Plaisted, M., and Zornberg, J.G. (2013). "Determination of the Swell-stress Curve of an Expansive Soil Using Centrifuge Technology." Proceedings of the 5^{th} *International Young Geotechnical Engineers' Conference - 5^{th} iYGEC 2013,* 31 August-01 September, Paris, France, pp. 443-446.

Edited by Y.-J. Cui et al. © 2013. The authors and IOS Press. doi:10.3233/978-1-61499-297-4-443

DETERMINATION OF THE SWELL-STRESS CURVE OF AN EXPANSIVE SOIL USING CENTRIFUGE TECHNOLOGY

DETERMINATION DE LA COURBE DE GONFLEMENT D'UN SOL GONFLANT A L'AIDE D'ESSAIS EN CENTRIFUGEUSE

Michael PLAISTED, Jorge ZORNBERG
The University of Texas at Austin, Austin, Texas, United States

ABSTRACT - A research project was conducted on the use of centrifuge technology to characterize the expansive properties of a highly plastic clay. The resulting testing procedure allowed the measurement of the one-dimensional swell of a soil sample in the centrifuge. This paper focuses on the analysis of testing results, specifically how to determine the relationship between swell and effective stress of a soil using centrifuge testing. Samples under centrifugation are subjected to wide range of effective stress across the sample height due to an increased unit weight. A method is initially proposed to define a representative effective stress of the centrifuge sample, which can be used to relate the swell of the sample to a single representative effective stress. A second, more robust method is also developed, which uses multiple centrifuge test results and curve fits a function to the data. Both methods were found to result in accurate swell-stress curves verified by comparison with curves determined by standard swell test methods.

1. Introduction

A research project was conducted in order to explore the use of centrifuge technology for the direct testing of expansive clays. During this research project the equipment and testing procedure for monitoring vertical strain of an expansive soil inside a centrifuge permeameter was developed. Previous research on the centrifuge testing of expansive soils (Frydman & Weisberg 1991; Gadre & Chandrasekaran 1994) focused on large, relatively expensive, research centrifuges. In contrast, the goal of this research project was to develop a quick, simple, and inexpensive method for directly determining the swelling potential of soils.

This paper focuses on the analysis of results from centrifuge testing. Two methods are proposed for analysis, both providing the ability to determine the relationship between swell and effective stress using centrifuge results.

2. Equipment and Testing Procedure

The equipment and testing procedure developed in this research project has been previously documented (Plaisted 2009, Plaisted & Zornberg 2010, Plaisted & Zornberg 2011) and will only be briefly discussed in this section. Centrifuge testing is conducted by compacting a soil specimen into acentrifuge cup and ponding water on top of the specimen. The specimen is then spun at high g-levels with the acceleration due to centrifugation forcing the ponded water to flow into the sample and promote swelling. A schematic of the testing setup can be seen in Figure 1.

Vertical deflections of the soil sample are monitored by linear position sensors resting on the surface of the soil specimen. The sensors are connected to a miniature battery powered data acquisition system (DAS) based on the open-source prototyping platform Arduino. The DAS wirelessly transmits sensor data to a nearby computer which records the values over time.

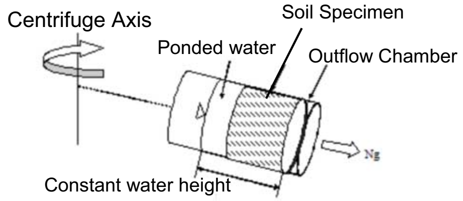

Figure 1 - Centrifuge permeameter

The procedure for compaction, loading, and wetting of specimens was developed in order to mimic the standard swell testing procedure as closely as possible. A typical plot of vertical deflection vs. time of a centrifuge test is shown in Figure 2. The shape of the curve is typical with the majority of the expansion occurring within the first 12 hours of centrifugation. Total test duration is typically between 24 and 48 hours with termination of testing occurring after the slope of secondary swell is well defined.

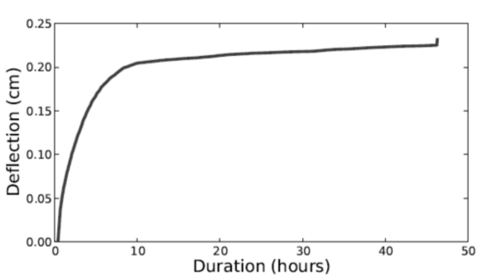

Figure 2 - Centrifuge test results

An excellent match was found between results from the centrifuge testing procedure and the standard procedure. Figure 3 includes results from six centrifuge tests performed on a highly expansive clay and eight results from standard swell tests. The centrifuge tests were analyzed using the procedures discussed in Section 3.

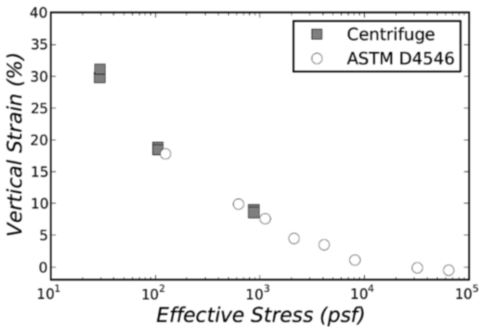

Figure 3 - Centrifuge vs. standard swell results

3. Analysis of Centrifuge Results

Results from standard swell tests can be directly related to a single effective stress because the sample is subject to an extremely narrow range of stresses. Instead, centrifuge test results correspond to a larger range of effective stresses. The increased stress range results from the high g-level induced during testing, which increases the unit weight of soil.

Two analysis methods are proposed in Sections 3.1-3.2, which are based on the concept an "equivalent stress". For a given stress-strain relationship, ε(σ'), the total strain for a sample with stresses ranging from σ_t' to σ_b' can be calculated as:

$$\varepsilon_{ave} = \frac{\int_{\sigma_t'}^{\sigma_b'} \varepsilon(\sigma')}{\sigma_b' - \sigma_t'} \qquad (1)$$

The equivalent stress is the stress value that would result in the same strain value that was calculated using Equation (1) but by using a single stress rather than a range in stresses. The equivalent stress can be calculated as:

$$\sigma'_{equiv} = \varepsilon^{-1}(\varepsilon_{ave}) \qquad (2)$$

In order for Equation (1) to be valid, a linear distribution of effective stress across the sample must be assumed. The distribution has been shown to actually be better represented by a polynomial distribution (Plaisted 2009), but errors between the two distributions were found to be less than 1%. The errors from this assumption will be further discussed in Section 3.3.1.

Both methods assume the capability of calculating the effective stress at the top and base of centrifuge specimens. The procedure and derivation for determining stresses in centrifuge samples can be found in Plaisted (2009) and McCartney & Zornberg (2010).

3.1. Representative Stress Method

The representative stress method was developed in order to approximately determine the equivalent stress of a centrifuge test using only the results from a single centrifuge test. The method is based on the assumption that the swell-stress relationship is log-linear across the range of stresses of a single sample. The errors associated with these assumptions will be discussed in Section 0.

In order to calculate the equivalent stress, a log-linear swell-stress curve is assumed as:

$$\varepsilon = A \ln(\sigma') + B \qquad (3)$$

The average strain across a range of stresses can be calculated using Equation (1). The equivalent stress cannot be directly calculated as the coefficients *A* and *B* are unknown. However it was found that the location of the equivalent stress relative to the stress range was independent of the coefficients of the assumed log-linear relationship. Therefore if the ratio of stresses at the base and top of the specimen is:

$$SR = \frac{\sigma_b'}{\sigma_t'} \qquad (4)$$

and the interpolation value is defined as:

$$IV = \frac{\sigma_t' - \sigma_{equiv}'}{\sigma_b' - \sigma_t'} \qquad (5)$$

then equations (1), (4) and (5) can be substituted into equation (2), and the terms rearranged and reduced in order to produce a relationship between the stress ratio (SR) and the interpolation value (IV) such that:

$$IV = \frac{\frac{1}{e} SR^{\left(\frac{1}{SR-1}+1\right)} - 1}{SR - 1} \qquad (6)$$

The resulting function is shown in Figure 4 over a range of stress ratios typical for centrifuge testing. Using Figure 4 or Equation (6) the interpolation value can be calculated for a single centrifuge test. By rearranging Equation (5), the equivalent stress can be determined from the interpolation value, as follows:

$$\sigma'_{equiv} = (\sigma_b' - \sigma_t')IV + \sigma_t' \qquad (7)$$

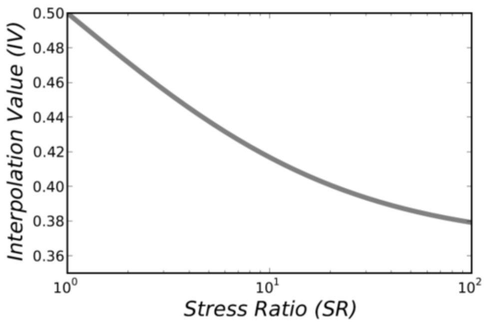

Figure 4 - Stress Ratio and Interpolation Value

The representative stress method was conducted for a set of tests conducted at g-levels of 5, 25, and 200 and the results are included in Table 1. The results show a well-defined trend between the equivalent stress and swell, as will be further discussed in Section 3.3.2.

Table 1 - Representative Stress Results

Swell (%)	σ_t' (psf)	σ_b' (psf)	SR	IV	σ_{equiv}' (psf)
8.99	32.5	219	6.59	0.43	908.8
8.58	32.6	219	6.54	0.43	909.9
18.87	268	1760	6.73	0.43	112.2
18.42	269	1760	6.72	0.43	112.3
29.81	9.03	62.4	6.91	0.43	31.84
31.12	9.02	62.7	6.94	0.43	31.93

3.2. Curve Fitting Method

The curve fitting method is used to solve for the function coefficients that result in the least error between the measured swell in centrifuge tests and the predicted swell based on the fitted swell function. The fitted function can then be used to accurately calculate the equivalent stress for each centrifuge test. While the procedure is similar to that used in general curve fitting, adjustments were made since the curve is being fit to data over a range of stresses rather than a point.

This method requires data from at least three centrifuge tests. A function is then chosen with baseline coefficients. The average swell is calculated for the range in stresses of each test using Equation (1) and the assumed function ε(σ'). Calculating the average will likely require numerical integration unless a simple function is found that fits the data.

The calculated average swell is compared with the measured swell for each test and total error calculated using least squares method such that:

$$Error = \sum_{i=0}^{n} (\varepsilon_{ave,i} - \varepsilon_{measured,i})^2 \quad (8)$$

Coefficients are refined and the process is repeated using the updated coefficients until the minimum error is found. Powell's method (SciPy 2013) is used in order to find minimum error of the function.

The process of determining the function to be used can be achieved in two ways. The first approach is to complete the analysis of all testing results using the Representative Stress Method in Section 3.1. The results from the representative stress method are plotted and a function is chosen that matches the general shape of the relationship found between swell and stress in the plotted data.

The second approach is to pre-select a variety of functions that typically represent well the relationship between swell and stress for expansive soils. The curve fitting method is then performed for all of the selected functions and the function with the lowest error is chosen.

Three functions that have been found to fit well swell-stress curves are listed below as equations (9) through (11). The procedure listed above was completed for each function using a Python script to automate the process. The resulting best-fit coefficients and the corresponding least-squares error is included in Table 2.

$$Swell(\sigma') = A\ln(\sigma') + B \quad (9)$$

$$Swell(\sigma') = A\ln(\mathrm{B}\ln(\sigma') + 1) + C \quad (10)$$

$$Swell(\sigma') = \frac{A}{\ln(B\sigma' + 1)} + C \quad (11)$$

All three functions provide a good correlation between effective stress and swell. The best-fit coefficients and errors for each equation are included in Table 2. The log-linear function performs worst, but for practical purposes would most likely be satisfactory for the small ranges in stress typical of the active zone of a soil profile. Equation (11) provides a very accurate representation of the swell-stress relation and was consistently found to be the best fit of the three functions for all data sets evaluated. The curve has been plotted in Figure 5 along with equivalent stresses determined using the representative stress method and the actual function.

Table 2 - Curve fitting results

Equation	A	B	C	Error (ε^2)
(9)	-7.55	56.39	N/A	39.5
(10)	-107.5	53113	322.7	14.2
(11)	128.8	0.714	-11.15	1.12

3.3. Accuracy of Methods

The proposed methods include inherent error due to their assumptions. Both methods assume a linear distribution of effective stress across centrifuge sample. The actual distribution has been shown to be polynomial (Plaisted 2009). The representative stress method also includes errors due to the assumption of a log-linear swell-stress curve. The errors associated with these assumptions will be discussed in the following sections.

3.3.1. Error due to assumed linear stress distribution

The difference between a linear stress distribution and the true polynomial distribution is minor, with the maximum error being approximately 1% at the center of the sample. The effect of this error on the analysis methods was evaluated by calculating the swell for samples using both the linear and polynomial distributions. This was accomplished using a finite difference approach with samples being divided into 1000 layers. The swell-stress curve calculated in Section 3.2 was used to predict the swell for each layer based on the two effective stress distributions (polynomial and linear).

It was found that the swell calculated using the assumed linear distribution was slightly higher than the polynomial. This result was expected as the polynomial distribution had slightly higher stresses in the middle of the sample. The error was calculated at under 0.25% (16.43% predicted for linear, 16.40% predicted for polynomial) and was considered acceptably small for practical applications given the simplification of analysis by assuming a linear stress distribution.

3.3.2. Error due to assumed log-linear swell-swell curve

The representative stress method assumes that the swell-stress relationship is log-linear across the range of stresses seen in a single sample. The errors associated with this assumption will depend on the true shape of the swell-stress relationship.

In order to evaluate the effects of this assumption the values of the equivalent stress were calculated using the best fit function from Section 3.2 compared with the values using the log-linear assumption. The location for the equivalent stress using the best-fit function and the log-linear were comparable with errors ranging from 4% to 8.5%. When the locations are plotted in Figure 5 the differences are very minor providing a nearly identical swell-stress relationship.

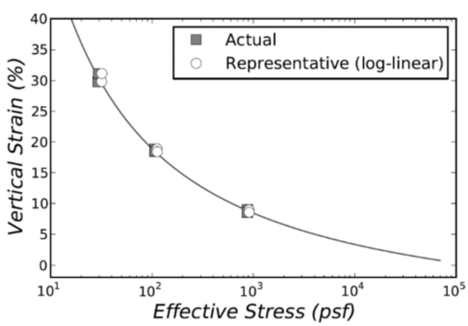

Figure 5 - Equivalent stress location

4. Conclusions

Two analysis methods are presented for analyzing centrifuge swell tests. The representative stress method was shown to provide a simple method to analyze single centrifuge tests. The results from analysis can be used in the same manner as results from standard swell test results. This method would be recommended for use in general practice when small errors are acceptable.

The second, more robust curve fitting method requires multiple centrifuge tests to be completed before analysis but provides a more accurate location of the equivalent stress and relationship between swell and effective stress. This method could also be used in practice but would require software or a complex spreadsheet to automate the process.

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