Technical Note by J.G. Zornberg and J.P. Giroud

UPLIFT OF GEOMEMBRANES BY WIND - EXTENSION OF EQUATIONS

ABSTRACT: This technical note presents an extension of earlier analytical methods published in 1995 by Giroud et al. for evaluating the uplift of geomembranes by wind. The extension incorporates: (i) the influence on wind uplift of the slope inclination of an exposed geomembrane; and (ii) a more accurate expression of the tension-strain relationship in a wind uplifted geomembrane. Use of the revised methods is particularly relevant for projects in which the exposed geomembrane mass per unit area is high, the slope inclination of the exposed geomembrane is steep, and the exposed geomembrane is subjected to strains due to mechanisms other than the wind (e.g. gravity, temperature) prior to wind uplift. Also, the revised equations are particularly appropriate for dimensioning a protective layer placed on top of the geomembrane to prevent wind uplift. The information provided in this technical note may be of particular significance for projects in which an exposed geomembrane is contemplated as the final cover for waste containment systems with steep slope configurations.

KEYWORDS: Geomembrane, Wind, Uplift, Design method, Steep slope.

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1 INTRODUCTION

A thorough evaluation of the uplift of geomembranes by wind is presented in a paper by Giroud et al. (1995). The paper presents approaches to assess the maximum wind velocity that an exposed geomembrane can withstand without being uplifted, the required thickness of a protective layer that would prevent the geomembrane from being uplifted, the tension and strain induced in the geomembrane by wind loads, and the geometry of the uplifted geomembrane. The evaluation of the uplift of geomembranes by wind is a design issue that deserves consideration not only for applications in which geomembranes are temporarily exposed (e.g. during construction of conventional landfill cover and liner systems), but also for challenging projects in which an exposed geomembrane without any protective layer is contemplated for final closure of a waste containment system.

This technical note presents revisions and extensions of the work by Giroud et al. (1995), including: (i) revision of a number of the original equations to account for the influence on wind uplift of the slope inclination of an exposed geomembrane; (ii) revision of the wind uplift tension-strain relationship to facilitate calculations and graphical applications in cases where the geomembrane is already subjected to an initial strain when wind uplifting begins; and (iii) methods for evaluating the geomembrane wind uplift under initial strain induced by gravity and temperature.

The revised equations may be of particular relevance when an exposed geomembrane is considered as the final cover for a waste containment landfill with steep slopes. An example of such an application is given in Section 5. This example is inspired by the feasibility evaluation of the use of an exposed geomembrane as the final cover for the Operating Industries, Inc. (OII) landfill located in southern California, which is in an area of high seismicity. The landfill slopes are up to 80 m high, with intermediate slopes between benches as steep as 1V:1.5H and up to 28 m high. The main reason for having considered an exposed geomembrane cover as a potential alternative for final closure of the landfill was the difficulty in demonstrating adequate slope stability, under static and seismic conditions, in the case of conventional covers where geosynthetics are overlain by soil layers. In contrast, an exposed geomembrane cover would be stable under both static and seismic conditions. Evaluation of the uplift by wind of the geomembrane becomes, however, a key consideration in the assessment of an exposed geomembrane as a final cover alternative.

2 INFLUENCE OF SLOPE INCLINATION ON GEOMEMBRANE UPLIFT BY WIND

The forces per unit area acting on a geomembrane exposed on a slope and subjected to wind-generated suction are shown in Figure 1. The suction, S, induced by the wind is normal to the exposed geomembrane and, therefore, its average direction is normal to the slope. A geomembrane exposed on a slope with inclination β will resist wind uplift by itself if the component of the geomembrane weight per unit area in the direction normal to the slope is greater than or equal to the suction S. Therefore, in order to prevent wind uplift, it should be verified that:

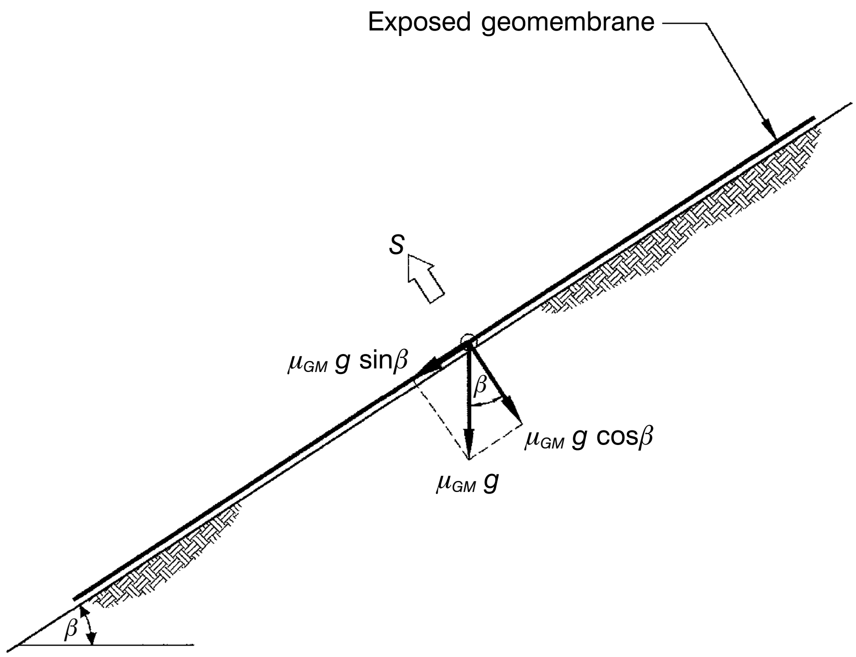

Figure 1. Geomembrane exposed on a slope of inclination β and subjected to wind-induced suction.

$$\mu_{GM}\, g\, \cos\beta \geq S \tag{1}$$

where: μ_{GM} = mass per unit area of the geomembrane; and g = acceleration due to gravity.

The earlier equations (Giroud et al. 1995) do not include the term cosβ in Equation 1. Although these earlier equations would rigorously be valid only for exposed horizontal geomembranes (cosβ = 1), they would be appropriate for practical purposes in many cases because: (i) many structures in which geomembrane uplift by wind is an issue of concern have slopes that are not steep (cos$\beta \approx 1$); and (ii) the geomembrane weight per unit area is generally much smaller than the wind-generated suction and, therefore, an error on the weight would have no significant impact on the calculated value of the effective suction. However, using cosβ = 1 would not be appropriate in cases where the slope of the exposed geomembrane is steep or in cases where the mass per unit area of the geomembrane is comparatively high (e.g. if bituminous geomembranes are used). The effect of the cosβ term is particularly significant when calculating the thickness of the protective layer on top of the geomembrane which is required to prevent wind uplift.

A revised version of the earlier equations, which incorporates the effect of the slope inclination β, is presented in the Appendix. The revised equations can be used to estimate, as a function of the slope inclination, the mass per unit area of geomembrane required to resist wind uplift, the threshold wind velocity below which a geomembrane should not be uplifted by wind, the required thickness of a protective layer that would prevent wind uplift, and the effective suction acting on an exposed geomembrane. The revised equations in the Appendix supersede the original equations (Giroud et al. 1995) and, to facilitate cross-referencing, the same numbering sequence is used for the revised and original equations (e.g. Equation A-14 in the Appendix is the revised version of the original Equation 14).

3 UPLIFT OF GEOMEMBRANES WITH INITIAL WRINKLES OR TENSION

3.1 Uplift Tension-Strain Relationship

The fundamental relationship for the geomembrane uplift problem is the "uplift tension-strain relationship" defined by Equation 47 in the paper by Giroud et al. (1995). In this relationship, the only strain component is the strain induced by the wind. Therefore, it is proposed to use the notation ε_w to distinguish this strain component from the total strain in the geomembrane, ε, which may result from multiple causes (e.g. wind, temperature, gravity). As the only cause of strain considered in the relationship presented by Giroud et al. (1995) is the wind, the notation ε is used in Equation 47 of the original paper. However, this notation becomes confusing when multiple causes of strain are considered in the analysis.

The new version of the original Equation 47 resulting from the use of the notation ε_w is:

$$\varepsilon_w = \frac{2T}{S_e L} \sin^{-1}\left[\frac{S_e L}{2T}\right] - 1 \qquad \text{(A-47)}$$

where: ε_w = geomembrane strain component induced by wind uplift; T = total geomembrane tension; S_e = effective suction; and L = length of geomembrane subjected to suction.

It is important to emphasize that the uplift tension-strain relationship (Equation A-47) relates the strain induced only by the wind, ε_w, with the total tension in the geomembrane, T, induced also by sources other than wind such as temperature or gravity. In other words, it should be noted that Equation A-47 is *not* a relationship between the wind-induced strain, ε_w, and the wind-induced tension, T_w. This is because, while the geometry of the uplifted geomembrane is governed by the wind-induced strain, ε_w, the effective suction acting over a length L is resisted by the total tension, T, in the geomembrane (Figure 2).

Equations A-45, A-46, A-51 and A-57 presented in the Appendix were also revised to explicitly use ε_w. The uplift tension-strain relationship expressed by Equation A-47 is represented by the curve in Figure 3. Note that the horizontal axis of the graph presented in Figure 3 corresponds to the wind-induced component of the geomembrane strain, ε_w.

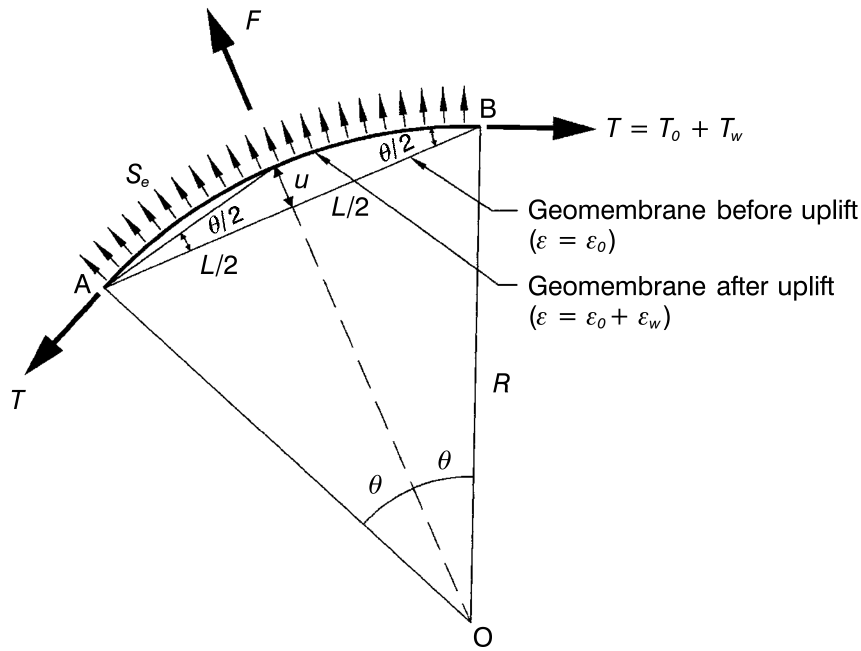

Figure 2. Schematic representation of an uplifted geomembrane (based on Figure 9 from Giroud et al. 1995).

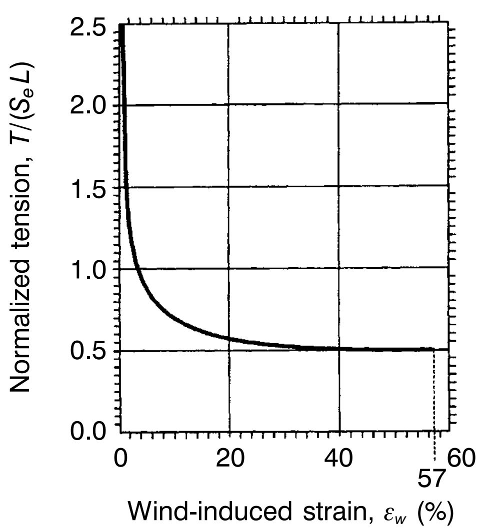

Figure 3. Normalized uplift tension-strain relationship.

Notes: This curve was established using Equation A-47. This is a revised version of Figure 10 presented by Giroud et al. (1995) in which the horizontal axis is the wind-induced component of the geomembrane strain, ε_w , instead of the total geomembrane strain, ε, used in the original figure.

3.2 Graphical Application

The representation of the uplift tension-strain relationship expressed by Equation A-47 and shown in Figure 3, where the initial horizontal axis is ε_w instead of ε, is useful in the evaluation of the effect of initial wrinkles or initial tension on the uplift by wind of geomembranes. A graphical determination of the total strain, ε, and total tension, T, in a geomembrane after wind uplift, for the case of a geomembrane with initial wrinkles or initial tension, is shown in Figure 4. For the purposes of the discussion presented herein, the initial strains in the geomembrane analyzed in Figure 4 are assumed to be induced by temperature changes. However, the procedure illustrated in Figure 4 can be equally used to evaluate the total tension and strain in the geomembrane if the initial strains are induced by other mechanisms.

Figure 4a shows a typical geomembrane tension-strain curve. The tension-strain behavior of the geomembrane shown in Figure 4 was extended to the "negative strain" portion of the curve in order to illustrate the behavior of a geomembrane with wrinkles. The axes of the geomembrane tension-strain curve are the total tension, T, and the total strain, ε, in the geomembrane. Since geomembranes do not sustain compression, the tension is zero on the negative side of the ε axis. Points A_0 , B_0 , and C_0 represent different possible initial state conditions of a geomembrane before it is subjected to wind-induced suction. State A_0 represents a geomembrane that has not undergone strains (neither positive nor negative) due to temperature changes, state B_0 represents a geomembrane with initial wrinkles due to thermal expansion (high temperature when uplifting begins), and state C_0 represents a geomembrane under initial tension due to thermal contraction (low temperature when uplifting begins).

Figure 4b illustrates the uplift tension-strain relationship for a specific value of $S_e\,L$, which is defined by the geometry of the exposed geomembrane and the design wind velocity. As discussed in Section 3.1, the axes of this uplift tension-strain relationship are the total tension, T, and the wind-induced component, ε_w , of the geomembrane strain.

Figure 4c illustrates how to estimate the final tension and strain in a geomembrane subjected to wind-induced suction for three cases representing possible initial states of a geomembrane when uplifting begins. The elements relevant for design provided by the graphical analysis illustrated in Figure 4c are the total tension, T, in the geomembrane, which should be less than the allowable tension, and the wind-induced strain component, ε_w , which defines the geometry of the uplifted geomembrane.

The three cases illustrated in Figure 4c are discussed below.

Case A. In this case, the geomembrane has no wrinkles or tension when uplifting begins. Points A_0 and A_1 along the geomembrane tension-strain curve represent the states before and after wind uplift and can be used to define the total tension, T_A , and the wind-induced strain, ε_{wA} , in the geomembrane. Point A_1 is the intersection of the geomembrane tension strain-curve, T - ε, with the uplift tension-strain curve, T - ε_w . The origin, O, of the uplift curve for this case (Curve A) is Point A_0, which represents the initial state in the tension-strain curve of the geomembrane.

Case B. In this case, due to thermal expansion prior to wind action, the geomembrane has wrinkles when uplifting begins, which are characterized by a "negative strain".

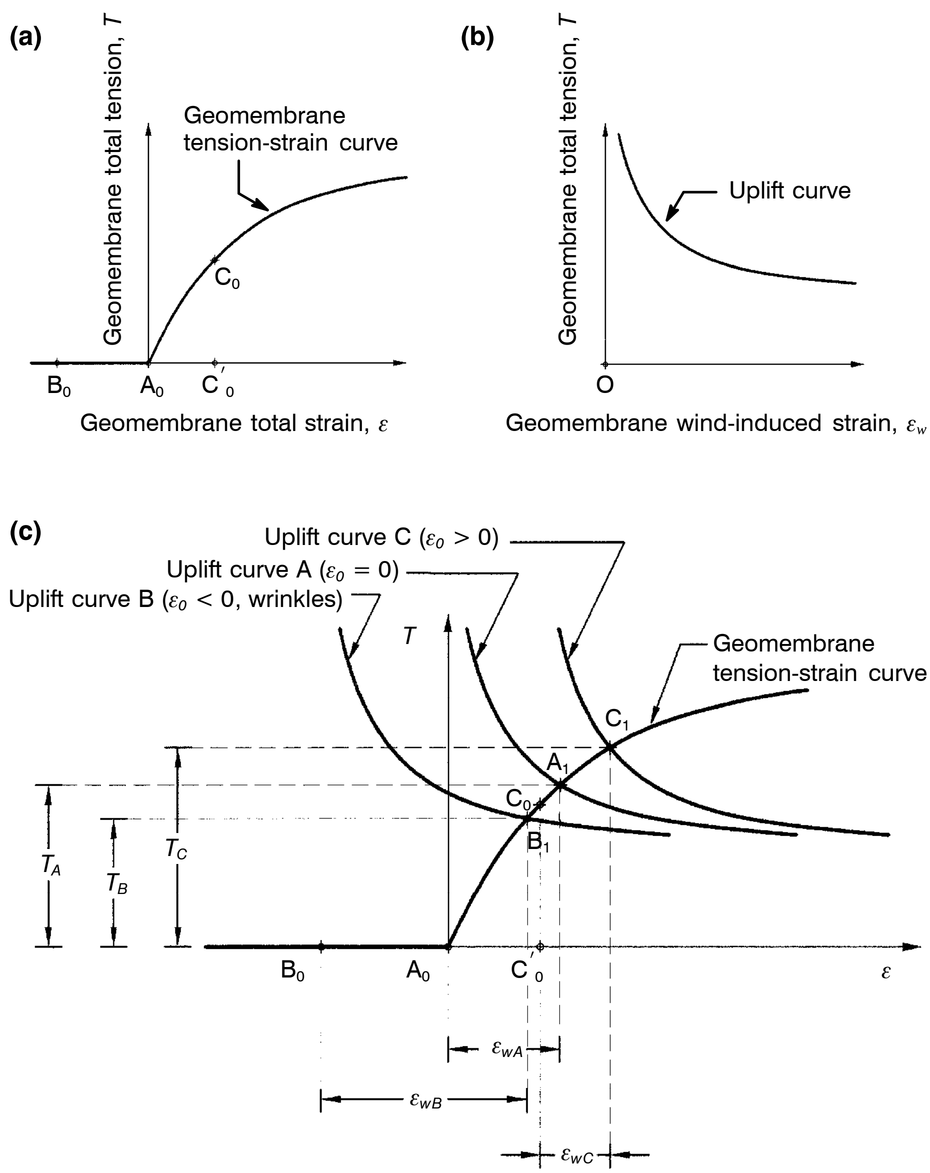

Figure 4. Uplift by wind of a geomembrane under initial strains: (a) schematic geomembrane tension-strain (T - ε) curve; (b) schematic geomembrane uplift tension-strain (T - ε_w) relationship; (c) uplift by wind of a geomembrane under no initial strain (from A_0 to A_1), with initial wrinkles (from B_0 to B_1), and under initial tension (from C_0 to C_1).

Points B_0 and B_1 along the geomembrane tension-strain curve represent the states before and after wind uplift and can be used to define the total tension, T_B , and the wind-induced strain, ε_{wB} . Point B_1 is the intersection of the geomembrane tension-strain curve,

T - ε, with the uplift tension-strain curve, T - ε_w . The origin, O, of the uplift curve for this case (Curve B) is Point B_0 , which represents the initial state in the tension-strain curve of the geomembrane. Figure 4c shows that the total tension, T_B , in the uplifted geomembrane is smaller than the total tension, T_A , in Case A and that the wind-induced strain, ε_{wB} , in the uplifted geomembrane is greater than the wind-induced strain, ε_{wA} , in Case A. In summary, as stated by Giroud et al. (1995), "if a geomembrane has wrinkles when uplifting begins, it is uplifted more, but with a smaller tension than if the geomembrane has no wrinkles when uplifting begins.".

Case C. In this case, due to thermal contraction prior to wind action, the geomembrane is under tension when uplifting begins. Points C_0 and C_1 along the geomembrane tension-strain curve represent the states before and after wind uplift and can be used to define the total tension, T_C , and the wind-induced strain, ε_{wC} , after wind uplift. Point C_1 is the intersection of the geomembrane tension-strain curve, T - ε, with the uplift tension-strain curve, T - ε_w . The origin, O, of the uplift curve, T - ε_w , for this case (Curve C) is point C'_0 , the projection on the ε axis of Point C_0 , which represents the initial state in the tension-strain curve of the geomembrane. Figure 4c shows that the total tension, T_C , in the uplifted geomembrane is greater than the total tension, T_A , in Case A and that the wind-induced strain, ε_{wC} , in the uplifted geomembrane is smaller than the strain, ε_{wA} , in Case A. In summary, as stated by Giroud et al. (1995), "if a geomembrane is under tension when uplifting begins, it is uplifted less, but with a greater tension than if the geomembrane has no wrinkles when uplifting begins.".

3.3 Discussion

Figure 4c of this technical note should be compared with Figure 25 in the paper by Giroud et al. (1995). Both figures illustrate the same approach, which consists of translating parallel to the strain axis one of the following two curves: the geomembrane tension-strain curve (Figure 4a) or the uplift tension-strain curve ("uplift curve") (Figure 4b). In Figure 4c, the uplift curve is translated, whereas in Figure 25 of the original paper the geomembrane tension-strain curve is translated. The conclusions drawn from Figure 4c in Section 3.2 are identical to those drawn by Giroud et al. (1995) from Figure 25. Experience has shown that some engineers prefer the approach in Figure 4c, whereas other engineers prefer the approach in Figure 25 (with the horizontal axis labeled ε_w instead of ε, as pointed out in Section 3.1). Therefore, it appears useful to have both approaches available. However, regardless of the approach selected, it is important that the horizontal axis of Figure 4b be labeled ε_w .

4 WIND UPLIFT OF GEOMEMBRANES UNDER INITIAL STRAINS INDUCED BY GRAVITY AND TEMPERATURE

The analysis presented in Section 3 regarding the uplift by the wind of a geomembrane under initial tension due to thermal contraction can also be applied for the case of a geomembrane under initial tension induced by other sources (e.g. gravity forces, seismic forces, tractive forces caused by surface water flow). Similarly, the analysis presented in Section 3 for wind uplift of a geomembrane initially with wrinkles due to thermal expansion can also be applied to the case of a geomembrane initially with wrinkles caused by other reasons (e.g. induced during construction).

Section 4 discusses the particular case of an exposed geomembrane under initial tension induced both by thermal contraction and gravity forces. These two sources of initial tension in geomembranes may be particularly relevant when evaluating the performance of geomembranes exposed on steep slopes. Geomembrane tensions induced by seismic forces and tractive water forces are not considered herein as it appears unreasonable, for design purposes, to consider that the design wind would occur simultaneously with a seismic event or with the design storm.

The initial strain, ε_0 , and initial tension, T_0 , in the geomembrane before uplifting begins can be estimated from:

$$\varepsilon_0 = \varepsilon_T + \varepsilon_g \tag{2}$$

$$T_0 = T_T + T_g \tag{3}$$

where: ε_T = geomembrane strain component induced by thermal contraction; ε_g = geomembrane strain component induced by gravity; T_T = geomembrane tension component induced by thermal contraction; and T_g = geomembrane tension component induced by gravity.

The strain component induced by thermal contraction can be calculated using the following equation, which is identical to Equation 58 in the paper by Giroud et al. (1995):

$$\varepsilon_T = \alpha(\Gamma - \Gamma_{base}) \tag{4}$$

where: α = coefficient of thermal contraction of the geomembrane; Γ = temperature of the geomembrane when uplift occurs; and Γ_{base} = temperature of the geomembrane when it rests on the supporting ground without wrinkles and without tension. After determining the strain component induced by thermal contraction, ε_T , the corresponding tension component, T_T , can be obtained using the nonlinear tension-strain curve of the geomembrane as shown in Figure 5.

If the geomembrane tension-strain curve, or a portion of it, can be assumed to be linear, determination of T_T does not require a graphical solution, but it can be estimated using the geomembrane tensile stiffness, J, as follows:

$$T_T = J\,\varepsilon_T \tag{5}$$

The tension component induced by gravity, T_g , is the component of the weight of the geomembrane and of the geomembrane protection layer, if any, in the direction of the slope (see Figure 1). Assuming that a geomembrane is properly anchored at the crest of the slope, the tension component induced by gravity increases from zero at the toe of the slope to a maximum tension at the crest of the slope. In the case of a

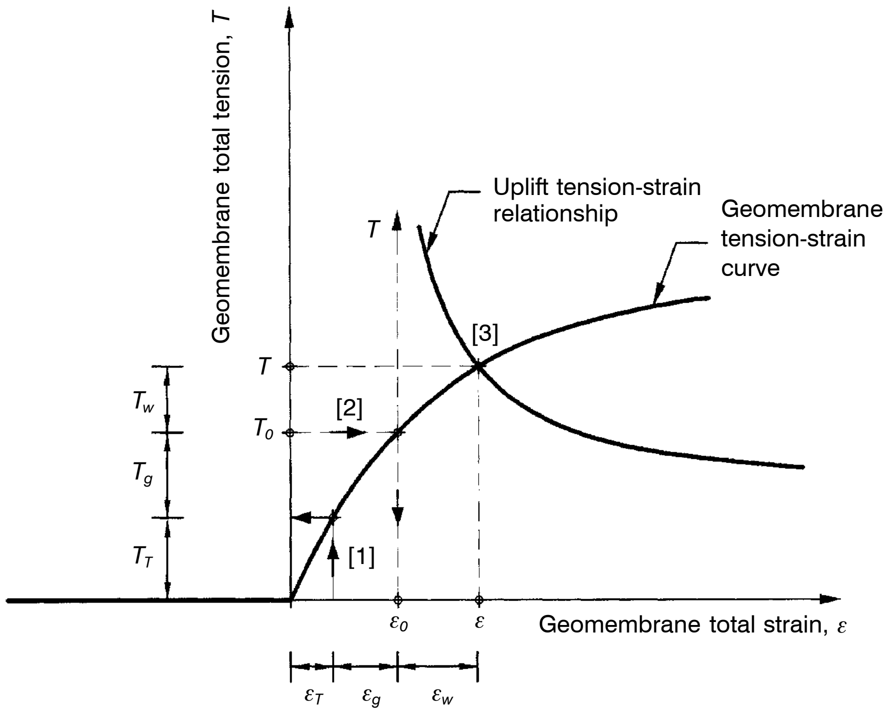

Figure 5. Uplift by wind of a geomembrane with nonlinear tension-strain behavior under initial tension induced by both thermal and gravity sources.

Notes: The nonlinear tension-strain curve is used to graphically define: T_T from the estimated ε_T (see [1] in figure); ε_g from the estimated value of T_g (see [2] in figure); and both T_w and ε_w from superimposing the geomembrane tension-strain curve on the uplift tension-strain curve (see [3] in figure).

geomembrane without a protection layer, the tension component induced by gravity at the crest of a slope of length L is:

$$T_g = \mu_{GM}\, g\, L \, \sin\beta \tag{6}$$

Since the tension component induced by gravity is not uniform along the geomembrane length, the average tension (i.e. half of the tension estimated using Equation 6) would be representative of the average condition of the geomembrane. However, the tension estimated using Equation 6 better represents the most critical section in the geomembrane. After determining the tension component induced by gravity, T_g , the corresponding strain component, ε_g , can be obtained from the nonlinear tension-strain curve of the geomembrane as shown in Figure 5. If the geomembrane tension-strain curve, or a portion of it, can be assumed to be linear, the strain component, ε_g , can be estimated using the geomembrane tensile stiffness, J, as follows:

$$\varepsilon_g = T_g / J \tag{7}$$

The procedure illustrated in Figure 5 assumes that thermal strains occur before the gravity-induced strains. It should be noted, however, that for the case of a geomembrane with a nonlinear tension-strain behavior, a different initial condition (i.e. ε_0 and T_0) would be obtained if the gravity-induced strains were assumed to occur before the thermal strains. The sequence illustrated in Figure 5 implies that gravity-induced strains occur at the moment wind uplift begins (i.e. after the thermal strains) and that, before wind uplift, the component of the geomembrane weight parallel to the slope was carried by shear stresses developed at the interface between the geomembrane and the side slope. A different assumption would be to consider that gravity-induced strains occur during placement of the geomembrane (i.e. before the thermal strains) and that shear stresses between the geomembrane and the side slope were not mobilized after construction. Nevertheless, the same initial condition (i.e. ε_0 and T_0) would be obtained, independently of the sequence in which thermal and gravity-induced strains are assumed to occur, if the geomembrane has a linear tension-strain relationship.

Once the initial state of the geomembrane before uplifting begins (i.e. ε_0 and T_0) has been defined, the geomembrane strain and tension components induced by wind uplift (ε_w and T_w , respectively) can be determined following the procedure described previously in Figure 4. These wind-induced strain and tension components should be obtained graphically if the geomembrane has a nonlinear tension-strain behavior.

If the geomembrane tension-strain curve, or a portion of it, can be assumed to be linear, determination of ε_w does not require a graphical solution, but it can be estimated using the geomembrane tensile stiffness J, the initial tension T_0 , the effective suction S_e , and the geomembrane length L by solving the following equation, which is adapted from Equation 57 by Giroud et al. (1995):

$$\frac{S_e\, L}{2\,(T_0 + J\,\varepsilon_w)} = \sin\left[\frac{S_e\, L}{2\,(T_0 + J\,\varepsilon_w)}\;(1 + \varepsilon_w)\right] \tag{A-57a}$$

The wind-induced strain component, ε_w , can also be estimated using the geomembrane tensile stiffness J, the initial strain ε_0 , and the term $S_e\, L$ by solving the following equivalent equation:

$$\frac{S_e\, L}{2\,J\,(\varepsilon_0 + \varepsilon_w)} = \sin\left[\frac{S_e\, L}{2\,J\,(\varepsilon_0 + \varepsilon_w)}\;(1 + \varepsilon_w)\right] \tag{A-57b}$$

Expressions A-57a or A-57b may be solved by trial and error in order to determine ε_w . An initial trial value can be defined using Table 4 from Giroud et al. (1995), which provides ε_w for the case of a geomembrane with a linear tension-strain behavior, but with no wrinkles or tension when uplifting begins ($\varepsilon_0 = 0$ and $T_0 = 0$).

After determining the wind-induced strain component, ε_w , the tension component, T_w , can also be estimated using the geomembrane tensile stiffness, J :

$$T_w = J\,\varepsilon_w \tag{8}$$

Finally, the total strain, ε, and total tension, T, in the geomembrane after wind uplift can be defined from:

$$\varepsilon = \varepsilon_0 + \varepsilon_w \tag{9}$$

$$T = T_0 + T_w \tag{10}$$

5 DESIGN EXAMPLE

Use of the equations presented in this technical note is illustrated in the following example, which evaluates the wind uplift of a geomembrane that is exposed on a steep landfill slope and which is initially under tension induced by both gravity and temperature before uplifting begins. These conditions are based on those considered in the feasibility evaluation of the use of an exposed geomembrane as a final cover for the OII landfill mentioned in Section 1.

Example. A reinforced geomembrane has a linear tension-strain curve characterized by a tensile stiffness of 310 kN/m and a strain at break of 23%. This geomembrane is installed and left exposed as part of the final cover system for a landfill site located 150 m above sea level, in an area where, during a certain season, winds with velocities up to 115 km/h can be expected. The geomembrane is exposed on a steep (1V:1.5H) slope, which is 28 m high between benches. The geomembrane has a mass per unit area of 1.41 kg/m^2 and a coefficient of thermal expansion of $1.2 \times 10^{-4}\,^{\circ}C^{-1}$. In order to have a factor of safety of 2, the allowable strain is 11.5%. Consider that the geomembrane is under initial tension when uplifting begins due to its own weight and to thermal contraction induced by a temperature change of 50°C. Assuming that the geomembrane is properly anchored at the crest of the slope and that no protective layer is used on top of the geomembrane, predict the total strain and tension in the geomembrane when it is uplifted by the considered wind. Calculate the thickness of a protective layer, with a density of 1600 kg/m^3, which would be required to prevent the uplift of the geomembrane.

The length of geomembrane subjected to wind-generated suction is the length of the slope, which is:

$$L = \sqrt{28^2 + (1.5 \times 28)^2} = 50.5 \text{ m}$$

The slope inclination is:

$$\beta = \tan^{-1}(1/1.5) = 33.69^{\circ}$$

The initial strain component induced by thermal contraction can be estimated using Equation 4 as follows:

$$\varepsilon_T = (1.2 \times 10^{-4})(50) = 0.0060 = 0.60\%$$

From the estimated strain component, the corresponding tension component induced by thermal contraction can be determined for a geomembrane with a linear tension-strain curve using Equation 5 as follows:

$$T_T = (310)(0.006) = 1.86 \text{ kN/m}$$

The initial tension component induced by gravity at the top of the side slope of length L can be estimated using Equation 6 as follows:

$$T_g = (1.41)(9.81)(50.5)\sin(33.69^\circ) = 387 \text{ N/m} = 0.387 \text{ kN/m}$$

From the estimated tension component, the corresponding strain component induced by gravity can then be determined using Equation 7 as follows:

$$\varepsilon_g = 0.387/310 = 0.00125 = 0.125\%$$

The initial strain and tension in the geomembrane before uplifting begins can be estimated from Equations 2 and 3, respectively, as follows:

$$\varepsilon_0 = 0.600 + 0.125 = 0.73\%$$

$$T_0 = 1.860 + 0.387 = 2.25 \text{ kN/m}$$

A value $\lambda = 0.7$ is recommended by Giroud et al. (1995) for the suction factor if the entire slope is considered, which is the case in this example. Using Equation A-41 presented in the Appendix, the effective suction on the side slope is calculated as follows:

$$\begin{aligned} S_e &= (0.050)(0.7)(115)^2 \, e^{-(1.252 \times 10^{-4})(150)} - (9.81)(1.41)\cos(33.69^\circ) \\ &= 454.26 - 11.51 = 442.75 \text{ Pa} \end{aligned}$$

It should be noted that, with $\cos\beta = 1$ instead of $\cos(33.69^\circ) = 0.83$, the error on S_e is only 0.5%. This confirms a comment made in Section 2 that, when the geomembrane weight per unit area is much smaller than the wind generated suction (which is often the case), using $\cos\beta = 1$ does not have a significant impact on the calculated value of the effective suction.

Next, $S_e L$ is calculated as follows:

$$S_e L = (442.75)(50.5) = 22{,}359 \text{ N/m} = 22.36 \text{ kN/m}$$

The strain component induced by wind-generated suction can be calculated using Equation A-57a as follows:

$$\frac{22.36}{2\,(2.25 + 310\,\varepsilon_w)} = \sin\left[\frac{22.36}{2\,(2.25 + 310\,\varepsilon_w)}\,(1 + \varepsilon_w)\right]$$

The equation above must be solved by trial and error in order to obtain ε_w . An initial trial can be defined using Table 4 from Giroud et al. (1995), which solves the wind uplift problem for a geomembrane with a linear tension-strain relationship, but with no wrinkles or tension when uplifting begins. Using a normalized tensile stiffness $(J/S_e L)$ = (310/22.36) = 13.86, the initial trial value defined using Table 4 is 6.4%. This initial trial value corresponds to the upper bound of the wind-induced strain on the considered geomembrane, which is under tension when uplifting begins (see Section 3.2). The solution obtained after solving Equation A-57a by trial and error is ε_w = 0.0585 = 5.85%.

The wind-generated tension component, T_w , can then be estimated using Equation 8 as follows:

$$T_w = (310)(0.0585) = 18.14 \text{ kN/m}$$

Finally, the total tension and strain in the geomembrane when it is uplifted by the considered wind can be estimated using Equations 9 and 10, respectively, as follows:

$$\varepsilon = 0.73 + 5.85 = 6.58\%$$

$$T = 2.25 + 18.14 = 20.39 \text{ kN/m}$$

The total strain in the geomembrane after it has been subjected to wind-generated suction is 6.58%, which is less than the allowable strain of 11.5%. Therefore the geomembrane should not fail in tension when it is uplifted by the wind. Although the total tension, T, in the geomembrane after wind uplift is lower than the allowable tension of the reinforced geomembrane, it is considerably higher than the allowable tensile strength of typical nonreinforced geomembranes. The initial tension, T_0 , in the exposed geomembrane before uplifting begins represents 11% of the total tension, T , in the uplifted geomembrane.

The thickness of the protective layer required to prevent uplift by wind of the geomembrane can be calculated using Equation A-33. Considering $\varrho_p =$ 1600 kg/m^3, μ_{GM} = 1.41 kg/m^2, λ = 0.7, V = 115 km/h, β = 33.69°, and z = 150 m, the required thickness, t_{req} , is calculated as follows:

$$t_{req} = \frac{1}{1600}\left[-1.41 + 0.005085 \, \frac{(0.7)(115)^2}{\cos(33.69^\circ)} \, e^{-1.252 \times 10^{-4}(150)} \right]$$

hence:

$$t_{req} = \frac{1}{1600}(-1.41 + 55.52) = 0.034 \text{ m} = 34 \text{ mm}$$

It should be noted that, with $\cos\beta = 1$ instead of $\cos(33.69^\circ) = 0.83$, an unconservative value of 28 mm would have been obtained for the required thickness of the protective layer, hence an 18% error. This confirms the comment made in Section 2 that the effect of $\cos\beta$ is particularly significant when calculating the thickness of the protective layer.

Finally, although the static and seismic stability of the 34 mm-thick protective layer on such a steep slope is not addressed herein, it should be recognized as an important design consideration.

END OF EXAMPLE

6 CONCLUSIONS

This technical note presents a revised version of equations as well as an extension of discussions initially presented by Giroud et al. (1995) which analyzes the phenomenon of uplift of geomembranes by wind. The following conclusions are drawn:

- The uplift effect of wind on geomembranes depends on the inclination of the side slope on which the geomembrane is exposed. The effect of the slope inclination is particularly relevant if the slope of the exposed geomembrane is steep and if heavy geomembranes are used. Also, the effect of slope inclination is significant for the calculation of the required thickness of a protective layer on top of the geomembrane to prevent wind uplift. Revised equations are provided to estimate, as a function of the slope inclination, the mass per unit area of geomembrane required to resist wind uplift, the threshold wind velocity below which a geomembrane should not be uplifted by wind, the required thickness of a protective layer that would prevent wind uplift, and the effective suction acting on an exposed geomembrane.
- The "uplift tension-strain relationship" (Equation A-47) that governs the uplift problem relates the geomembrane strain induced exclusively by wind action (and not the total geomembrane strain) to the total tension in the geomembrane induced not only by the wind but also by other sources such as temperature or gravity. Taking into account that the uplift relationship does not refer to the total geomembrane strain, a consistent approach is presented to assess the wind uplift phenomenon accounting for the effect of initial wrinkles or initial tension in the geomembrane.

- The effect of initial strains induced by multiple sources (e.g. temperature, gravity) on the uplift of geomembranes by wind can be evaluated either graphically, if the geomembrane has a nonlinear tension-strain relationship, or analytically, if the geomembrane tension-strain curve can be assumed to be linear. Methods are presented to evaluate the uplift of geomembranes under initial strains induced by multiple sources when uplifting begins.

ACKNOWLEDGMENTS

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REFERENCE

Giroud, J.P., Pelte, T. and Bathurst, R.J., 1995, "Uplift of Geomembranes by Wind", *Geosynthetics International*, Vol. 2, No. 6, pp. 897-952.

NOTATIONS

Basic SI units are given in parentheses.

A	=	area of geomembrane (m^2)
F	=	force applied on geomembrane by uplift suction, defined by Equation 42 of the paper by Giroud et al. (1995) (N/m)
g	=	acceleration due to gravity (m/s^2)
J	=	geomembrane tensile stiffness (N/m)
L	=	length of geomembrane subjected to suction (m)
p_0	=	atmospheric pressure at sea level (Pa)
R	=	radius of circular-shaped uplifted geomembrane (m)
S	=	suction (Pa)
S_e	=	"effective suction" defined by Equation A-35 (Pa)
T	=	total geomembrane tension (N/m)
T_A	=	total tension in uplifted geomembrane for base case (Case A) where geomembrane has no wrinkles and no tension when uplifting begins (N/m)
T_B	=	total tension in uplifted geomembrane that has wrinkles when uplifting begins (Case B) (N/m)

T_C	=	total tension in uplifted geomembrane that is under tension when uplifting begins (Case C) (N/m)
T_w	=	geomembrane tension component induced by wind uplift (N/m)
T_g	=	geomembrane initial tension induced by gravity (N/m)
T_T	=	geomembrane initial tension induced by temperature changes (N/m)
T_0	=	geomembrane initial tension (N/m)
t_{req}	=	required thickness of protective layer (m)
u	=	geomembrane uplift (m)
V	=	wind velocity (m/s)
V_{up}	=	wind velocity that causes geomembrane uplift (m/s)
W	=	weight of geomembrane (N)
z	=	altitude above sea level (m)
α	=	coefficient of thermal expansion-contraction of geomembrane ($^\circ C^{-1}$)
β	=	slope inclination ($^\circ$)
Γ	=	temperature of geomembrane when uplift occurs ($^\circ$C)
Γ_{base}	=	temperature of geomembrane when it rests on supporting ground without wrinkles and without tension ($^\circ$C)
ε	=	total geomembrane strain (dimensionless)
ε_T	=	geomembrane initial strain induced by temperature changes (dimensionless)
ε_g	=	geomembrane initial strain induced by gravity forces (dimensionless)
ε_w	=	geomembrane strain component induced by wind uplift (dimensionless)
ε_{wA}	=	strain component in uplifted geomembrane for base case (Case A) where geomembrane has no wrinkles and no tension when uplifting begins (dimensionless)
ε_{wB}	=	strain component in an uplifted geomembrane that has wrinkles when uplifting begins (Case B) (dimensionless)
ε_{wC}	=	strain component in an uplifted geomembrane that is under tension when uplifting begins (Case C) (dimensionless)
ε_0	=	geomembrane initial strain (dimensionless)
θ	=	angle between extremities of uplifted geomembrane and straight line passing through these extremities ($^\circ$)
λ	=	suction factor, defined by Equation 13 of the paper by Giroud et al. (1995) (dimensionless)
μ_{GM}	=	mass per unit area of geomembrane (kg/m^2)
μ_{GMreq}	=	mass per unit area of geomembrane required to resist wind uplift (kg/m^2)
ϱ	=	air density (kg/m^3)
ϱ_p	=	density of protective layer material (kg/m^3)
ϱ_0	=	air density at sea level (kg/m^3)

APPENDIX

The following are revised equations for the evaluation of wind uplift. In order to facilitate cross-referencing between the revised equations in this Appendix and the original equations presented by Giroud et al. (1995), the same numbering sequence is used in the revised and original sets of equations (e.g. Equation A-14 in this Appendix is the revised version of the original Equation 14). Equations presented by Giroud et al. (1995) which have not been revised are not repeated in this Appendix.

A-1 GEOMEMBRANE SENSITIVITY TO WIND UPLIFT[1]

$$(W/A)\cos\beta \geq S \tag{A-14}$$

$$\mu_{GM} \geq \mu_{GMreq} = \lambda \frac{\varrho_o V^2}{2g} e^{-\varrho_o g z/p_o} \frac{1}{\cos\beta} \tag{A-17}$$

$$\mu_{GM} \geq \mu_{GMreq} = 0.0659 \frac{\lambda V^2}{\cos\beta} \tag{A-18}$$

with $\mu_{GMreq}(\text{kg/m}^2)$ and $V(\text{km/h})$

$$\mu_{GM} \geq \mu_{GMreq} = 0.005085 \frac{\lambda V^2}{\cos\beta} \tag{A-19}$$

with $\mu_{GMreq}(\text{kg/m}^2)$ and $V(\text{km/h})$

$$\mu_{GM} \geq \mu_{GMreq} = 0.0659 \frac{\lambda V^2}{\cos\beta} e^{-(1.252 \times 10^{-4})z} \tag{A-20}$$

with $\mu_{GMreq}(\text{kg/m}^2)$, $V(\text{km/h})$ and $z(\text{m})$

$$\mu_{GM} \geq \mu_{GMreq} = 0.005085 \frac{\lambda V^2}{\cos\beta} e^{-(1.252 \times 10^{-4})z} \tag{A-21}$$

with $\mu_{GMreq}(\text{kg/m}^2)$, $V(\text{km/h})$ and $z(\text{m})$

$$V \leq V_{up} = \left[\frac{2g\mu_{GM}\cos\beta}{\lambda \varrho_o e^{-\varrho_o g z/p_o}}\right]^{1/2} \tag{A-22}$$

(Note: Equation 22 in the original paper contained a typographical error, p_o was used instead of ϱ_o after λ in the denominator.)

$$V \leq V_{up} = 3.895 \sqrt{\mu_{GM} \cos\beta / \lambda} \tag{A-23}$$

with V_{up}(m/s) and μ_{GM}(kg/m^2)

$$V \leq V_{up} = 14.023 \sqrt{\mu_{GM} \cos\beta / \lambda} \tag{A-24}$$

with V_{up}(m/s) and μ_{GM}(kg/m^2)

$$V \leq V_{up} = 3.895 e^{(6.259 \times 10^{-5})\, z} \sqrt{\mu_{GM} \cos\beta / \lambda} \tag{A-25}$$

with V_{up}(m/s), z(m) and μ_{GM}(kg/m^2)

$$V \leq V_{up} = 14.023 e^{(6.259 \times 10^{-5})\, z} \sqrt{\mu_{GM} \cos\beta / \lambda} \tag{A-26}$$

with V_{up}(m/s), z(m) and μ_{GM}(kg/m^2)

A-2 REQUIRED UNIFORM PRESSURE TO COUNTERACT WIND UPLIFT[1]

$$(\varrho_P g t_{req} + \mu_{GM} g) \cos\beta \geq S \tag{A-28}$$

$$t_{req} \geq \frac{1}{\varrho_P}\left(- \mu_{GM} + \frac{\varrho_o \lambda V^2}{2g \cos\beta} e^{-\varrho_o g z / p_o} \right) \tag{A-29}$$

$$t_{req} = \frac{1}{\varrho_P}\left(- \mu_{GM} + 0.0659 \frac{\lambda V^2}{\cos\beta} \right) \tag{A-30}$$

with t_{req} (m), ϱ_P (kg/m^3), μ_{GM} (kg/m^2) and V (m/s)

$$t_{req} = \frac{1}{\varrho_P}\left(- \mu_{GM} + 0.005085 \frac{\lambda V^2}{\cos\beta} \right) \tag{A-31}$$

with t_{req} (m), ϱ_P (kg/m^3), μ_{GM} (kg/m^2) and V (km/h)

$$t_{req} = \frac{1}{\varrho_P}\left(- \mu_{GM} + 0.0659 \frac{\lambda V^2}{\cos\beta} e^{-(1.252 \times 10^{-4}) z} \right) \tag{A-32}$$

with t_{req} (m), ϱ_P (kg/m^3), μ_{GM} (kg/m^2), V (m/s) and z (m)

$$t_{req} = \frac{1}{\varrho_P}\left(- \mu_{GM} + 0.005085 \frac{\lambda V^2}{\cos\beta} e^{-(1.252 \times 10^{-4}) z} \right) \tag{A-33}$$

with t_{req} (m), ϱ_P (kg/m^3), μ_{GM} (kg/m^2), V (km/h) and z (m)

A-3 EVALUATION OF EFFECTIVE SUCTION(1)

$$S_e = S - \mu_{GM}\, g \cos\beta \tag{A-35}$$

$$S_e = \lambda \varrho (V^2/2) - \mu_{GM} g \cos\beta \tag{A-36}$$

$$S_e = \lambda \varrho_o (V^2/2) \mathrm{e}^{-\varrho_o\, g\, z/p_o} - \mu_{GM} g \cos\beta \tag{A-37}$$

$$S_e = 0.6465 \lambda V^2 - 9.81 \mu_{GM} \cos\beta \tag{A-38}$$

with S_e(Pa), V(m/s) and μ_{GM}(kg/m^2)

$$S_e = 0.050 \lambda V^2 - 9.81 \mu_{GM} \cos\beta \tag{A-39}$$

with S_e(Pa), V(km/h) and μ_{GM}(kg/m^2)

$$S_e = 0.6465 \lambda V^2 \mathrm{e}^{-(1.252 \times 10^{-4})z} - 9.81 \mu_{GM} \cos\beta \tag{A-40}$$

with S_e(Pa), V(m/s), z(m) and μ_{GM}(kg/m^2)

$$S_e = 0.050 \lambda V^2 \mathrm{e}^{-(1.252 \times 10^{-4})z} - 9.81 \mu_{GM} \cos\beta \tag{A-41}$$

with S_e(Pa), V(km/h), z(m) and μ_{GM}(kg/m^2)

A-4 DETERMINATION OF GEOMEMBRANE TENSION AND STRAIN(2)

$$1 + \varepsilon_w = \frac{\text{arc AB}}{L} = \frac{2R\theta}{2R\sin\theta} \tag{A-45}$$

$$\varepsilon_w = \frac{\theta}{\sin\theta} - 1 \tag{A-46}$$

$$\varepsilon_w = \frac{2T}{S_e L} \sin^{-1}\left[\frac{S_e L}{2T}\right] - 1 \tag{A-47}$$

with $T = T_0 + T_w$

$$\varepsilon_w = \frac{1}{2}\left(\frac{2u}{L} + \frac{L}{2u}\right) \sin^{-1}\left[\frac{2}{\frac{2u}{L} + \frac{L}{2u}}\right] - 1 \tag{A-51}$$

$$\frac{S_e L}{2(T_0 + J\varepsilon_w)} = \sin\left[\frac{S_e L}{2(T_0 + J\varepsilon_w)}\left(1 + \varepsilon_w\right)\right] \tag{A-57a}$$

$$\frac{S_e L}{2J(\varepsilon_0 + \varepsilon_w)} = \sin\left[\frac{S_e L}{2J(\varepsilon_0 + \varepsilon_w)}\left(1 + \varepsilon_w\right)\right] \tag{A-57b}$$

Notes: (1) The revised equations incorporate the effect of slope inclination β through a term $\cos\beta$ that did not exist in the original equations. Also, the notation t_{req} is used, instead of D_{req} in the original paper, to make it clear that the dimension of the protective layer that is being calculated is the thickness (measured perpendicularly to the slope), whereas the notation D is commonly used for the depth (measured vertically). For horizontal slopes ($\beta = 0$) the revised equations are the same as those presented by Giroud et al. (1995).
(2) The revised equations incorporate considerations discussed in Section 3.1 of this technical note regarding the "uplift tension-strain" relationship, which relates the wind-induced strain component, ε_w , to the total tension, T, in the geomembrane ($T = T_0 + T_w$). If the initial strain, ε_0 , and initial tension, T_0 , in the geomembrane are zero (i.e. if the total strain, $\varepsilon = \varepsilon_w$, and the total tension, $T = T_w$), the revised equations are the same as those presented by Giroud et al. (1995).

Errata

UPLIFT OF GEOMEMBRANES BY WIND – EXTENSION OF EQUATIONS

TECHNICAL NOTE FOR ERRATA: Zornberg, J.G. and Giroud, J.P., 1997, "Uplift of Geomembranes by Wind – Extension of Equations", *Geosynthetics International*, Vol. 4, No. 2, pp. 187-207.

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REFERENCE FOR ERRATA: Zornberg, J.G. and Giroud, J.P., 1999, "Errata for 'Uplift of Geomembranes by Wind – Extension of Equations'", *Geosynthetics International*, Vol. 6, No. 6, pp. 521-522.

The authors inadvertently showed the wrong angle in Figure 2, p. 191 and used the wrong units in Equations A-18, A-20, A-24, and A-26, pp. 204-205, in their technical note, which appeared in *Geosynthetics International*, Vol. 4, No. 2.

ERRATUM FOR SECTION: 3.1 Uplift Tension-Strain Relationship

In Figure 2, p. 191 :

The angle at Point B (i.e. the angle between the geomembrane before uplift and the geomembrane after uplift) should be θ instead of $\theta/2$.

ERRATA FOR APPENDIX: A-1 GEOMEMBRANE SENSITIVITY TO WIND UPLIFT

Equation A-18, p. 204, replace km/h with m/s to give :

$$\mu_{GM} \geq \mu_{GMreq} = 0.0659 \frac{\lambda V^2}{\cos\beta} \tag{A-18}$$

with $\mu_{GMreq}(\mathrm{kg/m^2})$ and $V(\mathrm{m/s})$

Equation A-20, p. 204, replace km/h with m/s to give :

$$\mu_{GM} \geq \mu_{GMreq} = 0.0659 \frac{\lambda V^2}{\cos\beta} e^{-(1.252 \times 10^{-4}) z} \qquad \text{(A-20)}$$

$$\text{with } \mu_{GMreq}(\text{kg/m}^2),\ V(\text{m/s}) \text{ and } z(\text{m})$$

Equation A-24, p. 205, replace m/s with km/h to give :

$$V \leq V_{up} = 14.023 \sqrt{\mu_{GM} \cos\beta/\lambda} \qquad \text{(A-24)}$$

$$\text{with } V_{up}(\text{km/h}) \text{ and } \mu_{GM}(\text{kg/m}^2)$$

Equation A-26, p. 205, replace m/s with km/h to give :

$$V \leq V_{up} = 14.023 e^{(6.259 \times 10^{-5}) z} \sqrt{\mu_{GM} \cos\beta/\lambda} \qquad \text{(A-26)}$$

$$\text{with } V_{up}(\text{km/h}),\ z(\text{m}) \text{ and } \mu_{GM}(\text{kg/m}^2)$$