Geodesy, Map Projections and Coordinate Systems
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Besides this synopsis, please read the overview of Georeferencing and coordinate systems in the ArcGIS Resource Center:
and the section in the ArcGIS Help on Map Projections:
http://resources.arcgis.com/en/help/main/10.2/#/What_are_map_projections/003r00000001000000
For additional information on this subject, see National Research Council (2010).

*Geodesy* is “the branch of the earth sciences that deals with the measurement and representation of the earth, including its gravitational field” (definition from Wikipedia). Geodetic or geographic locations on the earth are defined by latitude (φ), longitude (λ) and elevation (z), where the horizontal coordinates (φ, λ) are measured using angular units, and the vertical coordinate (z) is measured using distance units.

A global positioning system (GPS) uses distance measurements from a collection of satellites to determine geodetic location. Distance is measured by taking the time difference between the time a signal is emitted from the satellite and the time it reaches the receiver and multiplying by the speed of light. A combination of signals from four satellites is sufficient to determine a point on the earth’s surface uniquely. The Continuously Operating Reference System (CORS) is a set of fixed GPS receivers whose position is known so accurately that it can be used to track the movement of tectonic plates, which is about 2-3 cm/year on the North American plate on which most of the continental United States rests, and a significantly large velocity on the Pacific plate on coastal California is located. This leads to adjustments in the location of survey benchmarks to reflect their position at a particular point in time.

Although we think of the earth as a sphere, it is in fact slightly flattened and best represented as an oblate spheroid, with an equatorial radius that is about 1/300 larger than its polar radius. This shape is a form of ellipsoid obtained by rotating an ellipse whose major and minor axes are sized to represent the earth’s shape, around an axis to form a three-dimensional shape. A horizontal earth datum is a combination of a particular ellipsoid shape and an axis of rotation. The two datums used most often in the United States are the North American Datum of 1927 (NAD27) and the North American Datum of 1983 (NAD83). NAD27 was designed so that the ellipsoid surface closely matches the land surface in North America; NAD83 was an adjustment with a different ellipsoid shape and a geocentric axis of rotation through the poles, which is a consequence of earth measurement from satellites. The World Geodetic System (WGS) is designed for military use and it is independent of movement of locations with tectonic motion, while adjustments to the civilian NAD datums take tectonic movement of fixed locations on earth into account.

The formal definition of the latitude of a point on the earth’s surface is that the latitude is the angle subtended in the equatorial plane of a line drawn normal to a tangent plane touching the earth’s spheroid at the point of interest. The formal definition of longitude is that longitude is the angle subtended at the polar axis between a plane drawn through the polar axis and the meridian passing through the point of interest, and the corresponding plane drawn through the prime meridian.

Distance measurement on the earth’s surface along meridians and parallels can be approximated for a spherical earth by using circular geometry, in other words arc length = radius of the earth multiplied by the angle subtended at the center in radians. The great circle distance, or shortest distance between
two points anywhere on the earth’s surface, is calculated using a more complicated formula, and
distance measurement on a spheroidal earth surface is even more complex.

Horizontal coordinates are defined geographically or geometrically, but elevation is defined
*gravitationally*. Newton’s law of gravitation defines the attractive force between two masses separated
by a distance, and *gravitational potential* is the potential energy that a unit mass possesses at a
particular location in a gravitational field. The *geoid* is a surface of constant gravitational potential at or
near the earth’s surface that is the reference location for defining a *vertical earth datum*. Two vertical
datums are used in the United States: the *National Geodetic Vertical Datum of 1929* (NGVD29), and the
*North American Vertical Datum of 1988* (NAVD88). Elevation is defined as the vertical height above the
geoid, or geodetic datum. While the ellipsoid forms a nice smooth mathematical surface, spatial
variability in the earth’s gravitational field causes the geoid to have a lumpy surface, in some cases
above the ellipsoid, in other cases below. The difference in heights between the geoid and the ellipsoid
is called the *gravity anomaly*. Gravity itself is not fixed in time but is continuously varying as the storage
of water changes seasonally and from year to year, as has been demonstrated by the GRACE (Gravity
Recovery and Climate Experiment) mission. Changes in the gravity anomaly for regions such as Texas
can measure how much water has been gained or lost through time, such as during the 2011 drought.

Elevation along the coast is defined using *tidal heights*, which are elevations measured using water
surface elevation gages in coastal waters relative to a statistical average tidal level calculated through
time. The best known reference tidal height is the *mean sea level*, and land surface elevations are often
quoted as being “feet above mean sea level” as if the sea were to extend under the land surface. The
concept of mean sea level is outdated for two reasons – first, tidal measurements over long periods of
time show consistent and pronounced trends in tidal heights, rising on the order of one foot per century
along the US coastline; second, the ocean itself is constantly moving, and has spatial and temporal
trends in its temperature and salinity that affect its surface elevation. Elevation measurement is a
complicated matter that actually has three different measurement systems – height above ellipsoid
(used by GPS), height above geoid (the legal standard used by land surveyors), and tidal heights (used
for coastal management). Further complicating all this is that the gravitational field of the earth changes
in time because of the season accumulation and release of water storage in snow, reservoirs and
aquifers. We live in a complicated world!

*Map projection* is a method of producing a flat map representation of a portion of the earth’s surface.
The resulting horizontal coordinates are termed the *Easting* ($x$) and *Northing* ($y$), and map projection
is actually carried out by a mathematical transformation between ($\phi, \lambda$) and ($x,y$). There are three types of
map projections, each of which can be visualized by placing a sheet of paper around or on a globe, in the
form of a cone for a *conic projection*, as a cylinder for a *cylindrical projection*, and as a plane for an
*azimuthal projection*. Conic projections are used for regions oriented in the East-West direction, such as
the continental United States; cylindrical projections used for regions oriented in the North-South
direction, such as for California; azimuthal projections are used for particular regions such as the poles
where all the meridians come together.

All map projections have an associated set of *projection parameters*, which specify the particular way
the projection will be applied, typically including a reference latitude and meridian ($\phi_0, \lambda_0$) that locate a
point of origin on the land surface near the center of the map, and a *false easting* and *false northing*
($x_0,y_0$) that represent the same origin point on the map. The false easting and northing are sometimes
specified as large numbers in meters or feet so that all points on the map end up having positive ($x,y$)
coordinates. Since latitude and longitude vary slightly depending on which horizontal earth datum is a
complete description of a map projection includes the earth datum, the projection type, and the projection parameters.

There are four properties that map projections seek to preserve: area, shape, orientation, and distance. The Albers equal area projection preserves true earth surface area by distorting distance in the North-South direction at just the inverse rate that it distorts distance in the East-West direction. We use this projection extensively in hydrology because preserving true earth area is important when computing mass balances such as from rainfall to runoff. The transverse Mercator projection places a cylinder around the earth with its axis through the poles. Meridians and parallels form rectangles in this projection. A variant of this, called the web Mercator projection is used for cached basemaps. A coordinate system is a method of establishing standard values of (x,y) for particular locations on the land surface. Within the United States, legal locations for land boundaries are defined using the state plane coordinate system, in which each state is described by one more zones, with a particular set of projection parameters associated with each zone. For example, Texas has five state plane zones (North, North-Central, Central, South-Central and South) to minimize distortion of true earth properties within each zone, and in each zone, map units in feet are used to measure distance.

Globally, the best known coordinate system is the Universal Transverse Mercator (UTM) system, which was devised during the US military operations during World War II. In UTM coordinates the map units are in meters. The world is divided into thin vertical strips bounded by meridians, each such zone being six degrees of longitude in width, and stretching from pole to pole in latitude. The zones are numbered beginning with zone 1 at 180°W and increasing eastward. Central Texas lies in zone 14. A central meridian (λ₀) runs through the middle of each zone, and for all zones the reference latitude (φ₀) is the equator. In the Northern Hemisphere, the corresponding false easting and northing (x₀,y₀) are (500,000, 0). Hence, any point on earth can be located given its UTM zone and (x,y) coordinates. You can imagine how important this was in conducting military operations during World War II using paper maps! Many of the advances in earth mapping have arisen first from military needs, including the global positioning system we now use so much in the civilian world. The Web Mercator projection is used for Google, Bing and ESRI basemaps.

In ArcGIS, the spatial reference of a feature dataset or feature class contains its datum, projection and coordinate system. This is defined uniquely for a feature dataset within a geodatabase, and serves to ensure that all feature classes within the feature dataset are represented in the same coordinate system.

Reference