CE 394K.2 Hydrology

Homework Problem Set #2

Due Tues March 6

Problems in "Applied Hydrology"

3.5.1 Evaporation from a lake

3.6.3 Evapotranspiration computation

4.1.3 Soil water flux

3.5.1.

The computations are summarized in Table 3.5.1. For example, in winter (first row of the table), from Eq. (2.7.6) of the textbook, the latent heat of vaporization of water is $\ell_v = 2500 - 2.36 \times 5 = 2489$ kJ/kg as shown in Col. (3) of Table 3.5.1. From Table 2.5.2 of the textbook, $\rho_w = 1000$ kg/m³ as shown in Col. (4) of Table 3.5.1. Then, the evaporation rate by the energy balance method is given by Eq. (3.5.10) of the textbook,

$$E_r = R_n/(\ell_v \rho_w) = 50/(2489 \times 10^3 \times 1000) = 2.01 \times 10^{-8} \text{ m/s} = 1.7 \text{ mm/d}$$

as shown in Col. (5) of Table 3.5.1. From Eq. (3.2.9) of the textbook, the saturated vapor pressure is

$$e_s = 611 \exp[17.27T/(237.3+T)] = 611 \exp[17.27 \times 5/(237.3 + 5)] = 873 \text{ Pa}$$

and the winter gradient of the saturated vapor pressure curve is, from Eq. (3.2.10) of the textbook,

$$\Delta = 4098 e_s/(237.3+T)^2 = 4098 \times 873/(273.3 + 5)^2 = 60.9 \text{ Pa/}^\circ\text{C}$$

as shown in Col. (6) of Table 3.5.1. The psychrometric constant γ is given by Eq. (3.5.24) of the textbook, with $C_p = 1005$ J/kg/°K, $P = 101.3$ kPa and $K_h/K_w = 1$,

$$\gamma = C_p K_h P/(0.622 \ell_v K_w) = 1005 \times 101.3 \times 10^3 \times 1/(0.622 \times 2489 \times 10^3)$$
$$= 65.8 \text{ Pa/}^\circ\text{C}$$

as shown in Col. (7) of Table 3.5.1. Then, the Priestley-Taylor evaporation is given by Eq. (3.5.27) of the textbook, with $\alpha = 1.3$,

$$E = \alpha\Delta/(\Delta+\gamma)\, E_r = 1.3 \times 60.9/(60.9 + 65.8) \times 1.74 = 1.1 \text{ mm/d}$$

as shown in Col. (8) of the table. For summer the evaporation may be similarly computed, giving E = 9.1 mm/d.

	(1) Temp. T (°C)	(2) Net Rad. R_n (W/m²)	(3) Latent Heat ℓ_v (kJ/kg)	(4) Water Dens. ρ_w (kg/m³)	(5) Energy Evap. E_r (mm/d)	(6) Δ (Pa/°C)	(7) γ (Pa/°C)	(8) Priestley-Taylor E (mm/d)
Winter	5	50	2489	1000	1.7	60.9	65.8	1.1
Summer	30	250	2430	996	8.9	243.4	67.4	9.1

Table 3.5.1. Evaporation computations by the Priestley-Taylor method.

3.6.3.

(a) Energy method. The computations are summarized in Table 3.6.3. For example, for May (Col. 3) of the table), the temperature is $T = 17°C$, so the latent heat of vaporization of water is $\ell_v = 2500 - 2.36 \times 17 = 2460$ kJ/kg in Row (2) of Table 3.6.3, from Eq. (2.7.6) of the textbook. From Table 2.5.2 of the textbook, $\rho_w = 998.6$ kg/m³ as shown in Row (3). Then, the evaporation rate for May by the energy balance method is given by Eq. (3.5.10) of the textbook, with $R_n = 169$ W/m²

$$E_r = R_n/(\ell_v \rho_w) = 169/(2460 \times 10^3 \times 998.6) = 6.88 \times 10^{-8} \text{ m/s} = 5.9 \text{ mm/d}$$

as shown in Row (12) of Table 3.6.3.

(b) Aerodynamic method. From Eq. (3.2.9) of the textbook, the saturated vapor pressure for May is

$$e_{as} = 611 \exp[17.27T/(237.3+T)] = 611 \exp[17.27 \times 17/(237.3 + 17)]$$

$$= 1938 \text{ Pa}$$

as shown in Row (6) of Table 3.6.3, and the vapor transfer coefficient B is given by Eq. (3.6.2) from the textbook, with wind run $u = 167$ km/d,

$$B = 0.0027 (1 + u/100) = 0.0027 (1 + 167/100) = 7.21 \times 10^{-3} \text{ mm/d/Pa}$$

as shown in Row (10) of Table 3.6.3. The vapor pressure is $e_a = 1100$ Pa, so the evapotranspiration computed by the aerodynamic method for May is, by Eq. (3.5.17) from the textbook,

$$E_a = B(e_{as} - e_a) = 7.21 \times 10^{-3} (1938 - 1100) = 6.0 \text{ mm/d}$$

as shown in Row (13) of Table 3.6.3.

(c) Combination method. For May, the gradient of the saturated vapor pressure curve is, from Eq. (3.2.10),

$$\Delta = 4098 e_s/(237.3+T)^2 = 4098 \times 873/(273.3 + 17)^2 = 122.8 \text{ Pa/°C}$$

as shown in Row (7) of Table 3.6.3. The psychrometric constant γ is given by Eq. (3.5.24) of the textbook, with $C_p = 1005$ J/kg/°K, $p = 101.3$ kPa and $K_h/K_w = 1$,

$$\gamma = C_p K_h p/(0.622 \ell_v K_w) = 1005 \times 101.3 \times 10^3 \times 1/(0.622 \times 2433 \times 10^3)$$

$$= 66.5 \text{ Pa/°C}$$

as shown in Row (10) of Table 3.6.3. Then, E_r and E_a may be combined according to Eq. (3.5.26) of the textbook to give the evapotranspiration rate for May

$$E = \Delta/(\Delta+\gamma) E_r + \gamma/(\Delta+\gamma) E_a$$

$= 122.8/(122.8+66.5)\ 5.9 + 66.5/(122.8+66.5)\ 6.0 = \underline{6.0 \text{ mm/d}}$

as shown in Row (14) of Table 3.6.3.

(d) Priestley-Taylor method. The evapotranspiration for May is given by Eq. (3.5.27) of the textbook, with $\alpha = 1.3$,

$$E = \alpha\Delta/(\Delta+\gamma)\ E_r = 1.3 \times 122.8/(122.8 + 66.5) \times 5.9 = \underline{5.0 \text{ mm/d}}$$

as shown in Row (15) of Table 3.6.3. Values of evapotranspiration for June and July may be similarly computed.

(1)	(2) Units	(3) May	(4) July	(5) Sept.
(1) Temperature T	(^oC)	17	23	20
(2) Latent heat, l_v	(kJ/kg)	2460	2446	2453
(3) Water density	(kg/m^3)	998.6	997.4	998
(4) Net radiation, R_n	(W/m^2)	169	189	114
(5) Wind run, u	(km/d)	167	121	133
(6) Sat. vapor presure, e_{as}	(Pa)	1938	2810	2339
(7) Δ	(Pa/^oC)	122.8	170.0	144.8
(8) Air pressure, p	(kPa)	101.3	101.3	101.3
(9) Vapor pressure, e_a	(Pa)	1100	1400	1200
(10) B	(10^{-3} mm/d/Pa)	7.21	5.97	6.29
(11) γ	(Pa/^oC)	66.5	66.9	66.7
Method		**Evapotranspiration rate**		
(12) Energy balance, E_r	(mm/d)	5.9	6.7	4.0
(13) Aerodynamic method, E_a	(mm/d)	6.0	8.4	7.2
(14) Combination method	(mm/d)	6.0	7.2	5.0
(15) Priestley Taylor method	(mm/d)	5.0	6.2	3.6

Table 3.6.3. Evapotranspiration at Davis, California.

<u>3.6.4.</u>

The computations are identical to those in Problem 3.6.3. The results are summarized in Table 3.6.4.

soil moisture flux in week 1 is

$$q = -K\,S_f = -0.041 \times 0.75 = -0.030 \text{ cm/d}$$

as shown in Col. (9) of Table 4.1.1. The flux q is negative because moisture is flowing downwards in the soil.

4.1.2.

The moisture flux between 1.0 and 1.2 m depth may be computed following the method outlined in Problem 4.1.1. For example, for week 1 and depth $z_1 = -100$ cm, $h_1 = -160$ cm so $\psi_1 = h_1 - z_1 = -160 - (-100) = -60$ cm, and, similarly, $\psi_2 = -190 - (-120) = -70$ cm as shown in Cols. (4) and (5) of Table 4.1.2. The hydraulic conductivity varies with ψ, so an approximate average value may be found corresponding to the average of the ψ values at $z_1 = 100$ cm and $z_2 = 120$ cm; $\psi = [-60 + (-70)]/2 = -65$ cm as shown in Col. (6), and the corresponding hydraulic conductivity is $K = 250(-\psi)^{-2.11} = 250 \times 65^{-2.11} = 0.037$ cm/d in Col. (7). In week 1, the hydraulic gradient in Col. (8) of Table 4.1.2 is $S_f = (h_1 - h_2)/(z_1 - z_2) = [-190 - (-160)]/[-120 - (-100)] = 1.5$, so the soil moisture flux in week 1 is

$$q = -K\,S_f = -0.037 \times 1.5 = -0.056 \text{ cm/d}$$

as shown in Col. (9) of Table 4.1.2. The flux q is negative because moisture is flowing downwards in the soil.

4.1.3.

The moisture fluxes may be computed between different depths following the method outlined in Problems 4.1.1 and 4.1.2. Table 4.1.3-1 shows the hydraulic heads measured from Fig. 4.1.5(b) of the texbook at different depths. The resulting fluxes are summarized in Table 4.1.3-2. The values of the flux at 3m depth are very high because the soil is saturated most of the time and the relationship between hydraulic conductivity and suction head is no longer applicable; the flow is driven by gravity alone.

Fig. 4.1.3 shows curves of moisture flux versus time between different depths in the soil. It is clear from the figure that rainfall drives the infiltration process. The response of the soil to precipitation is very rapid in the upper layers of the soil. For example, between 0.4 and 0.8m, infiltration increases abruptly after storms followed by a decay later. As we move deeper into the soil, the response is more damped and a single storm is no longer influential to the same degree; longer rainy periods are required to increase the moisture flux, as shown by the 1.5 to 1.8m profile.

During the summer months, suction heads are very high throughout the soil profile. The effect of precipitation in moisture flux is negligible, except in the upper sections of the soil. Between 0.4 and 0.8m, the direction of flow is eventually reversed as moisture moves upwards to leave the soil as evapotranspiration. The calculations shown here are approximate as they do not account for the variation of soil properties with depth i.e. the same relationship between K and ψ is used for all depths in the soil.

Week	Station depth							
	-40	-80	-100	-120	-150	-180	-240	-300
1	-145	-145	-160	-190	-	-230	-265	-
2	-200	-165	-180	-205	-	-235	-265	-
3	-110	-130	-150	-190	-220	-240	-265	-310
4	-120	-140	-170	-200	-220	-240	-265	-310
5	-80	-125	-160	-190	-215	-240	-265	-310
6	-60	-105	-130	-160	-200	-230	-265	-310
7	-135	-135	-150	-165	-190	-215	-255	-310
8	-145	-150	-170	-190	-210	-230	-255	-310
9	-155	-165	-190	-205	-225	-240	-260	-315
10	-240	-190	-210	-220	-235	-245	-265	-315
11	-240	-220	-230	-235	-250	-255	-270	-315
12	-285	-230	-250	-250	-260	-265	-275	-320
13	-	-255	-265	-265	-270	-275	-285	-330
14	-	-280	-285	-275	-285	-285	-295	-

Table 4.1.3-1. Total soil water head h (cm) in a loam soil at Deep Dean, Sussex, England.

	Soil moisture flux between depth						
Week	40-80	80-100	100-120	120-150	150-180	180-240	240-300
1	0.000	-0.030	-0.056	-	-	-0.069	-
2	0.008	-0.016	-0.028	-	-	-0.052	-
3	-0.022	-0.065	-0.088	-0.031	-0.024	-0.038	-0.446
4	-0.015	-0.056	-0.041	-0.018	-0.024	-0.038	-0.446
5	-0.103	-0.102	-0.056	-0.028	-0.033	-0.038	-0.446
6	-0.394	-0.286	-0.207	-0.108	-0.065	-0.069	-0.446
7	0.000	-0.044	-0.054	-0.076	-0.099	-0.187	-1.110
8	-0.002	-0.031	-0.031	-0.024	-0.035	-0.067	-1.110
9	-0.003	-0.024	-0.014	-0.016	-0.017	-0.034	-0.546
10	0.007	-0.012	-0.006	-0.008	-0.009	-0.027	-0.374
11	0.002	-0.003	-0.002	-0.006	-0.003	-0.014	-0.262
12	0.005	-0.006	0.000	-0.003	-0.002	-0.007	-0.172
13	-	-0.002	0.000	-0.001	-0.002	-0.005	-0.089
14	-	-0.000	0.002	-0.002	0.000	-0.004	-

Table 4.1.3-2. Soil moisture flux q (cm/day) at Deep Dean, Sussex, England.

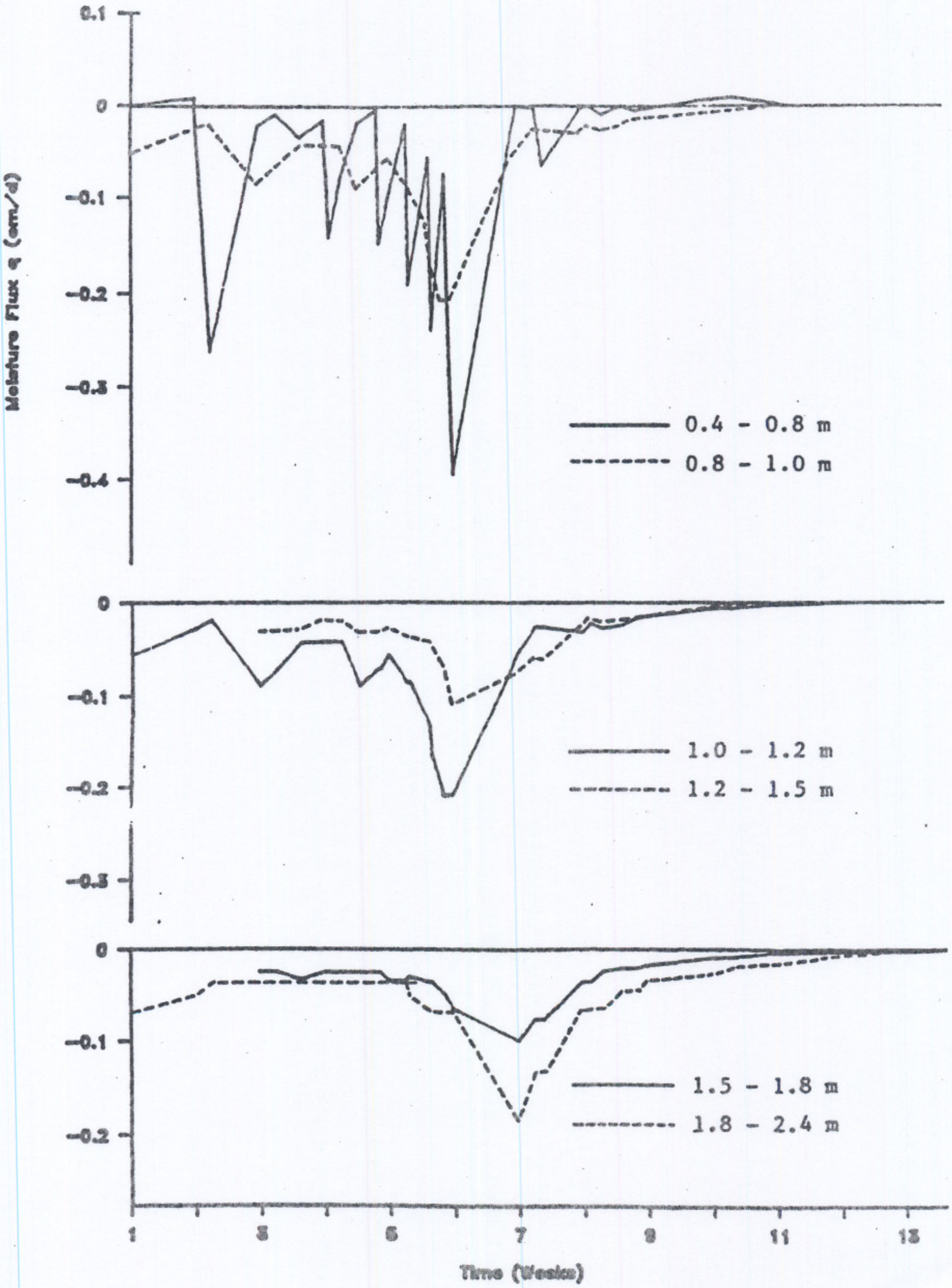

Fig. 4.1.3. Soil moisture flux between different depths in a loam soil in Deep Dean, Sussex, England. Negative sign indicates flow into the ground.