GIS in Water Resources   Exercise #3   Solution

Part 1. Slope Calculations

1.1 Hand Calculation (Point A only):

(i) ESRI Slope

45.4	46.1	47	48.6	47.7
45	46.1	46.4 B	47.9	47.4
45.1	45.8	46.8 A	48.6	47.6
47.5	48	47.7	50.6	48.3

Cell Referencing

a	b	c
d	e	f
g	h	i

Equations:

c_size = 10m

dz/dx = ((a+2d+g)-(c+2f+i))/8*c_size

= ((46.1+2*45.8+48)-(47.9+2*48.6+50.6))/(8*10) = -0.125

dz/dy = ((g+2h+i)-(a+2b+c))/8*c_size

= ((46.1+2*46.4+47.9)-(48+2*47.7+50.6))/(8*10) = 0.09

These represent the x and y components of the slope vector shortened as follows

Δx = dz/dx = -0.125

Δy = dz/dy = 0.09

slope (rise/run) = sqrt(Δx^2+Δy^2) = sqrt((-0.125)^2+(0.09)^2) = 0.154

slope (angle) = atan(slope (rise/run)) = atan(0.154) = 0.153 rads = 8.76 degrees

Note: degrees = rads * 180/π. Calculators can be set to return rads or degrees. Excel and computer programs usually return rads.

aspect = atan(Δx/Δy) = atan(-0.125/0.09) = -0.95 rads = -54.2 degrees

This is an angle in the NW quadrant since x component is negative and y component positive. Add 360 degrees to get the angle clockwise from north

aspect = 360+(-54.2) = 305.75

The following Excel Object includes the formulae. You can double click on this to open this object in Excel.

(i) ESRI Standard Slope Function								
Grid size	10	m						
45.4	46.1	47	48.6	47.7		dz/dx=	-0.125	
45	46.1	46.4	47.9	47.4		dz/dy=	0.0900	
45.1	45.8	**46.8**	48.6	47.6				
47.5	48	47.7	50.6	48.3		rise/run=	0.154029	
						Slope=	0.152828	radians
							8.756408	**degree**
						Aspect	-0.94677	radians
							-54.2461	degree
			Result as angle clockwise from North				**305.7539**	degree
(This is an Excel Object so you can click on it to see the formulas)								

(ii) The 8 direction pour point model D8

Slope is calculated separately to each adjacent grid cell using the formula
Slope = (Center elevation - Side elevation)/Distance
Distance to diagonal side cells is the diagonal distance $\sqrt{2}$ * cell size

The following Excel object includes these calculations.

ii) D8	Center cell	46.8						
Distances	Side	10	Diagonal	14.14214				
Direction	Value	Distance	Slope					
1	48.6	10	-0.180		Direction Encoding			
2	50.6	14.142	-0.269		32	64	128	
4	47.7	10	-0.090		16	←	1	
8	48	14.142	-0.085		8	4	2	
16	45.8	10	**0.100**	Maximum (positive down) slope to cell in direction 16				
32	46.1	14.142	0.049					
64	46.4	10	0.040					
128	47.9	14.142	-0.078					
(This is an Excel Object so you can click on it to see the formulas)								

Note that the steepest 8 direction pour point model slope in direction 16 is:

$$\frac{\text{center cell} - \text{side cell 16}}{\text{cell size}} = \frac{46.8 - 45.8}{10} = 0.10$$

D8 Slope: 0.1
D8 Direction: 16

Differences

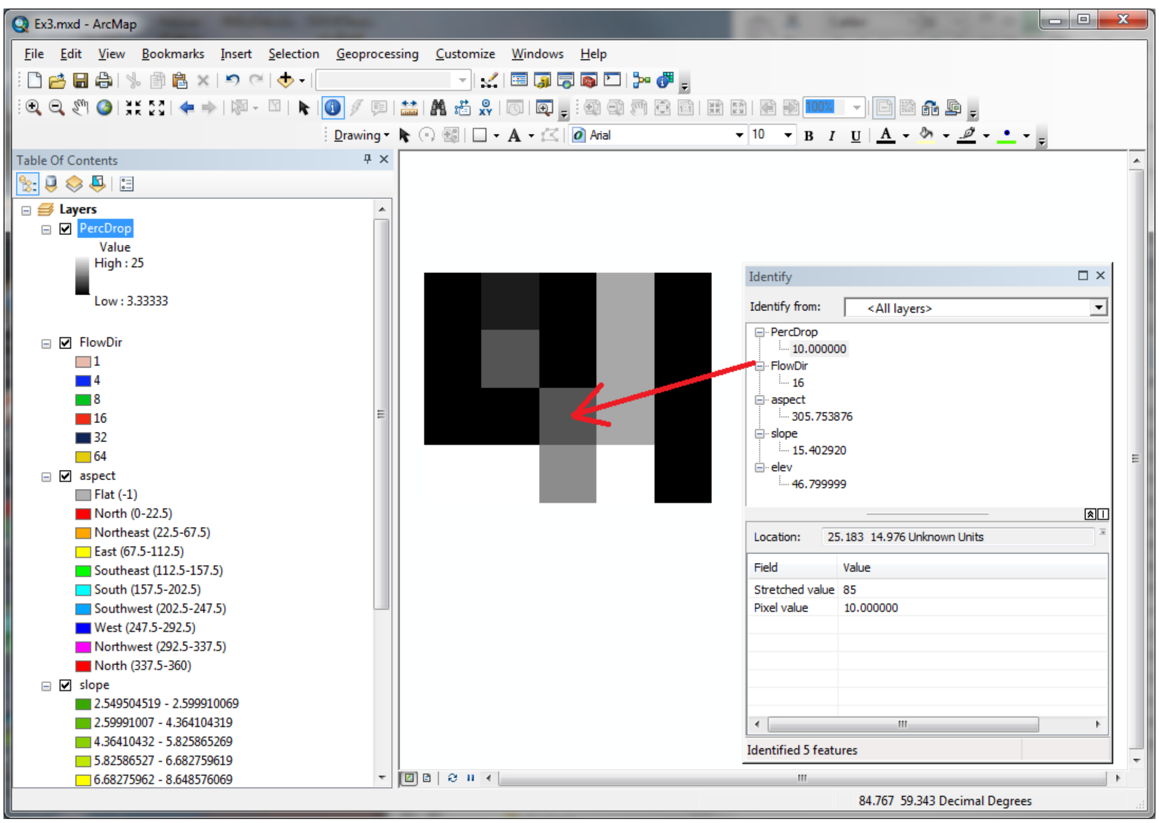
Represented as an aspect the D8 direction would be 270 degrees, but simply stating the direction as 16 or to the W is sufficient.

The main difference is that the ESRI slope considers all 8 surrounding grid cell values, and curiously, not the actual grid cell value. It represents the slope of a polynomial surface fit to all these grid cells. The D8 method only considers adjacent elevations lower than the center cell which is consistent with the assumption of where water would flow not being influenced by adjacent neighbors that are higher.

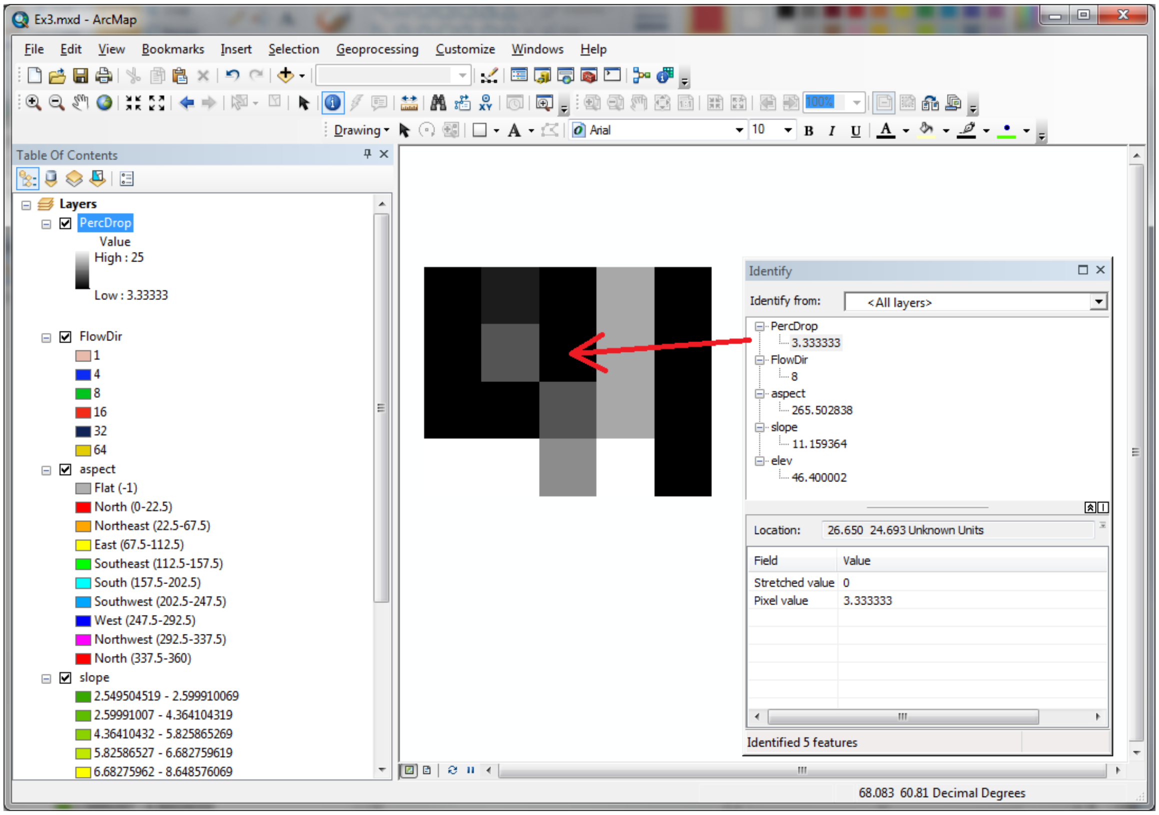
The D8 flow direction is to the W, while the ESRI slope aspect is to the NW significantly influenced by the cell with value of 50.6 to the SE. The ESRI slope is probably most appropriate for computation of quantities such as illumination due to sunlight in energy balance calculations where the slope of the surface fit based on all surrounding values seems best, but for the flow of water, the D8 method is better.

1.2 Verifying calculations using ArcGIS

The values at cell A of Slope = 15.4%, Aspect = 305.8 deg, PercDrop = 10% and FlowDir=16 correspond to the hand calculations

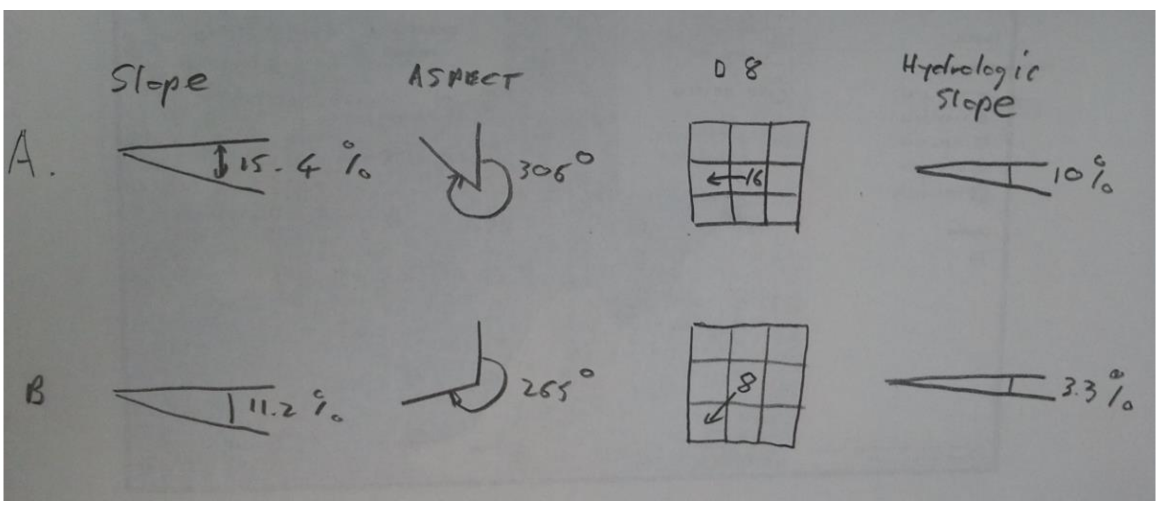


At cell B



<u>Summary of ArcGIS Calculated:</u>

Point	Slope (%)	Aspect (deg)	D8 Slope (%)	Flow Dir (D8)
A	15.4	305.8	10	16
B	11.2	265.5	3.33	8



Note that if you look at the data underlying D8 slope at B you have

46.1	47	48.6
46.1	46.4	47.9
45.8	46.8	48.6

The percentage drop in direction 8 (indicated with arrow) should thus be (46.4-45.8)/(SQRT(2)*10)=0.0424 = 4.24%
The fact that the ArcGIS function is reporting 3.33% is, I believe, a bug. Buyer beware!

1.3 Model Builder Output

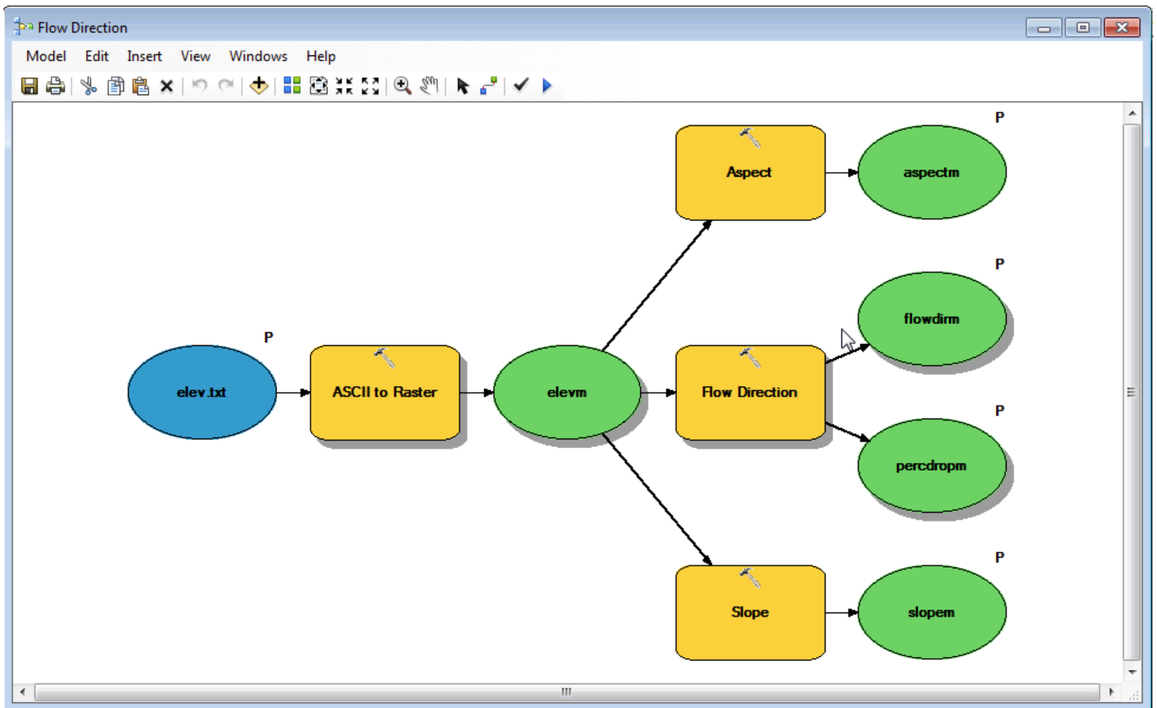


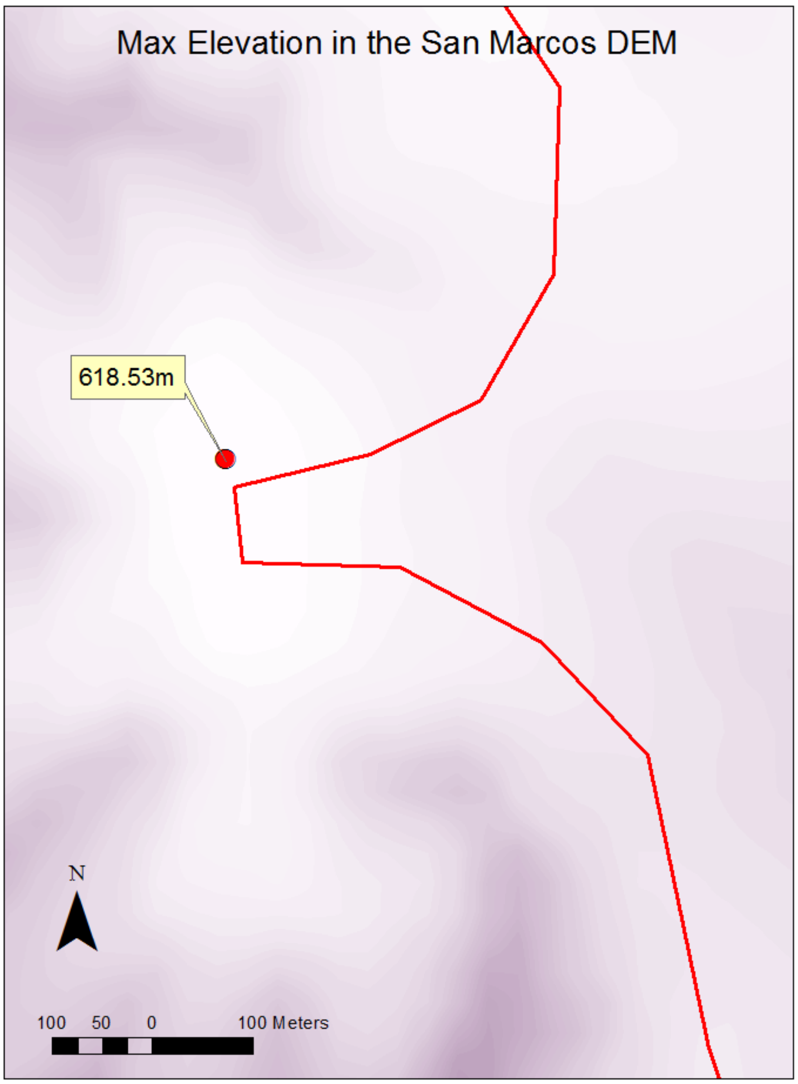
Table: Summary of Demo.asc Ouputs

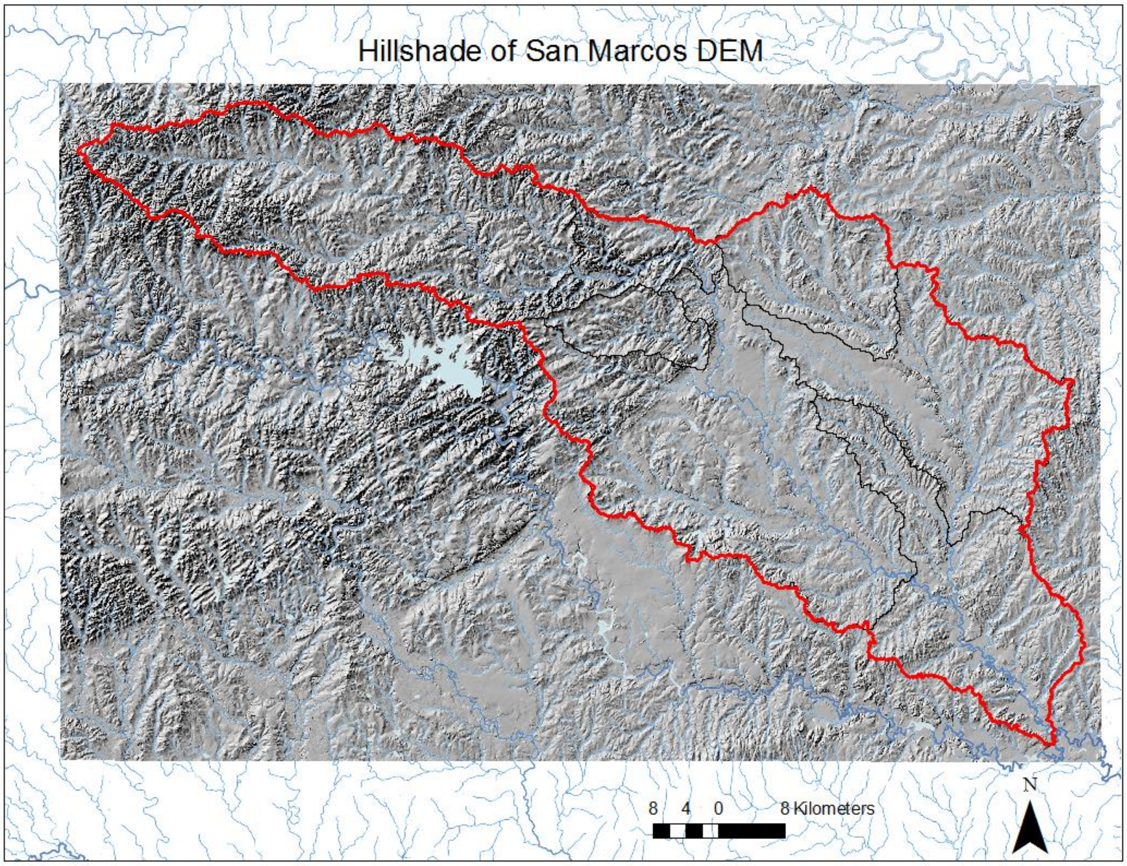
Layer	Min	Max
Slope	0	149
Aspect	-1	360
Flow Dir	1	128
PercDrop	0.066	146.6

-1 for aspect is used to represent flat grid cells

Part 2 San Marcos

DEM Summary (projdem.tif)
Rows: 2745
Columns: 4222
Cell Size: 30 x 30
Min: 69.7651
Max: 618.532





Subwatershed Elevation Summary

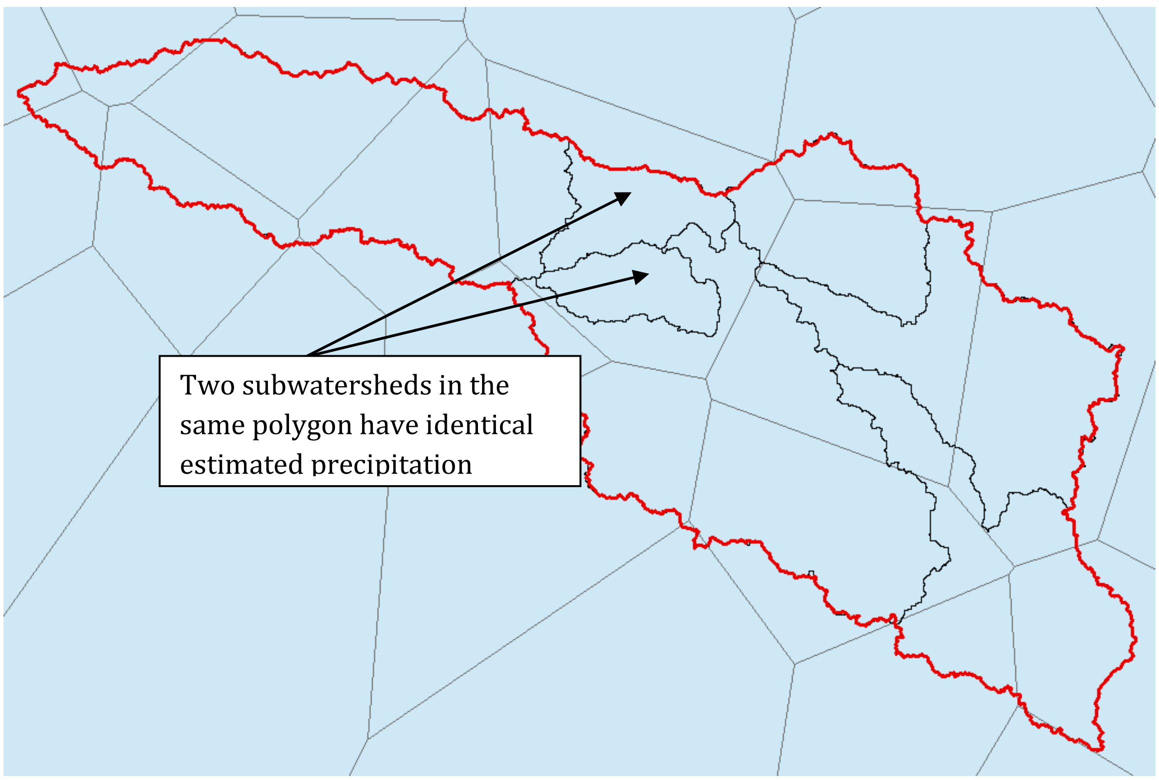
HydroID	SiteName	Elev. Range (m)	Elev Mean (m)
330	Plum Ck at Lockhart, Tx	137.71	189.94
331	Blanco Rv at Wimberley, Tx	372.97	418.56
332	Blanco Rv nr Kyle, Tx	216.83	288.60
333	San Marcos Rv at San Marcos, Tx	218.73	266.31
334	Plum Ck nr Luling, Tx	115.95	151.96
335	San Marcos Rv at Luling, Tx	311.83	183.54

Highest: Blanco Rv at Wimberley, TX
Largest Range: Blanco Rv at Wimberley, TX

Area Average Precipitation using Thiessen Polygons

HydroID	SiteName	SubW Precip (in)
330	Plum Ck at Lockhart, Tx	36.37
331	Blanco Rv at Wimberley, Tx	37.83
332	Blanco Rv nr Kyle, Tx	40.48
333	San Marcos Rv at San Marcos, Tx	40.48
334	Plum Ck nr Luling, Tx	36.52
335	San Marcos Rv at Luling, Tx	37.59

The highest mean precipitation is found for the San Marcos River at San Marcos and Blanco River near Kyle watersheds. These are identical, because they are both in the same polygon.



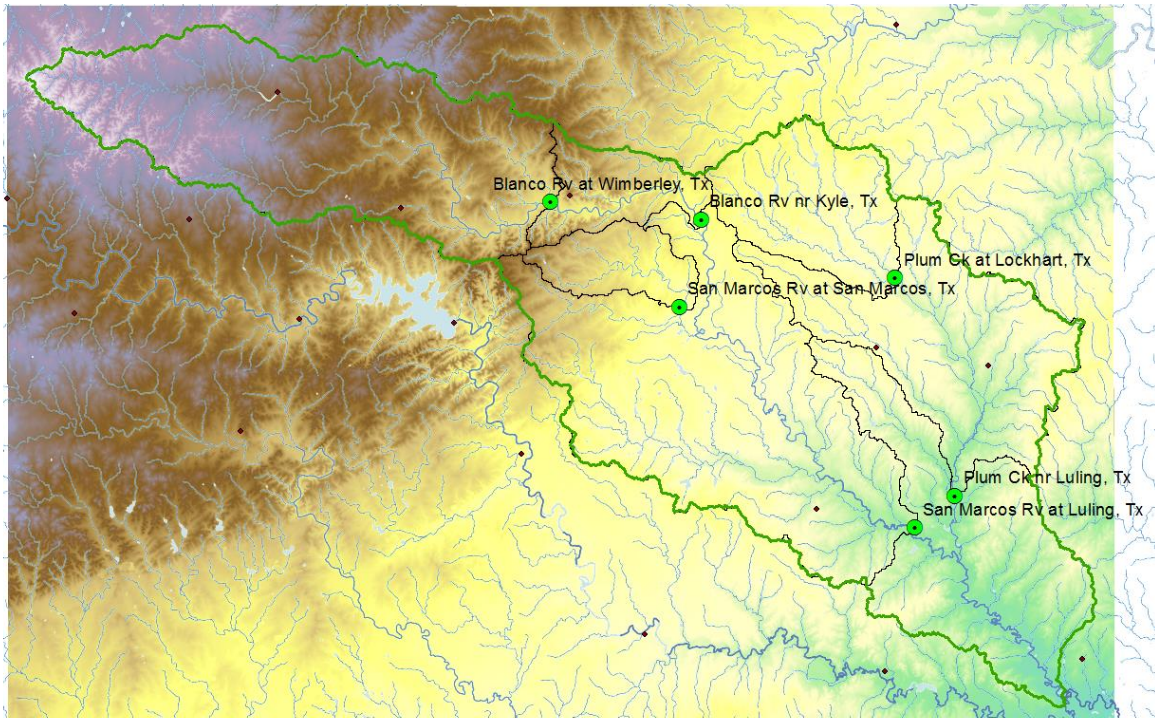
Area average mean annual precipitation using Spatial Interpolation/Surface fitting (Tension Spline Method)

HYDROID	SiteName	Precip (inches)
330	Plum Ck at Lockhart, Tx	36.22
331	Blanco Rv at Wimberley, Tx	37.89
332	Blanco Rv nr Kyle, Tx	39.79
333	San Marcos Rv at San Marcos, Tx	39.66
334	Plum Ck nr Luling, Tx	36.46
335	San Marcos Rv at Luling, Tx	37.99

Blanco Rv nr Kyle, TX has the highest mean precipitation estimated from Tension Spline Interpolation.

Runoff Coefficients

The following map shows stream gages at the outlet of each subwatershed



This indicates the following subwatersheds which comprise each watershed

Watershed	**Subwatersheds**
Plum Ck at Lockhart, TX	Plum Ck at Lockhart, TX
Blanco Rv at Wimberley, TX	Blanco Rv at Wimberley, TX
Blanco Rv nr Kyle, TX	Blanco Rv nr Kyle, TX Blanco Rv at Wimberley, TX
San Marcos Rv at San Marcos, TX	San Marcos Rv at San Marcos, TX
Plum Ck nr Luling, TX	Plum Ck nr Luling, TX Plum Ck at Lockhart, TX
San Marcos Rv at Luling, TX	Blanco Rv nr Kyle, TX Blanco Rv at Wimberley, TX San Marcos Rv at San Marcos, TX San Marcos Rv at Luling, TX

Table determining precipitation volume for each subwatershed. In this table Precip volume is Mean precip * Area divided by 12 x 0.3048^2 to obtain volume in ft^3.

Subwatershed Precip Volume from Thiessen Polygons				
HydroID	Name	Area (m^2)	Mean Precip (in)	Precip Volume (ft^3)
330	Plum Ck at Lockhart, Tx	290770000	36.37	9.485E+09
331	Blanco Rv at Wimberley, Tx	921160000	37.83	3.125E+10
332	Blanco Rv nr Kyle, Tx	149160000	40.48	5.416E+09
333	San Marcos Rv at San Marcos, Tx	126660000	40.48	4.599E+09
334	Plum Ck nr Luling, Tx	521280000	36.52	1.708E+10
335	San Marcos Rv at Luling, Tx	980250000	37.59	3.305E+10

Flow volume is obtained from flow in cfs by multiplying by 365.25*24*3600*3600. The subwatersheds that comprise each watershed are identified and precip volume obtained by summing these. Watersheds that are comprised of multiple subwatersheds are grouped together to facilitate totaling of Precip volume, Runoff ratio is then flow volume/precip volume.

Watersheds (with contributing subwatersheds indented above)							
Hydro-ID	Name		Flow (cfs)	Flow Volume (ft^3)	Precip volume subwater-shed	Precip volume total	**Runoff ratio**
330	Plum Ck at Lockhart, Tx		49.00	1.546E+09	9.485E+09	9.485E+09	**0.1630**
331	Blanco Rv at Wimberley, Tx		142.00	4.481E+09	3.125E+10	3.125E+10	**0.1434**
	331	Blanco Rv at Wimberley, Tx (subwatershed)			3.125E+10		
	332	Blanco Rv nr Kyle, Tx (subwatershed)			5.416E+09		
332	Blanco Rv nr Kyle, Tx (watershed)		165.00	5.207E+09		3.667E+10	**0.1420**
333	San Marcos Rv at San Marcos, Tx		176.00	5.554E+09	4.599E+09	4.599E+09	**1.2078**
	330	Plum Ck at Lockhart, Tx (subwatershed)			9.485E+09		
	334	Plum Ck nr Luling, Tx (subwatershed)			1.708E+10		
334	Plum Ck nr Luling, Tx (watershed)		114.00	3.598E+09		2.656E+10	**0.1354**
	331	Blanco Rv at Wimberley, Tx (subwatershed)			3.125E+10		
	332	Blanco Rv nr Kyle, Tx (subwatershed)			5.416E+09		
	333	San Marcos Rv at San Marcos, Tx (subwatershed)			4.599E+09		
	335	San Marcos Rv at Luling, Tx (subwatershed)			3.305E+10		
335	San Marcos Rv at Luling, Tx (watershed)		408.00	1.288E+10		7.432E+10	**0.1732**

The runoff ratio for the San Marcos river at San Marcos is anomalously high due to flow from springs that are fed by precipitation that recharges the Edwards Aquifer outside the watershed. This anomalous high flow attenuates downstream. Plum Creek at Lockhart is also in the vicinity of where the Edwards aquifer outcrops and has a slightly higher runoff ratio so likely gets some spring contributions too. Over all the other watersheds, runoff ratio is pretty consistent between 0.11 and 0.15, which seems about right for this region.