

# EFFICIENT APPROACH TO SOLVING TRANSIENT UNSATURATED FLOW PROBLEMS. I: ANALYTICAL SOLUTIONS

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**Abstract:** Richard's equation governs the migration of moisture in the soil under unsaturated conditions. Although this differential equation provides a rigorous approach to simulating important infiltration problems, obtaining analytical and numerical solutions to this equation has been a particularly challenging task. This is largely due to the highly nonlinear nature of the soil hydraulic properties, including the moisture retention curve and the hydraulic conductivity function. Whereas analytical solutions of Richard's equation have been reported for problems involving steady-state conditions and simple hydraulic models, solutions for transient conditions have rarely been obtained. However, such analytical solutions would be particularly valuable, for example, to validate the accuracy of numerical schemes, as well as to facilitate parametric evaluations. A series of analytical solutions of Richard's equation for unsaturated flow under transient conditions have been developed as part of this study. The solutions involve a variety of initial and boundary conditions. The analytical solutions in this study could be obtained after expressing the governing equation as the addition of advective and diffusive flow components. The solutions consider logarithmic and linear models to represent the soil moisture retention and the hydraulic conductivity functions, respectively. Solutions are also provided for special cases in which either the advective or the diffusive components dominate the flow process, as well as for the steady-state cases. A parametric evaluation was found to provide insight into important characteristics of infiltration problems. In particular, relevant features of an unsaturated flow problem can be explained by evaluating the trends in its advective and diffusive flow components.

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Full reference:

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