2.3.1.

(a) The daily maximum air temperature is a single value which occurs once in a day, therefore it is recorded as sample data.

(b) The daily precipitation is the sum of all rainfall occurring within the day, so it is recorded as pulse data.

(c) The daily wind speed is the average speed measured in the day, so it is recorded as pulse data.

(d) The annual precipitation is the sum of all rainfall occurring in a year, so it is recorded as pulse data.

(e) The annual maximum discharge in a river is the maximum of the instantaneous flowrates occurring during the year, so it is recorded as sample data.

2.3.2.

The precipitation input is recorded as a pulse data sequence as shown in column (3) of Table 2.3.2, in which the precipitation shown is the incremental depth which has occurred during the preceeding time interval. The streamflow output is recorded as a sample data sequence in which the value shown is the instantaneous flow rate occurring at that moment. To apply the discrete-time continuity equation, the streamflow must be converted to a pulse data sequence. For each 0.5 hr. interval, the volume of streamflow which occurred is found by averaging the streamflow rates at the ends of the interval and multiplying by $\Delta t = 0.5$ hr $= 1800$ s. The equivalent depth over the watershed is then calculated by dividing the streamflow volume by the watershed area $A = 7.03$ mi^2 $= 7.03 \times 5280^2$ ft^2 $= 1.96 \times 10^8$ ft^2.

The computations follow those in Example 2.3.1 in the textbook. For example, during the first time interval, between 0 and 0.5 hrs, the streamflows are $Q(0) = 25$ cfs and $Q(0.5) = 27$ cfs, so the streamflow volume in this interval is $\Delta t \times (25 + 27)/2 = 1800 \times 26$ ft^3 $= 46{,}800$ ft^3. The equivalent depth over the watershed is $Q_1 = 46{,}800 / 1.96 \times 10^8$ ft $= 2.38 \times 10^{-4}$ ft $= 0.003$ in, as shown in Col. (5) of Table 2.3.2.

The precipitation depth for the first time interval is 0.18 in, so the incremental change in storage is found as

$$\Delta S_j = I_j - Q_j \qquad (2.3.8\text{-}1)$$

giving for $j = 1$

$$\Delta S_1 = I_1 - Q_1 = 0.180 - 0.003 = 0.177 \text{ in}$$

as shown in Col. (6) of the table. The cumulative storage in the watershed is found by adding the incremental changes in storage (Eq. 2.3.3 from the textbook) with initial storage $S_0 = 0$. Then, with $j = 1$

$$S_1 = S_0 + \Delta S_1 = 0 + 0.177 = 0.177 \text{ in}$$

as shown in Col. (7) of Table 2.3.2. The calculations for successive time intervals $j = 2,3,\ldots,16$ follow similarly. The total amount of precipitation was 0.97 in. The total amount of runoff in eight hours was 0.872 in, or 90 % of the total precipitation. The remaining 0.098 in. remained stored in the watershed at the end of the eight hour period. The maximum storage occurred 2 hours after the beginning of the storm and was 0.932 in. Fig. 2.3.2(a) shows the time distribution of precipitation intensity (obtained by dividing the precipitation increments by $\Delta t = 0.5$ hr) and streamflow (in equivalent units of in/hr over the watershed). The change in storage for 0.5 hr intervals and the total storage in the watershed are plotted in Fig. 2.3.2(b).

(1) Time Interval j	(2) Time t (hrs)	(3) Precip. Input, Ij (in)	(4) Stream-flow Q (cfs)	(5) Stream-flow Qj (in)	(6) Increm. Stor. ΔSj (in)	(7) Storage Sj (in)
	0		25			0
1	0.5	0.18	27	0.003	0.177	0.177
2	1	0.42	38	0.004	0.416	0.594
3	1.5	0.21	109	0.008	0.202	0.795
4	2	0.16	310	0.023	0.137	0.932
5	2.5		655	0.053	-0.053	0.879
6	3		949	0.088	-0.088	0.791
7	3.5		1060	0.111	-0.111	0.680
8	4		968	0.112	-0.112	0.568
9	4.5		1030	0.110	-0.110	0.458
10	5		826	0.102	-0.102	0.356
11	5.5		655	0.082	-0.082	0.274
12	6		466	0.062	-0.062	0.213
13	6.5		321	0.043	-0.043	0.169
14	7		227	0.030	-0.030	0.139
15	7.5		175	0.022	-0.022	0.117
16	8		160	0.018	-0.018	0.098
Total		0.97		0.872		

Table 2.3.2. Time distribution of storage for the May 12, 1980 storm on the Shoal Creek watershed calculated using the discrete-time continuity equation.

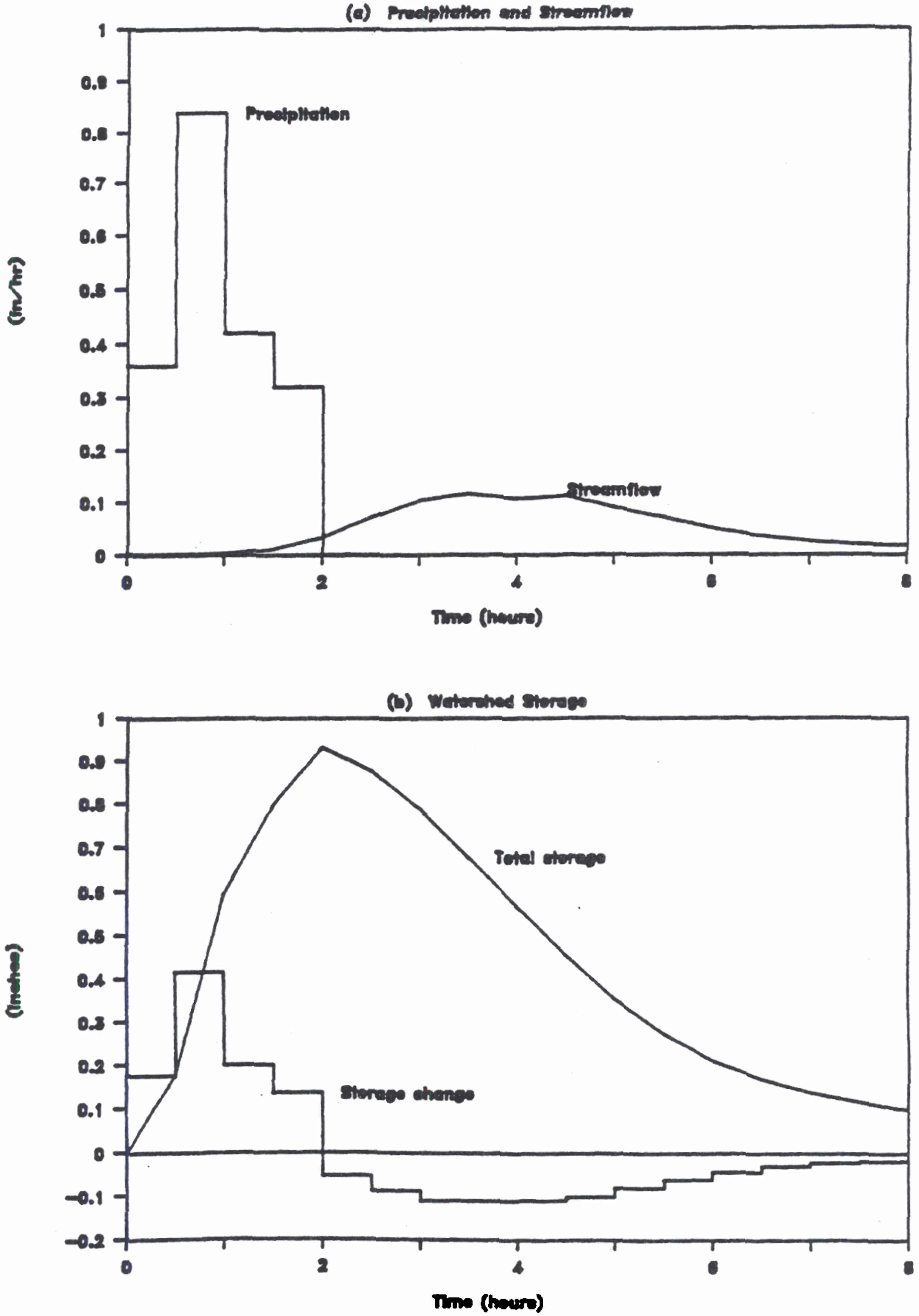

Fig. 2.3.2. Time distribution of precipitation and streamflow (divided by the watershed area). Storm of May 12, 1980 on Shoal Creek, Austin, Texas. at Northwest Park.

$= S_0 = 0.01$ for uniform flow

$$v = (1.49/n)\ R^{2/3}\ S_f^{1/2} = (1.49/0.035) \times 2.75^{2/3} \times 0.01^{1/2}\ \text{ft/s}$$
$$= 8.35\ \text{ft/s}$$

The flow rate is $Q = v\ A = 8.35 \times 327$ cfs $= 2{,}731$ cfs. The criterion for fully turbulent flow is calculated from Eq. (2.5.9a) in the textbook

$$n^6\ (RS_f)^{1/2} = 0.035^6\ (2.75 \times 0.01)^{1/2} = 3.05 \times 10^{-10}$$

which is larger than 1.9×10^{-13} so the criterion is satisfied and Manning's equation is applicable.

2.5.4.

The area of the channel is $A = (30 + 3 \times 1) \times 1\ m^2 = 33\ m^2$. The wetted perimeter is

$$P = 30 + 2 \times 1 \times (1^2 + 3^2)^{1/2}\ m = 33.16\ m$$

so the hydraulic radius is $R = A/P = 33/33.16 = 0.995$ m.

The flow velocity is given by Manning's equation with $n = 0.035$ and $S_f = S_0 = 0.01$ for uniform flow

$$v = (1/n)\ R^{2/3}\ S_f^{1/2} = (1/0.035) \times 0.995^{2/3} \times 0.01^{1/2}\ m/s = 2.85\ m/s$$

The flow rate is $Q = v\ A = 2.85 \times 33\ m^3/s = 94\ m^3/s$. The criterion for fully turbulent flow is calculated from Eq. (2.5.9b) in the textbook

$$n^6\ (RS_f)^{1/2} = 0.035^6\ (0.995 \times 0.01)^{1/2} = 1.83 \times 10^{-10}$$

which is larger than 1.1×10^{-13} so the criterion is satisfied and Manning's equation is applicable.

2.5.5.

Shallow flow over a parking lot is equivalent to flow in an infinite width channel; in this case, the flow can be analyzed in a portion of channel of unit width. For a flow depth of 1 in, the area of this channel portion is $A = 1/12 \times 1\ ft^2 = 0.083\ ft^2$. The wetted perimeter corresponds only to the channel bottom, $P = 1$ ft, so the hydraulic radius is $R = A/P = 0.083/1$ ft $= 0.083$ ft, equal to the flow depth.

[illegible]e flow velocity is given by Manning's equation with $n = 0.015$ and $S_f = S_0$ [illegible]0.5 % $= 0.005$ for uniform flow

$$[illegible] = (1.49/n)\ R^{2/3}\ S_f^{1/2} = (1.49/0.015) \times 0.083^{2/3} \times 0.005^{1/2}\ \text{ft/s} =$$

$= 1.34$ ft/s

The flow rate per unit width of channel is $Q = v\,A = 1.34 \times 0.083$ cfs/ft = 2,731 cfs/ft. The criterion for fully turbulent flow is calculated from Eq. (2.5.9a) in the textbook

$$n^6\,(RS_f)^{1/2} = 0.015^6\,(0.083 \times 0.005)^{1/2} = 2.32 \times 10^{-13}$$

which is larger than 1.9×10^{-13} so the criterion is satisfied and Manning's equation is applicable.

2.5.6.

For a flow depth of 1 cm, the area of a unit width channel portion is $A = 0.01 \times 1\ m^2 = 0.01\ m^2$. The wetted perimeter corresponds only to the channel bottom, $P = 1$ m, so the hydraulic radius is $R = A/P = 0.01/1$ m = 0.01 m, equal to the flow depth.

To check whether Manning's equation is applicable, the criterion for fully turbulent flow is calculated from Eq. (2.5.9b) in the textbook

$$n^6\,(RS_f)^{1/2} = 0.015^6\,(0.01 \times 0.005)^{1/2} = 0.81 \times 10^{-13}$$

which is smaller than 1.1×10^{-13} so the criterion is not satisfied (although the flow is very close to fully turbulent); Manning's equation is not applicable and the Darcy-Weisbach equation should be used instead.

The relative roughness ε is computed using Eq. (2.5.15) from the textbook, as a function of the hydraulic radius $R = 0.01$ m and the Manning's coefficient $n = 0.015$, with the factor $\Phi = 1$ for SI units. Then

$$\varepsilon = 3 \times 10^{-\Phi R^{1/6}/[4n(2g)^{1/2}]} = 0.054$$

The flow velocity v and the Darcy-Weisbach friction factor f have to be calculated in an iterative fashion. For a given value of v, the Reynolds number Re can be computed using Eq. (2.5.10) from the textbook. For a value of the kinematic viscosity $\nu = 10^{-6}\ m^2/s$, we have

$$Re = 4vR/\nu = 4v \times 0.01/10^{-6} = 40{,}000\ v \qquad (2.5.6\text{-}1)$$

The friction factor f can then be updated using the modified Moody diagram in Fig. 2.5.1 of the textbook or the Colebrook-White equation (Eq. 2.5.13 from the textbook). In this case the latter method will be preferred since it is more accurate and easier to implement in a computer code. For flow in the transition zone,

$$1/\sqrt{f} = -2\,\log_{10}[\varepsilon/3 + 2.5/Re \times 1/\sqrt{f}] =$$

$$= -2\,\log_{10}[0.054/3 + 2.5/Re \times 1/\sqrt{f}] \qquad (2.5.6\text{-}2)$$

3.2.1.

The saturated vapor pressure at $T = 25\ ^{o}C$ is given by Equation (3.2.9) of the textbook

$$e_s = 611\ \exp[17.27T/(237.3+T)] = 611\ \exp[17.27 \times 25/(237.3 + 25)]$$

so $e_s = 2598$ Pa. The actual vapor pressure, e, is calculated by the same method substituting the dew point temperature $T_d = 20\ ^{o}C$ for T

$$e = 611\ \exp[17.27T_d/(237.3+T_d)] = 611\ \exp[17.27 \times 20/(237.3 + 20)]$$

so e = 1984 Pa.

The relative humidity, from Equation (3.2.11) of the textbook, is

$$R_h = e/e_s = 1984/2598 = 0.76$$

and the specific humidity is given by Equation (3.2.6) of the texbook, with air pressure $p = 101.1 \times 10^3$ Pa

$$q_v = 0.622\ e/p = 0.622 \times 1984/(101.1 \times 10^3) = 0.012$$

The gas constant for air, R_a, is given by Equation (3.2.8) of the texbook

$$R_a = 287\ (1 + 0.608\ q_v) = 287\ (1 + 0.608 \times 0.012) = 289\ J/(kg.^{o}K)$$

and the air density is calculated from the ideal gas law (Equation 3.2.7 of the textbook) with temperature $T = 273 + 25 = 298\ ^{o}K$, so that

$$\rho_a = p/(R_aT) = 101.1 \times 10^3/(289 \times 298) = 1.17\ kg/m^3$$

3.2.2.

The temperature T_2 at elevation $z_2 = 1500$ m is given by Equation (3.2.16) of the textbook with $T_1 = 25^{o}C$, $z_1 = 0$, and lapse rate $\alpha = 9\ ^{o}C/km$, so that

$$T_2 = T_1 - \alpha\ (z_2 - z_1) = 25 - 9 \times 10^{-3}\ (1500 - 0) = 11.5\ ^{o}C = 284.5\ ^{o}K$$

This temperature is below the dew point temperature for surface conditions, $T_d = 20\ ^{o}C$, so the vapor pressure at 1500 m elevation corresponds to the saturated vapor pressure given by Equation (3.2.9) of the textbook

$$e_s = 611\ \exp[17.27T/(237.3+T)] = 611\ \exp[17.27 \times 11.5/(237.3 + 11.5)]$$

so $e = e_s = 1357$ Pa.

The air pressure p_2 at $z_2 = 1500$ m is given by Equation (3.2.15) from the textbook, with $p_1 = 101.1$ kPa, so that

$$p_2 = p_1\ (T_2/T_1)^{g/(\alpha R_d)} = 101.1 \times (284.5/298)^{9.81/(0.009 \times 289)} = 84.9\ kPa$$

where the gas constant R_d = 289 J/kg°K has been taken for surface conditions, since its variation with specific humidity is small (this assumption can be checked later).

The specific humidity q_v is given by Equation (3.2.6) of the textbook, with air pressure p = 84.9 x 10³ Pa

$$q_v = 0.622\ e/p = 0.622 \times 1357/(84.9 \times 10^3) = 0.010$$

We can now check the value of the gas constant used before. For specific humidity q_v = 0.01

$$R_d = 287\ (1 + 0.608\ q_v) = 287\ (1 + 0.608 \times 0.010) = 288.7\ J/(kg.^oK)$$

which is very close to R_d = 289 J/(kg.°K) assumed initially so no correction is necessary.

The air density is given by the ideal gas law (Equation 3.2.7 of the textbook)

$$\rho_a = p/(R_a T) = 84.77 \times 10^3/(289 \times 284.5) = 1.03\ kg/m^3$$

3.2.3.

The saturated vapor pressure at T = 15 °C is given by Equation (3.2.9) of the textbook

$$e_s = 611\ \exp[17.27T/(237.3+T)] = 611\ \exp[17.27 \times 15/(237.3 + 15)]$$

so e_s = 1706 Pa. The actual vapor pressure is, according to Equation (3.2.11) from the textbook, with relative humidity R_h = 0.35,

$$e = e_s R_h = 1706 \times 0.35 = 597\ Pa$$

The specific humidity is, with air pressure p = 101.3 kPa,

$$q_v = 0.622\ e/p = 0.622 \times 597/(101.3 \times 10^3) = 0.0037$$

The gas constant for air, R_a, is given by Equation (3.2.8) of the textbook

$$R_a = 287\ (1 + 0.608\ q_v) = 287\ (1 + 0.608 \times 0.0037) = 287.6\ J/(kg.^oK)$$

and the air density is given by the ideal gas law (Equation 3.2.7 of the textbook) with temperature T = 273 + 15 = 288 °K, so that

$$\rho_a = p/(R_a T) = 101.3 \times 10^3/(287.6 \times 288) = 1.22\ kg/m^3$$

Surface Temperature (^oC)	Precipitable Water (mm)
0	9.95
10	20.77
20	41.01
30	77.03
40	138.39

Table 3.2.6. Variation of precipitable water depth with surface temperature.

3.3.2.

From Equation (3.3.4) of the textbook, the terminal velocity v_t of a falling raindrop of diameter D = 2 mm = 0.002 m, with C_d = 0.517 from Table 3.3.1 of the textbook, is

$$v_t = [4gD/(3C_d)\ (\rho_w/\rho_a - 1)]^{1/2}$$

$$= [4 \times 9.81 \times 0.002/(3 \times 0.517)\ (998/1.20 - 1)]^{1/2} = 6.48 \text{ m/s}$$

this is the drop velocity relative to the surrounding air. If the air is rising with velocity v_a = - 5 m/s, the absolute velocity of the drop is

$$v_{rel} = v_t + v_a = 6.48 - 5 = 1.48 \text{ m/s}$$

and the drop is falling.

For drop diameter D = 0.2 mm = 0.0002 m, with C_d = 4.2, the terminal velocity can be calculated similarly, resulting v_t = 0.72 m/s and

$$v_{rel} = v_t + v_a = 0.72 - 5 = - 4.28 \text{ m/s}$$

and the drop is rising.

3.3.3.

Three vertical forces act on a falling raindrop: a gravity force F_w due to its weight, a buoyancy force F_b due to the air displaced by the drop and a drag force F_d caused by the friction between the drop and the surrounding air. If v is the vertical fall velocity of the drop of mass m, from Newton's law

$$m\ dv/dt = F_w - F_b - F_d \qquad (3.3.3\text{-}1)$$

The maximum rainfall depth recorded in 10, 20 and 30 min. intervals is found by computing the running totals in Cols. (4), (5) and (6) of Table 3.4.3, respectively, through the storm, then selecting the maximum value of the corresponding series, as shown in Table 3.4.3. For example, for a 30 minute time interval, the maximum 30 minute depth is 1.16 in, recorded between 5 and 35 min. The rainfall intensity (depth divided by time) corresponding to this depth is 1.16 in/0.5 hr = 2.32 in/hr. This value is less than 60 % of the 30 min intensity experienced at gage 1-Bee for the same storm (see Table 3.4.1 from the textbook).

<u>3.4.4.</u>

The computations follow those in Problem 3.4.3. and are summarized in Table 3.4.4. The storm hyetograph is shown in Fig. 3.4.4(a). The cumulative rainfall hyetograph, or rainfall mass curve, is obtained in Col. (3) of Table 3.4.4, and plotted in Fig. 3.4.4(b).

The maximum rainfall depth or intensity (depth divided by time) recorded in 10, 30, 60, 90 and 120 min. intervals is found by computing the running totals in Cols. (4)-(8) of the table, through the storm, then selecting the maximum value of the corresponding series, as shown in Table 3.4.4. These intensities are about 60-70 % of the intensities observed at gage 1-Bee for the same storm (see Table 3.4.1 from the textbook), which experienced more severe rainfall.

<u>3.4.5.</u>

(a) Arithmetic mean method. Raingages numbers 1, 4, 6 and 8 are located outside the watershed and will not be considered in the computation of the arithmetic mean. The areal average rainfall is, therefore,

$$\bar{P} = (P_2 + P_3 + P_5 + P_7 + P_9)/5$$

$$= (59 + 41 + 105 + 60 + 81)/5 = 69.2 \text{ mm}$$

(b) Thiessen method. The Thiessen polygon network is shown in Fig. 3.4.5-1. The areas assigned to each station are shown in Col. (3) of Table 3.4.5-1. The watershed area is $A = 125 \text{ km}^2$ and the areal average rainfall is given by Eq. (3.4.1) of the textbook

$$\bar{P} = 1/A \sum_{j=1}^{J} A_j P_j = 8746/125 = 70.0 \text{ mm}$$

(c) Isohyetal method. The isohyetal map is shown in Fig. 3.4.5-2. The average rainfall is found by adding the weighted rainfall values in Col. (4) of Table 3.4.5-2,

$$\bar{P} = 1/A \sum_{j=1}^{J} A_j P_j = 8638/125 = 69.1 \text{ mm}$$

(1) Time	(2) Rainfall	(3) Cumulative Rainfall	(4)	(5)	(6)	(7)	(8)
			Runing totals (in)				
(min)	(in)	(in)	10 min.	30 min.	60 min.	90 min.	120 min.
0	-	0.000					
5	0.009	0.009					
10	0.000	0.009	0.009				
15	0.030	0.039	0.030				
20	0.130	0.169	0.160				
25	0.100	0.269	0.230				
30	0.130	0.399	0.230	0.399			
35	0.210	0.609	0.340	0.600			
40	0.370	0.979	0.580	0.970			
45	0.220	1.199	0.590	1.160			
50	0.300	1.499	0.520	1.330			
55	0.200	1.699	0.500	1.430			
60	0.100	1.799	0.300	1.400	1.799		
65	0.130	1.929	0.230	1.320	1.920		
70	0.140	2.069	0.270	1.090	2.060		
75	0.120	2.189	0.260	0.990	2.150		
80	0.160	2.349	0.280	0.850	2.180		
85	0.140	2.489	0.300	0.790	2.220		
90	0.180	2.669	0.320	0.870	2.270	2.669	
95	0.250	2.919	0.430	0.990	2.310	2.910	
100	0.480	3.399	0.730	1.330	2.420	3.390	
105	0.400	3.799	0.880	1.610	2.600	3.760	
110	0.390	4.189	0.790	1.840	2.690	4.020	
115	0.240	4.429	0.630	1.940	2.730	4.160	
120	0.410	4.839	0.650	2.170	3.040	4.440	4.839
125	0.440	5.279	0.850	2.360	3.350	4.670	5.270
130	0.270	5.549	0.710	2.150	3.480	4.570	5.540
135	0.170	5.719	0.440	1.920	3.530	4.520	5.680
140	0.170	5.889	0.340	1.700	3.540	4.390	5.720
145	0.140	6.029	0.310	1.600	3.540	4.330	5.760
150	0.100	6.129	0.240	1.290	3.460	4.330	5.730
Max. Depth	0.480		0.880	2.360	3.540	4.670	5.760 (in)
Max. Int.	5.760		5.280	4.720	3.540	3.113	2.880 (in/hr)

Table 3.4.4. Computation of rainfall depth and intensity

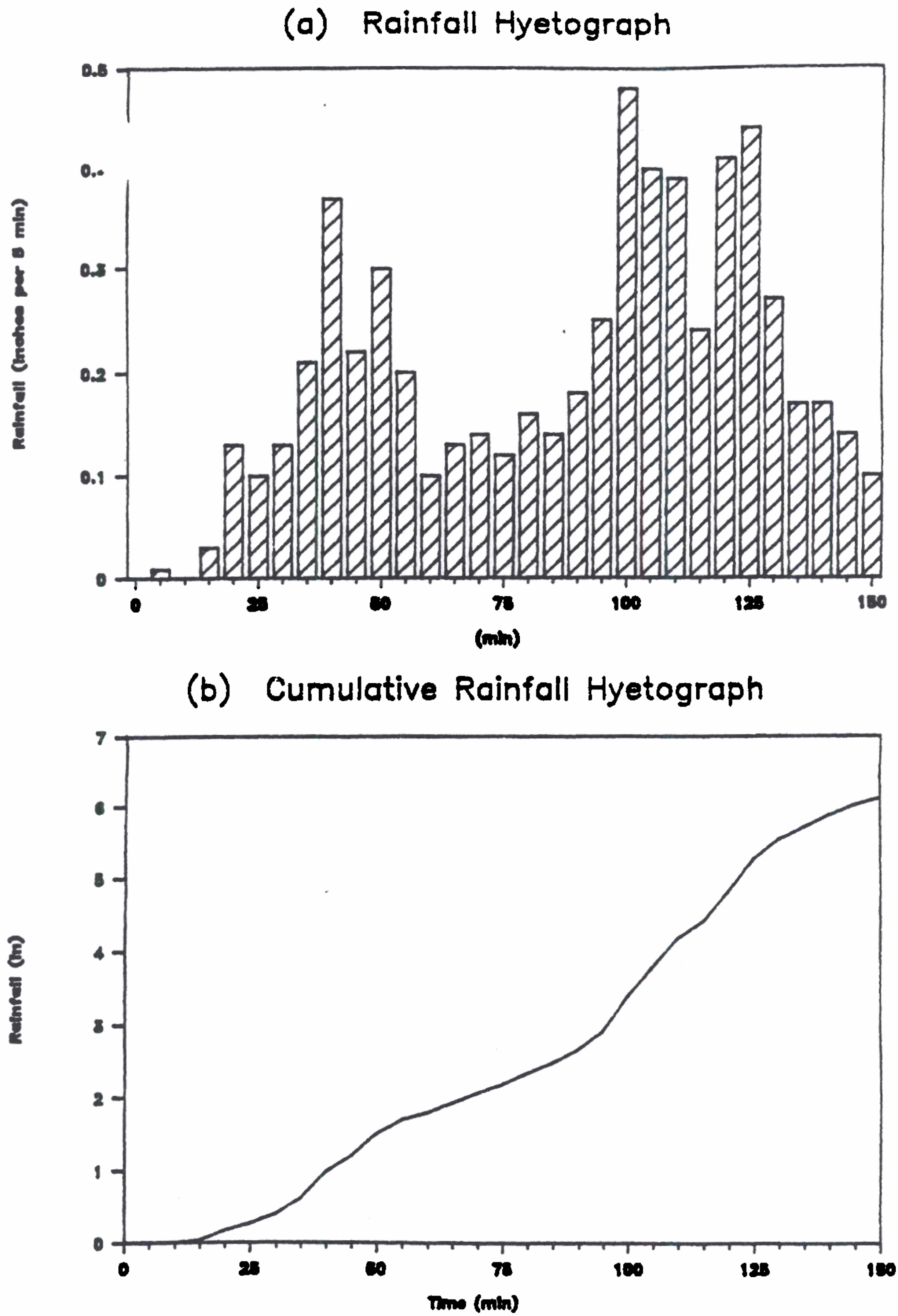

Fig. 3.4.4. Rainfall hyetograph (in 5 min. increments) and cumulative rainfall hyetograph at gage 1-WLN for the storm of May 24-25, 1981 in Austin, Texas.

CHAPTER 4. SUBSURFACE WATER

4.1.1.

Equation (4.1.10) of the textbook may be written for the average moisture flux q between measurement points 1 and 2 as

$$q = -K\,S_f = -K\,(h_1 - h_2)/(z_1 - z_2)$$

where $z_1 = -80$ cm, $z_2 = -100$ cm and h_1, h_2 can be measured from Fig. 4.1.5.(b) of the textbook. The suction head ψ at each depth may be found from Eq. (4.1.9) of the textbook as $\psi = h - z$. For example, for week 1 at $z_1 = -80$ cm, $h_1 = -145$ cm so $\psi_1 = h_1 - z_1 = -145 - (-80) = -65$ cm, and, similarly, $\psi_2 = -160 - (-100) = -60$ cm as shown in Cols. (4) and (5) of Table 4.1.1.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Total head		Suction head			Hydraul. conduc.	Hydraul. gradient	Moisture flux
Week	h_1 (cm)	h_2 (cm)	ψ_1 (cm)	ψ_2 (cm)	ψ (cm)	K (cm/d)	S_f	q (cm/d)
1	-145	-160	-65	-60	-62.5	0.041	0.75	-0.030
2	-165	-180	-85	-80	-82.5	0.023	0.75	-0.017
3	-130	-150	-50	-50	-50.0	0.065	1.00	-0.065
4	-140	-170	-60	-70	-65.0	0.037	1.50	-0.056
5	-125	-160	-45	-60	-52.5	0.059	1.75	-0.103
6	-105	-130	-25	-30	-27.5	0.230	1.25	-0.287
7	-135	-150	-55	-50	-52.5	0.059	0.75	-0.044
8	-150	-170	-70	-70	-70.0	0.032	1.00	-0.032
9	-165	-190	-85	-90	-87.5	0.020	1.25	-0.025
10	-190	-210	-110	-110	-110.0	0.012	1.00	-0.012
11	-220	-230	-140	-130	-135.0	0.008	0.50	-0.004
12	-230	-250	-150	-150	-150.0	0.006	1.00	-0.006
13	-255	-265	-175	-165	-170.0	0.005	0.50	-0.002
14	-280	-285	-200	-185	-192.5	0.004	0.25	-0.001

Table 4.1.1. Soil moisture flux between 0.8 and 1.0 m at Deep Dean, Sussex, England.

The hydraulic conductivity varies with ψ, so an approximate average value may be found corresponding to the average of the ψ values at $z_1 = 80$ cm and $z_2 = 100$ cm, $\psi = [-65 + (-60)]/2 = -62.5$ cm as shown in Col. (6), and the corresponding hydraulic conductivity is $K = 250(-\psi)^{-2.11} = 250 \times 62.5^{-2.11} = 0.041$ cm/d in Col. (7). In week 1, the hydraulic gradient in Col. (8) is $S_f = (h_1 - h_2)/(z_1 - z_2) = [-145 - (-160)]/[-80 - (-100)] = 0.75$, so the soil moisture flux in week 1 is

$$q = -K\, S_f = -0.041 \times 0.75 = -0.030 \text{ cm/d}$$

as shown in Col. (9) of Table 4.1.1. The flux q is negative because moisture is flowing downwards in the soil.

4.1.2.

The moisture flux between 1.0 and 1.2 m depth may be computed following the method outlined in Problem 4.1.1. For example, for week 1 and depth $z_1 = -100$ cm, $h_1 = -160$ cm so $\psi_1 = h_1 - z_1 = -160 -(-100) = -60$ cm, and, similarly, $\psi_2 = -190 -(-120) = -70$ cm as shown in Cols. (4) and (5) of Table 4.1.2. The hydraulic conductivity varies with ψ, so an approximate average value may be found corresponding to the average of the ψ values at $z_1 = 100$ cm and $z_2 = 120$ cm; $\psi = [-60 + (-70)]/2 = -65$ cm as shown in Col. (6), and the corresponding hydraulic conductivity is $K = 250(-\psi)^{-2.11} = 250 \times 65^{-2.11} = 0.037$ cm/d in Col. (7). In week 1, the hydraulic gradient in Col. (8) of Table 4.1.2 is $S_f = (h_1-h_2)/(z_1-z_2) = [-190 -(-160)]/[-120 -(-100)] = 1.5$, so the soil moisture flux in week 1 is

$$q = -K\, S_f = -0.037 \times 1.5 = -0.056 \text{ cm/d}$$

as shown in Col. (9) of Table 4.1.2. The flux q is negative because moisture is flowing downwards in the soil.

4.1.3.

The moisture fluxes may be computed between different depths following the method outlined in Problems 4.1.1 and 4.1.2. Table 4.1.3-1 shows the hydraulic heads measured from Fig. 4.1.5(b) of the texbook at different depths. The resulting fluxes are summarized in Table 4.1.3-2. The values of the flux at 3m depth are very high because the soil is saturated most of the time and the relationship between hydraulic conductivity and suction head is no longer applicable; the flow is driven by gravity alone.

Fig. 4.1.3 shows curves of moisture flux versus time between different depths in the soil. It is clear from the figure that rainfall drives the infiltration process. The response of the soil to precipitation is very rapid in the upper layers of the soil. For example, between 0.4 and 0.8m, infiltration increases abruptly after storms followed by a decay later. As we move deeper into the soil, the response is more damped and a single storm is no longer influential to the same degree; longer rainy periods are required to increase the moisture flux, as shown by the 1.5 to 1.8m profile.

During the summer months, suction heads are very high throughout the soil profile. The effect of precipitation in moisture flux is negligible, except in the upper sections of the soil. Between 0.4 and 0.8m, the direction of flow is eventually reversed as moisture moves upwards to leave the soil as evapotranspiration. The calculations shown here are approximate as they do not account for the variation of soil properties with depth i.e. the same relationship between K and ψ is used for all depths in the soil.

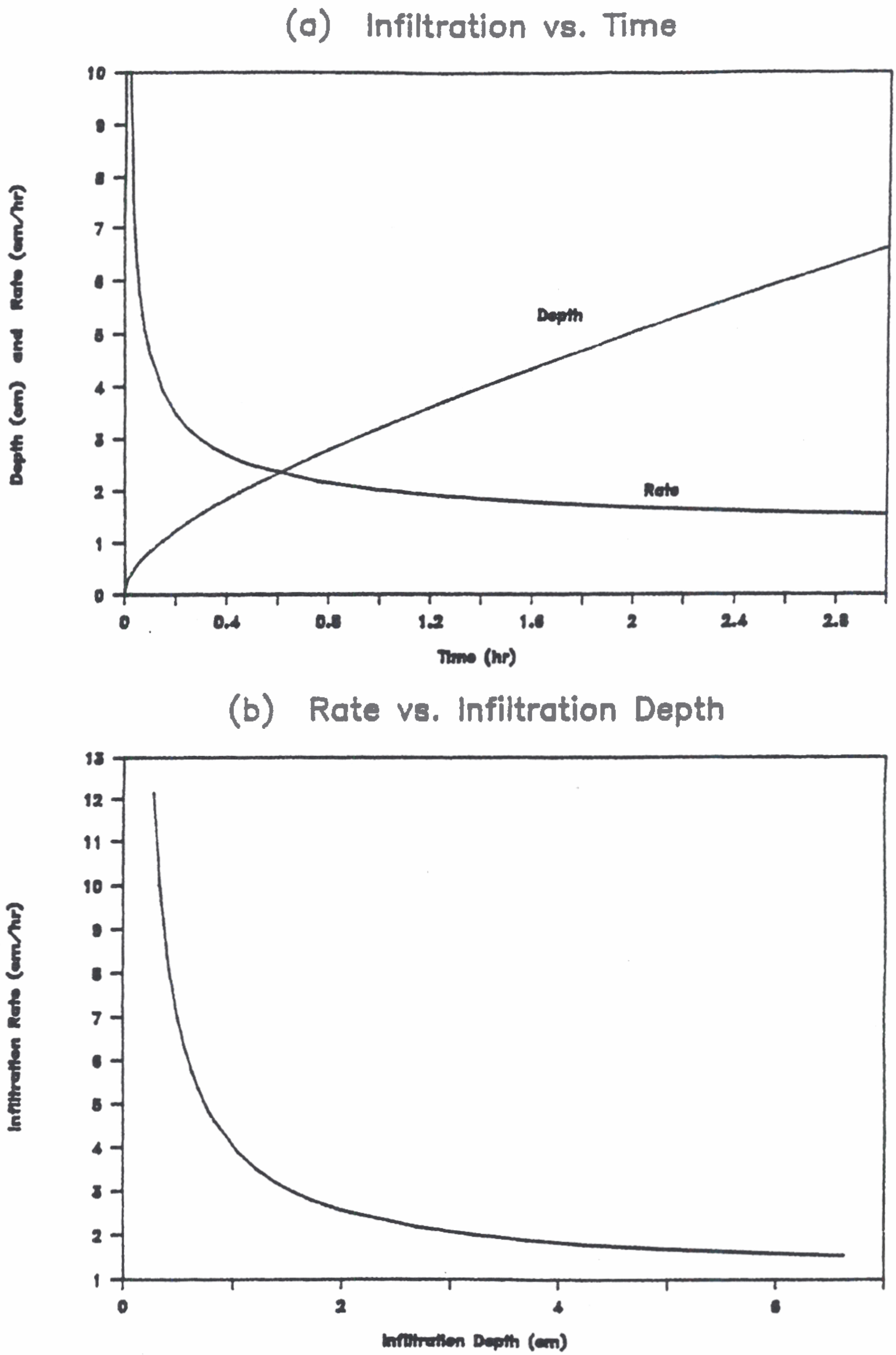

Fig. 4.3.2. Infiltration rate and cumulative infiltration depth computed by the Green-Ampt method.

4.3.2.

The infiltration rate f and the cumulative infiltration F at time t = 0, 0.5, 1, 1.5, 2, 2.5 and 3 hrs may be computed following the method outlined in Problem 4.3.1. The cumulative infiltration is first computed using Eq. (4.3.8) of the textbook

$$F(t) = kt + \psi\Delta\theta \, \ln[1 + F/(\psi\Delta\theta)]$$

$$= 1.09\ t + 2.72\ \ln[1 + 3/2.72]$$

which may be solved by successive approximation for each value of t. The infiltration rate is then computed using Eq. (4.3.7) of the textbook

$$f(t) = k\ (\psi\Delta\theta/F + 1) = 1.09\ (2.72/F + 1)$$

The results are listed in Table 4.3.2. Fig. 4.3.2(a) shows a plot of the infiltration rate and cumulative infiltration versus time. Fig. 4.3.2(b) shows the variation of the infiltration rate f with the infiltration depth F.

	Infiltration	
Time t (hr)	Rate f (cm/hr)	Depth F (cm)
0.0	-	0.00
0.5	2.51	2.10
1.0	2.01	3.21
1.5	1.80	4.16
2.0	1.68	5.03
2.5	1.60	5.85
3.0	1.54	6.63

Table 4.3.2. Infiltration computedby theGreen-Ampt method.

4.3.3.

From Table 4.4.1 from the textbook, for a silty clay soil, $\theta_e = 0.423$, $\psi = 29.22$ cm and $k = 0.05$ cm/hr. The initial effective saturation is $S_e = 0.2$, so $\Delta\theta = (1 - S_e)\theta_e = (1 - 0.20)0.423 = 0.338$, and $\psi\Delta\theta = 29.22 \times 0.338 = 9.888$ cm. Assuming continuous ponding, the cumulative infiltration F is found by successive substitution in Eq. (4.3.8) from the textbook

$$F = kt + \psi\Delta\theta \, \ell n[1 + F/(\psi\Delta\theta)]$$

$$= 0.05\,t + 9.888 \, \ell n[1 + F/9.888]$$

For example, at time t = 0.1 hr, the cumulative infiltration converges to a final value F = 0.29. The infiltration rate f is then computed using Eq. (4.3.7) of the textbook

$$f = k(1 + \psi\Delta\theta/F) = 0.05(1 + 9.89/F)$$

For example, at time t= 0.1 hr, f = 0.05(1 + 9.89/0.29) = 1.78 cm/hr. The infiltration rate and the cumulative infiltration are similarly computed between 0 and 6 hours at 0.1 hr intervals; the results are shown in Table 4.3.3.

4.3.4.

The cumulative infiltration may be computed by the method outlined in Problem 4.3.1, with $\theta_e = 0.423$, $\psi = 29.22$ cm and $k = 0.05$ cm/hr from Table 4.3.1 of the textbook. The cumulative infiltration F after t = 1 hr is shown in Col. (4) of Table 4.3.4 and the variation of F with the initial value of S_e is plotted in Fig. 4.3.4. For example, for effective saturation $S_e = 0$, $\Delta\theta = (1 - S_e)\theta_e = (1 - 0)0.423 = 0.423$, and $\psi\Delta\theta = 29.22 \times 0.423 = 12.36$ cm as shown in Col. (2) of Table 4.3.4. The infiltrated depth is computed solving Eq. (4.3.8) of the textbook by the method of successive approximation. For example, for $S_e = 0$ the cumulative infiltration after 1 hr converges to a final value F = 1.14 cm, as shown in Col. (3) of the table.

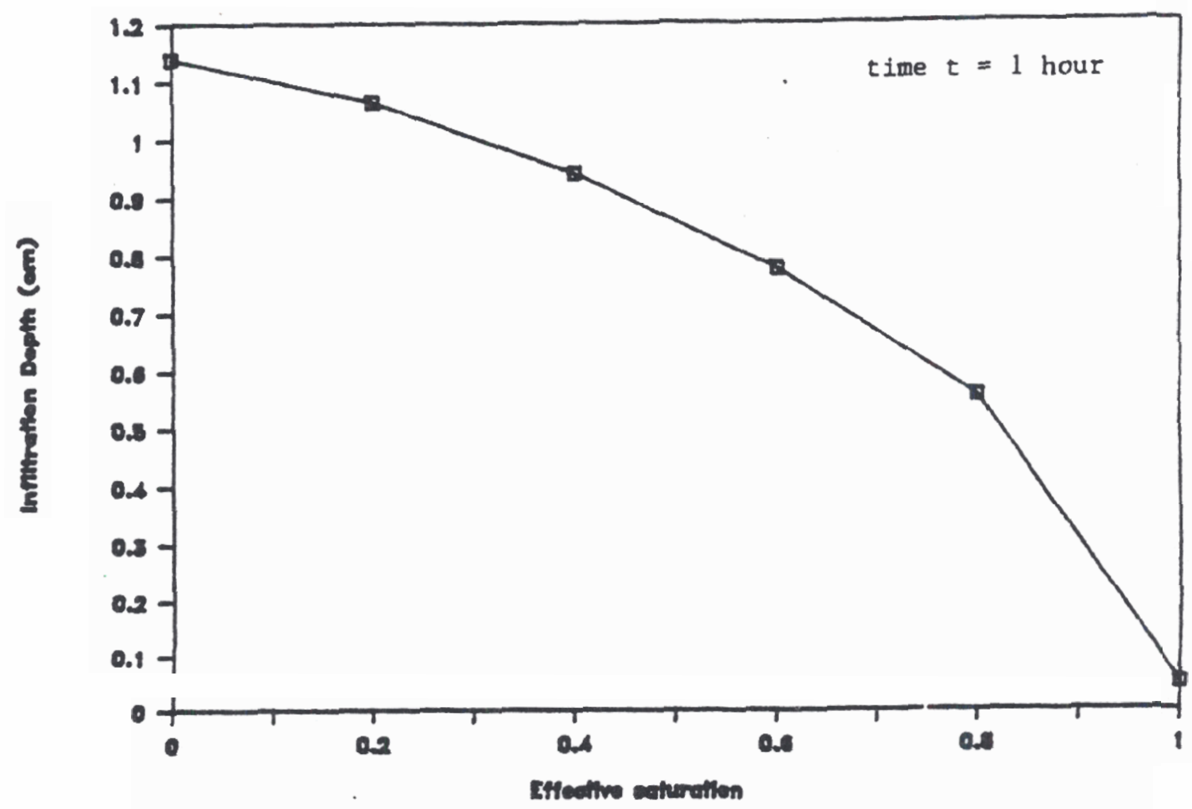

Fig. 4.3.4. Cumulative infiltration depth, computed by the Green-Ampt method, as a function of the initial effective saturation of the soil.

(1) Effective Saturation Se	(2) $\psi \Delta\theta$ (cm)	(3) Cumulative Infiltration F (cm)
0.0	12.360	1.14
0.2	9.888	1.07
0.4	7.416	0.94
0.6	4.944	0.78
0.8	2.472	0.56
1.0	0.000	0.05

Table 4.3.4. Cumulative infiltration after 1 hr.

4.3.5.

The analysis follows the derivation in Chapter 4.3 of the textbook for a single-layer Green-Ampt equation. A control volume is defined, as in Fig. 4.3.2 of the textbook, around a vertical column of soil with unit crossectional area between the surface and depth $L = H_1 + L_2$, where H_1 is the upper layer thickness and L_2 is the depth of the wet front in the lower layer. The continuity equation gives Eq. (4.3.13) of the textbook

$$F = \int_0^L \ddot{\Delta\theta}\, dz = \ddot{\Delta\theta}_1 H_1 + \ddot{\Delta\theta}_2 L_2$$

and differentiating this equation gives

$$f = dF/dt = \ddot{\Delta\theta}_2\, dL_2/dt \qquad (4.3.5\text{-}1)$$

The momentum equation (Darcy's law) applied between the soil surface and the wetting front yields, with ponded depth h_0 on the surface,

$$f = K\,[h_0 - (-\psi_2 - H_1 - L_2)]/(H_1 + L_2)$$

where $K = [H_1/K_1 + L_2/K_2]^{-1}$ is the equivalent hydraulic conductivity for flow across two layers of thicknesses H_1 and L_2, and conductivities K_1 and K_2. Substituting K into the previous equation yields, neglecting h_0, Eq. (4.3.12) of the textbook.

29.22 cm, while, for t = 1 hr, the term it - F is less than it = 1 cm, much smaller than ψ. An approximate solution may then be obtained by assuming h_0 is constant. In this case, Eq. (4.4.7-2) may be written as

$$dF/dt = K[\Delta\theta(h_0 + \psi) + F]/F$$

so that

$$dF\ F/[\Delta\theta(h_0 + \psi) + F] = K\ dt$$

which can be integrated between t_p and t following the method outlined in Section 4.3 of the textbook to yield

$$F = F_p + \Delta\theta(h_0 + \psi)\ \ell n\{[\Delta\theta(h_0 + \psi) + F]/[\Delta\theta(h_0 + \psi) + F_p] + K(t - t_p)$$

The values of F and h_0 may be approximated using an iterative procedure as follows. For h_0 = 0, F = 0.88 cm at time t = 1 hr, from Problem 4.4.6. Since all excess rainfall constitutes ponded water, this yields a new value of h_0 = it - F = 1 x 1 - 0.88 = 0.12 cm. Then, the previous equation may be solved by the method of successive substitution with t = 1 hr, K = 0.05 cm/hr, $\psi\Delta\theta$ = - 9.89 cm, t_p = 0.52 hr, F_p = 0.52 and h_0 = 0.12 cm. The new value of F is F = 0.88 cm and h_0 = it - F = 1 x 1 - 0.88 = 0.12 cm. This shows that the depth h_0 is in effect negligible at time t = 1hr.

4.4.8.

For a clay loam soil, θ_e = 0.309, ψ = 20.88 cm and K = 0.1 cm/hr, from Table 4.3.1 of the textbook. The ponding time is, from Table 4.4.1 of the textbook,

$$t_p = K\psi\Delta\theta/[i(i-K)] = K\psi(1-S_e)\theta_e/[i(i-K)]$$

so that the initial effective saturation is, with t_p = 5 min = 0.083 hr and rainfall intensity i = 2 cm/hr,

$$S_e = 1 - t_p i(i-K)/(K\psi\theta_e)$$

$$= 1 - 0.083 \times 2\ (2 - 0.1)/(0.1 \times 20.88 \times 0.309) = 0.509$$

Then, for a sandy loam soil, θ_e = 0.412, ψ = 11.01 cm and K = 1.09 cm/hr, so the ponding time under rainfall intensity i = 2 cm/hr is

$$t_p = K\psi(1-S_e)\theta_e/[i(i-K)] = 1.09 \times 11.01\ (1-0.509)\ 0.412/[2(2-1.09)]$$

$$= 1.33\ hr$$

4.4.9.

The ponding time for the Philip's equation is given in Table 4.4.1 of the textbook. For S = 5 cm $hr^{-1/2}$, K = 0.4 cm/hr and rainfall intensity i = 6 cm/hr, this gives