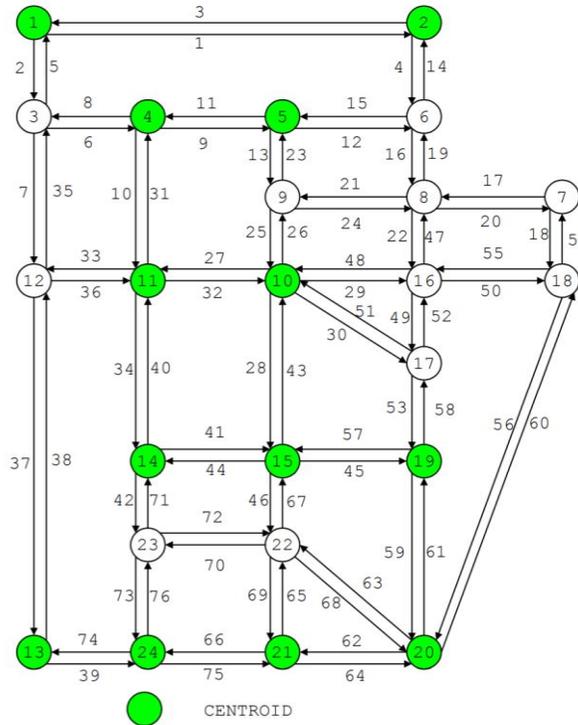


FIGURE 2 Sioux falls network

Scenarios Analyzed

Of course, different cost and behavioral settings will generally deliver different EVCS siting and sizing decisions. Table 1 describes the various levels of key assumptions used to define the 15 distinctive scenarios tested here. The variety in these settings allow one to examine how BEV owners’ demand levels and EVCS owners’ costs, fee choices, cord count constraints, and time horizons should impact decision-making, and to ascertain whether there is some meaningful stability or robustness in profit-maximizing decisions. As shown in Table 2, Table 1’s Level 2 values are pivoted off of, as a Base Case, to provide 15 scenarios total (by testing values shown in Table 1’s Level 1 and 3 columns).

TABLE 1 Scenario Design Settings

	<i>Level 1</i>	<i>Level 2 (Base Case)</i>	<i>Level 3</i>
Time horizon (years)	2 yr	3	4
Max cord # per station	1 cord	2	3
Charging time (minutes)	20 min	30	40
Fee per charge (\$)	\$6/charge	8	10
%BEV users needing to charge	5% of BEVs	10%	100%

EVCS Siting and Design Results

Table 2 shows the results of these 15 scenarios. Total number of stations developed (to maximize EVCS investor profits) ranges from 8 to all 24 of this coarse network’s nodes. The typical investment decision is a rather reliable 17 sites, representing a spatially-extensive investment. However, with a less coarse network (i.e., a greater number of nodes) and more fixed costs in setting up each site (and no constraints on expanding each site), a much lower ratio of sites per network node is expected to be optimal (e.g., 1 to 10% of nodes, rather than 33 to 100% and averaging about 70%).

Different scenarios can be compared to the base case, which is shown in Table 2’s second row. The base case is defined as allowing no more than two cords per station, assuming a 30-minute charge time, and

1 carrying an \$8 fee for each BEV charging event, over a 3-year investment period with 10% of BEV
 2 owners hoping to charge en route. With a longer (or shorter) time horizon, profits rise (or fall) roughly
 3 \$0.5 million per year for this very specific network case. There are no queues expected in this static
 4 assignment setup under these first 3 scenarios (3-year base case tested with 2- and 4-year periods),
 5 assuming EVCS demand remains stable over time. However, PEV and BEV ownership levels are
 6 growing, most everywhere in the world (Schefter and Knox, 2018; Kiser et al., 2018). With increasing
 7 BEV use, especially by those who do not have good charging options at home or work or school, queues
 8 can emerge, and 2 or 3 cords per station may be inadequate, especially at central stations, in more complex
 9 and realistic networks, where relatively few nodes are assigned an EVCS. Fortunately, advances in power
 10 delivery and on-site battery storage may also increase power delivery rates, thereby reducing charging
 11 durations for similar range delivery. The algorithms developed here can handle such settings, assuming
 12 computing power exists for those cases.

13 If no more than one cord can be provided at each station, queues are expected at many stations and
 14 times of day, averaging 13.4 BEVs, stifling profitability. Allowing 3 cords per station results in fewer
 15 EVCS sites needed. Queues also emerge if one alters the base case scenario to have 40 minute charging
 16 times. Longer turnover between BEV customers results in lower revenues and profits (assuming the same
 17 fee is used, per charging event).

18 Of course, fee or pricing decisions also affect station siting and sizing decisions. Higher fees lead
 19 to more stations being opened across the network and more cords being added to many sites, to maximize
 20 EVCS owner profits. A fare of \$6 per charge is not enough to return positive profits, and also results in
 21 queuing at many stations. More BEV owners deciding to charge en route, combined with rather low caps
 22 on cord counts, delivers more queues, but also a healthy return on investment.

23
 24 **TABLE 2 Station Results for Different Scenarios**

Time Horizon (years)	Max # Cords	Charging Time (minutes/charge)	Fee (\$ per charge)	% BEV Owners Seeking Charge En Route	Profits (\$ Million)	# Queued BEVs Expected Per Day	Total # Cords	Profit Max'g # Stations
2 yrs	2 cords	30 min	\$8/charge	10%	\$0.49M	0 BEVs	24 cords	17 sites
3					0.96	0	22	17
4					1.44	0	23	16
3	1	30	8	10%	0.86	13.4	16	16
	2				0.96	0	22	17
	3				1.13	0	20	11
3	2	20	8	10%	1.97	0	16	15
		30			0.96	0	22	17
		40			0.15	11.7	19	12
3	2	30	10	10%	2.20	0	25	17
			8		0.96	0	22	17
			6		-0.05	11.3	13	8
3	2	30	8	5%	0.66	0	18	13
				10%	0.96	0	22	17
				100%	5.06	381.1	48	24

25 Table 3 specifies the profit-maximizing nodes chosen for EVCSs – by listing their cord counts. Nodes not
 26 assigned a station or cords are denoted “-”. Important spatial patterns can be seen when combining these
 27 values with Figure 2’s network. For example, when cord count can be as high as three, only nodes 10 (at
 28 the city’s center) and 20 (on the southeast corner of the network, which near highway with a park there)

merit this kind of capacity, while the optimal station count falls by 5. Essentially, if power supply (and site space) permits, several stations with several cords can profitably compensate for many fewer station sites (though BEV owners may not prefer such setups, due to longer detours to arrive at an EVCS). Stations 11 and 16 consistently play important roles, while nodes 6 and 23 are less relevant. Interstate Highway 29 has nodes 1, 3, 12 and 13 as major interchanges, with each consistently receiving a station (though not always with maximum cord counts). If a larger region and external trips had been permitted, station and cord counts on such perimeter highways would presumably rise.

As expected, longer charging times and higher shares of BEVs seeking to charge en route result in more cords or stations being provided. Interestingly, three cords seems like a common investment decision, given that year US Department of Energy (2018) statistics for year 2018 suggest that the US has 51,766 charging outlets (all types of electric vehicle chargers) across 18,489 public charging stations (thus averaging 2.8 cords per station). Of course, most of those are not Level 3 or DCFC charging stations, but many may eventually upgrade. Maximum profits appear to emerge under longer time horizons with more cords permitted. Of course, competition is always a possibility, and a single owner is unlikely to control all sites.

TABLE 3. Optimal Cord Counts by Network Node (EVCS Site) across 15 Scenarios

Station Index (Fig. 2)		Profit-Maximizing # Cords Placed across the 24 Potential EVCS Sites																							
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Time Horizon (years)	2	2	1	1	1	-	-	-	1	-	2	1	1	1	1	2	2	2	-	-	2	1	2	1	-
	3	2	1	1	1	-	-	-	1	1	2	2	1	1	1	2	2	-	1	-	1	1	1	-	-
	4	2	1	1	-	1	-	-	2	-	2	1	2	1	1	-	2	2	-	-	2	1	1	1	-
Max # Cords	1	1	1	1	1	-	1	1	1	1	1	1	-	-	1	1	1	1	-	-	1	-	1	-	-
	2	2	1	1	1	-	-	-	1	1	2	2	1	1	1	2	2	-	1	-	1	1	1	-	-
	3	-	-	-	-	-	-	-	-	1	3	2	1	1	1	2	2	-	-	-	3	-	2	-	2
Charging Time (min per charge)	20	-	-	-	1	-	-	1	1	1	2	1	1	1	1	1	1	1	-	1	1	-	1	-	-
	30	2	1	1	1	-	-	-	1	1	2	2	1	1	1	2	2	-	1	-	1	1	1	-	-
	40	-	1	2	-	2	-	1	2	-	2	-	2	2	-	-	1	-	-	-	1	1	-	-	2
Fare (\$ per charge)	10	2	-	1	2	-	-	1	-	1	2	2	1	1	1	2	2	2	-	1	1	-	2	-	1
	8	2	1	1	1	-	-	-	1	1	2	2	1	1	1	2	2	-	1	-	1	1	1	-	-
	6	2	2	-	1	-	-	2	-	1	2	-	2	1	-	-	-	-	-	-	-	-	-	-	-
% BEV Users seeking charge	5%	0	1	0	0	0	0	0	0	2	2	1	1	1	1	0	1	2	0	0	1	1	2	0	2
	10%	2	1	1	1	-	-	-	1	1	2	2	1	1	1	2	2	-	1	-	1	1	1	-	-
	100%	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2

CONCLUSIONS

This work design, applies, and iteratively solves a complex charging station location-and-sizing problem to maximize EVCS owner profits across a region for PEV owners who wish to charge en route. The model is the first to allow for congested-travel and congested-station feedback into travelers' route choices and BEV owners' station choices, as well as elastic demand for all road users (BEVs and non-BEVs). The method's results deliver specific station locations and cord-count details, while reflecting all Level 3 charging station costs. The Sioux Falls application results suggest that profit-maximizing EVCS sites are generally located largely in the city center and alongside major highways. When assuming that 10% of the town's PEV users (or just 0.16% of all vehicle-trips) will seek en-route charging, a 30-minute charging fee of \$6 is not enough to deliver a profit in 3 years time (with power-delivery costs of 12¢/kWh, land rental costs of \$10,000 per station per year, and O&M costs of \$10,000 per station per year). In contrast, \$8 and \$10 charging fees deliver reasonable profits. Providing no more than 2 cords at each station can accommodate most PEV owner demands for en-route charging, but lowers EVCS profitability. Profits rise when operators take a longer-term perspective, but this complicates the solution, requiring

1 longer computing times (on fast supercomputers). Shorter charging sessions, higher fees, and/or allowing
2 for more cords per site also increase profits, everything else constant.

3 Enhancements to this work may include calibration of the demand-elasticity parameter and wait-
4 time (at charging stations) disutility, through survey work and actual station use and queuing observations.
5 A much larger network application would also be helpful, with station counts limited and station location
6 costs much more variable over space and position. However, for larger, more realistic applications, faster
7 algorithms will key to achieving profit-maximizing user equilibria in reasonable computing times.
8 Improved UE algorithms could be path-based (Jayakrishnan et al., 1994) or bush-based (Dial, 1971), as
9 they track paths or bushes instead of links in the algorithm. The GA-solution assumptions and parameters
10 used here may also be improved through machine learning techniques. Site specific variations will also
11 exist, on power delivery rates and costs, land rental and site maintenance costs, and cord supply costs.
12 And pricing or fees may work best per minute and per kWh delivered, further complicating the solution
13 process. Finally, not all PEVs can use the same charging facilities yet - due to cross-manufacturer design
14 incompatibilities. Currently, nearly all BEVs that offer DCFC capability in the U.S. use one of three
15 standards: CHAdeMO, Combined Charging System (CCS), or a Tesla Supercharger. Experts expect that
16 all three standards will continue coexisting in the U.S. and many other places (McDonald, 2016).
17 Interoperability, for maximum demand uptake at EVCS sites, may require more investment expense than
18 assumed here, especially if one wishes to reduce charging durations to become more competitive with
19 conventional vehicle refueling. Fortunately, technologists are tackling such issues, and prices continue to
20 fall, making EV futures more and more likely.

21 **AUTHOR CONTRIBUTION STATEMENT**

22 The authors confirm the contribution to the paper as follows: study conception and design: Huang, Y.;
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