CE 394K.2 Hydrology

Homework Problem Set #3

Due Thurs March 29

Problems in "Applied Hydrology"

4.2.3 Infiltration by Horton's method

4.3.2 Infiltration by Green-Ampt method

4.4.3 Ponding time and cumulative infiltration at ponding

4.4.11 Theoretical study of infiltration at ponding time using Philip's equation

(1)	(2)	(3)
	Infiltration	
Time t (hr)	Rate f (in/hr)	Cumulative F (in)
0.0	3.00	0.00
0.5	0.84	0.78
1.0	0.57	1.11
1.5	0.53	1.38
2.0	0.53	1.65

Table 4.2.1. Infiltration computed by Horton's equation

4.2.2.

Assuming continuously ponded conditions, the cumulative infiltration at time $t = 0.75$ hrs for Horton's equation is given by Eq. (2) from Table 4.4.1 of the textbook

$$F = f_c t + (f_o - f_c)(1 - e^{-kt})/k$$

so that

$$F(0.75) = 0.53 \times 0.75 + (3 - 0.53)[1 - \exp(-4.182 \times 0.75)]/4.182$$

$$= 0.96 \text{ in.}$$

The cumulative infiltration at time $t = 2$ hr can be similarly computed, giving $F(2) = 1.65$ in. Therefore, the incremental depth of infiltration between time $t = 0.75$ and $t = 2$ hrs. is $1.65 - 0.96 = 0.69$ in.

4.2.3.

Assuming continuously ponded conditions, the values of the infiltration rate f and the cumulative infiltration F are computed using Eqs. (1) and (2) from Table 4.4.1 of the textbook,

$$f(t) = f_c + (f_o - f_c)\, e^{-kt}$$

$$F(t) = f_c t + (f_o - f_c)(1 - e^{-kt})/k$$

with $f_c = 1$ cm/hr, $f_o = 5$ cm/hr and $k = 2$ hr^{-1}.

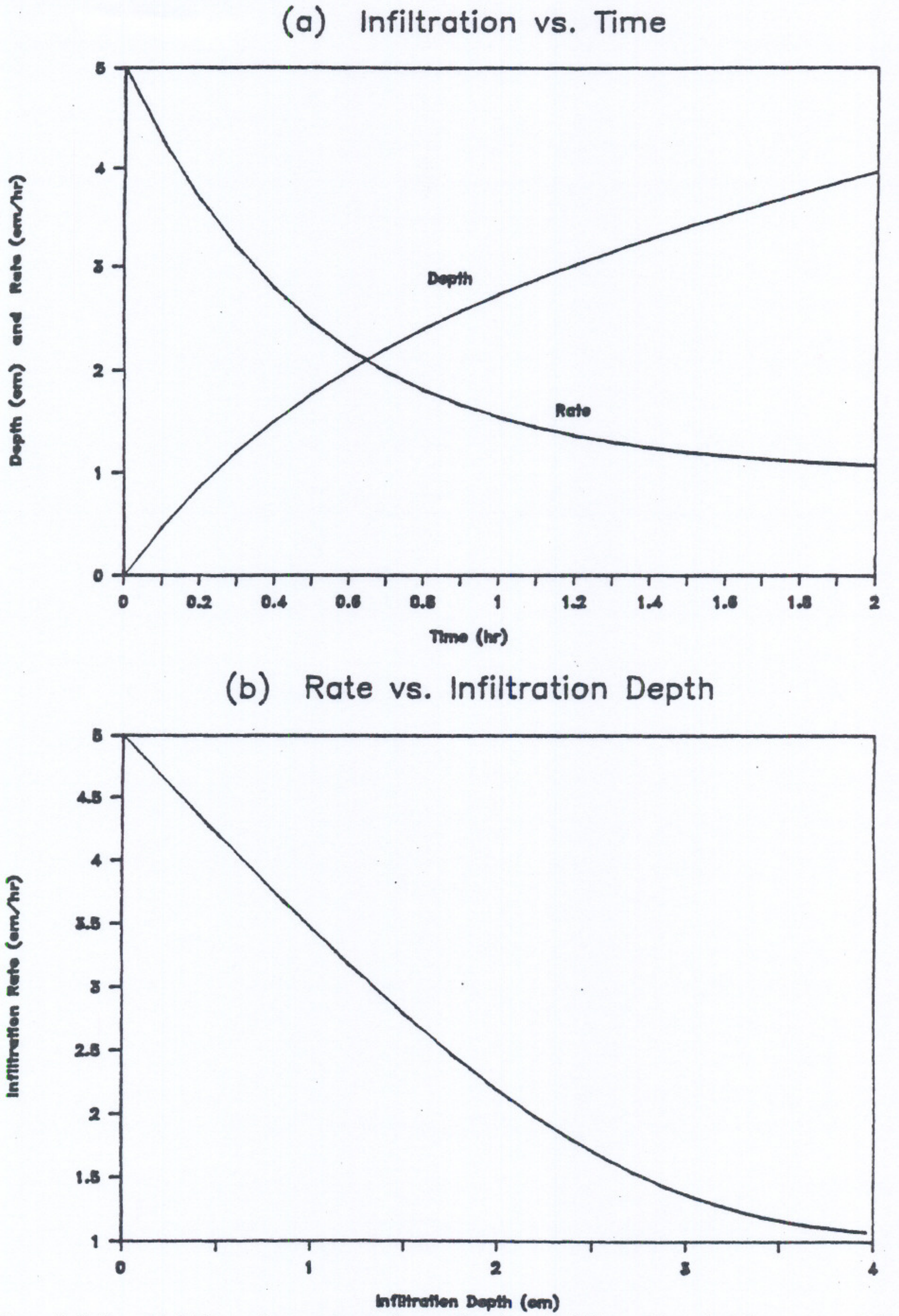

Fig. 4.2.3. Infiltration rate and cumulative infiltration depth computed by Horton's equation.

(1)	(2)	(3)
	Infiltration	
Time t (hr)	Rate f (cm/hr)	Cumulative F (cm)
0.0	5.00	0.00
0.5	2.47	1.76
1.0	1.54	2.72
1.5	1.19	3.40
2.0	1.07	3.96

Table 4.2.3. Infiltration computed by Horton's equation.

The results are shown in Table 4.2.3 for time t = 0, 0.5, 1, 1.5 and 2 hrs. For example, for time t = 0.5 hrs, f(0.5) = 1 + (5 - 1) exp(-2 x 0.5) = 0.84 in/hr, as shown in Col. (2) of Table 4.2.3 and F(0.5) = 1 x 0.5 + (5 - 1)[1 - exp(-2 x 0.5)]/2 = 0.78 in, as shown in Col. (3) of the table.

The infiltration rate and cumulative infiltration rate are plotted versus time in Fig. 4.2.3(a). Fig. 4.2.3(b) shows the infiltration rate as a function of cumulative infiltration.

4.2.4.

Assuming continuously ponded conditions, the infiltration rate is, according to Horton's equation (Eq. 4.2.3 from the textbook)

$$f = f_c + (f_o - f_c)\, e^{-kt}$$

so that

$$f_c = (f - f_o\, e^{-kt})/(1 - e^{-kt})$$

The cumulative infiltration for Horton's equation is given by

$$F = f_c t + (f_o - f_c)(1 - e^{-kt})/k$$

Substituting f_c in the previous equation yields

4.3.2.

The infiltration rate f and the cumulative infiltration F at time t = 0, 0.5, 1, 1.5, 2, 2.5 and 3 hrs may be computed following the method outlined in Problem 4.3.1. The cumulative infiltration is first computed using Eq. (4.3.8) of the textbook

$$F(t) = kt + \psi\Delta\theta \; \ell n[1 + F/(\psi\Delta\theta)]$$

$$= 1.09\ t + 2.72\ \ell n[1 + 3/2.72]$$

which may be solved by successive approximation for each value of t. The infiltration rate is then computed using Eq. (4.3.7) of the textbook

$$f(t) = k\ (\psi\Delta\theta/F + 1) = 1.09\ (2.72/F + 1)$$

The results are listed in Table 4.3.2. Fig. 4.3.2(a) shows a plot of the infiltration rate and cumulative infiltration versus time. Fig. 4.3.2(b) shows the variation of the infiltration rate f with the infiltration depth F.

$S_e = 0.4$

Sandy loam

$\theta_e = 0.412$

$\psi = 11.01$

$K = 1.09$

$\Delta\theta = \theta_e(1-S_e)$

$= 0.412(1-0.4)$

$= 0.2472$

$\psi\Delta\theta = 11.01 \times 0.2472$

$= 2.72$

Data from Table 4.3.1 p. 115 of text.

Infiltration		
Time t (hr)	Rate f (cm/hr)	Depth F (cm)
0.0	∞	0.00
0.5	2.51	2.10
1.0	2.01	3.21
1.5	1.80	4.16
2.0	1.68	5.03
2.5	1.60	5.85
3.0	1.54	6.63

Table 4.3.2. Infiltration computedby theGreen-Ampt method.

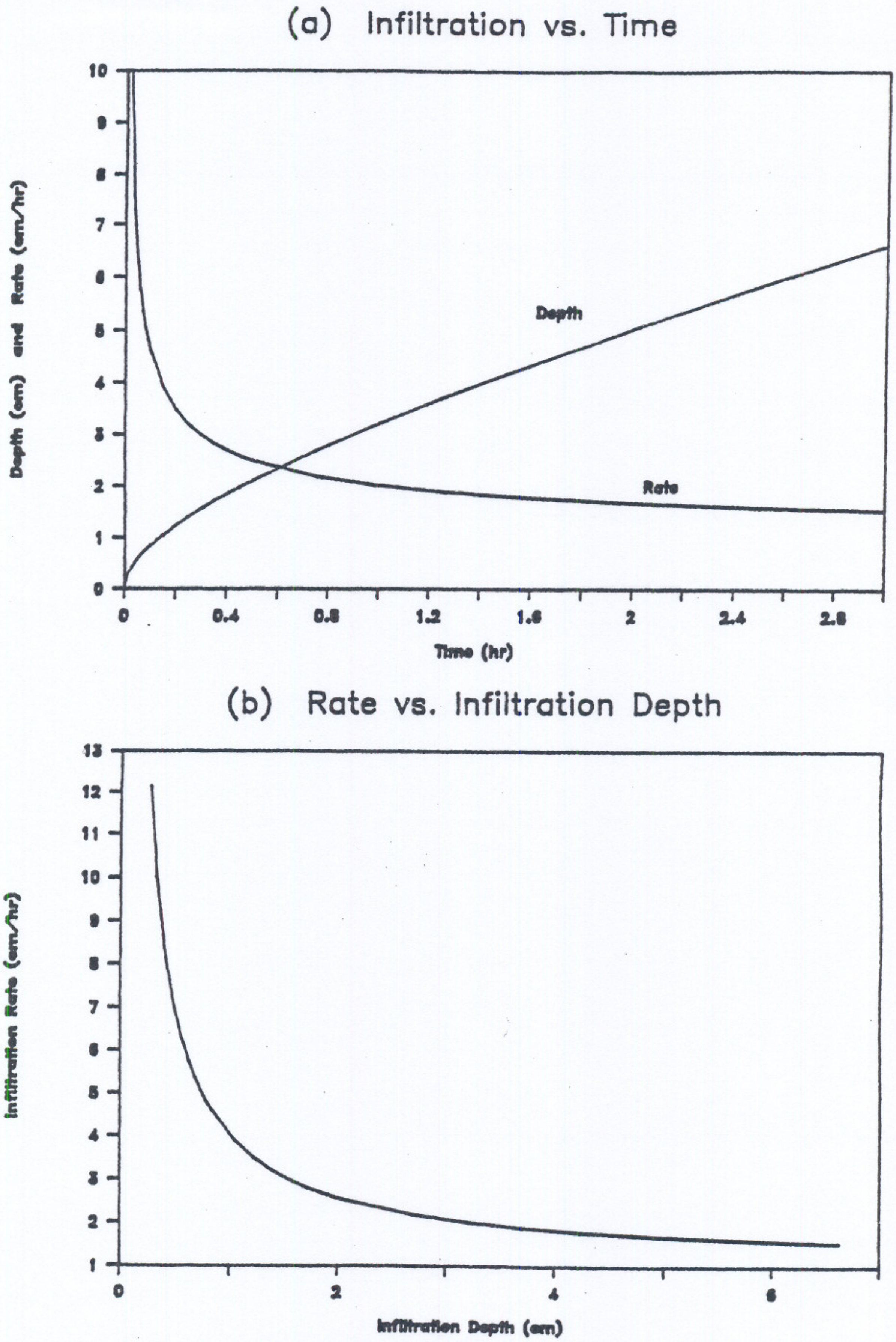

Fig. 4.3.2. Infiltration rate and cumulative infiltration depth computed by the Green-Ampt method.

example, starting with $F = 1$ cm gives a new value $F = 0.97 + 6.48 \ln[(6.48+1)/(6.48+0.97)] + 0.65(1 - 0.19) = 1.52$ cm. This value is then substituted in the right hand side of the previous equation and a new value $F = 1.96$ cm is obtained. After 17 iterations, the solution converges to $F = 3.17$ cm. The corresponding infiltration rate is given by Eq. 4.3.7 from the textbook

$$f = K(1 + \psi\Delta\theta/F) = 0.65\ (1 + 6.48/3.17) = 1.98 \text{ cm/hr}$$

4.4.3.

For a clay loam soil, from Table 4.3.1 of the textbook, $\theta_e = 0.309$, $\psi = 20.88$ cm and $K = 0.1$ cm/hr. The initial effective saturation is $S_e = 0.25$ so from Eq. (4.3.10) from the textbook, $\Delta\theta = (1 - S_e)\theta_e = (1 - 0.25)0.309 = 0.232$ and $\psi\Delta\theta = 20.88 \times 0.232 = 4.84$ cm. For $i = 1$ cm/hr, the ponding time is given by Eq. (4.4.2) of the textbook

$$t_p = K\psi\Delta\theta/[i(i-K)] = 0.1 \times 4.84/[1\ (1 - 0.1)] = 0.54 \text{ hr}$$

and the infiltrated depth at ponding is $F_p = t_p i = 0.54 \times 1 = 0.54$ cm.

For $i = 3$ cm/hr, t_p and F_p may be similarly computed to yield $t_p = 0.06$ hr and $F_p = 0.17$ cm.

4.4.4.

From Problem 4.4.3, $K = 0.1$ cm/hr, $\psi\Delta\theta = 4.84$ cm, $t_p = 0.06$ hr and $F_p = 0.17$ cm under rainfall intensity $i = 3$ cm/hr. For $t = 1$ hr, the infiltration depth is given by Eq. (4.4.5) from the textbook

$$F = F_p + \psi\Delta\theta \ln[(\psi\Delta\theta+F)/(\psi\Delta\theta+F_p)] + K(t - t_p)$$

$$= 0.17 + 4.84 \ln[(4.84+F)/(4.84+0.17)] + 0.1(1 - 0.06)$$

The solution F may be found by the method of successive substitution. For example, starting with $F = 1$ cm gives a new value $F = 0.17 + 4.84 \ln[(4.84+1)/(4.84+0.17)] + 0.1(1 - 0.06) = 1.01$ cm. This value is then substituted in the right hand side of the previous equation and a new value of F is obtained. The solution converges to $F = 1.04$ cm. The corresponding infiltration rate is given by Eq. (4.3.7) from the textbook

$$f = K(1 + \psi\Delta\theta/F) = 0.1\ (1 + 4.84/1.04) = 0.57 \text{ cm/hr}$$

$t_p = S^2(i-K/2)/[2i(i-K)^2] = 5^2\ (6 - 0.4/2)/[2 \times 6\ (6-0.4)^2] = 0.385$ hr

The cumulative infiltration at ponding is $F_p = it_p = 6 \times 0.385 = 2.31$ cm.

4.4.10.

The ponding time for the Horton's equation is given in Table 4.4.1 of the textbook. For $f_0 = 10$ cm/hr, $f_c = 4$ cm/hr, $k = 2\ hr^{-1}$ and rainfall intensity $i = 6$ cm/hr, this gives

$$t_p = \{f_0 - i + f_c\ \ln[(f_0 - f_c)/(i - f_c)]\}/(ik)$$

$$= \{10 - 6 + 4\ \ln[(10-4)/(6-4)]\}/(6 \times 2) = 0.70 \text{ hr}$$

The cumulative infiltration at ponding is $F_p = it_p = 6 \times 0.70 = 4.2$ cm.

4.4.11.

For Philip's equation (Table 4.4.1 of the textbook), the infiltration rate is

$$f = (S/2)\ t^{-1/2} + K$$

so that the time t may be expressed as

$$t = S^2/[4(f-K)^2]$$

Substituting t into the equation for the cumulative infiltration yields

$$F = St^{1/2} + Kt = S^2/[2(f-K)] + KS^2/[4(f-K)^2] = S^2(f-K/2)/[2(f-K)^2]$$

The cumulative infiltration at ponding time is $F_p = it_p$ and the infiltration rate is $f_p = i$, where i is the constant rainfall rate (see Fig. 4.4.2 from the textbook). Substituting F_p and f_p into the previous equation yields

$$it_p = S^2(i-K/2)/[2(i-K)^2]$$

so that

$$t_p = S^2(i-K/2)/[2i(i-K)^2]$$

4.4.12.

The infiltration rate for Horton's equation is, from Table 4.4.1 of the textbook,

$$f = f_c + (f_0 - f_c)\ e^{-Kt}$$

so that the time t may be expressed as