Technical Paper by J.P. Giroud, M.H. Gleason and J.G. Zornberg

DESIGN OF GEOMEMBRANE ANCHORAGE AGAINST WIND ACTION

ABSTRACT: This paper provides a method for designing anchor benches and trenches used to secure geomembranes exposed to wind action. Only the cases where anchorage is provided by gravity (i.e. by the weight of the material in the bench or trench) are considered. Anchorage by tensile members is not considered. Three potential failure mechanisms are identified for the case of anchor benches: (i) sliding of the anchor bench in the downslope direction; (ii) sliding of the anchor bench in the upslope direction; and (iii) uplifting of the anchor bench. It is shown that the first mechanism is the most likely and that the third mechanism is the least likely. Criteria are provided to determine the governing potential failure mechanism as a function of the geometry of the slope on which the geomembrane is resting and the geomembrane tensions induced by wind action. Equations are provided to calculate the required size of anchor benches for each of the three identified potential failure mechanisms. It is shown that the usual method, which consists of only checking the resistance of anchor benches against uplifting, is unconservative because, in the case of anchor benches, lateral sliding is more likely to occur than uplifting. The use of the proposed method as a conservative approach for the design of anchor trenches is discussed. Practical recommendations are made and design examples are provided.

KEYWORDS: Geomembrane, Wind, Uplift, Anchor, Trench, Bench.

AUTHORS: J.P. Giroud, Chairman Emeritus, GeoSyntec Consultants, 621 N.W. 53rd Street, Suite 650, Boca Raton, Florida 33487, USA, Telephone: 1/561-995-0900, Telefax: 1/561-995-0925, E-mail: jpgiroud@geosyntec.com; M.H. Gleason, Senior Project Engineer, GeoSyntec Consultants, 532 Great Road, Acton, Massachusetts 01720, USA, Telephone: 1/978-263-9588, Telefax: 1/978-263-9594, E-mail: markg@geosyntec.com; and J.G. Zornberg, Assistant Professor, Department of Civil, Environmental and Architectural Engineering, University of Colorado at Boulder, Campus Box 428, Boulder, Colorado 80309-0428, USA, Telephone: 1/303-492-4699, Telefax:1/303-492-7317, E-mail: zornberg@colorado.edu.

PUBLICATION: *Geosynthetics International* is published by the Industrial Fabrics Association International, 1801 County Road B West, Roseville, Minnesota 55113-4061, USA, Telephone: 1/651-222-2508, Telefax: 1/651-631-9334. *Geosynthetics International* is registered under ISSN 1072-6349.

DATES: Original manuscript received 13 July 1999, revised version received 29 December 1999 and accepted 31 December 1999. Discussion open until 1 September 2000.

REFERENCE: Giroud, J.P., Gleason, M.H. and Zornberg, J.G., 1999, "Design of Geomembrane Anchorage Against Wind Action", *Geosynthetics International*, Vol. 6, No. 6, pp. 481-507.

1 INTRODUCTION

1.1 Purpose

The uplift of geomembranes by wind is discussed in two papers by Giroud et al. (1995) and Zornberg and Giroud (1997). The current paper complements the 1995 and 1997 papers by presenting a method for designing anchors to secure geomembranes that are exposed to wind action.

1.2 Configuration of a Geomembrane Uplifted by Wind

Figure 1 shows the configuration of a geomembrane that is uplifted between two anchors by wind action. The angle, θ, and the tension, T, can be calculated as a function of the distance between anchors, L, the slope angle, β, and other parameters including the velocity of the wind and the tensile characteristics of the geomembrane, using the method developed by Giroud et al. (1995) and extended by Zornberg and Giroud (1997).

1.3 Types of Anchors

Anchorage is assumed to be provided by gravity, i.e. by the weight of the material (e.g. soil, concrete) located in an anchor trench (Figures 2a and 2b), or an anchor bench (Figure 2c). The case of tensile anchor members that are driven or screwed into the ground is not considered herein.

1.4 Scope

In the current paper, equations are developed for the case of anchor benches. However, these equations can also be used as a conservative approach for the case of anchor trenches (Section 3.7).

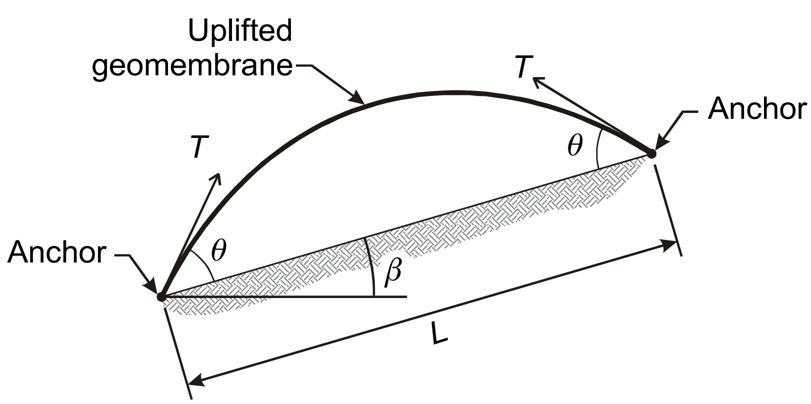

Figure 1. Configuration of an uplifted geomembrane.

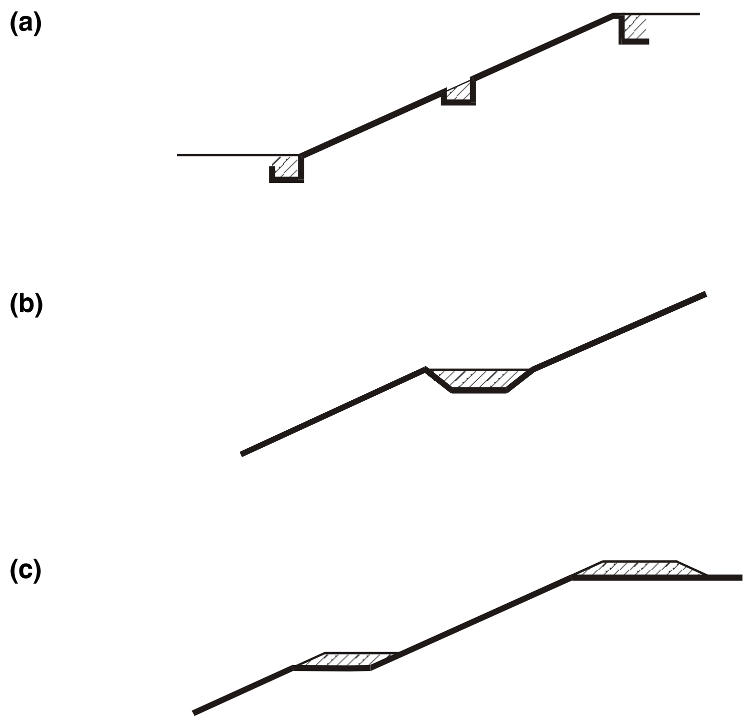

Figure 2. Examples of anchor trenches (a, b) and benches (c).

2 ANALYSIS

2.1 Assumptions

The anchor bench shown in Figure 3 is considered. It is assumed that the geomembrane is continuous through the anchor bench (i.e. the geomembrane is not interrupted under the anchor bench). The interface shear strength at the interface between the geomembrane and the underlying soil is assumed to be purely frictional. Accordingly, it is characterized by an interface friction angle, δ, and zero adhesion.

The bottom of the anchor bench is sloping at an angle β_a . This angle, which is generally small (e.g. tanβ_a = 2%), is used to provide drainage. The angle β_a is assumed to be less than the interface friction angle, δ, between the geomembrane and the underlying soil, as required to ensure stability of the anchor bench when there is no wind. The angle β_a can be positive (as shown in Figure 3) if water is allowed to run off along the downslope or if there is a collector pipe or swale at Point A. The angle β_a can be negative if there is a collector pipe or swale at Point B. The angle β_a can be zero if no drainage is needed or if drainage is provided otherwise.

The soil supporting the geomembrane is sloping at an angle β_d on the downslope side of the anchor bench and at an angle β_u on the upslope side of the anchor bench (Figure 3). These two angles are often equal. If the geomembrane is on horizontal ground, the angles β_a , β_d , and β_u are equal to zero.

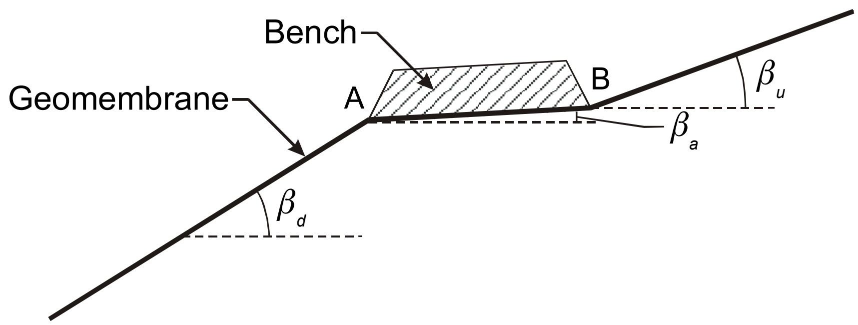

Figure 3. Anchor bench geometry.
Note: The angle β_a is positive in the figure.

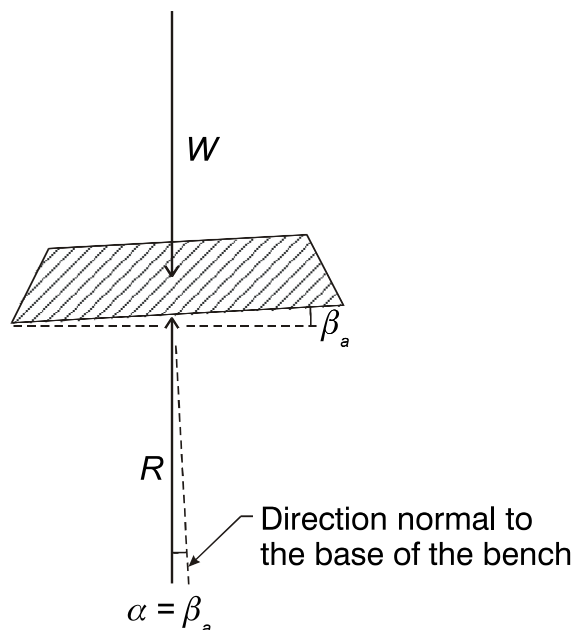

Figure 4. Forces acting on an anchor bench when there is no wind.

2.2 Forces Involved

When there is no wind, the only forces involved are the weight, W, of the anchor bench and the soil reaction, R, at the interface between the geomembrane and the underlying soil (Figure 4). The soil reaction, R, is equal and opposite to the weight, W. As a result, the angle α between the direction perpendicular to the base of the anchor bench and the reaction R is equal to β_a . Because β_a is less than δ, as indicated in Section 2.1, the anchor bench does not slide along its base when there is no wind. More generally, if only vertical forces act on the anchor bench, no sliding occurs along the base of the anchor bench because β_a is less than δ.

When the wind blows, the geomembrane is uplifted, and, as a result, it is under tension. The forces involved are shown in Figure 5. They include: the weight, W, of the anchor bench; the soil reaction, R, at the interface between the geomembrane and the underlying soil; the geomembrane tension, T_d , which is applied on the downslope side of the anchor; and the geomembrane tension, T_u , which is applied on the upslope side of the anchor. The orientations of the two geomembrane tensions are characterized by the angles θ_d and θ_u . The magnitude and orientation of the two tensions depend on sev-

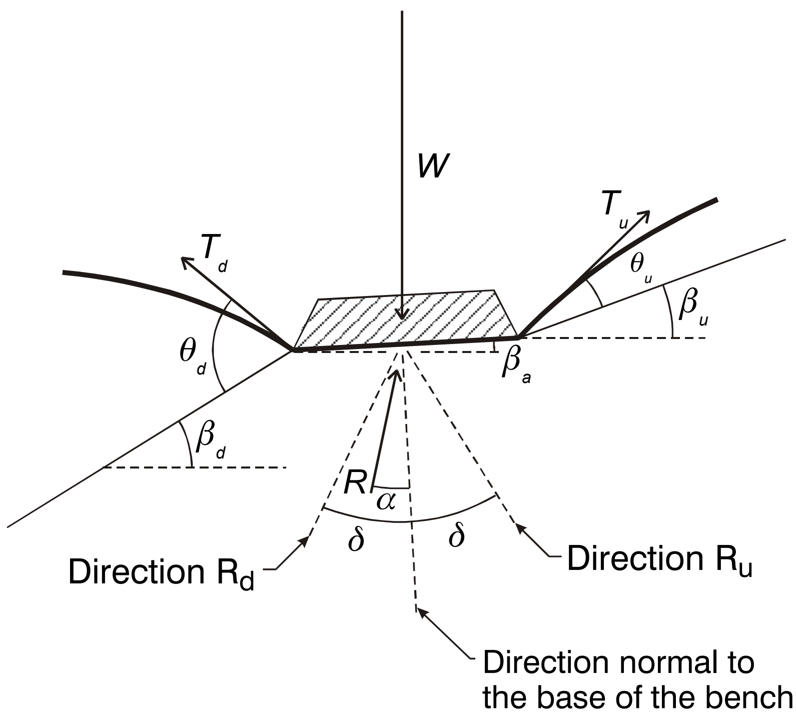

Figure 5. Forces acting on an anchor bench when the geomembrane is uplifted by wind action.

Note: The soil reaction, R, is located either in R_d if sliding occurs in the downslope direction (see Figure 7a) or in R_u if sliding occurs in the upslope direction (see Figure 8a).

eral parameters, such as the wind velocity, the length of geomembrane exposed on each side of the anchor, and the tensile stress-strain curve of the geomembrane (which depends on temperature) (Giroud et al. 1995).

The reaction, R, of the soil located beneath the geomembrane at the base of the bench forms an angle α with the direction normal to the base of the bench (Figure 5). The anchor bench does not slide along its base as long as α is smaller than the interface friction angle, δ. If the forces W, T_d, and T_u are such that the angle α is equal to δ, sliding occurs (because it was assumed in Section 2.1 that the interface shear strength between the geomembrane and the underlying soil is purely frictional). There are two possible directions of sliding depending on whether R is on the downslope or upslope side of the direction normal to the base of the bench. If R is on the downslope side, i.e. if R is located on the direction R_d in Figure 5, sliding occurs in the downslope direction. If R is on the upslope side, i.e. if R is located on the direction R_u in Figure 5, sliding occurs in the upslope direction. The direction of sliding is further discussed in Section 2.3 where criteria for the determination of the sliding direction are developed.

2.3 Criteria for Sliding Direction

2.3.1 *Development of Criteria*

As shown in Figure 6, all the forces involved can be decomposed into two components, a vertical component (with a subscript 1) and a component parallel to the potential sliding plane, i.e. the plane of the base of the anchor bench (with a subscript 2). Figure 6 shows the situation where the projection, R_2, of the reaction R on the potential sliding plane is in the upslope direction, i.e. the situation where sliding tends to occur in the downslope direction. In this case:

$$T_{d2} > T_{u2} \tag{1}$$

where: T_{d2} = projection of T_d on the potential sliding plane; T_{u2} = projection of T_u on the potential sliding plane; T_d = geomembrane tension on the downslope side of the anchor; and T_u = geomembrane tension on the upslope side of the anchor.

As seen in Figure 6, Equation 1 is equivalent to:

$$T_{dH} > T_{uH} \tag{2}$$

where: T_{dH} = horizontal projection of T_d; and T_{uH} = horizontal projection of T_u. These two projections are expressed as follows, based on Figures 5 and 6:

$$T_{dH} = T_d \cos(\theta_d - \beta_d) \tag{3}$$

$$T_{uH} = T_u \cos(\theta_u + \beta_u) \tag{4}$$

where: θ_d = angle of the geomembrane with the slope on the downslope side of the anchor; θ_u = angle of the geomembrane with the slope on the upslope side of the anchor; β_d = slope angle on the downslope side of the anchor; and β_u = slope angle on the upslope side of the anchor. Equations 3 and 4 can be used with any set of coherent units.

Similarly, sliding tends to occur in the upslope direction if:

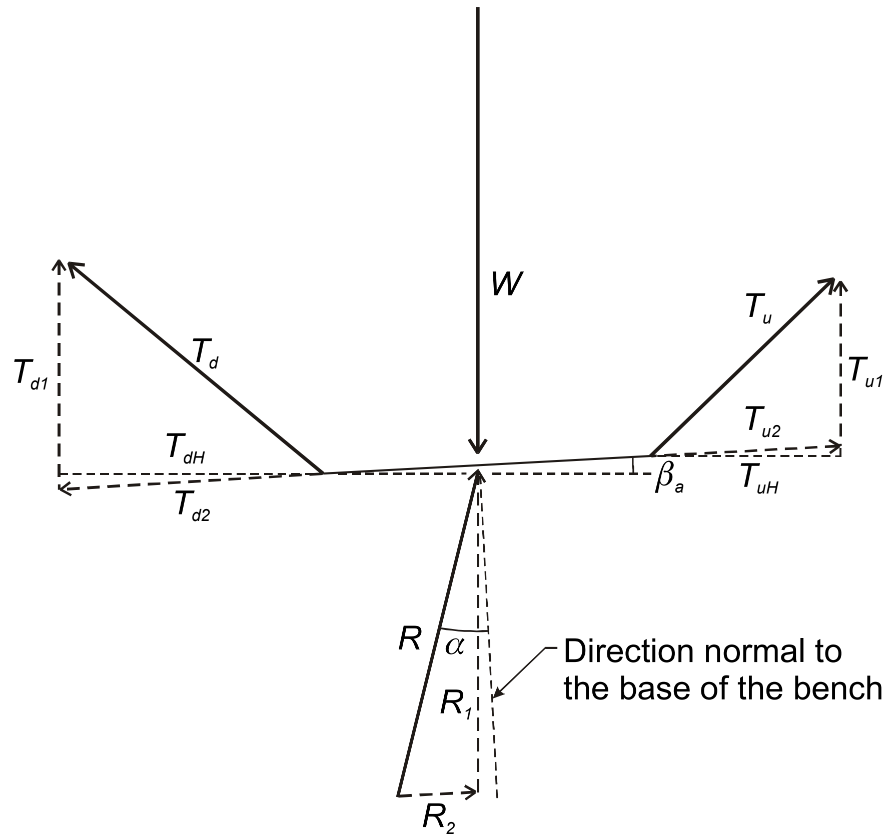

Figure 6. Decomposition of the forces acting on an anchor bench into vertical components (subscript 1) and components parallel to the base of the anchor bench (subscript 2).

$$T_{dH} < T_{uH} \qquad (5)$$

Finally, if:

$$T_{dH} = T_{uH} \qquad (6)$$

sliding cannot occur and the only potential mode of failure of the anchor bench is by uplifting.

2.3.2 *Comments*

It is important to note that Equations 2, 5, and 6 are independent of the slope of the geomembrane under the anchor bench, β_a. In other words, the direction of sliding (downslope or upslope) can be defined using the horizontal projections, T_{dH} and T_{uH}, of the geomembrane tensions, T_d and T_u, and not the projections of these tensions on the base of the anchor bench. It is also important to note that Equations 2, 5, and 6 are independent of *W*, which is the unknown in the design of an anchor bench.

It should be noted that the case of sliding in the downslope direction is more likely than the case of sliding in the upslope direction because $\cos(\theta_d - \beta_d)$ is generally significantly greater than $\cos(\theta_u + \beta_u)$, and, as a result, the criterion expressed by Equation 2 is more likely to be satisfied than the criterion expressed by Equation 5. However, due to the numerous parameters that affect geomembrane tension (Giroud et al. 1995), there are cases where T_u is significantly greater than T_d, which may result in the fact that the criterion expressed by Equation 5 is satisfied and the criterion expressed by Equation 2 is not. Finally, it should be noted that the case of uplifting is rare because it occurs only when Equation 6 is satisfied, i.e. when, by chance, the horizontal projections of T_d and T_u are equal.

2.4 Development of Equations for Sizing Anchor Benches

2.4.1 *Anchor Failure by Sliding in the Downslope Direction*

The forces involved in the case where an anchor bench is at the verge of failure due to sliding in the downslope direction are shown in Figure 7a. Balancing these forces can be done by projecting the forces on the direction XX′ perpendicular to the direction of the soil reaction, *R*, in order to eliminate the unknown magnitude of *R* (Figure 7b). The following equation is thus obtained for the minimum required weight of the bench, i.e. the weight that corresponds to failure of the anchor bench:

$$W_{min} = W_{min\,downsliding} = \frac{T_d \cos(\theta_d - \beta_d - \delta + \beta_a) - T_u \cos(\theta_u + \beta_u + \delta - \beta_a)}{\sin(\delta - \beta_a)} \quad (7)$$

where: W_{min} = minimum required weight of the anchor material per unit length perpendicular to the plane of the cross section; $W_{min\,downsliding}$ = value of W_{min} in the case where sliding tends to occur in the downslope direction; δ = interface friction angle between the geomembrane and the underlying soil; and β_a = slope of the geomembrane in the anchor bench. The other symbols were defined after Equation 4. The angle δ is always positive. The angle β_a is positive when the anchor bench is sloping in the downslope direction (as shown in Figure 3) and is negative when the anchor bench is sloping in the upslope direction. Equation 7 (as well as the following Equations 8 and 9) can be used with any set of coherent units.

2.4.2 *Anchor Failure by Sliding in the Upslope Direction*

The forces involved in the case where an anchor bench is at the verge of failure due to sliding in the upslope direction are shown in Figure 8a. Balancing these forces can be done by projecting the forces on the direction XX′ perpendicular to the direction of the soil reaction, *R*, in order to eliminate the unknown magnitude of *R* (Figure 8b). The following equation is thus obtained for the minimum required weight of the bench, i.e. the weight that corresponds to failure of the anchor bench:

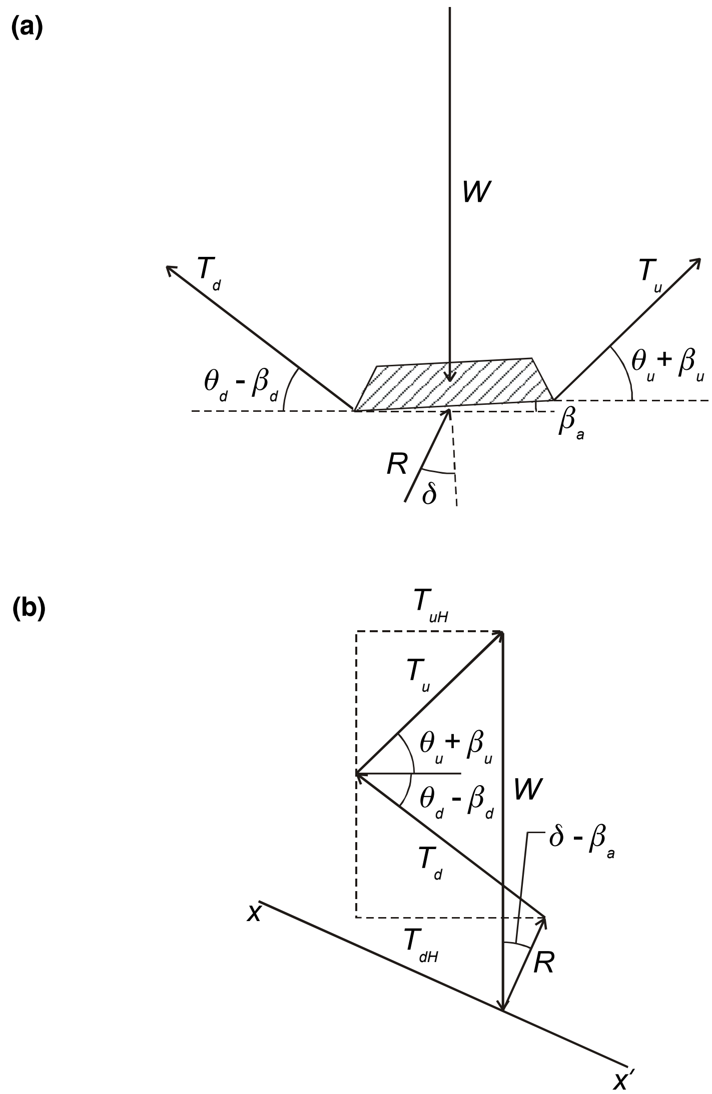

Figure 7. Case of anchor bench failure by sliding in the downslope direction: (a) forces acting on the anchor bench; (b) force diagram.

$$W_{min} = W_{min\,upsliding} = \frac{-\,T_d\,\cos(\theta_d - \beta_d + \delta + \beta_a) + T_u\,\cos(\theta_u + \beta_u - \delta - \beta_a)}{\sin(\delta + \beta_a)} \quad (8)$$

where $W_{min\ upsliding}$ is the value of W_{min} in the case where sliding tends to occur in the upslope direction.

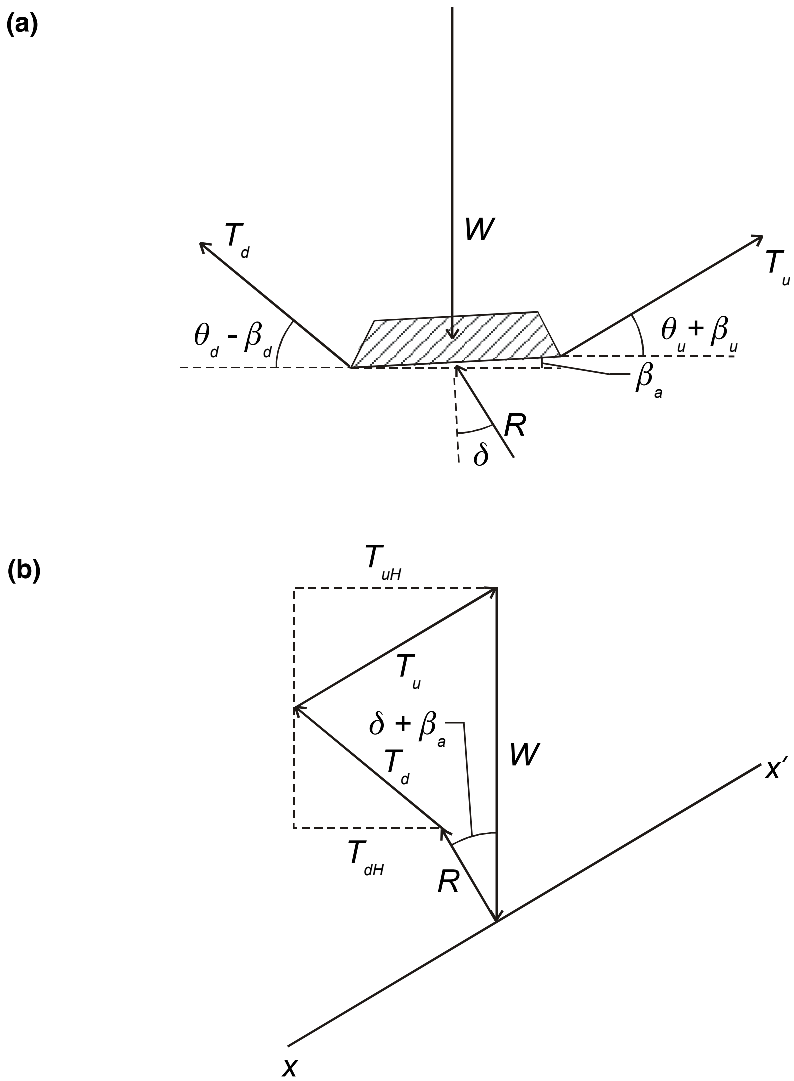

Figure 8. Case of anchor bench failure by sliding in the upslope direction: (a) forces acting on the anchor bench; (b) force diagram.

2.4.3 *Anchor Failure by Uplifting*

The forces involved in the case where an anchor bench is at the verge of failure due to uplifting are shown in Figure 9a. Balancing these forces, considering that the interface between the geomembrane and the underlying soil is purely frictional (Figure 9b),

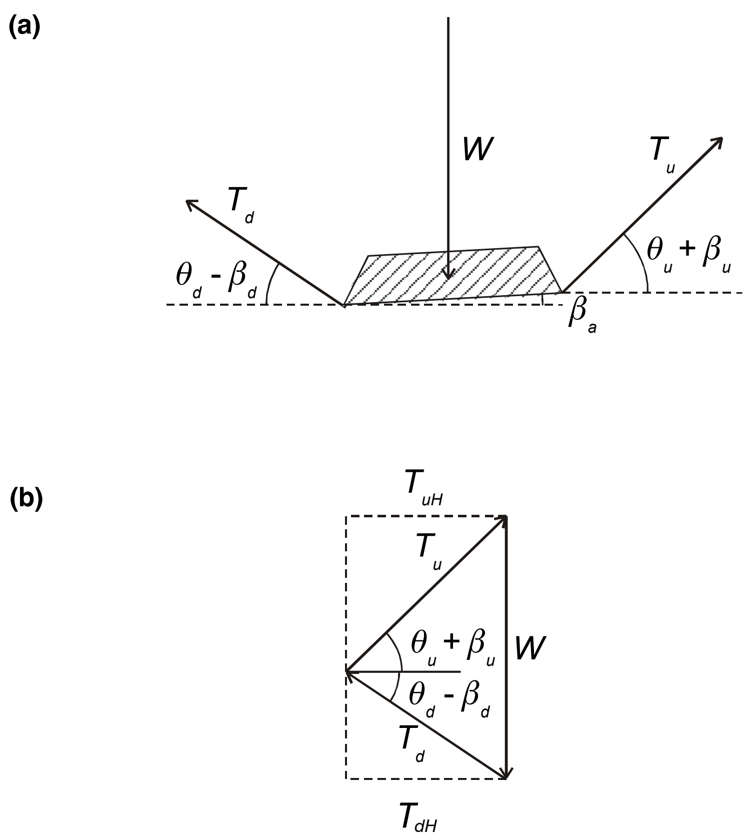

Figure 9. Case of anchor bench failure by uplifting: (a) forces acting on the anchor bench; (b) force diagram.

gives the following equation for the minimum required weight of the bench, i.e. the weight that corresponds to failure of the anchor bench:

$$W_{min} = W_{min\,uplifting} = T_d \sin(\theta_d - \beta_d) + T_u \sin(\theta_u + \beta_u) \tag{9}$$

where $W_{min\,uplifting}$ is the value of W_{min} in the case where anchor bench failure results from uplifting.

2.4.4 *Special Case*

In the common case where the soil slope is the same on both sides of the anchor, $\beta_d = \beta_u = \beta$ in Equations 7, 8, and 9. On the other hand, based on information provided by Giroud et al. (1995), it is unlikely that the angle θ and the geomembrane tension, T, would be the same on both sides of the anchor bench.

2.5 Discussion of the Equations

For the sake of analytical completeness, it is interesting to demonstrate that there is continuity between the three failure mechanisms presented in Section 2.4. This can be demonstrated by showing that: (i) Equation 7 tends progressively toward Equation 9 when the criterion expressed by Equation 2 tends toward the criterion expressed by Equation 6; and (ii) Equation 8 tends progressively toward Equation 9 when the criterion expressed by Equation 5 tends toward the criterion expressed by Equation 6. This is shown below.

Equation 7 can be transformed as follows using the classical relationships for $\cos(x + y)$ and $\cos(x - y)$:

$$W_{min} = W_{min\,downsliding} = \frac{T_d\,[\cos(\theta_d - \beta_d)\,\cos(\delta - \beta_a) + \sin(\theta_d - \beta_d)\sin(\delta - \beta_a)]}{\sin(\delta - \beta_a)} - \frac{T_u\,[\cos(\theta_u + \beta_u)\,\cos(\delta - \beta_a) - \sin(\theta_u + \beta_u)\,\sin(\delta - \beta_a)]}{\sin(\delta - \beta_a)} \tag{10}$$

hence:

$$W_{min\,downsliding} = \frac{T_d\,\cos(\theta_d - \beta_d) - T_u\,\cos(\theta_u + \beta_u)}{\tan(\delta - \beta_a)} + T_d\,\sin(\theta_d - \beta_d) + T_u\,\sin(\theta_u + \beta_u) \tag{11}$$

If Equation 2 tends progressively toward Equation 6, the first term of Equation 11 tends progressively toward zero. Therefore, $W_{min\,downsliding}$ tends progressively toward the value given by Equation 9. This demonstrates that there is continuity between Equations 7 and 9. Furthermore, Equation 2 shows that the first term of Equation 11 is positive. Therefore, comparison of Equations 9 and 11 shows that:

$$W_{min\,downsliding} > W_{min\,uplifting} \tag{12}$$

Similar calculations show that there is continuity between Equations 8 and 9 and that:

$$W_{min\,upsliding} > W_{min\,uplifting} \tag{13}$$

Equations 12 and 13 will be useful for the discussion presented in Section 3.6.

2.6 Influence of Parameters

2.6.1 *Influence of the Slope of the Geomembrane in the Anchor Bench*

The influence of the slope of the geomembrane in the anchor bench on the value of the minimum required anchor weight can be evaluated by calculating the derivative of

W_{min} with respect to β_a . After lengthy calculations, the following values are obtained that are remarkably simple:

$$\frac{\partial W_{min\,downsliding}}{\partial \beta_a} = \frac{T_d \cos(\theta_d - \beta_d) - T_u \cos(\theta_u + \beta_u)}{\sin^2(\delta - \beta_a)} \tag{14}$$

$$\frac{\partial W_{min\,upsliding}}{\partial \beta_a} = \frac{T_d \cos(\theta_d - \beta_d) - T_u \cos(\theta_u + \beta_u)}{\sin^2(\delta + \beta_a)} \tag{15}$$

From Equation 2, it is known that the numerator of Equation 14 is positive. Because the denominator is also positive (since it is a square), the derivative expressed by Equation 14 is positive. Therefore, $W_{min\ downsliding}$ increases if β_a increases. In other words, increasing β_a is detrimental to the anchor's performance in the case where anchor failure results from sliding in the downslope direction, which is physically obvious.

From Equation 5, it is known that the numerator of Equation 15 is negative. Because the denominator is positive (since it is a square), the derivative expressed by Equation 15 is negative. Therefore, $W_{min\ upsliding}$ decreases if β_a increases. In other words, increasing β_a is beneficial to the anchor's performance in the case where anchor failure results from sliding in the upslope direction, which is physically obvious.

Finally, Equation 9 shows that $W_{min\ uplifting}$ does not depend on β_a .

It should be noted that the influence of β_a on the value of the minimum required anchor weight, as evaluated in Equations 14 and 15 using derivatives, is consistent with what could be physically expected. This confirms the validity of the equations presented in the current paper.

2.6.2 *Influence of the Interface Friction Angle*

The influence of the interface friction angle between the geomembrane and the underlying soil on the value of the minimum required anchor weight can be evaluated by calculating the derivative of W_{min} with respect to δ. Lengthy calculations are avoided by noting that δ plays the same role as $-\beta_a$ in Equation 7 and the same role as β_a in Equation 8. Therefore, the derivatives of W_{min} with respect to δ can be derived from the derivatives of W_{min} with respect to β_a , hence:

$$\frac{\partial W_{min\,downsliding}}{\partial \delta} = \frac{- T_d \cos(\theta_d - \beta_d) + T_u \cos(\theta_u + \beta_u)}{\sin^2(\delta - \beta_a)} \tag{16}$$

$$\frac{\partial W_{min\,upsliding}}{\partial \delta} = \frac{T_d \cos(\theta_d - \beta_d) - T_u \cos(\theta_u + \beta_u)}{\sin^2(\delta + \beta_a)} \tag{17}$$

From Equation 2, it is known that the numerator of Equation 16 is negative and, from Equation 5, it is known that the numerator of Equation 17 is negative. Because the denominators of both equations are positive (since they are squares), the two above derivatives are negative. Therefore, both $W_{min\ downsliding}$ and $W_{min\ upsliding}$ decrease if δ increases. In other words, increasing δ is beneficial to the anchor's performance, regardless of the direction of sliding, which is physically obvious.

It should be noted that, in the case of an anchor bench, passive pressure may exist on the upslope side of the bench, such as in the case of the lower bench in Figure 2c. Therefore, in this case, Equation 8 (i.e. the equation for the case where sliding occurs in the upslope direction) gives an upper boundary of the required weight of the anchor bench material, because Equation 8 does not account for passive pressure.

Finally, Equation 9 shows that $W_{min\ uplifting}$ does not depend on δ .

It should be noted that the influence of δ on the value of the minimum required anchor weight, as evaluated in Equations 16 and 17 using derivatives, is consistent with what could be physically expected. This confirms the validity of the equations presented in the current paper.

3 APPLICATION AND DISCUSSION

General information on the use of equations presented in Section 2 is provided in Sections 3.1 to 3.3. Then, the sizing of anchor benches is addressed in Sections 3.4 to 3.6, and the sizing of anchor trenches in Section 3.7. Finally, practical recommendations are made in Section 3.8.

3.1 Preliminary Calculations

Prior to using the equations presented in Section 2 to design an anchor bench or trench, the design engineer should calculate the geomembrane tensions (T_d and T_u) and the angles between the uplifted geomembrane and the supporting soil (θ_d and θ_u) using the method developed by Giroud et al. (1995) and extended by Zornberg and Giroud (1997). For a given geomembrane on a given slope with a given length, the largest values of the geomembrane tensions and angles are obtained for the highest wind velocity. Based on this fact, inspection of Equation 9 (i.e. the equation for the case where anchor failure results from uplifting) reveals that the higher the wind velocity, the higher the minimum required value of the weight of the anchor, W_{min} . On the other hand, it does not seem possible, just from inspection of Equations 7 and 8, to draw a general conclusion regarding the variation of W_{min} as a function of wind velocity for the cases where anchor failure results from sliding (which are the most likely cases, as indicated in Section 2.3.2). Some guidance is provided by Figures 7b and 8b. If these figures are redrawn with increased values of the four wind-related parameters (T_d , T_u , θ_d , and θ_u), the minimum required weight of the anchor, W_{min} , generally increases, but it is possible to find cases where W_{min} decreases. However, it is not possible to know from the force diagram (i.e. without performing lengthy wind uplift calculations) if these cases correspond to the same wind velocity increase on both sides of the anchor bench. To completely determine the variation of the minimum required value of the weight of the anchor, W_{min} , as a function of wind velocity would require an exhaustive parametric study; this is beyond the scope of the current paper. In the special case where $T_d = T_u$ and $\theta_d = \theta_u$, an analysis presented in the Appendix shows that the minimum required value of the weight of the anchor is approximately proportional to the square of the wind velocity. This confirms that W_{min} generally increases when the wind velocity increases.

Based on the foregoing discussion, it is sufficient, in most cases, to perform calculations for the maximum wind velocity. However, a careful designer may calculate the values of θ_d , T_d , θ_u , and T_u for several wind velocities, as an attempt to find if there is a case more critical than the case where the maximum wind velocity is used. Also, a careful designer may try potential "worst cases" by calculating θ_d and T_d for a certain wind velocity and θ_u and T_u for a different wind velocity to account for different gusts of wind on the downslope portion and the upslope portion of the geomembrane. For example, as an extreme case, the design engineer may use a zero wind velocity on one side of the anchor and the maximum wind velocity at the site on the other side of the anchor. Comparing Equations 7 and 8, it appears that this scenario is mostly severe if the maximum wind velocity is considered on the downslope side of the anchor. This is contrary to what happens in general: the wind generated suction is generally greater in the upper part of a slope than in the lower part (Giroud et al. 1995).

3.2 Factor of Safety

Equations 7, 8, and 9 do not include a factor of safety. Design engineers using these equations can chose between two approaches to calculate the factored value of the minimum required anchor weight, i.e. the value of the weight that is increased through the use of a factor of safety.

The first approach consists of using a global factor of safety on the value of W_{min} calculated using Equations 7, 8, or 9:

$$W_{factored} = FS\,W_{min} \tag{18}$$

A value of 1.5 is suggested for the global factor of safety, *FS*.

The second approach consists of using partial factors of safety on (or ranges of values for) the parameters likely to be affected by uncertainties, namely: T_d , T_u , θ_d , θ_u , and δ. While the use of a partial factor of safety on δ is straightforward, it should be noted that the direct use of partial factors of safety on the four wind-related parameters (T_d , T_u , θ_d , and θ_u) is not possible because these parameters are not independent: T_d and θ_d are not independent because they are a function of the wind velocity considered on the downslope side of the anchor; and T_u and θ_u are not independent because they are a function of the wind velocity considered on the upslope side of the anchor. Therefore, a partial factor of safety should not be used directly on these four parameters, but could be used on the two wind velocities (or on the wind velocity if the same wind velocity is considered on both sides of the anchor). The four wind-related parameters (T_d , T_u , θ_d , and θ_u) also depend on variables such as the geomembrane characteristics and the temperature (Giroud et al. 1995; Zornberg and Giroud 1997); therefore, partial factors of safety could be used on these variables (or ranges of values could be used for these variables).

The second approach, although more complex, is preferable because it is more logical. The symbol $W_{factored}$ is also used for the factored minimum required weight obtained using partial factors of safety on (or ranges of values for) the relevant parameters.

3.3 Required Size of the Anchor

Assuming that the anchor bench or trench is continuous in the direction perpendicular to the plane of the cross section, the required cross-sectional area of the anchor bench or trench is given by:

$$A_{req} = W_{factored} / \gamma \tag{19}$$

where: $W_{factored}$ = factored minimum required weight (per unit length perpendicular to the plane of the cross section) of the anchor bench or trench, i.e. the weight (per unit length) including a global factor of safety or partial factors of safety, as explained in Section 3.2; and γ = unit weight of the anchor material. Equation 19 can be used with any set of coherent units.

To ensure that the pressure applied by the anchor bench or trench is not too small near the edges of the anchor, the ratio between the height and the width of the anchor bench should be greater than a certain minimum value (0.25 is suggested).

3.4 Design Examples

Three design examples are presented to illustrate the three failure mechanisms of anchor benches described in Section 2.4.

Example 1. A geomembrane is exposed to a high-velocity wind on a 25° slope. The following values were calculated using the method developed by Giroud et al. (1995) and extended by Zornberg and Giroud (1997): θ_d = 47°, T_d = 17 kN/m, θ_u = 36°, and T_u = 26 kN/m. The interface friction angle between the geomembrane and the underlying soil is 21°. The anchor bench slope is β_a = +2°. Calculate the required cross-sectional area of the anchor bench.

First, the criteria expressed by Equations 2, 5, and 6 should be checked to determine which of the equilibrium equations (Equations 7 to 9) should be used. Equation 3 gives:

$$T_{dH} = 17 \cos(47^\circ - 25^\circ) = 15.76 \text{ kN/m}$$

and Equation 4 gives:

$$T_{uH} = 26 \cos(36^\circ + 25^\circ) = 12.61 \text{ kN/m}$$

Therefore, the criterion expressed by Equation 2 is satisfied, and sliding tends to occur in the downslope direction. As a result, Equation 7 should be used to calculate the minimum required weight of the anchor bench:

$$W_{min} = \frac{17 \cos(47^\circ - 25^\circ - 21^\circ + 2^\circ) - 26 \cos(36^\circ + 25^\circ + 21^\circ - 2^\circ)}{\sin(21^\circ - 2^\circ)}$$

$$= 38.28 \text{ kN/m}$$

Then, Equation 18 can be used, for example with a factor of safety of 1.5, to obtain the factored weight:

$$W_{factored} = 1.5 \times 38.28 = 57.42 \text{ kN/m}$$

Finally, assuming that the unit weight of the compacted soil used as anchor material is 18 kN/m^3, Equation 19 gives the following value for the required cross-sectional area of the anchor:

$$A_{req} = 57.42 / 18 = 3.19 \text{ m}^2$$

END OF EXAMPLE 1

Example 2. A geomembrane is exposed to a high-velocity wind on a 25° slope. The following values were calculated using the method developed by Giroud et al. (1995) and extended by Zornberg and Giroud (1997): $\theta_d = 47°$, $T_d = 12$ kN/m, $\theta_u = 36°$, and $T_u = 26$ kN/m. The interface friction angle between the geomembrane and the underlying soil is 21°. The anchor bench slope is $\beta_a = +2°$. Calculate the required cross-sectional area of the anchor bench.

First, the criteria expressed by Equations 2, 5, and 6 should be checked to determine which of the equilibrium equations (Equations 7 to 9) should be used. Equation 3 gives:

$$T_{dH} = 12 \cos(47° - 25°) = 11.13 \text{ kN/m}$$

and Equation 4 gives:

$$T_{uH} = 26 \cos(36° + 25°) = 12.61 \text{ kN/m}$$

Therefore, the criterion expressed by Equation 5 is satisfied, and sliding tends to occur in the upslope direction. As a result, Equation 8 should be used to calculate the minimum required weight of the anchor bench:

$$W_{min} = \frac{-12 \cos(47° - 25° + 21° + 2°) + 26 \cos(36° + 25° - 21° - 2°)}{\sin(21° + 2°)}$$

$$= 30.72 \text{ kN/m}$$

Then, Equation 18 can be used, for example with a factor of safety of 1.5, to obtain the factored weight:

$$W_{factored} = 1.5 \times 30.72 = 46.08 \text{ kN/m}$$

Finally, assuming that the unit weight of the compacted soil used as anchor material is 18 kN/m^3, Equation 19 gives the following value for the required cross-sectional area of the anchor:

$$A_{req} = 46.08 / 18 = 2.56 \text{ m}^2$$

END OF EXAMPLE 2

Example 3. A geomembrane is exposed to a high-velocity wind on a 25° slope. The following values were calculated using the method developed by Giroud et al. (1995) and extended by Zornberg and Giroud (1997): $\theta_d = 47°$, $T_d = 13.6$ kN/m, $\theta_u = 36°$, and $T_u = 26$ kN/m. The interface friction angle between the geomembrane and the underlying soil is 21°. The anchor bench slope is $\beta_a = +2°$. Calculate the required cross-sectional area of the anchor bench.

First, the criteria expressed by Equations 2, 5, and 6 should be checked to determine which of the equilibrium equations (Equations 7 to 9) should be used. Equation 3 gives:

$$T_{dH} = 13.6 \ \cos(47° - 25°) = 12.61 \ \text{kN/m}$$

and Equation 4 gives:

$$T_{uH} = 26.0 \ \cos(36° + 25°) = 12.61 \ \text{kN/m}$$

Therefore, the criterion expressed by Equation 6 is satisfied, and failure of the anchor bench tends to occur by uplifting. As a result, Equation 9 should be used to calculate the minimum required weight of the anchor bench:

$$W_{min} = 13.6 \ \sin(47° - 25°) + 26.0 \ \sin(36° + 25°) = 27.83 \ \text{kN/m}$$

Then, Equation 18 can be used, for example with a factor of safety of 1.5, to obtain the factored weight:

$$W_{factored} = 1.5 \times 27.83 = 41.75 \ \text{kN/m}$$

Finally, assuming that the unit weight of the compacted soil used as anchor material is 18 kN/m^3, Equation 19 gives the following value for the required cross-sectional area of the anchor:

$$A_{req} = 41.75 \ / \ 18 = 2.32 \ \text{m}^2$$

END OF EXAMPLE 3

3.5 Influence of Parameters

3.5.1 *Influence of the Slope of the Geomembrane in the Anchor Bench*

In Example 1, the slope of the geomembrane in the anchor bench is $\beta_a = +2°$ (where the + sign indicates that the geomembrane slope has the same orientation as shown in Figure 3). Calculations done using Equation 7 for the same case with different values of β_a give the following values for the minimum required weight of the anchor material:

$$W_{min} = 36.55 \text{ kN/m for } \beta_a = -2^\circ$$
$$W_{min} = 37.33 \text{ kN/m for } \beta_a = 0^\circ$$
$$W_{min} = 38.28 \text{ kN/m for } \beta_a = +2^\circ$$

It is interesting to compare these values with those obtained using the derivative calculations presented in Section 2.6.1. For $\beta_a = +1^\circ$ (i.e. average between 0° and +2°), Equation 14 gives:

$$\frac{\partial W_{min\,downsliding}}{\partial \beta_a} = 26.99 \text{ kN/m}$$

An increase in β_a of +2° (i.e. 0.349 radians) results in an increase of $W_{min\,downsliding}$ of:

$$\Delta W_{min\,downsliding} = 26.99 \times 0.0349 = 0.94 \text{ kN/m}$$

This is consistent with the difference 38.28 - 37.33 = 0.95. (One should not expect the derivative and the difference to give exactly the same result because 2° is not an infinitesimal increment.)

In Example 2, the slope of the geomembrane in the anchor bench is $\beta_a = +2^\circ$ (where the + sign indicates that the geomembrane slope has the same orientation as shown in Figure 3). Calculations done using Equation 8 for the same case with different values of β_a give the following values for the minimum required weight of the anchor material:

$$W_{min} = 31.53 \text{ kN/m for } \beta_a = -2^\circ$$
$$W_{min} = 31.09 \text{ kN/m for } \beta_a = 0^\circ$$
$$W_{min} = 30.72 \text{ kN/m for } \beta_a = +2^\circ$$

It is interesting to compare these values with those obtained using the derivative calculations presented in Section 2.6.1. For $\beta_a = +1^\circ$ (i.e. average between 0° and +2°), Equation 15 gives:

$$\frac{\partial W_{min\,upsliding}}{\partial \beta_a} = -10.54 \text{ kN/m}$$

An increase in β_a of +2° (i.e. 0.349 radians) results in a decrease of $W_{min\,upsliding}$ of:

$$\Delta W_{min\,upsliding} = -10.54 \times 0.0349 = -0.37 \text{ kN/m}$$

This is consistent with the difference 30.72 - 31.09 = -0.37.

In the above examples, the influence of β_a is not very large. However, the influence of β_a can be large if the numerator of Equation 14 or 15 is large.

3.5.2 *Influence of the Interface Friction Angle*

Using the values of the parameters of Example 1, with the exception of taking $\beta_a = 0^\circ$ and considering δ a variable, the following values are obtained for W_{min} using Equation 7:

$$W_{min} = 37.33 \text{ kN/m} \text{ for } \beta_a = 0^\circ \text{ and } \delta = 21^\circ$$
$$W_{min} = 36.55 \text{ kN/m} \text{ for } \beta_a = 0^\circ \text{ and } \delta = 23^\circ$$

It is interesting to compare these values with those obtained using the derivative calculations presented in Section 2.6.2. For $\delta = 22°$, which is average of 21° and 23°, Equation 16 gives:

$$\frac{\partial W_{min\,downsliding}}{\partial \delta} = -22.50 \text{ kN/m}$$

An increase in δ of $+2°$ (i.e. 0.349 radians) results in an increase of $W_{min\,downsliding}$ of:

$$\Delta W_{min\,downsliding} = -22.50 \times 0.0349 = -0.79 \text{ kN/m}$$

This is consistent with the difference 36.55 – 37.33 = –0.78. (One should not expect the derivative and the difference to give exactly the same result because 2° is not an infinitesimal increment.)

In the above examples, the influence of δ is not very large. However, the influence of δ can be large if the numerator of Equation 16 or 17 is large.

3.6 Influence of the Failure Mechanism Considered

The senior author has designed anchor trenches to secure geomembranes subjected to wind action since the 1970s. This was always done by evaluating the required weight of the anchor trench to prevent uplifting of the anchor trench, as recommended in a paper by Giroud and Huot (1977). Apparently, no paper was published on the subject in the English language technical literature until 1998, when Gleason et al. (1998) published a paper where the use of an equation identical to Equation 9 of the current paper (i.e. the equation for uplifting) for the design of an anchor trench is mentioned. While it may be legitimate to design anchor trenches by considering only the mode of failure by uplifting (as discussed in Section 3.7), this practice is not appropriate in the case of anchor benches, as discussed below.

In the case of anchor benches, considering a purely frictional interface between the geomembrane and the underlying soil, it has been shown in Section 2.5 that, for any given case, Equation 9 (i.e. the equation for uplifting) gives values of the minimum required weight that are lower than the values obtained using Equations 7 and 8 (i.e. the equations for sliding). Therefore, in the case of anchor benches, it is unconservative to follow the practice that consists of only considering the mode of failure by uplifting (except, of course, in the case where uplifting is the mode of failure of the anchor bench, but this case is rare, as indicated in Section 2.3.2). The magnitude of the error thus made may be significant, as illustrated by the two examples below.

In Example 1, if Equation 9 had been used instead of Equation 7 the following value would have been obtained for the minimum required value of the anchor bench weight:

$$W_{min} = W_{min\,uplifting} = 17 \sin(47^\circ - 25^\circ) + 26 \sin(36^\circ + 25^\circ) = 29.11 \text{ kN/m}$$

Using 29.11 kN/m instead of 38.28 kN/m results in a 24% underestimation of the required weight of the anchor bench.

In Example 2, if Equation 9 had been used instead of Equation 8 the following value would have been obtained for the minimum required value of the anchor bench weight:

$$W_{min} = W_{min\,uplifting} = 12\ \sin(47^\circ - 25^\circ) + 26\ \sin(36^\circ + 25^\circ) = 27.24\ \text{kN/m}$$

Using 27.24 kN/m instead of 30.72 kN/m results in an 11% underestimation of the required weight of the anchor bench.

From the foregoing discussion and examples, it may be concluded that, in the case of anchor benches, it is unconservative to follow the practice that consists of only considering the mode of failure by uplifting. Therefore, the equations presented in the current paper, which make it possible to design anchor benches not only against uplifting but also against sliding, should lead to better designs.

3.7 Application to the Case of Anchor Trenches

The equations presented in Section 2 correspond to the case of an anchor bench. The only mechanism of resistance to lateral sliding considered in the development of the equations presented in Section 2 is the interface friction between the geomembrane and the underlying material (i.e. no cohesive component was considered for the interface shear strength). If anchorage is provided by an anchor trench (Figures 2a and 2b), the passive pressure on the trench side, opposite to the direction of potential sliding, adds to the sliding resistance provided by the interface friction between the geomembrane and the underlying material. Therefore, in the case of an anchor trench, Equations 7 and 8 (i.e. the equations for sliding) give an upper boundary of the required weight of the anchor trench material. However, if, for a specific anchor trench, the design engineer estimates that there is no risk of sliding (as discussed below), the use of Equations 7 and 8 may not be necessary.

If the edges of an anchor trench provide sufficient passive pressure against lateral sliding (which may well be the case with most anchor trenches), the likely mode of failure of the anchor trench is uplifting. In this case, it is adequate to only consider uplifting. Equation 9 (developed for the uplifting of anchor benches) can then be used for anchor trenches because, when failure results from uplifting, an anchor trench and an anchor bench that have the same geometry are equivalent since the passive pressure at the edges of the trench is not mobilized.

3.8 Practical Recommendations

The required cross-sectional areas obtained in Examples 1 to 3 are of the order of 3 m^2. If the width available for the bench is only 3 or 4 m, there is not enough space to construct a soil bench with a cross-sectional area of 3 m^2 due to the soil angle of repose. Furthermore, a soil bench with sloping edges may not perform well because the pressure applied on the geomembrane near the edges is small, which may result in uplift of the edge of the bench and partial destruction of the bench when the wind uplifts the exposed portion of the geomembrane. It is important that the bench act as a monolith, thereby applying a pressure as uniform as possible on the geomembrane

Based on the foregoing discussion, it is recommended that: (i) the ratio between the height and the width of the anchor bench be greater than a certain minimum value (0.25 is suggested); and (ii) the soil in the bench be confined and/or reinforced in order to construct a bench with vertical or quasi-vertical edges. Methods that can be used to confine and/or reinforce the soil include the following: (i) soil bags can be used, at least at the periphery of the bench; (ii) the entire bench can be encapsulated in a geomembrane, which prevents the desiccation of compacted soil, thus maintaining the integrity of the bench, a technique known as membrane-encapsulated soil layer (MESL); (iii) the edges of the bench can be constructed quasi-vertical using "wrapped-around" geotextiles, geomembranes, or geogrids (a technique similar to that used for the construction of some reinforced soil walls); (iv) layers of soil-filled geocells can be stacked on top of each other; and/or (v) layers of geosynthetics, such as geotextiles, geogrids, and polymeric straps, can be used to provide internal reinforcement to the bench. Several of these methods can be combined. Alternatively, concrete could be used to construct an anchor bench, or an anchor trench could be used.

If an anchor trench is used, it is important that the edges be as vertical as possible. This will ensure that the pressure applied on the geomembrane at the bottom of the trench is as uniform as possible, thereby minimizing the risk of uplift of the edge of the trench and partial destruction of the trench when the wind uplifts the exposed portion of the geomembrane (a potential problem already mentioned above for anchor benches). An example of anchor trench with quasi-vertical edges, specifically designed to resist geomembrane uplift by wind action, is described in a paper by Gleason et al. (1998). Also, as for anchor benches, the ratio between the height and the width of the anchor bench should be greater than a certain minimum value (0.25 is suggested).

4 CONCLUSIONS

Designing an exposed geomembrane against wind action requires accurate prediction of the tensions induced by wind in the geomembrane and proper sizing of the anchors used to secure the geomembrane. Accurate prediction of the tensions induced by wind in the geomembrane can be done by using the method developed by Giroud et al. (1995) and extended by Zornberg and Giroud (1997). This method has already been used to design several exposed geomembranes (see, for example, the paper by Gleason et al. 1998).

Sizing of the anchors used to secure the geomembrane is the subject of the current paper. Only cases where anchorage is provided by gravity (i.e. by the weight of the material in a bench or a trench) are considered. Anchorage by tensile members that are driven or screwed into the ground is not considered. The interface shear strength between the geomembrane and the underlying soil is assumed to be purely frictional. The equations presented were developed for anchor benches (Figure 2c); however, they can also be used for anchor trenches (Figures 2a and 2b), as discussed below.

The analysis presented in the current paper shows that anchor benches are more likely to fail by lateral sliding than by uplifting. The analysis also shows that sliding is more likely to occur in the downslope direction than in the upslope direction. However, sliding in the upslope direction is possible; accordingly, criteria are provided in the current paper to determine what is the likely failure mechanism of the anchor bench depending on the geomembrane tensions induced by wind. Identifying the governing failure mechanism makes it possible to select the appropriate equation for sizing the considered anchor bench.

The equations presented in the current paper (which make it possible to size anchor benches based on both lateral sliding and uplifting) are an improvement compared to the current state of practice, which consists of designing anchor benches based only on resistance to uplifting. Indeed, the discussion presented in Section 3.6 shows that sizing anchor benches based only on resistance to uplifting is unconservative.

The method presented in the current paper can also be used for the case when the geomembrane is anchored using an anchor trench. In this case, the equations presented tend to overestimate (which is conservative) the required cross-sectional area of the anchor trench. This is because the lateral passive pressure developed on the trench sides is not accounted for in the equations that were developed for anchor benches.

As geomembranes are increasingly used without any protective layer in landfill caps and other applications, designing against wind action becomes more important than ever before. Addressing this important problem, the current paper provides a rational, though still simple, design method for dimensioning anchor benches and trenches used to secure geomembranes exposed to wind action.

ACKNOWLEDGMENTS

The support of GeoSyntec Consultants is acknowledged. Also, the authors are grateful to K.L. Soderman and B.S. Clark for their review and valuable comments and to R. Ortiz for assistance in the preparation of the paper.

REFERENCES

Giroud, J.P. and Huot, P., 1977, "Conception des barrages en terre et en enrochements munis d'étanchéité par feuille mince", *Proceedings of the Eleventh Conference Européenne de la Commission Internationale de l'Irrigation et du Drainage,* CIID, Theme 3, Rome, Italy, May 1977, 8 p. (in French)

Giroud, J.P., Pelte, T. and Bathurst, R.J., 1995, "Uplift of Geomembranes by Wind", *Geosynthetics International*, Special Issue on Design of Geomembrane Applications, Vol. 2, No. 6, pp. 897-952.

Gleason, M.H., Houlihan, M.F. and Giroud, J.P., 1998, "An Exposed Geomembrane Cover System for a Landfill", *Proceedings of the Sixth International Conference on Geosynthetics*, IFAI, Vol. 1, Atlanta, Georgia, USA, March 1998, pp. 211-218.

Zornberg, J.G. and Giroud, J.P., 1997, "Uplift of Geomembranes by Wind – Extension of Equations", *Geosynthetics International*, Vol. 4, No. 2, pp. 187-207 (Errata: 1999, Vol. 6, No. 6, pp. 521-522).

NOTATIONS

Basic SI units are given in parentheses.

A_{req}	=	required cross-sectional area of anchor (m^2)
FS	=	global factor of safety (dimensionless)
L	=	distance between anchors (m)
R	=	reaction at interface between geomembrane and underlying soil (N/m)
R_1	=	vertical projection of R (Figure 6) (N/m)
R_2	=	projection of R on the direction of sliding plane (Figure 6) (N/m)
T	=	geomembrane tension (N/m)
T_d	=	geomembrane tension on downslope side of anchor bench (N/m)
T_{dH}	=	horizontal projection of T_d (N/m)
T_{d1}	=	vertical projection of T_d (N/m)
T_{d2}	=	projection of T_d on a direction parallel to base of anchor bench (N/m)
T_u	=	geomembrane tension on upslope side of anchor bench (N/m)
T_{uH}	=	horizontal projection of T_u (N/m)
T_{u1}	=	vertical projection of T_u (N/m)
T_{u2}	=	projection of T_u on a direction parallel to base of anchor bench (N/m)
W	=	weight of anchor material per unit length perpendicular to plane of cross section (N/m)
$W_{factored}$	=	factored minimum required weight of anchor material per unit length perpendicular to plane of cross section (N/m)
W_{min}	=	minimum required weight of anchor material per unit length perpendicular to plane of cross section (N/m)
$W_{min\ downsliding}$	=	value of W_{min} in the case where sliding tends to occur in downslope direction (N/m)
$W_{min\ uplifting}$	=	value of W_{min} in the case where anchor bench failure tends to occur as result of uplifting (N/m)
$W_{min\ upsliding}$	=	value of W_{min} in the case where sliding tends to occur in upslope direction (N/m)
α	=	angle between direction perpendicular to base of anchor bench and reaction R (°)
β	=	slope angle (°)
β_a	=	slope angle of geomembrane in anchor bench or trench (°)
β_d	=	slope angle on downslope side of anchor (°)
β_u	=	slope angle on upslope side of anchor (°)

γ = unit weight of anchor material (N/m^3)
δ = interface friction angle between geomembrane and underlying material (°)
θ = angle of uplifted geomembrane with slope at edge of anchor (°)
θ_d = angle of uplifted geomembrane with slope on downslope side of anchor (°)
θ_u = angle of uplifted geomembrane with slope on upslope side of anchor (°)

APPENDIX

An analysis of the influence of wind velocity on the minimum required weight of an anchor bench is presented in this Appendix for a special case defined by the following conditions:

$$\beta_d = \beta_u = \beta \qquad \beta_a = 0 \tag{A-1}$$

It is also assumed that the length of exposed geomembrane, L, and the wind velocity, V, are the same on both sides of the anchor bench. Based on these assumptions and Equation A-1, the following relationships exist according to Giroud et al. (1995):

$$T_d = T_u = T \qquad \theta_d = \theta_u = \theta \tag{A-2}$$

In this case, the criterion expressed by Equation 2 is met and Equation 7 should be used to obtain the minimum required weight of the bench. Combining Equations 7, A-1, and A-2 gives:

$$W_{min\,downsliding} = \frac{T\cos(\theta - \beta - \delta) - T\cos(\theta + \beta + \delta)}{\sin\delta} \tag{A-3}$$

Using classical trigonometric relationships, Equation A-3 becomes:

$$W_{min\,downsliding} = \frac{T}{\sin\delta}\left\{\left[\cos\theta\cos(\beta+\delta) + \sin\theta\sin(\beta+\delta)\right] - \left[\cos\theta\cos(\beta+\delta) - \sin\theta\sin(\beta+\delta)\right]\right\} \tag{A-4}$$

hence:

$$W_{min\,downsliding} = 2\,T\sin\theta\,\frac{\sin(\beta + \delta)}{\sin\delta} \tag{A-5}$$

From Giroud et al. (1995), the following relationship exists:

$$\frac{T}{S_e L} = \frac{1}{2\sin\theta} \tag{A-6}$$

where: S_e = effective suction exerted on the geomembrane by the atmospheric depression caused by the wind; and L = length of geomembrane exposed to the wind on each side of the anchor bench.

Combining Equations A-5 and A-6 gives:

$$W_{min\,downsliding} = S_e\, L\, \frac{\sin(\beta + \delta)}{\sin \delta} \qquad \text{(A-7)}$$

From Giroud et al. (1995) and Zornberg and Giroud (1997), the following relationship exists:

$$S_e \approx \lambda\, \varrho\, \frac{V^2}{2} \qquad \text{(A-8)}$$

where: λ = wind suction factor defined by Giroud et al. (1995); ϱ = air density; and V = wind velocity.

Combining Equations A-7 and A-8 gives:

$$W_{min\,downsliding} = \lambda\, \varrho\, \frac{V^2}{2}\, L\, \frac{\sin(\beta + \delta)}{\sin \delta} \qquad \text{(A-9)}$$

Equation A-9 shows that W_{min} increases approximately proportionally to V^2.