CHANGES IN THE FLOW-DENSITY RELATION DUE TO ENVIRONMENTAL, VEHICLE, AND DRIVER CHARACTERISTICS

by

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ABSTRACT:

This research explores the idea that weather conditions and driver- and vehicle-population characteristics affect a homogenous roadway segment's flow-versus-density relationship. Third-order-polynomial regressions of flow on powers of density interacted with a variety of explanatory variables suggest that driver, vehicle, and environmental attributes significantly influence the flow-density relation and conform in substantial part with intuitive expectations. For example, higher flows are predicted across most densities for more mature and more male traveler groups, as well as for non-rainy conditions with fewer long vehicles and trucks. Moreover, under highly congested conditions, braking is associated with slightly higher flows than those predicted for accelerating vehicles.

KEY WORDS:

Traffic models, flow and density

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INTRODUCTION:

A critical assumption of many continuum models of traffic behavior is the uniqueness of a flow-density relationship for any homogenous section of roadway, under stationary flow conditions. A generally unstated qualification of this assumption is that the drivers and their environment are held constant.

Yet, if one considers the influence of, for example, trucks and other large vehicles on flow conditions, one may expect, *a priori* and *ceteris paribus*, that increased vehicle lengths constrict flow due to such things as reduced sight distances and diminished opportunities for maneuvering. So, even though the count density of vehicles may remain the same, flow is inhibited. One can test this hypothesis by acquiring data on vehicle lengths in traffic and including these descriptors as explanatory variables in a flow-versus-density regression.

Environmental conditions, such as sunny versus rainy and dawn/dusk versus midday-sun versus nighttime, and other explanatory variables can be controlled for as well. Simple tests of significance on these variables' coefficients help indicate whether such factors influence traffic flows, and the resulting elasticity estimates suggest strength of influence.

This research will explore the idea that weather conditions and driver- and vehicle-population characteristics affect a homogenous roadway segment's flow-versus-density relationship. After a discussion of research related to this topic and a description of the data sets used, statistical examinations of multiple hypotheses based on a wide range of flow-density data are performed and conclusions are made.

RELATED RESEARCH:

The best-known early descriptions of the bivariate relationships assumed to underlie traffic density "k" (vehicles per length of lane), space-mean speed "v_s", and flow "q" come from Greenshield's 1935 linear density-speed and parabolic flow-density and flow-speed models (*1*). Although based on minimal data, Greenshield's models offer strong first-order approximations to the actual relationships. In fact, the 1965 version of the *Highway Capacity Model* uses the parabola to define roadway flow conditions. Over the past 30 years, researchers have proposed a variety of functional forms and have fitted these to data (see, *e.g.*, *2*, *3*, *4*, *5*, *6*, *7*, *8*), yet there remains significant debate as to the form and even the continuity of the relations (*e.g.*, *9*, *10*, *11*).

Notably missing from the debate on the shape of the bivariate relations are effects on the shape due to variations in drivers, their vehicles, and their environment. May (*12*) comments on how *capacity* may change with merging or incidents, and the *Highway Capacity Manual* (*9*) quantitatively acknowledges the effects of very general driver and vehicle factors (*e.g.*, commuter vs. non-commuter and heavy-vehicle fraction) on capacity. However, no mention is made of the fact that these and other factors may affect the *entire* flow-density-speed relation. When flow, speed, and density data are plotted and when a single relationship is sought to link these, there is an implicit assumption that other factors have not changed. In fact, there are plenty of non-random factors that may be varying across observations and adding scatter – and one can control for several of these. This is the purpose of this research.

MODEL CONCEPTION:

There are a variety of manners in which one may approach a statistical analysis of the flow-density relation; the results then imply flow-speed and density-speed relations by way of Lighthill and Whitham's equation (flow equals density times space-mean speed, *13*). A linear-in-parameters regression model of flow versus density interacted with other, measurable and continuous explanatory variables is the model applied here; but it is worth first mentioning what a behaviorally-based hypothesis would imply for function form. One hypothesis is that the observed flow-density relationship is just a weighted average of the different drivers' individual flow-density curves. In fact, such a hypothesis makes considerable sense under *uncongested* conditions when vehicle interaction is not so important and each driver picks his or her own comfortable speed. Under such a situation a regression of flow on density interacted with proportions of distinct driver/vehicle classes is likely to yield strong estimates of free-flow speeds for such classes. Equation 1 illustrates this relation.

$$
\begin{aligned}
&\text{Flow}_{i,\,Uncongested} = q_{i,\,Uncongested}(x,t) = v_{\max,i}k_i(x,t) + \varepsilon_i(x,t),\\
&q_{Total,Uncongested}(x,t) = \sum_i q_i = \sum_i v_{\max,i}k_i + \varepsilon_i(x,t),\\
&q_{Total,Uncongested} = \sum_i v_{\max,i}Proportion_i k_{Total} + \varepsilon_i(x,t), \qquad \textbf{Equation (1)}\\
&\text{where } v_{\max,i} = \text{Free-flow speed for driver / vehicle class "i"},\\
&\text{and } k_i = \text{ Density of vehicles per unit length of roadway.}
\end{aligned}
$$

Under uncongested conditions, traffic data suggest that speeds are relatively constant (*i.e.*, the flow-density relation is linear, as illustrated in Figure 1 and in data presented in *2*, *3*, *4*, *5*, *6*, and *7*), so modeling total flow as a linear function of density interacted with proportions appears to be quite reasonable. However, as one moves to analysis of traffic data under *congested* conditions, speeds are no longer constant for differing densities and the relationship becomes more complex. One hypothesis is that under a congested regime, a class of driver's spacing is a linear function of the speed of all vehicles. Scatterplots of the inverse of density (*i.e.*, average spacing) versus speed using the data at hand and elsewhere (*e.g.*, *14*) suggest such a simple relation is reasonable for the congested regime. Unfortunately, the resulting functional form for flow versus density is highly non-linear in class proportions. Equation 2 describes this situation.

$$
\begin{aligned}
&\text{Under Congested Conditions:}\\
&Spacing_i = s_i = a_i + b_i v = \ Car\ front_i\text{-}to\text{-}front\ of\ previous\ vehicle,\\
&and\ Density_i = k_i = p_i k = \frac{p_i}{\bar{s}} = \frac{p_i}{\sum_i p_i s_i} = \frac{p_i}{\sum_i p_i(a_i + b_i v)},\\
&so\ v = \left(\frac{p_i}{k_i} - \sum_i p_i a_i\right) \Big/ \left(\sum_i p_i b_i\right), where\ p_i = proportion\ of\ drivers\ of\ type\ i. \qquad \textbf{Eq. (2)}\\
&Thus,\ q_{Total,Congested} = vk = \frac{v}{\bar{s}} = \frac{v}{\sum_i p_i s_i} = \frac{v}{\sum_i p_i(a_i + b_i v)} = \frac{v}{\bar{a} + \bar{b}v},\\
&so\ q_{Total,Congested} = \left(\frac{p_i}{k_i} - \sum_i p_i a_i\right) \Big/ \left(\sum_i p_i b_i * \sum_i p_i \frac{1}{k}\right) = \left(1 - k\sum_i p_i a_i\right) \Big/ \left(\sum_i p_i b_i\right).
\end{aligned}
$$

Notice how a given vehicle/driver mix implies a linear (negatively sloped) congested flow-density relation, which is consistent with much of the data and theory (*e.g.*, the "inverted lambda" hypothesis). However, this function is non-linear in the proportions data making it more difficult to estimate than that of the uncongested case just described. And the sharp contrast in expected behavior and, therefore, functional form across the congested and uncongested traffic regimes effectively implies two distinct regression models, which are difficult to statistically link in a continuous and relatively flexible fashion. Moreover, there is richer information available for use as explanatory variables than just the proportions of broad-based traveler types. For these reasons, the models described in Equations 1 and 2 are not strictly followed here. The following section provides a description of the data used, and the sections on statistical methodology and results describe the actual analysis employed.

DATA:

The data set analyzed for this research consists of freeway traffic observations from loop detectors, characteristics of automobile users by time period, and levels of precipitation in the vicinity of the freeway section studied. The set of variables is provided in Table 1, and the data are discussed in more detail here.

Traffic Data:

Count, occupancy, and speed data were obtained from the Freeway Service Patrol Project (*15*) for paired induction-loop detectors on Interstate Highway 880's five-lane northbound section in Hayward, California, about 20 minutes south of Oakland. All data made publicly available come from weekdays in February and March of 1993, between the hours of 5 and 10 am and 2 and 8 pm; however, only a couple days' data for Lane 2, the second innermost lane of the five in this section, are analyzed here. This second lane is thought to be substantially shielded from merging effects from upstream and downstream ramps while not being as specialized and exclusive as Lane 1 (which is an HOV lane during the rush hours, from 6 to 9 am and from 4 to 7 pm). Of the five lanes, Lane 2 is not too unusual, except that it is the most heavily used (in total number of vehicles) of all five lanes. This lane carries vehicles typically at high speeds, with 50% of the data points exhibiting speeds over 100 kph.

A quick look at the flow-density data for Lane 2 gives one a very good idea of the minimum speed which can be considered as "free flow" in these data; 77 kph forms a lower boundary for the distinctly straight and dense band of uncongested observations, as visible in Figure 1. There are not many data points falling below this speed; in fact, only 14% of the 24 days' data exhibit unstable flow/congestion.

So that the regression models would not depend too greatly on uncongested data, the first two days in the data set which have distinct weather patterns and a significant share of congested-flow data points were chosen for analysis. These two days represent a rainy Friday and a dry Thursday, and 24% of their roughly 2,600 observations exhibit congested conditions.

The explanatory variable of density was derived very carefully, *without* dividing flow by space-mean speed. Inconsistent coefficient estimates would be the result of using explanatory variables calculated using the model's dependent variable (*i.e.*, flow in these models). Duncan (*16*, *17*) warns of this statistical difficulty. To avoid this, occupancy and speed data, along with covariance of speed and vehicle length, were combined to determine density, as illustrated in Equation 3.

$$Occupancy = \frac{\sum \left(t_i \frac{1}{l_i}\right) l_i}{T} = \frac{\sum \frac{1}{v_i} l_i}{T} = \frac{N}{T} E\left(\frac{1}{v_i} l_i\right) = Flow * \left(\frac{E(l)}{v_{space}} + COV\left[l, \frac{1}{v}\right]\right),$$

$$\left(since\ E(XY) = E(X)E(Y) + COV(X,Y)\ \&\ E\left(\frac{1}{v_i}\right) = 1/v_{space}\right) \qquad \textbf{Eq. (3)}$$

$$so, \frac{Flow}{v_{space}} = Density = \frac{Occupancy}{\bar{l} + v_{space} COV\left(l, \frac{1}{v}\right)} \approx \frac{Occupancy}{Avg.Length}\ since\ COV\ is\ negligible\ here,$$

$$where\ l_i = vehicle\ i\text{'s}\ length,\ t_i = time\ of\ vehicle_i\ over\ detector,\ and\ v_i = speed\ of\ vehicle_i.$$

As indicated, the covariance between lengths and the inverse of speeds of individual vehicles is negligible here, exhibiting a sample correlation of just -.00375 over the 24-day study period. For lanes with a greater truck population, this relation is expected to be stronger as well as positive, since, for example, trucks face lower speed limits than passenger cars and may generally travel at lower speeds, in order to avoid braking.

In addition to density, other explanatory variables were computed from the traffic data. Individual vehicle-length data (computed from continuous occupancy pulses and inter-detector times and distance, *18*) produced length distribution information. The mean, 20th-percentile, and 80th-percentile vehicle lengths per 30-second interval were computed as explanatory variables, as was the fraction of an interval's vehicles that were presumed to be trucks, *i.e.*, those over 6.1 meters in length. However, less than one percent (0.895%) of the vehicles counted in Lane 2 happen to meet this length-based definition of trucks.

Additionally, researchers have observed and hypothesized traffic to exhibit a hysteresis effect of denser conditions upon braking, versus looser/less dense traffic upon acceleration – for a given speed – in the congested regime. (*19*, *20*, *21*) To examine the effects of braking versus accelerating under congested conditions, two dummy variables were defined. If the speeds across adjacent 30-second observations in the congested region differed by more than 4.0 kilometers per hour, a binary variable of congested-acceleration or congested-deceleration (depending on the direction of speed change) is given a value of one. For example, if one 30-second observation's space mean speed is 60 kph and the next observation's is 65 kph, the first observation is assigned a congested-acceleration value of one because the vehicles appear to be accelerating in the congested zone. In the two-day data set used here, 7.4% and 8.4% of the observations are able to be labeled as congested-acceleration (CongAcel) and congested-deceleration (CongDecel), respectively. These two dummy variables may help distinguish different types of driver behavior in unstable traffic streams, if they exist, although the choice of 4.0 kph for determining inclusion of an observation under this definition is somewhat arbitrary.

Traveler Population Characteristics and Precipitation:

The San Francisco Bay Area's Metropolitan Transportation Commission surveyed over 9,000 Bay Area households in its 1990 Bay Area Travel Surveys. All households' members age five and over completed detailed travel diaries for a single weekday and these data were compiled here to determine many characteristics of travelers taking relatively long personal-vehicle trips (with "long" defined here to be over 4.0 Euclidean kilometers). By time of day, one can estimate the percentage of long personal-vehicle trips by purpose, the age and gender distributions of trip-makers, and the vehicle availability for these travelers.

Weather data were taken from a variety of sources, including newspaper reports and NOAA reports (*22*, *23*). A dummy variable called "Rain" describes observations that occurred during rainfall.

STATISTICAL METHODOLOGY:

The intent of these models is predictive in nature; thus, the explanatory variables (*e.g.*, density) do not need to be strictly exogenous. In reality, density is more reasonably considered to be a function of flow, since typically drivers "demand" space on a roadway and the density at which they have to drive is a function of how many are trying to occupy the same roadspace. However, if no conclusions are to be drawn with respect to causation, a predictive model for flow is appropriate. This direction of modeling is particularly suitable in the flow-density relationship, because a *density vs. flow* relation is not one-to-one and thus creates a duality of possible values, rendering ordinary least squares (OLS) regression useless and calling instead for much more sophisticated methods, such as a "switching model" (*24*).

Researchers have suggested a variety of functional forms to describe flow-density relations, such as a parabola (*1*) and non-linear-in-parameters multiplicative logarithmic and exponential forms (*25*, *26*, *27*). And, as discussed in the model-conception section of this paper, the modeling of uncongested and congested flows as linear and non-linear functions, respectively, of density and driver/vehicle class proportions appears to be very reasonable. However, the *a priori* partitioning of the available and largely continuous data into distinct classes may not be wise, given the loss of information that can result (*e.g.*, drawing a line between "old" and "young" drivers). And, the piece-wise analysis of the two "regimes" neglects a very possible continuity in the flow-density relation (*11*).

To take advantage of the continuous nature of many of the explanatory variables and accommodate functional continuity, while providing adequate parametric flexibility and utilizing the relatively simple method of OLS estimation, different third-order-polynomial models were analyzed here. The base model incorporates explanatory variables suspected to influence the actual flow-density (q-k) relation and its functional structure is shown in Equation 4.

$$\begin{aligned} & q = \beta_o + \beta_1 k + \beta_2 k^2 + \beta_3 k^3 + \varepsilon, \\ & where\ \beta_i = \sum_j \beta_{ij} x_j . \end{aligned} \qquad \textbf{Equation (4)}$$

Note that the parameters giving the flow-density (q-k) curve its shape, the β_i's, are defined to be linear functions of exogenous, explanatory variables "x_j". The variables described in Table 1 were used as the x_j's in all initial models. A variety of variations on this general model were studied.

The term "Basic" models refers to the category of models analyzed initially; these included only a constant as the intercept term, rather than allowing β_o to be a function of more interesting traffic characteristics, such as traveler age and vehicle lengths. This was done in an effort to minimize any spurious correlation between flow and traffic characteristics from influencing the coefficient estimates. Such correlation may arise in a variety of ways. For example, congested conditions from high demand exceeding downstream bottleneck capacities generally do not occur outside of commute hours, so a coefficient on WorkFraction (*i.e.*, the fraction of travelers making work trips) may be biased low. The bias is due to the correlation of some variables with the error term, which includes all unobserved information affecting the curves.

Due to the concern for spurious correlation, a set of "Modified-Variables Models" was also examined, wherein explanatory variables suspected of providing such correlation were regressed on the dependent variable of "flow" (*i.e.*, count in a 30-second period) and the residuals of these simple linear regressions were used in the formal models in lieu of these variables. Thus, there is no simple linear correlation to be found in these modified variables with the dependent variable. The modified variables are the following: Age, Male, RecrTrip, WorkFraction, CongAccel, and CongDecel.

Note that theoretically there should be *no* intercept term, β_o, under the third-order-polynomial structure studied here, because at zero density there can be no flow. The inclusion of a simple intercept term provides more degrees of freedom for estimating a curve through the scatter of observations, so the (unadjusted) R-squared values of these regressions are naturally higher. However, models without any intercept term whatsoever are also analyzed and provide a convenient reference to the initial models.

RESULTS:

Due to space constraints, only the results for the "full" and "reduced" forms of the *Modified Variables* models are shown in Tables 2 and 3; "reduced" models are those whose variables are all highly statistically significant following a process of stepwise deletion. The signs on the estimated coefficients in both the Basic and Modified models are similar and generally correspond to expectations. For example, Density, Density^2, and Density^3 have positive, negative, and positive coefficients, respectively. However, some of the other variables' coefficients are not as anticipated.

For example, the variable of vehicle ownership (VehOwn) exerted a very strong positive influence on flow even though its variation is rather minimal (VehOwn falls between 0.97 and 1.01 over the ten hours of observations); moreover, vehicle ownership was suspected of causing estimation problems due to collinearity with the work-fraction variable (correlations are +0.63 for VehOwn and WorkFraction, and +0.61 for VehOwnResid and WorkFractionResid). For these reasons a series of Modified-Variables models were run without this variable, and the reduced version of this set of models is shown in Table 3.

In Tables 2 and 3's tabulated results, notice how the models explain a large fraction of variation found in count levels. R-squareds are consistently above 0.77 for the modified-variables models, and they are above 0.86 for the "basic" models (though these results are not shown). In contrast, the R-squared for a regression of flow on just a constant and the three powers of density is 0.66. Moreover, most of the variables are found to be highly statistically significant for purposes of flow/count prediction, suggesting that more than simple density is at work in determining the level of flow – and thus the shape of flow-density relation (and, therefore, the flow-speed and speed-density relations).

Since most variables appear in a slightly different form three or more times in a model (*i.e.*, multiplied by different powers of density), a single measure of a variable's influence proves useful. Elasticities estimated at *mean* values of the variables are provided in Table 4 – along with elasticities that incorporate the *capacity* values of flow and density. Capacity runs around 2,500 vphpl (20.8 vehicles in a 30-second interval) and critical density (k^*) is about 28 vehicles/kilometer; this "critical" traffic condition is actually found to occur with significant frequency in the two days' data.

The sign on the elasticity estimate for Male changes when one moves from the Basic to the Modified Variables Models. The author's *a priori* expectation for a positive sign on this variable seems supported by the Modified Models, suggesting benefits of having removed simple linear correlation with count in this variable.

While most of the models' results are plausible for most of the explanatory variables used, several deserve closer inspection. For example, one may believe that workers travel more aggressively so that WorkFrxn should have a positive flow elasticity associated with it, instead of the consistently negative one shown; its elasticity remains negative even after removal of simple correlation with flow, as evidenced by the modified-variables models. One may also believe that recreational travelers drive less aggressively, leading to a negative flow elasticity for RecrTrip (and RecrTripResid), rather than the positive elasticities computed. The modified-variables results suggest that travelers may not travel in accordance with such expectations; perhaps workers are not in a great rush or are not very attentive and alert when they travel – relative to recreational travelers? However, the recreational-travel variable (RecrTrip) exhibits minimal range in the data set, varying between 0% and 1.1%, so not much extrapolation should be performed using this variable. And, the final model examined, which leaves out VehOwn to diminish its collinear effect on WorkFraction causes WorkFraction's elasticity to drop to a effectively negligible level of roughly -2.2%.

The vehicle-ownership variable (VehOwn) produces very high elasticity estimates consistently, suggesting that network planners in regions of high-vehicle ownership, where travelers are likely to be more familiar with driving cars, can design their region's roadway networks less generously than their counterparts in less auto-intensive regions. However, the range of this variable in the data set (from an average 0.97 to 1.01 vehicles per household member over five years of age) is minimal, so one should be wary of estimates using values outside this range.

Also of interest is the fact that several of the length-related variables did not turn out to be statistically significant. The variables describing variable length were particularly weak and often excluded from the reduced models (which require statistical significance). Primary reasons for the lack of significance are the variety of variables available which provide this information (*i.e.*, MeanLength, 20%Length, 80%Length, and TruckFraction) and the lack of much length variation in Lane 2's observations; recall that trucks make up less than one percent of the vehicle population observed in the sample. As expected, however, length contributes negatively to the flow-density relation, with the sum of the length elasticities consistently negative.

Table 4's elasticity estimates for the congested-conditions speed-change variables (CongAcel and CongDecel) do not appear to conform with expectations of braking offering higher flows at a given speed. However, given that these two types of observations only occur under congested conditions, their elasticity estimates should not be based on mean or capacity levels of density and flow. When values of 1,400 vphpl and 62 vehicles/km. – representing *congested* conditions – are used for flow and density, the relative elasticities on CongAcel and CongDecel meet expectations by being estimated to be -2.97% and -1.19%, respectively. Thus, under situations where this behavior is possible, the relative magnitudes conform to expectations, lending support to Newell's (*19*, *20*) and Edie's (*21*) conjectures. However, the magnitudes of these elasticities are not so high that the difference can be confidently presumed clear or pronounced.

To provide additional illustration of how the models predict flow, Figures 2 and 3 plot predicted flows versus density for the reduced, no-intercept modified model without the vehicle-ownership variable. Plots are provided for mean conditions, as well as for 75% males vs. 35% males, travelers with average ages of 35 and 45, rain vs. no-rain conditions, and 10% trucks vs. no trucks scenarios. The implications of these variables seem reasonable as plotted, and these graphical results are generally shared across all models studied. However, while it may be intuitive that rainy conditions and high numbers of long vehicles or trucks are associated with lower flow levels (as evidenced in Figure 3), the implications of Figure 2 – that older, males drive/travel more aggressively (*e.g.*, at shorter headways) – is not as obvious and requires more careful consideration.

Also, in interpreting these plots one should recall that the fitted models are third-order polynomials; so these results will imply negative flows over certain density ranges (*e.g.*, at negative densities), and at sufficiently high densities the plots show flows increasing towards infinity. These results are an outcome of the functional form, so one should not predict flows for traffic conditions outside the actual ranges of observed data. By the same token, one should not interpret every part of every plot literally. For example, Figure 2's plot of 35-year average traveler age only intersects the density-axis once, while, in reality, flow would have to go to zero a second time; the difficulty in interpretation here may be due to the fact that the age data available only vary between 38 and 44 years. And estimates for a 100% or 0% male traveling population or 50% truck population are likely to lead to highly implausible estimates of flow. Much more variation in the actual data used is needed for a researcher to confidently estimate the flow effects of such conditions.

AREAS FOR IMPROVEMENT:

While the research conducted here offers valuable information and represents a significant departure from much of the research in this area, there are a variety of ways in which the data and analytical methods can be improved.

For example, better information on roadway-section users – particularly drivers – would be useful, rather than the averages for an entire hour across the region's long-trip personal-vehicle users that were used here. However, such data can be quite costly to acquire, particularly if one needs to survey the actual users observed in the data set.

Models run across all lanes' data and/or information on nearby lanes' users should prove useful, since most modeling is done at the roadway-section level (rather than lane by lane) and flow variation may be better explained since information on variables affecting more than a single lane (*e.g.*, presence of trucks) would be included. Also, lanes experiencing greater volumes of trucks would be of interest; the Lane 2 data analyzed here show minimal truck presence.

CONCLUSIONS:

The research methodology and results presented here offer insight into fundamental traffic relations, suggesting that the flow-density curve can be substantially influenced by roadway users, their vehicles, and weather conditions. As shown here, a flow-density model's predictive power rises significantly when one supplements density data with other relevant information, and most variables examined produce results that are highly statistically significant and intuitively acceptable.

Models of this form promise to refine the debate on the form of the flow-density relation and improve the modeling of traffic flows. With more information about the effects travelers, their vehicles, and weather conditions have on traffic flows, planners and engineers can better design roadways, estimate service levels, and advocate policy. For example, corridors which attract younger travelers, travelers owning fewer vehicles, and/or travelers with longer vehicles may need to be designed more generously to achieve desired service levels; and congestion tolls can be more appropriately set for trucks and other vehicle and driver types which constrain flow.

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TABLE 1. Description of Variables

Dependent Variable:

Flow (q)	Count of Vehicles in a 30-second interval

Explanatory Variables:

Density (k)	Density (vehicles per lane-kilometer)
Density^2 (k2)	Density Squared
Density^3 (k3)	Density Cubed
MeanLength	Average Vehicle Length in 30-second interval (meters)
20%Length	20th Percentile for Vehicle Lengths in 30-second interval (meters)
80%Length	80th Percentile for Vehicle Lengths in 30-second interval (meters)
TruckFraction	Fraction of Counted Vehicles with Length > 6.1 meters
WorkFraction	Fraction of Personal-Vehicle Users on "Long" Work-Related Trips (*i.e.*, work is the trip purpose at either the origin or destination zone) ("Long" is defined as ≥ 4.0 Euclidean kilometers in distance between origin and destination tract)
RecrTrip	Fraction of Personal-Vehicle Users on "Long" Recreational Trips
Male	Fraction of Personal-Vehicle Users on "Long" Trips who are Male
AutoOwn	Avg. #Personal Vehicles per Household Member for PV Users on "Long" Trips
Age	Avg. Age of Personal-Vehicle Users on "Long" Trips
Rain	Rain falling in the vicinity during the hour
CongAccel	Speeds increasing during Congested Conditions
CongDecel	Speeds falling during Congested Conditions
*varname*Resid	Residuals of Linear Regressions (with constant terms) of Variable "*varname*" on Count (Used to remove simple linear correlation between non-density variables and flow.)

TABLE 2. Regression Results of Modified-Variables Models – with Intercepts

Dependent Variable = Count of Vehicles in 30-second Interval

	Full Model, with Intercept			Reduced Model, with Intercept		
Adjusted R-Squared:		0.9864			0.9863	
Variable:	**Beta**	**T-Statistic**	**P-Value**	**Beta**	**T-Statistic**	**P-Value**
Intercept	368.55	17.297	0.000	368.74	20.46	0.00
Rain	2.999	8.031	0.000	2.832	12.11	0.00
AgeResid	0.058	0.463	0.643			
MaleResid	-63.907	-11.737	0.000	-68.856	-14.45	0.00
VehOwnership	-378.93	-17.556	0.000	-384.72	-21.05	0.00
RecrTripResid	-1675.63	-10.568	0.000	-1705.79	-13.77	0.00
WorkFrxnResid	56.913	15.221	0.000	59.420	23.07	0.00
AvgLength	-1.962	-1.197	0.232			
20%Length	2.719	2.697	0.007	0.964	2.45	0.01
80%Length	-0.924	-1.032	0.302			
TruckFraction	7.648	1.840	0.066			
CongAccelResid	4.431	3.150	0.002	4.240	3.19	0.00
CongDecelResid	3.717	2.022	0.043	3.091	1.75	0.08
Density	-31.17	-15.888	0.000	-31.11	-20.88	0.00
k*Rain	-0.286	-8.356	0.000	-0.263	-17.50	0.00
k*AgeResid	0.001	0.077	0.939			
k*MaleResid	7.454	14.202	0.000	7.915	16.49	0.00
k*VehOwnership	32.93	16.676	0.000	33.40	22.19	0.00
k*RecrTripResid	132.4	8.577	0.000	133.4	12.45	0.00
k*WorkFrxnResid	-4.870	-14.435	0.000	-5.079	-27.83	0.00
k*AvgLength	0.296	1.826	0.068	0.122	4.47	0.00
k*20%Length	-0.317	-3.065	0.002	-0.132	-3.84	0.00
k*80%Length	0.115	1.239	0.215			
k*TruckFraction	-1.099	-2.241	0.025	-0.380	-3.43	0.00
k*CongAccelResid	-0.647	-7.346	0.000	-0.630	-7.74	0.00
k*CongDecelResid	-0.604	-5.190	0.000	-0.561	-5.08	0.00
Density^2	0.468	8.660	0.000	0.467	13.79	0.00
k2*Rain	4.24E-03	4.856	0.000	3.51E-03	18.68	0.00
k2*AgeResid	1.02E-03	2.641	0.008	1.23E-03	16.69	0.00
k2*MaleResid	-0.238	-15.181	0.000	-0.250	-17.21	0.00
k2*VehOwnership	-0.488	-8.983	0.000	-0.499	-14.64	0.00
k2*RecrTripResid	-0.731	-1.607	0.108	-0.691	-2.52	0.01
k2*WorkFrxnResid	0.0641	6.970	0.000	0.0687	25.54	0.00
k2*AvgLength	-7.27E-03	-1.730	0.084	-2.77E-03	-4.94	0.00
k2*20%Length	6.89E-03	2.457	0.014	1.92E-03	3.78	0.00
k2*80%Length	-4.83E-03	-1.840	0.066	-1.95E-03	-4.40	0.00
k2*TruckFraction	2.48E-02	1.753	0.080	6.91E-03	3.59	0.00
k2*CongAccelResid	1.62E-02	9.243	0.000	1.58E-02	9.87	0.00
k2*CongDecelResid	1.53E-02	6.622	0.000	1.44E-02	6.66	0.00
Density^3	-7.00E-04	-1.596	0.111	-6.54E-04	-2.95	0.00
k3*Rain	-5.97E-06	-0.926	0.355			

k3*AgeResid	-2.14E-05	-6.604	0.000	-2.38E-05	-20.47	0.00
k3*MaleResid	2.31E-03	16.438	0.000	2.41E-03	18.77	0.00
k3*VehOwnership	6.84E-04	1.556	0.120	7.16E-04	3.25	0.00
k3*RecrTripResid	-2.02E-02	-5.050	0.000	-2.09E-02	-9.41	0.00
k3*WorkFrxnResid	2.33E-05	0.303	0.762			
k3*AvgLength	3.41E-05	1.076	0.282			
k3*20%Length	-3.67E-05	-1.676	0.094			
k3*80%Length	5.11E-05	2.389	0.017	3.32E-05	5.07	0.00
k3*TruckFraction	-1.22E-04	-1.142	0.253			
k3*CongAccelResid	-1.07E-04	-9.848	0.000	-1.05E-04	-10.51	0.00
k3*CongDecelResid	-1.00E-04	-7.060	0.000	-9.45E-05	-7.23	0.00

TABLE 3. Regression Results of Modified-Variables Models – without Intercepts

Dependent Variable = Count of Vehicles in 30-second Interval

	Reduced Model, w/out Intercept without Intercept			Reduced Model, w/out Intercept & w/out VehOwnership		
Adjusted R-Squared:		0.8291			0.7718	
Variable:	**Beta**	**T-Statistic**	**P-Value**	**Beta**	**T-Statistic**	**P-Value**
Density	-2.717	-4.635	0.000	1.148	135.874	0.000
k*Rain	-0.0526	-4.869	0.000	-0.0408	-9.398	0.000
k*AgeResid	0.0128	3.393	0.001			
k*MaleResid	2.271	15.208	0.000			
k*VehOwnership	4.034	6.785	0.000	na	na	na
k*RecrTripResid	-24.94	-6.026	0.000	-30.82	-10.579	0.000
k*WorkFrxnResid	-0.2298	-2.298	0.022	0.2633	4.785	0.000
k*AvgLength	-0.0291	-4.065	0.000			
k*20%Length						
k*80%Length						
k*TruckFraction						
k*CongAccelResid	-0.2023	-8.281	0.000	-0.1472	-10.827	0.000
k*CongDecelResid	-0.1932	-7.197	0.000	-0.2732	-9.563	0.000
Density^2	-0.1707	-6.161	0.000	-0.0123	-18.946	0.000
k2*Rain	-1.01E-03	-2.267	0.023			
k2*AgeResid	6.96E-04	3.370	0.001	1.19E-03	15.196	0.000
k2*MaleResid	-0.1140	-14.570	0.000	0.0169	7.320	0.000
k2*VehOwnership	0.1542	5.464	0.000	na	na	na
k2*RecrTripResid	3.6851	17.386	0.000	2.6741	18.878	0.000
k2*WorkFrxnResid	-0.0444	-9.364	0.000	-0.0181	-7.249	0.000
k2*AvgLength				-9.16E-04	-4.156	0.000
k2*20%Length						
k2*80%Length				-5.91E-04	-2.983	0.003
k2*TruckFraction						
k2*CongAccelResid	4.72E-03	5.870	0.000	2.04E-03	9.199	0.000
k2*CongDecelResid	4.08E-03	4.462	0.000	6.12E-03	6.511	0.000
Density^3	3.49E-03	12.180	0.000			
k3*Rain	2.71E-05	6.713	0.000	5.90E-06	5.793	0.000
k3*AgeResid	-1.72E-05	-7.871	0.000	-1.64E-05	-13.751	0.000
k3*MaleResid	1.37E-03	14.868	0.000	-2.17E-04	-5.611	0.000
k3*VehOwnership	-3.49E-03	-11.928	0.000	na	na	na
k3*RecrTripResid	-5.45E-02	-21.473	0.000	-3.30E-02	-21.363	0.000
k3*WorkFrxnResid	7.44E-04	14.255	0.000	1.69E-04	6.623	0.000
k3*AvgLength						
k3*20%Length						
k3*80%Length				1.32E-05	13.655	0.000
k3*TruckFraction						
k3*CongAccelResid	-2.47E-05	-3.835	0.000			
k3*CongDecelResid	-1.61E-05	-2.164	0.031	-2.99E-05	-4.069	0.000

TABLE 4. Estimated Elasticities

Basic Models

	Primary Model		No-Intercept Model	
	Mean	Capacity	Mean	Capacity
Variable:				
Rain	-3.09%	-2.26%	-3.13%	-2.29%
Age	210.68%	163.70%	222.87%	174.00%
Male	-9.34%	-12.99%	-17.30%	-17.62%
VehOwnership	353%	253.8%	404.2%	279.3%
RecrTrip	19.65%	15.85%	20.44%	16.75%
WorkFraction	-29.59%	-22.59%	-27.64%	-21.12%
AvgLength	-7.9%	-7.34%	-7.59%	-7.09%
20%Length	na	na	na	na
80%Length	-10.00%	-10.73%	-10.00%	-10.11%
TruckFraction	na	na	na	na
CongAccel	-2.81%	-1.75%	-2.50%	-1.58%
CongDecel	-3.06%	-1.88%	-2.77%	-1.72%

Models with Modified ("Residual") Variables

	Primary Model		No-Intercept Model		No-Intercept and No-VehOwn Model	
	Mean	Capacity	Mean	Capacity	Mean	Capacity
Variable:						
Rain	-5.71%	-4.29%	-5.61%	-4.00%	-3.43%	-2.43%
AgeResid	5.30%	3.85%	6.54%	4.61%	6.54%	5.03%
MaleResid	5.11%	3.06%	3.50%	1.35%	3.46%	2.68%
VehOwnership	1078.6%	828.3%	1038.3%	745.3%	na	na
RecrTripResid	20.94%	16.25%	19.94%	15.72%	9.18%	8.04%
WorkFractionRes.	-26.88%	-20.74%	-23.99%	-17.85%	-2.24%	-2.23%
AvgLength	41.25%	25.08%	-22.37%	-16.33%	-17.41%	-14.37%
20%Length	-31.91%	-22.42%	na	na	0.00%	0.00%
80%Length	-23.11%	-17.27%	na	na	-5.38%	-3.72%
TruckFraction	-0.42%	-0.28%	na	na	na	na
CongAccelResid	-6.70%	-4.40%	-5.07%	-3.30%	-4.87%	-3.31%
CongDecelResid	-6.69%	-4.32%	-4.93%	-3.23%	-6.77%	-4.42%

Notes:

Elasticities are provided for estimates from "reduced", rather than "full" models; thus, all are statistically significant estimates.

Mean elasticities are computed at variables' basic mean values (*e.g.*, flow = 1,614 vphpl = 13.45 veh./30 sec., density = 24.73 vpkm) for all variables except those of estimated residuals (following a linear regression on flow). Since the means of residuals are necessarily zero and would cause an elasticity-at-mean to be zero, one standard deviation was used as the level at which to estimate the elasticity of these variables.

Capacity elasticities are essentially mean elasticities, but with the values of flow and density taken as the (approximate) capacity conditions in Lane 2 of the roadway section studied; the capacity values used here are 2,500 vphpl and 28 vehicles/kilometer.

FIGURE 1: Plot of Observed Counts vs. Density

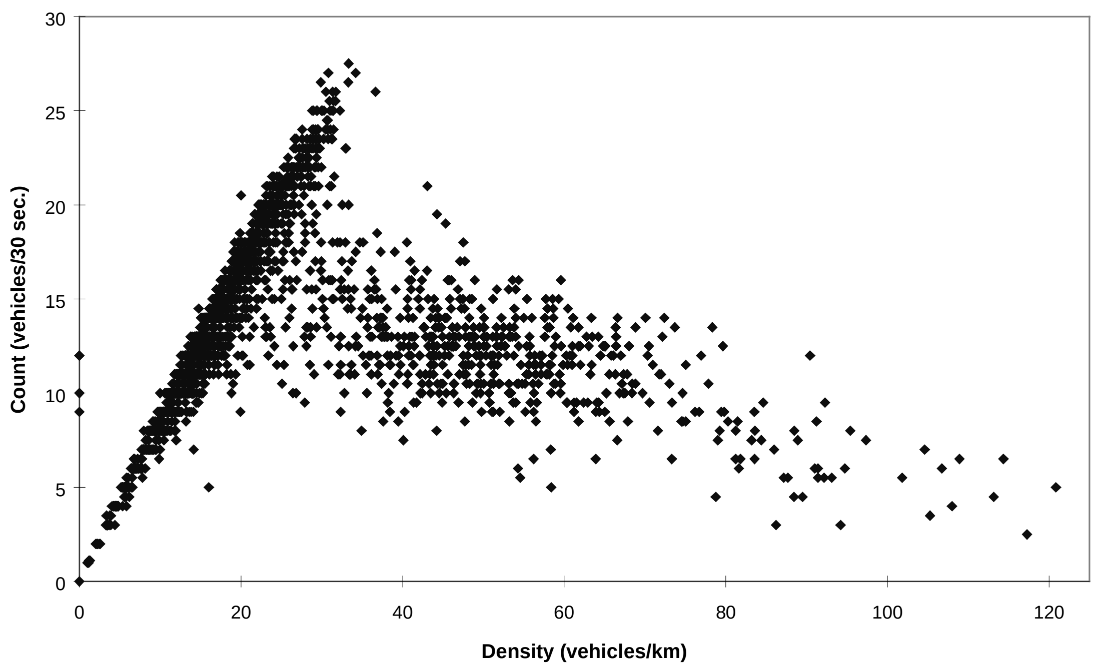


FIGURE 2: Plot of Predicted Flow vs. Density
over Different Traveler Characteristics,
for the Modified No-Intercept, No-Vehicle-Ownership Model

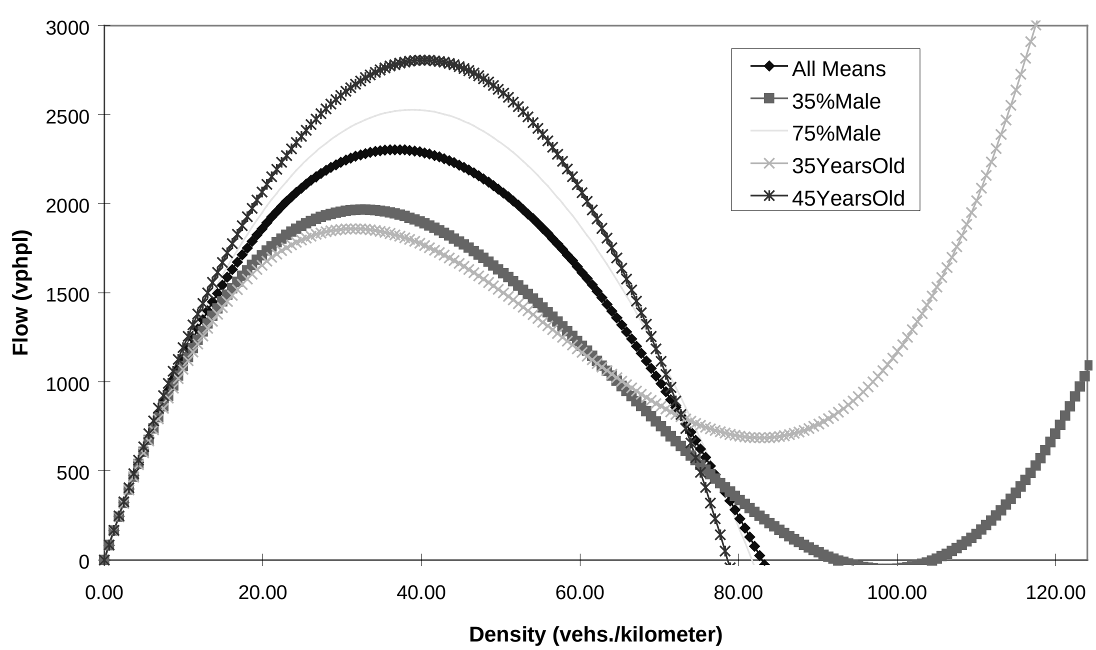
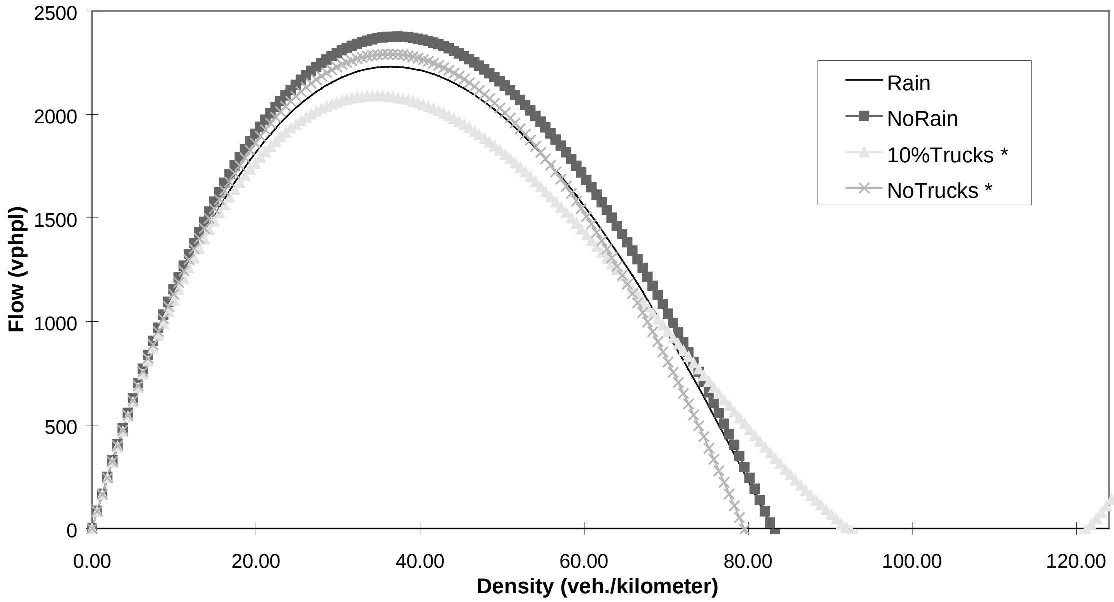


FIGURE 3: Plot of Predicted Flow vs. Density over Different Vehicle & Weather Conditions for the Modified No-Intercept and No-Vehicle-Ownership Model



* Note: 10% Trucks condition is defined with higher vehicle lengths than data averag
and No Truck condition constrains 80% Length to be equal to Average.