part III

Psychrometrics

chapter 7

Thermodynamic Properties of Moist Air

.1 ATMOSPHERIC AIR

The earth's atmosphere is a mixture of gases, including nitrogen, oxygen, argon, carbon dioxide, water vapor, and traces of other gases. Atmospheric air usually contains various particulate matter. Additional vapors are often present. Dust particles and condensible vapors such as water vapor are usually concentrated in the atmosphere only within a few thousand feet of the earth's surface. Above an altitude of about 20,000 ft (6100 m), the atmosphere consists essentially of dry air.

Barometric pressure is the force per unit area due to the weight of the atmosphere. Standard sea-level pressure is 14.696 psia (101.325 kPa). As one proceeds vertically into the atmosphere, pressure decreases. Temperature also decreases until the tropopause is reached.

The U.S. Standard Atmosphere provides a reference standard with respect to barometric pressure for an air conditioning engineer. The *ASHRAE Handbook of Fundamentals* [1] has given the following information which forms the definition of the U.S. Standard Atmosphere:

1. The atmosphere consists of dry air which behaves as a perfect gas; thus
$$Pv = RT \tag{7.1}$$
2. Gravity is constant at 32.174 ft/sec^2 (9.80665 m/s^2)
3. At sea level, pressure is 29.921 in. Hg and temperature is 59 °F (101.325 kPa, 18 °C).
4. Temperature t decreases linearly with altitude z up to the lower limit of the isothermal atmosphere according to the relation
$$t = t_0 - 0.00356z \tag{7.2}$$
with t and t_0 in °F and z in ft. Or
$$t = t_0 - 0.0065z$$
with t and t_0 in °C and z in meters.

The isothermal atmosphere, −69.7 °F (−56.5 °C) begins at 36,152 ft (11,000 m). Equations (7.1) and (7.2) can be combined to give the relation

$$\frac{P}{P_0} = \left(1 - \frac{Az}{T_0}\right)^{5.266} \tag{7.3}$$

with

$$A = 0.00356 \text{ °R/ft } (0.0065 \text{ K/m})$$

Table 7.1 shows variation of pressure and temperature for the U.S. standard atmosphere. Although the actual atmosphere above a locality would not correspond precisely to the U.S. standard atmosphere, Table 7.1 provides a convenient means for estimating barometric pressure for a given altitude above sea level.

7.2 FUNDAMENTAL DISCUSSION OF MOIST AIR

The composition of atmospheric air is variable, particularly with regard to amounts of water vapor and particulate matter. Before we can discuss thermodynamic properties, the substance must be precisely defined. The working substance in air conditioning problems is called *moist air*. Moist air is defined as a binary mixture of dry air and water vapor. Goff [2], in a final report of the Working Subcommittee, International Joint Committee on Psychrometric Data, has defined dry air as shown in Table 7.2.

Although somewhat arbitrary, this composition is regarded as exact, by definition. The molecular weights for dry air and water vapor are 28.966 and 18.016, respectively. The respective gas constants can be obtained by dividing the universal gas constant, $\overline{R}$ = 1.986 Btu/lbmole · °R = 1545 ft-lbf/lbmole · °R (8.314 kJ/kmole · K), by the appropriate molecular weight.

$$\text{dry air:} \quad R_a = \frac{\overline{R}}{28.966} = 0.0686 \text{ Btu/lbm} \cdot \text{°R} = 53.35 \text{ ft-lbf/lbm} \cdot \text{°R} = 287 \text{ J/kg} \cdot \text{°K}$$

$$\text{water vapor;} \quad R_v = \frac{\overline{R}}{18.016} = 0.110 \text{ Btu/lbm} \cdot \text{°R} = 85.78 \text{ ft-lbf/lbm} \cdot \text{°R}$$
$$= 462 \text{ J/kg} \cdot \text{°K}$$

Moist air may contain variable amounts of water vapor from zero (dry air) to that of saturated moist air. Goff [3] has defined *saturation of moist air* as that condition where moist air may coexist in neutral equilibrium with associated condensed water, presenting a flat surface to it.

The humidity ratio, W, is defined as the mass of water vapor per unit mass of dry air in a moist air mixture.

$$W = \frac{m_v}{m_a}$$

Two measures of humidity relative to saturation conditions are commonly used. *Degree of saturation* is defined by the relation

$$\mu = \frac{W}{W_s} \tag{7.4}$$

TABLE 7.1 Standard Atmospheric Data for Altitudes to 60,000 ft

Altitude, ft	*Temperature, °F*	*Pressure*	
		in. Hg	*psia*
−1000	62.6	31.02	15.236
−500	60.8	30.47	14.966
0	59.0	29.921	14.696
500	57.2	29.38	14.430
1000	55.4	28.86	14.175
2000	51.9	27.82	13.664
3000	48.3	26.82	13.173
4000	44.7	25.82	12.682
5000	41.2	24.90	12.230
6000	37.6	23.98	11.778
7000	34.0	23.09	11.341
8000	30.5	22.22	10.914
9000	26.9	21.39	10.506
10,000	23.4	20.58	10.108
15,000	5.5	16.89	8.296
20,000	−12.3	13.76	6.758
30,000	−47.8	8.90	4.371
40,000	−69.7	5.56	2.731
50,000	−69.7	3.44	1.690
60,000	−69.7	2.14	1.051

Standard Atmospheric Data for Altitudes to 10,000 m

Altitude, m	*Temperature, °C*	*Pressure, kPa*
−500	18.2	107.478
0	15.0	101.325
500	11.8	95.461
1000	8.5	89.874
2000	2.0	79.495
3000	−4.5	70.108
4000	−11.0	61.640
5000	−17.5	54.020
6000	−24.0	47.181
7000	−30.5	41.061
8000	−37.0	35.600
9000	−43.5	30.742
10,000	−50.0	26.436

SOURCE: Abstracted by permission from *ASHRAE Handbook of Fundamentals 1993*, ch. 6.

where W_s is the humidity ratio at saturation for the same temperature and pressure as those of the actual state.

Relative humidity ϕ is defined by the relation

$$\phi = \frac{x_w}{x_{w,s}} \tag{7.5}$$

TABLE 7.2 Composition of Dry Air

Substance	*Molecular Weight*	*Mol-fraction Composition in Dry Air*	*Partial Molecular Weight in Dry Air*
Oxygen (O_2)	32.000	0.2095	6.704
Nitrogen (N_2)	28.016	0.7809	21.878
Argon (A)	39.944	0.0093	0.371
Carbon dioxide (CO_2)	44.01	0.0003	0.013
		1.0000	28.966

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where x_w is the mole fraction of the water vapor in the mixture and $x_{w,s}$ is the mole fraction of water vapor at saturation for the same temperature and pressure as those of the actual state. We may convert Eq. (7.5) to the forms

$$\phi = \frac{1 + \dfrac{0.622}{W_s}}{1 + \dfrac{0.622}{W}} = \mu \frac{0.622 + W_s}{0.622 + W} \tag{7.6}$$

It is important to observe that neither μ nor ϕ are defined when the temperature of moist air exceeds the saturation temperature of pure water corresponding to the moist air pressure. For sea-level pressure, W_s approaches infinity at 212 °F (100 °C). Thus for 14.696 psia (101.325 kPa), μ and ϕ are undefined for temperatures higher than 212 °F (100 °C).

Three moist-air properties are associated with temperature. The dry-bulb temperature, t, is the true temperature of moist air at rest. The dew-point temperature, t_d, is defined as the solution $t_d(P, W)$ of the equation

$$W_s(P, t_d) = W \tag{7.7}$$

In words, the dew-point temperature of a moist air state is the saturation temperature corresponding to the humidity ratio and pressure of the state. Another way to describe the dew-point temperature is to consider a moist air mixture defined by P, W, and t. If you slowly reduce the temperature of the mixture while holding P and W constant, then the temperature at which saturation is reached is the dew-point temperature, t_d.

The third temperature associated with moist air property is the *thermodynamic wet-bulb temperature, t^**, which is also referred to as the *adiabatic saturation temperature.* However, before defining this temperature it is necessary to introduce the enthalpy of the moist air mixture, H, which is the sum of the enthalpies of the dry air, H_a, and the water vapor, H_w.

$$H = H_a + H_w = m_a h_a + m_w h_w \tag{7.8}$$

For convenience, the specific enthalpy of moist air, h, is defined per unit mass of dry air.

$$h \equiv \frac{H}{m_a} \tag{7.9}$$

(Note that this form of the definition is particularly convenient for humidification and dehumidification processes where the flow rate of dry air remains constant as the moist air passes through the humidifier or dehumidifier.)

Thus

$$h = h_a + \frac{m_w}{m_a} h_w$$

or

$$h = h_a + W h_w \tag{7.10}$$

3 THERMODYNAMIC WET-BULB TEMPERATURE

Adiabatic saturation temperature is that temperature at which water, by evaporating into air, can bring the air to saturation adiabatically at the same temperature. It is unnecessary to inject practical details into a discussion of adiabatic saturation, since the results of the process are given by definition. However, to help us understand it better, we may consider how such a process could be approached.

Figure 7.1 schematically shows a device which may provide adiabatic saturation. The chamber could be indefinitely long and perfectly insulated. The total quantity of water present could be large compared to that added to the air in a given length of time. We may assume no temperature gradients within the water body. Regardless of the initial temperature of the water, we would expect that after sufficient time the water would assume a constant temperature. This limiting temperature of water should be less than the entering air dry-bulb temperature, but greater than the entering air dew-point temperature.

In order to maintain steady conditions within the chamber, makeup water at temperature t_2 is supplied at a rate $\dot{m}_l$, which is equal to the rate at which vapor is added to the moist air. Thus, if the mass flow rate of dry air passing through the chamber is $\dot{m}_a$, the rate of makeup water is given by

$$\dot{m}_l = \dot{m}_a (W_{s,2} - W_1) \tag{7.11}$$

The chamber is adiabatic, and no work is done by or on the apparatus. Therefore, the steady-flow energy equation for the process is

$$\dot{m}_a h_{a,1} + \dot{m}_a W_1 h_{w,1} + \dot{m}_l h_{f,2} = \dot{m}_a h_{a,2} + \dot{m}_a W_{s,2} h_{w,2} \tag{7.12}$$

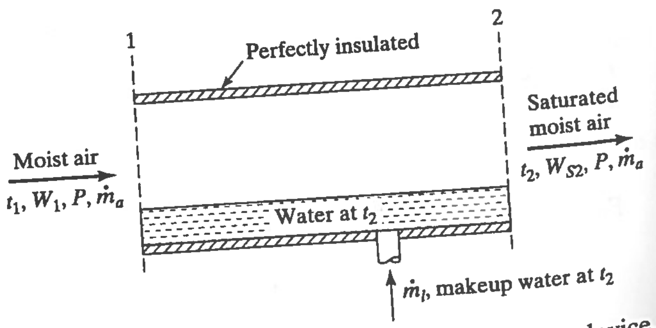

Figure 7.1 Schematic of an adiabatic saturation device.

By applying Eq. (7.10) in order to write the energy balance in terms of the moist-air enthalpy, substituting Eq. (7.11) for the rate of makeup water and noting that saturation conditions exist at the exit, the steady-flow energy equation can be written as

$$h_1 + (W_{s,2} - W_1) h_{f,2} = h_{s,2} \tag{7.13}$$

Since the leaving air is saturated, and since we assume a constant pressure P, the quantities $W_{s,2}$, $h_{s,2}$, and h_{f2}, are sole functions of temperature t_2. We may then deduce that t_2 is a function of h_1, W_1, and P, or that t_2 is a function of state 1. Therefore, t_2 is a thermodynamic property of state 1. We call this property the thermodynamic wet-bulb temperature t^*. Denoting all properties evaluated at t^* with the superscript *, Eq. (7.13) becomes

$$h + (W_s^* - W) h_f^* = h_s^* \tag{7.14}$$

Thus, for given values of h, W, and P (given moist air state), the thermodynamic wet-bulb temperature t^* is that value of temperature which satisfies Eq. (7.14). There are numerous practical problems where the concept of thermodynamic wet-bulb temperature is useful.

7.4 THE TABLES FOR MOIST AIR

By applying fundamental procedures of statistical mechanics, Goff and Gratch [4] calculated accurate thermodynamic properties of moist air for standard sea-level pressure. More recently, new formulations were developed by Hyland and Wexler [5]. Tables A.4E and A.4SI, extracted from the *ASHRAE Handbook, 1993 Fundamentals Volume* [6], presents properties for dry air and saturated moist air based on the new formulations. In the following list, brief explanations of the data in Tables A.4E and A.4SI are shown.

W_s = humidity ratio of saturated air, mass of water vapor per mass of dry air.
v_a = specific volume of dry air under 14.696 psia (101.325 kPa) pressure, ft^3/lbm$_a$ (m^3/kg$_a$).
v_s = volume of saturated air, ft^3/lbm dry air (m^3/kg dry air).
$v_{as} = v_s - v_a$, ft^3/lbm dry air (m^3/kg dry air).
h_a = specific enthalpy of dry air, Btu/lbm$_a$ (kJ/kg dry air). Zero enthalpy for dry air is taken at 0 °F (0 °C).
h_s = enthalpy of saturated air, Btu/lbm of dry air (kJ/kg dry air).
$h_{as} = h_s - h_a$, Btu/lbm of dry air (kJ/kg dry air).
s_a = specific entropy of dry air, Btu/lbm$_a$ · °R (kJ/kg$_a$-K). Zero entropy for dry air is taken at 0 °F (0 °C).
s_s = entropy of saturated air, Btu/lbm of dry air · °R (kJ/kg dry air · K).
$s_{as} = s_s - s_a$, Btu/lbm of dry air · °R (kJ/kg dry air · K).

Calculations for volume, enthalpy, and entropy of unsaturated moist air states are closely given by the relations

$$v = v_a + \mu v_{as} \tag{7.15}$$

$$h = h_a + \mu h_{as} \tag{7.16}$$

$$s = s_a + \mu s_{as} \tag{7.17}$$

The relations for specific volume and enthalpy would be exact if both the water vapor and dry air were perfect gases. At a temperature of approximately 100 °F (38 °C) real gas effects result in maximum corrections to Eqs. (7.15) and (7.16) of 0.003 percent and 0.015 percent, respectively. As the temperature increases, the real gas effects increase. However, for the purposes of most moist air calculations the corrections to Eqs. (7.15) and (7.16) are still small enough to be neglected. For example, at approximately 190 °F (88 °C) the corrections to the two equations are 0.16 percent and 0.11 percent, respectively.

The values of entropy calculated using Eq. (7.17) closely approximate the true values up to approximately 100 °F (38 °C). Above that temperature the mixing entropy not accounted for in Eq. (7.17) is significant. For example, Eq. (7.17) is in error by 1.8 percent, 3.5 percent, and 4.2 percent at temperatures of 96 °F (36 °C), 144 °F (62 °C) and 192 °F (89 °C), respectively. Additional information regarding correction factors for Eqs. (7.15), (7.16), and (7.8), including formulae for calculating the corrections, can be found in the *ASHRAE Handbook, 1977 Fundamentals Volume* [7].

EXAMPLE 7.1

Moist air exists at 80 °F dry-bulb temperature, 60 °F dew-point temperature, and 14.696 psia pressure. Determine (a) the humidity ratio, $\text{lbm}_w/\text{lbm}_a$, (b) degree of saturation, (c) relative humidity, (d) enthalpy, Btu/lbm_a, and (e) the volume in ft^3/lbm_a.

Solution: **(a)** Since sea-level pressure exists, Table A.4E will be used. By Eq. (7.7), $W = W_s$ at 60 °F. Thus $W = 0.011087\ \text{lbm}_w/\text{lbm}_a$.

(b) At 80 °F, $W_s = 0.02234\ \text{lbm}_w/\text{lbm}_a$. By Eq. (7.4)

$$\mu = 0.011087/0.02234 = 0.496 \text{ or } 49.6 \text{ percent}$$

(c) By Eq. (7.6)

$$\phi = \frac{(0.496)(0.622 + 0.02234)}{0.622 + 0.01108} = 0.505 \text{ or } 50.5 \text{ percent}$$

(d) We may calculate the enthalpy by Eq. (7.16). Using Table A.4E, we have $h = 19.222 + (0.496)(24.479) = 31.36\ \text{Btu/lbm}_a$

(e) We may calculate the volume by Eq. (7.15). Using Table A.4E, we have $v = 13.602 + (0.496)(0.487) = 13.84\ \text{ft}^3/\text{lbm}_a$

EXAMPLE 7.2

Moist air exists at 66 °C dry-bulb temperature and 30 percent degree of saturation. Pressure is 101.325 kPa. Determine (a) the enthalpy, kJ/kg_a, and (b) the specific volume, m^3/kg_a.

Solution: **(a)** By Table A.4SI, $W_s = 0.21848\ \text{kg}_w/\text{kg}_a$. By Eq. (7.16),

$$h = 66.455 + (0.30)(572.116) = 238.09\ \text{kJ/kg}_a$$

(b) By Eq. (7.15),

$$v = 0.9608 + (0.30)(0.3350) = 1.061\ \text{m}^3/\text{kg}_a$$

EXAMPLE 7.3

Moist air exists at 80 °F dry-bulb temperature, 60 °F thermodynamic wet-bulb temperature, and 14.696 psia pressure. Through use of Tables A.4E and A.1E, determine (a) the degree of saturation, and (b) the enthalpy.

Solution: **(a)** By Eqs. (7.16) and (7.14),

$$h_a + \mu h_{as} = h_s^* - (W_s^* - W)h_f^* = h_s^* - W_s^* h_f^* + \mu W_s h_f^*$$

Thus

$$\mu = \frac{h_s^* - W_s^* h_f^* - h_a}{h_{as} - W_s h_f^*} = \frac{26.467 - (0.011087)(27.63) - 19.222}{24.479 - (0.02234)(27.63)}$$

$$= 0.291$$

(b) By Eq. (7.16),

$$h = 19.222 + (0.291)(24.479) = 26.34\ \text{Btu/lbm}_a$$

7.5 PERFECT-GAS RELATIONSHIPS FOR APPROXIMATE CALCULATIONS

The methods of Sec. 7.4 allow accurate calculation of moist air properties through use of Table A.4. However, Table A.4 is restricted to standard atmospheric pressure. Basic relationships shown in Secs. 7.2 and 7.3 may be applied for any existing pressure. The discussion presented in Chapter 2 regarding low-pressure water vapor showed that for water-vapor pressure below 1 psia (6.9 kPa) the vapor exhibits approximate perfect-gas behavior. Therefore, in this section we will assume that perfect-gas relations exist and formulations for this special situation will be developed for the expressions presented in Secs. 7.2 and 7.3.

A useful expression for the humidity ratio can be derived by substituting the perfect-gas equations into the definition of W:

$$W = \frac{m_w}{m_a} = \frac{P_w V/R_w T}{P_a V/R_a T} = \frac{R_a}{R_w}\frac{P_w}{P_a}$$

It is convenient to substitute for the partial pressure of dry air, P_a, by noting that the total pressure of the mixture, P, is the sum of the partial pressures. In addition, the ratio of the gas constants can be replaced by the inverse ratio of the molecular weights. These substitutions result in the equation

$$W = 0.622\frac{P_w}{P - P_w} \tag{7.18}$$

This equation is also useful in establishing a method of evaluating the dew-point temperature, since it shows that a process in which W and P are constant corresponds to a process in which P_w is constant. Therefore, from Eqs. (7.7) and (7.18) we may deduce that the dew-point temperature, t_d, is equal to the saturation temperature corresponding to the vapor pressure P_w.

For a perfect gas the enthalpy is a function of temperature only. Thus

$$h_a = \int c_{pa}\, dT$$

$$h_w = \int c_{pw}\, dT$$

where c_{pa} and c_{pw} are the specific heats at constant pressure for dry air and water vapor, respectively. If the specific heat of dry air is assumed to be a constant and if we select the same reference states used in Tables A.4E and A.4SI, (i.e., $h_a = 0$ at zero degrees Fahrenheit or Celsius), the enthalpy of dry air can be expressed as

$$h_a = c_{pa} t \tag{7.19}$$

Similarly, if the specific heat at constant pressure for water vapor is assumed constant and we select the same reference states as those used in Tables A.1E and A.1SI, the enthalpy of the water vapor can be expressed as (see Sec. 2.14)

$$h_w = h_g = c_{pw} t + h_{g0} \tag{7.20}$$

where h_{g0} is the enthalpy of saturated water vapor at zero degrees Fahrenheit or Celsius.

The values of the quantities used in Eqs. (7.19) and (7.20) are given in Table 7.3.

The substitution of Eqs. (7.19) and (7.20) into (7.10) provides an expression for the specific enthalpy of the mixture

$$h = c_{pa} t + W h_g \tag{7.21}$$

$$h = c_{pa} t + W(c_{pw} t + h_{g0}) \tag{7.22}$$

Another form of the enthalpy equation can be written by defining a specific heat of the mixture, c_p, as

$$c_p = c_{pa} + W c_{pw} \tag{7.23}$$

which results in

$$h = c_p t + W h_{g0} \tag{7.24}$$

The specific volume of the mixture, v, is defined as the volume of the mixture per unit mass of dry air. However, since the mixture, the dry air, and the water vapor all occupy the same volume,

$$v = \frac{R_a T}{P_a}$$

TABLE 7.3 Quantities Used in Perfect-Gas Approximations for Enthalpy

t	*English Units* °F	*S.I. Units* °C
c_{pa}	0.240 Btu/lbm$_a$ · °F	1.00 kJ/kg$_a$ · °C
c_{pw}	0.444 Btu/lbm$_w$ · °F	1.86 kJ/kg$_w$ · °C
h_{g0}	1061 Btu/lbm$_w$	2501 kJ/kg$_w$

or

$$v = \frac{R_a T}{P - P_w} \tag{7.25}$$

The perfect-gas approximation for the relative humidity is obtained by observing that for a perfect gas

$$x_w = \frac{P_w}{P} \tag{7.26}$$

and thus from Eqs. (7.5) and (7.26)

$$\phi = \frac{P_w}{P_{ws}} \tag{7.27}$$

Equation (7.14) may be altered through use of the perfect-gas relations. By Eqs. (7.14), (7.23), and (7.24)

$$(W_s^* - W)h_{fg}^* = c_p(t - t^*) \tag{7.28}$$

By Eqs. (7.14) and (7.21)

$$W = \frac{W_s^* h_{fg}^* - c_{pa}(t - t^*)}{h_g - h_f^*} \tag{7.29}$$

The total entropy of a moist air mixture, S, which is at a pressure and temperature P and T, can be written as

$$S = S_{a,0} + \Delta S_{a,T,P} + \Delta S_{a,\text{mix}} + S_{w,0} + \Delta S_{w,T,P} + \Delta S_{w,\text{mix}} \tag{7.30}$$

The quantities $S_{a,0}$ and $S_{w,0}$ are the entropy values at the reference conditions for the dry air and water vapor, respectively. The terms $\Delta S_{a,T,P}$ and $\Delta S_{w,T,P}$ are the entropy changes that result in going from the reference states to the state P, T and the terms $\Delta S_{a,\text{mix}}$ and $\Delta S_{w,\text{mix}}$ are the mixing entropies for the dry air and water vapor. These latter two mixing terms account for the fact that it is the partial pressures, P_a and P_w, not the total pressure, P, that are needed in evaluating the entropies of the components.

For the case of perfect gases with constant specific heats and with P_0, T_0 as the reference state

$$\Delta S_{a,T,P} = m_a\left(c_{pa} \ln \frac{T}{T_0} - R_a \ln \frac{P}{P_0}\right) \tag{7.31}$$

$$\Delta S_{a,\text{mix}} = -m_a\left(R_a \ln \frac{P_a}{P}\right) \tag{7.32}$$

$$\Delta S_{w,T,P} = m_w\left(c_{pw} \ln \frac{T}{T_0} - R_w \ln \frac{P}{P_0}\right) \tag{7.33}$$

$$\Delta S_{w,\text{mix}} = -m_w\left(R_w \ln \frac{P_w}{P}\right) \tag{7.34}$$

The temperatures and pressures in Eqs. (7.31) thru (7.34) must be absolute values.

In the English system of units the reference condition is taken as $T_0 = 0$ °F and $P_0 = 14.696$ psia. It will be convenient to select $S_{a,0}$ and $S_{w,0}$ such that the values of S will correspond to tabulated values. The dry air tables are constructed with zero entropy at 0 °F, 14.696 psia. Therefore, $S_{a,0} = 0$. However, the steam tables are constructed with the entropy of saturated liquid equal to zero at 32 °F, and a value for $S_{w,0}$ must be calculated. Although $S_{w,0}$ is at a fictitious state, it can be evaluated by extrapolating the ideal-gas relations to the reference state at T_0, P_0. Thus

$$S_{w,0} = m_w\left[s_g(T_0, P_{w,s,T_0}) - R_w \ln \frac{P_0}{P_{w,s,T_0}}\right] \qquad (7.35)$$

where P_{w,s,T_0} is the saturation pressure at T_0, the reference temperature. Substitution in to Eq. (7.35) results in

$$S_{w,0} = m_w(1.5937\ \text{Btu/lbm}_w \cdot {}^\circ\text{R}) = m_w s_{w,0}$$

In the S.I. system of units the reference condition is selected as $T_0 = 0$ °C and $P_0 = 101.325$ kPa. As before, if we select values of $S_{a,0}$ and $S_{a,w}$ in order that entropy values correspond to tabulated values, we find

$$S_{a,0} = 0$$

$$S_{w,0} = m_w(6.7975\ \text{kJ/kg}_\text{w} \cdot \text{K}) = m_w s_{w,0}$$

Substituting Eqs. (7.31), (7.32), (7.33), and (7.34) into Eq. (7.30), setting $S_{a,0} = 0$, dividing through by m_a, and combining terms results in the following expression for the specific entropy for the mixture:

$$\frac{S}{m_a} = s = c_{pa} \ln \frac{T}{T_0} - R_a \ln \frac{P_a}{P_0} + W\left(s_{w,0} + c_{pw} \ln \frac{T}{T_0} - R_w \ln \frac{P_w}{P_0}\right) \qquad (7.36)$$

or

$$s = (c_{pa} + c_{pw} W) \ln \frac{T}{T_0} - R_a \ln \frac{P_a}{P_0} + W\left(s_{w,0} - R_w \ln \frac{P_w}{P_0}\right) \qquad (7.37)$$

As stated earlier, equations of this section are only approximate. Figure 7.2 shows percent error in calculation of humidity ratio W_s', enthalpy h_s', and volume v_s' by Eqs. (7.18), (7.21), and (7.25), respectively, for saturated air at 14.696 psia (101.325 kPa).

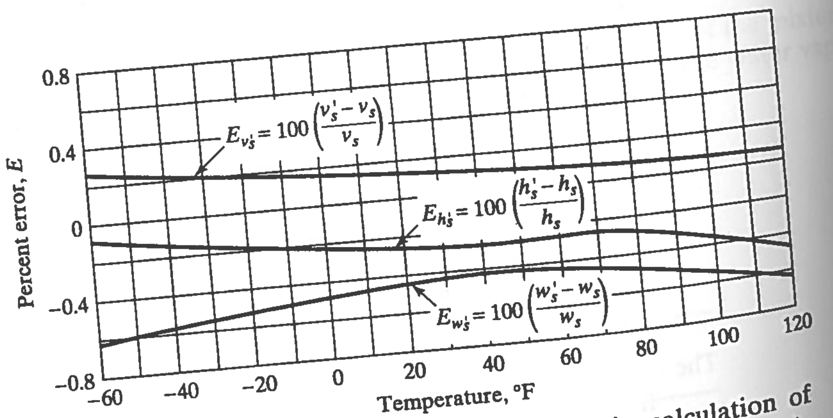

Figure 7.2 Error of perfect-gas relationships in calculation of humidity ratio, enthalpy, and volume of saturated air at 14.696 psia pressure (101.324 kPa).

Correct values W_s, h_s, and v_s were taken from Table A.4E. Figure 7.2 shows that the error in calculation of humidity ratio by Eq. (7.18) is less than about 0.6 percent in the range of −50 °F to 110 °F (−46 °C to 43 °C). Except for temperatures above about 100 °F (38 °C), the error in calculation of enthalpy by Eq. (7.21) is less than about 0.4 percent. For the temperature range of −30 °F to 120 °F (−34 °C to 49 °C), the volume may be calculated by Eq. (7.25) with an error of less than 0.2 percent.

EXAMPLE 7.4

Rework Ex. 7.1, using perfect-gas relations and steam-table data.

Solution:

(a) We may deduce from Eq. (7.18) that the dew-point temperature t_d is the saturation temperature corresponding to the partial pressure of water vapor P_w. By Table A.1E, at 60 °F, $P = 0.25629$ psi. By Eq. (7.18),

$$W = \frac{(0.622)(0.256)}{14.696 - 0.256} = 0.01103\ \text{lbm}_w/\text{lbm}_a$$

(b) By Table A.1E, at 80 °F, $P_{w,s} = 0.50734$ psia. By Eq. (7.18) for saturated air

$$W_s = \frac{(0.622)(0.507)}{14.696 - 0.507} = 0.02223\ \text{lbm}_w/\text{lbm}_a$$

By Eq. (7.4),

$$\mu = \frac{0.01103}{0.02223} = 0.496$$

(c) By Eq. (7.27),

$$\phi = \frac{0.25618}{0.50701} = 0.505$$

(d) By Table A.1E, at 80 °F, $h_g = 1096.20$ Btu/lbm$_w$. By Eq. (7.21),

$$h = (0.240)(80) + (0.01103)(1096.20) = 31.29\ \text{Btu/lbm}_a$$

(e) By Eq. (7.25),

$$v = \frac{(53.35)(540)}{(14.44)(144)} = 13.85\ \text{ft}^3/\text{lbm}_a$$

We observe that the answers of Example 7.4 differ but slightly from those of Example 7.1.

EXAMPLE 7.5

Moist air exists at 30 °C dry-bulb temperature, 40 percent relative humidity, and 86.60 kPa barometric pressure. Determine (a) the dew-point temperature, (b) the specific enthalpy, and (c) the specific entropy.

Solution: **(a)** By Table A.1SI, at 30 °C, $P_{w,s} = 4.246$ kPa. By Eq. (7.27),

$$P_w = (0.40)(4.246) = 1.698 \text{ kPa}$$

Interpolation in Table A.1SI gives $t_d = 14.9$ °C.

(b) By Eq. (7.18),

$$W = \frac{(0.622)(1.698)}{86.60 - 1.698} = 0.0124 \text{ kg}_w/\text{kg}_a$$

By Table A.1SI, at 30 °C, $h_g = 2555.80$ kJ/kg$_w$. By Eq. (7.21),

$$h = (1.0)(30) + (0.0124)(2555.80) = 61.79 \text{ kJ/kg}_a$$

(c) The specific entropy can be calculated using Eq. (7.37). First find the partial pressure of dry air:

$$P_a = P - P_w = 86.600 - 1.698 = 84.902 \text{ kPa}$$

By Eq. (7.37),

$$\begin{aligned} s &= [1.0 + 1.86(0.0124)] \ln \frac{303}{273} - 0.287 \ln \frac{84.902}{101.325} \\ &\quad + (0.0124)\left(6.7975 - 0.461 \ln \frac{1.698}{101.325}\right) \\ &= 0.2651 \text{ kJ/kg}_a \cdot \text{K} \end{aligned}$$

There is always the temptation to perform the calculations using tables and charts for standard atmospheric pressure. The pressure in this example corresponds to that which would occur at an altitude of approximately 1300 meters (see Table 7.1) and is approximately 15 percent below standard atmospheric pressure. If we had neglected that the pressure was lower and had used Table A.4SI along with Eqs. (7.6), (7.16), and (7.17), the results would have been

$W = 0.0107$ kg/kg$_a$ (14% lower than the correct value)
$h = 57.39$ kJ/kg$_a$ (7% lower than the correct value)
$s = 0.1996$ kJ/kg$_a \cdot$ K (25% lower than the correct value)

Thus it is apparent that the moist air property tables can give results that are considerably different from the correct values when the pressure differs from standard atmospheric pressure.

EXAMPLE 7.6

Moist air exists at 100 °F dry-bulb temperature, 80 °F thermodynamic wet-bulb temperature, and 13.86 psia pressure. Determine (a) the humidity ratio and (b) the relative humidity.

Solution: **(a)** By Table A.1E, at 80 °F, $P^*_{w,s} = 0.50734$ psi. By Eq. (7.18),

$$W^*_s = \frac{(0.622)(0.507)}{13.86 - 0.507} = 0.02362 \text{ lbm}_w/\text{lbm}_a$$

By Table A.1E, at 80 °F, $h^*_{fg} = 1048.5$ Btu/lbm$_w$, $h^*_f = 47.70$ Btu/lbm$_w$. At 100 °F, $h_g = 1104.83$ Btu/lbm$_w$. By Eq. (7.29)

$$W = \frac{(0.02362)(1048.5) - (0.240)(20)}{1104.83 - 47.70} = 0.01889 \text{ lbm}_w/\text{lbm}_a$$

(b) Equation (7.18) may be changed to the form

$$P_w = \frac{1.608PW}{1 + 1.608W}$$

Thus

$$P_w = \frac{(1.608)(13.86)(0.01889)}{1 + (1.608)(0.01889)} = 0.408 \text{ psi}$$

By Table A.1E, at 100 °F, $P_{w,s} = 0.95034$ psi. By Eq. (7.27),

$$\phi = \frac{0.408}{0.950} = 0.430 \text{ or } 43.0 \text{ percent}$$

In the United States, air conditioning engineers sometimes use an approximate procedure in performing psychrometric process calculations. The concept followed is to treat any general psychrometric process as the sum of a sensible and a latent process and to use approximate relationships to evaluate these processes.

In order to illustrate the procedure, consider the enthalpy change in a general process that goes from state 1 to state 2.

$$\Delta h = h_2 - h_1$$

Applying Eq. (7.22),

$$\Delta h = (c_{pa}t_2 + W_2 c_{pw} t_2 + W_2 h_{g0}) - (c_{pa}t_1 + W_1 c_{pw} t_1 + W_1 h_{g0}) \tag{7.38}$$

If
$\bar{t} = (t_1 + t_2)/2$
$\Delta t = t_2 - t_1$
$\overline{W} = (W_1 + W_2)/2$
$\Delta W = W_2 - W_1$

then Eq. (7.38) can be rewritten as

$$\Delta h = (c_{pa} + \overline{W} c_{pw})\, \Delta t + (c_{pw}\bar{t} + h_{g0})\Delta W \tag{7.39}$$

The terms on the right-hand side of Eq. (7.39) can be thought of as the enthalpy changes for sensible and latent processes, respectively. Thus the total enthalpy change is divided into sensible and latent components.

$$\Delta h = \Delta h_S - \Delta h_L \tag{7.40}$$

where

$$\Delta h_S = (c_{pa} + \overline{W} c_{pw})\, \Delta t: \qquad \text{sensible} \tag{7.41}$$

$$\Delta h_L = (c_{pw}\bar{t} + h_{g0})\, \Delta W: \qquad \text{latent} \tag{7.42}$$

The coefficient $(c_{pa} + \overline{W} c_{pw})$ is the average specific heat for the moist air mixture for the process from state 1 to state 2:

$$\bar{c}_p = c_{pa} + \overline{W} c_{pw} \tag{7.43}$$

The coefficient $(c_{pw}\bar{t} + h_{g0})$ can be interpreted as the average enthalpy of water vapor for the process from state 1 to state 2:

$$\bar{h}_g = c_{pw}\bar{t} + h_{g0} \tag{7.44}$$

Therefore Eqs. (7.41) and (7.42) can be written as

$$\Delta h_s = \bar{c}_p \, \Delta t \tag{7.45}$$

$$\Delta h_L = \bar{h}_g \, \Delta W \tag{7.46}$$

No new approximations, beyond the assumptions, of perfect gas and constant specific heat, have been used in the development of Eqs. (7.45) and (7.46). The approximate nature of the procedure is introduced in selecting the coefficients for the two equations. The numerical values of $\bar{c}_p$ and $\bar{h}_g$ are relatively insensitive to the humidity ratios and temperatures, respectively, that commonly occur in HVAC processes. Therefore, constant values are generally selected. The specific heat, $\bar{c}_p$, is commonly taken as 0.245 Btu/lbm$_a$ · °F (1.02 kJ/kg$_a$ · °C). Depending upon the reference, values selected for the average enthalpy of water vapor, $\bar{h}_g$, vary from about 1050 to 1150 Btu/lbm$_w$ (2500 to 2700 kJ/kg$_w$). The use of these constant coefficients makes the procedure easy to use but also approximate.

ONSTRUCTION OF THE PSYCHROMETRIC CHART

In previous sections of this chapter we considered various thermodynamic properties of moist air and the equations relating them. With these relations we may accurately solve problems concerning moist air. However, the calculations can be tedious and time consuming.

It is of considerable advantage to plot the relations to give a nomograph called a *psychrometric chart*. Such a chart not only allows graphical reading of the various properties but also provides for convenient graphical solutions to many process problems.

A thermodynamic state for moist air is uniquely fixed if the barometric pressure and two independent properties are known. A psychrometric chart may be constructed for some single value of barometric pressure. Traditionally, standard sea-level pressure has been used. The choice of coordinates is, of course, arbitrary. Many psychrometric charts used in the United States have employed dry-bulb temperature and humidity ratio as the basic coordinates. In 1923, Richard Mollier [8] of Dresden, Germany, introduced a chart using enthalpy and humidity ratio as the coordinates. This chart received wide acceptance in Europe.

The use of enthalpy and humidity ratio as basic coordinates presents many advantages. Thermodynamic wet-bulb temperature lines are identically straight. A majority of the common psychrometric processes appear as straight lines on *h-W* coordinates. In general, the Mollier type of chart allows the most fundamentally consistent treatment of air conditioning problems with a minimum of approximations.

Through use of the psychrometric relations of this chapter we may readily construct an *h-W* chart. Experience has shown that the best intersections result when enthalpy is used as an oblique coordinate and humidity ratio as a rectangular coordinate.

The construction method used here has been adapted from Goodman's [9] procedure. Figure 7.3 shows the basic geometry. The enthalpy lines are inclined at an angle β to the horizontal humidity ratio lines. The line $\overline{12}$ may represent any straight line. The vertical scalar distance representing $(W_2 - W_1)$ is L_W, and the horizontal scalar distance representing $(h_2 - h_1)$ is L_h.

By the law of sines, we may write

$$\frac{L_h}{\sin\alpha} = \frac{b}{\sin\beta} \tag{7.47}$$

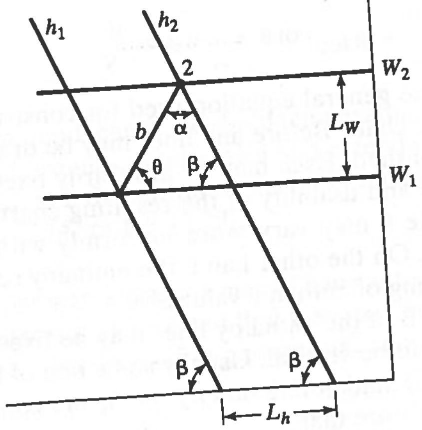

Figure 7.3 Fundamental geometry of psychrometric chart.

Also

$$\alpha = 180 - (\theta + \beta)$$

$$\sin\alpha = \sin(\theta + \beta) = \sin\theta\cos\beta + \cos\theta\sin\beta \tag{7.48}$$

and

$$b = \frac{L_w}{\sin\theta} \tag{7.49}$$

By Eqs. (7.47)–(7.49) we have

$$\cot\beta + \cot\theta = \frac{L_h}{L_W} \tag{7.50}$$

We may define an enthalpy scale factor s_h by

$$s_h = \frac{h_2 - h_1}{L_h}$$

and a humidity-ratio scale factor s_W by

$$s_W = \frac{W_2 - W_1}{L_W}$$

Then

$$\frac{L_h}{L_W} = \frac{s_W(h_2 - h_1)}{s_h(W_2 - W_1)} = \frac{q'}{S} \tag{7.51}$$

where

$$S = \frac{S_h}{S_W} = \text{chart scale factor in Btu per lbm}_w\text{(kJ/kg}_w\text{)}$$

and

$$q' = \frac{h_2 - h_1}{W_2 - W_1} = \text{enthalpy-moisture ratio in Btu/lbm}_w\text{(kJ/kg}_w\text{)}$$

By Eqs. (7.50) and (7.51),

$$\cot\theta + \cot\beta = \frac{q'}{S} \tag{7.52}$$

Equation (7.52) is the general equation used for constructing the various straight lines on the psychrometric chart. Before any lines may be drawn, the scale factor S and the angle β must be established. Each may be arbitrarily fixed, but their choice greatly influences the appearance and usability of the resulting chart. A large scale factor S is desirable so that the angle θ may vary more uniformly with uniform changes in the enthalpy-moisture ratio q'. On the other hand, the enthalpy scale factor s_h can be made too large for accurate reading of enthalpy values.

The inclination angle β of the enthalpy lines may be fixed by choosing some property line and considering it to be vertical. Usually some one of the dry-bulb temperature lines is chosen. By Eq. (7.10), and noting that h_a and h_w are constants, it is true for a line of constant dry-bulb temperature that

$$h_2 - h_1 = (W_2 - W_1)h_w$$

or

$$q' = h_w = h_g \tag{7.53}$$

Thus the dry-bulb temperature lines are straight but not parallel, since h_g varies with temperature. If we arbitrarily choose some dry-bulb temperature line t to be vertical, by Eqs. (7.52) and (7.53) we have

$$\tan\beta = \frac{S}{h_{g,t}} \tag{7.54}$$

Equation (7.54) establishes the inclination of the enthalpy lines. The grid of h and W lines may then be constructed.

The remaining procedure for constructing a complete psychrometric chart will now be reviewed.

Saturation Curve. The saturation curve is a locus of points representing saturated air. For standard barometric pressure, values of h_s and W_s at various temperatures may be read from Table A.4 and the points plotted. For other barometric pressures, Eq. (7.18) and Eq. (7.21) may be used.

Dry-Bulb Temperature Lines. We have already shown that, within the accuracy of the perfect-gas approximation, these lines are straight. By Eqs. (7.52) and (7.53),

$$\cot\theta = \frac{h_g}{S} - \cot\beta$$

For various dry-bulb temperatures we may determine h_g and calculate $\cot\theta$. One point on each line may be conveniently located at $W = 0$, and knowledge of the angle θ then lets us draw the line.

Thermodynamic Wet-Bulb Temperature Lines. By Eq. (7.14),

$$q' = h_f^* = \frac{h_s^* - h}{W_s^* - W}$$

For a line of constant wet-bulb temperature, Eq. (7.52) becomes

$$\cot\theta = \frac{h_f^*}{S} - \cot\beta$$

Thus thermodynamic wet-bulb temperature lines are identically straight. For various values of t^*, we may determine θ and extend the lines through one known point. Locations are known on the saturation curve. For unsaturated air, a point may be calculated by solving Eq. (7.14) or Eq. (7.29).

Volume Lines. Lines of constant specific volume, v, are not strictly straight, but their curvature is so slight that they may be drawn as straight lines. For a line of constant volume, with the perfect-gas approximation, we may show that

$$q' = \frac{dh}{dW} = \left(\frac{c_{pw} - 1.608\, c_{pa}}{R_a}\right)\left[\frac{Pv}{(1 + 1.608W)^2}\right] - c_{pw}T_0 + h_{g0} \tag{7.55}$$

The temperature T_0 is the reference temperature in absolute units in defining the enthalpy of the mixture, i.e., T_0 = 460 °R (273 K). An average W may be used in Eq. (7.55) with little error. (For example, a value of $W = 0.01$ is acceptable.) This leads to the result that the slope of the constant-volume line expressed by Eq. (7.55) is only a function of the chosen v and the pressure, which is constant for the entire chart. This slope, when substituted into Eq. (7.52), yields a value for the angle θ. For the chosen v, a value of dry-bulb temperature may be found at $W = 0$. Through knowledge of θ, we may then extend the volume line.

Relative-Humidity Lines. By Eqs. (7.18) and (7.27), we have

$$W = 0.622\frac{\phi P_{ws}}{P - \phi P_{ws}}$$

For chosen values of ϕ and t, we may calculate values of W and plot points for a line of constant relative humidity.

Enthalpy-Moisture Ratio Protractor. A convenient aid in many psychrometric-chart problems is a protractor showing the direction of straight lines for various values of the enthalpy-moisture ratio q'. Such a protractor may be directly calculated from Eq. (7.52).

Sensible-Heat-Ratio Protractor. Many psychrometric charts have a sensible-heat-ratio (SHR) protractor. For a given process from state 1 to state 2 the *sensible-heat ratio* is defined as the sensible enthalpy change divided by the total enthalpy change

$$\mathrm{SHR} = \frac{\Delta h_s}{\Delta h} \tag{7.56}$$

The relationship between the SHR and the enthalpy-moisture ratio can be found by combining Eqs. (7.56), (7.40), and (7.46):

$$\mathrm{SHR} = \frac{\Delta h - \Delta h_L}{\Delta h} = 1 - \frac{\overline{h}_g\,\Delta W}{\Delta h}$$

Thus

$$\mathrm{SHR} = 1 - \frac{\bar{h}_g}{q'} \tag{7.57}$$

or

$$q' = \frac{\bar{h}_g}{1 - \mathrm{SHR}} \tag{7.58}$$

In order to draw a SHR protractor, a constant value of $\bar{h}_g$ is selected. Therefore, these lines are approximate.

THE ASHRAE PSYCHROMETRIC CHARTS

Eight psychrometric charts are included with this text. These are the English and S.I. versions of the three ASHRAE sea-level pressure charts (low, normal, and high temperature) and the ASHRAE charts for high elevation (5000 ft and 1500 m) at normal temperatures.

The three ASHRAE psychrometric charts are similar in format. We will limit further discussion in this section to Fig. C-8E and SI. Humidity-ratio lines are horizontal and are shown for the range from zero (dry air) to 0.03 lbm moisture/lbm dry air or to 30 g of moisture/kg dry air. Enthalpy lines are obliquely drawn across the chart in intervals of 5 Btu/lbm dry air or 10 kJ/kg dry air. All enthalpy lines are precisely parallel. Edge scales for enthalpy are shown above the saturation curve and at the bottom and right-hand margins.

Dry-bulb temperature lines are shown at intervals of 1 °F or 1 °C. The dry-bulb temperature lines are drawn straight, are inclined slightly from the vertical position, and are not strictly parallel to one another. Thermodynamic wet-bulb temperature lines are obliquely drawn across the chart in intervals of 1 °F or 1°C. Their directions differ but slightly from that of the enthalpy lines. The thermodynamic wet-bulb temperature lines are exactly straight but are not strictly parallel to each other.

Relative-humidity lines are shown in intervals of 10 percent from zero ($W = 0$) to 100 percent (saturation curve). Volume lines are obliquely drawn straight lines in intervals of 0.5 ft³/lbm dry air or 0.01 m³/kg dry air. The volume lines are not strictly parallel to one another.

A narrow region above the saturation curve has been developed for fog conditions. A *fog* is a mechanical mixture of saturated moist air and water droplets, both at the same temperature. A fog is equivalent to the condition produced by taking saturated moist air and adiabatically supersaturating it with water at the same temperature. Thus an isotherm in the fog region is an extension of a thermodynamic wet-bulb temperature line.

A protractor is shown to the left of the chart body. The protractor shows two scales—one for the ratio of enthalpy difference to humidity-ratio difference ($q' = \Delta h/\Delta W$) and one for the sensible-total heat ratio (SHR). The S.I. version includes a second sensible-heat-ratio protractor. The protractor scale is along the right-hand side of the chart, and the center is at the small circle located at 24 °C dry-bulb temperature and 50 percent relative humidity. From Eq. (7.57) and examination of the protractors, we see that values of approximately $\bar{h}_g = 1100$ Btu/lbm$_w$ and $\bar{h}_g = 2500$ kJ/kg$_w$ were used in the construction of the protractors in Figs. C-8E and C-8SI, respectively.

EXAMPLE 7.7

Moist air exists at a condition of 100 °F dry-bulb temperature, 65 °F thermodynamic wet-bulb temperature, and 14.696 psia pressure. Determine (a) the humidity ratio, (b) enthalpy, (c) dew-point temperature, (d) relative humidity, and (e) the volume.

Solution: The state-point may be located on Fig. C-8E at the intersection of the 100 °F dry-bulb temperature line and the 65 °F thermodynamic wet-bulb temperature line.

(a) Read $W = 0.00523$ lbm water vapor/lbm dry air.

(b) Through use of two triangles, draw a line parallel to nearest enthalpy line (30 Btu/lbm dry air) through the state-point to the nearest edge scale. Read $h = 29.80$ Btu/lbm dry air.

(c) The dew-point temperature may be read at the intersection of $W = 0.00523$ lbm water vapor per lbm dry air with the saturation curve. Thus, $t_d = 40.1$ °F.

(d) Read $\phi = 13$ percent.

(e) The volume may be accurately found by linear interpolation between the volume lines for 14.0 and 14.5 ft³/lbm$_a$ air. Thus, $v = 14.22$ ft³/lbm$_a$.

ENDNOTES

1. *ASHRAE Handbook, Fundamentals Volume* (Atlanta: American Society of Heating, Refrigerating and Air Conditioning Engineers, 1993), 6.1 (IP & SI Editions).
2. J. A. Goff, "Standardization of Thermodynamic Properties of Moist Air," *Trans. ASHVE*, 55 (1949), 463–464.
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4. J. A. Goff and S. Gratch, "Thermodynamic Properties of Moist Air," *Trans. ASHVE*, 51 (1945), 125–164.
5. R. W. Hyland and A. Wexler, "Formulations for the Thermodynamic Properties of Dry Air from 173.15 K to 473.15 K and of Saturated Moist Air from 173.15 K to 372.15 K, at Pressures to 5 MPa," *ASHRAE Trans.*, 89, pt. 2 (1983).
6. *ASHRAE Handbook, Fundamentals Volume* (Atlanta: American Society of Heating, Refrigerating and Air Conditioning Engineers, 1993).
7. *ASHRAE Handbook, Fundamentals Volume* (Atlanta: American Society of Heating, Refrigerating and Air Conditioning Engineers, 1977), 5.5–5.6.
8. Richard Mollier, "Ein neues Diagram für Dampfluftgemische," *ZVDI*, 67 (September 8, 1923), 869–872.
9. William Goodman, *Air Conditioning Analysis* (New York: The Macmillan Company, 1943), 271–276.

PROBLEMS

7.1 Moist air exists at 80 °F dry-bulb temperature, 0.0150 lbm$_w$/lbm$_a$ humidity ratio, and 14.696 psia pressure. Through the use of Table A.4 and fundamental relations, determine

(a) the dew-point temperature,
(b) relative humidity,
(c) volume in ft³/lbm$_a$, and
(d) the enthalpy in Btu/lbm$_a$.

2 Calculate values of humidity ratio, enthalpy, and volume for saturated air at 14.696 psia pressure, using perfect-gas relations and Table A.1, for temperatures of
(a) 70 °F, and (b) −20 °F.
Compare your results with those shown in Table A.4.

.3 The atmosphere within a room is at 70 °F dry-bulb temperature, 50 percent degree of saturation, and 14.696 psia pressure. The inside surface temperature of the windows is 40 °F. Will moisture condense out of the air upon the window glass?

.4 Assume that the dimensions of the room of Prob. 7.3 are 30 ft by 15 ft by 8 ft high. Calculate the number of pounds of water vapor in the room.

.5 Calculate values of humidity ratio, enthalpy, and volume for saturated air at 101.325 kPa pressure using perfect-gas relations and Table A.1, for temperatures of
(a) 20 °C, and (b) −25 °C.
Compare your results with those shown in Table A.4.

7.6 Moist air exists at a dry-bulb temperature of 40 °C, relative humidity of 20 percent, and 101.325 kPa pressure. Find the enthalpy in $\mathrm{kJ/kg}_a$. Base the solution on Table A.4 and fundamental relations. Do not use perfect-gas expressions.

7.7 Moist air exists at a dry-bulb temperature of 100 °F, relative humidity of 20 percent, and 14.696 psia pressure. Find the enthalpy in $\mathrm{Btu/lbm}_a$. Base the solution on Table A.4 and fundamental relations. Do not use perfect-gas expressions.

7.8 Moist air exists at a dew-point temperature of 65 °F, a relative humidity of 60.3 percent, and a pressure of 14.00 psia. Determine
(a) the humidity ratio in $\mathrm{lbm}_w/\mathrm{lbm}_a$, and
(b) the volume in $\mathrm{ft}^3/\mathrm{lbm}_a$.

7.9 Moist air exists at a dew-point temperature of 20 °C, a relative humidity of 60.3 percent, and a pressure of 96.5 kPa. Determine
(a) the humidity ratio, and
(b) the volume in $\mathrm{m}^3/\mathrm{kg}_a$.

7.10 The inside surface temperature of a window in a house is 5 °C. The dry-bulb temperature of air in the house is 20 °C, and the pressure is 101.325 kPa. What is the maximum relative humidity allowable in the house if no condensation is to form on the window?

7.11 Determine the relative humidity and dew-point temperature of moist air at 95 °F dry-bulb temperature, 80 °F thermodynamic wet-bulb temperature, and 13.20 psia pressure.

7.12 Determine the relative humidity and dew-point temperature of moist air at 35 °C dry-bulb temperature, 25 °C thermodynamic wet-bulb termperature, and 91.0 kPa pressure.

7.13 Calculate the dry-bulb temperature of moist air at 30 °C thermodynamic wet-bulb temperature, 0.020 $\mathrm{kg}_w/\mathrm{kg}_a$ humidity ratio, and 90.0 kPa pressure.

7.14 Calculate the enthalpy in Btu per lbm_a of moist air at 70 °F thermodynamic wet-bulb temperature, 34 °F dew-point temperature, and 14.696 psia pressure.

7.15 Calculate the dry-bulb temperature of moist air at 80 °F thermodynamic wet-bulb temperature, 0.01250 $\mathrm{lbm}_w/\mathrm{lbm}_a$ humidity ratio, and 13.00 psia pressure.

7.16 Develop the complete derivation for Eq. (7.3).

7.17 Through the use of basic definitions and perfect-gas relations, derive the following equations:

(a) $v = \frac{R_a T}{P}(1 + 1.608W)$

(b) $\phi = 1.608 \frac{P}{P_{ws}}\left(\frac{W}{1 + 1.608W}\right)$

7.18 Through the use of basic definitions and perfect-gas expressions, show that

$$v = v_a + \mu v_{as}$$

reduces to

$$v = \frac{R_a T}{P - P_w}$$

7.19 It is planned to construct an *h*-*W* diagram for sea-level pressure from Table A.4E. Scale factor *S* for the diagram is to be 1000 Btu/lbm water. The 80 °F dry-bulb temperature line is to be vertical. Determine the slope (tan θ) of the 60 °F thermodynamic wet-bulb temperature line.

7.20 For a line of constant volume in $\mathrm{ft}^3/\mathrm{lbm}_a$ on the Mollier psychrometric chart, and assuming that moist air is a perfect-gas mixture, derive Eq. (7.55). [*Hint:* A good starting place is Eqs. (7.24) and (7.23). Also, the results of Prob. 7.17(a) may be useful in substituting for *t* in Eq. (7.24), keeping in mind that the temperature *t* is in °F (°C) while *T* is in absolute units.]

7.21 Work Prob. 7.1 using the psychrometric chart and compare the answers to those found in Prob. 7.1.

7.22 Work Prob. 7.6 using the psychrometric chart and compare answers.

7.23 What errors would have resulted if you had neglected the deviation from standard atmospheric pressure and had used the psychrometric chart in working Prob. 7.12?

7.24 Moist air exists under conditions of 85 °F dry-bulb temperature, 40 percent relative humidity, and 14.696 psia pressure. By the psychrometric chart, determine
(a) the dew-point temperature,
(b) thermodynamic wet-bulb temperature,
(c) humidity ratio,
(d) enthalpy, and
(e) the volume.

7.25 Moist air exists under conditions of 25 °C dry-bulb temperature, 40 percent relative humidity, and 101.325 kPa pressure. By the psychrometric chart, determine
(a) the dew-point temperature,
(b) thermodynamic wet-bulb temperature,
(c) humidity ratio,
(d) enthalpy, and
(e) the volume.

7.26 In a psychrometric process, moist air at standard atmospheric pressure goes from state 1 of 55 °F dry-bulb temperature and 50 °F dew-point temperature to state 2 of 100 °F dry-bulb temperature and 30 percent relative humidity. Use the psychrometric chart and answer the following:
(a) What is the enthalpy-humidity ratio, *q*′, for the process?
(b) What is the sensible-heat ratio, SHR, for the process?
(c) For each pound of dry air undergoing the process, how much water vapor, lbm_w, is added?

7.27 In a psychrometric process, moist air at standard atmospheric pressure goes from state 1 of 15 °C dry-bulb temperature and 10 °C wet-bulb temperature to state 2 of 35 °C dry-bulb temperature and 10 °C dew-point temperature. By the psychrometric chart, determine
(a) the enthalpy-humidity ratio, *q*′, for the process, and
(b) the sensible-heat ratio, SHR, for the process.
Notice that the SHR scale on the right side of the chart can be read with much greater accuracy than the one on the protractor to the left of the chart.

7.28 Saturated moist air at 15 °C and standard atmospheric pressure undergoes a process which has a sensible-heat ratio of SHR = 0.70. After the process, the dry-bulb temperature is 40 °C. By the psychrometric chart, determine the following at the end of the process:
(a) humidity ratio,
(b) dew-point temperature,
(c) relative humidity, and
(d) wet-bulb temperature.

SYMBOLS

A	Coefficient in Eq. (7.3).
c_p	Specific heat of moist air at constant pressure, $\mathrm{Btu/lbm}_a \cdot$ °F or $\mathrm{kJ/kg}_a \cdot$ °C.
$\bar{c}_p$	Average specific heat of moist air at constant pressure for a process, $\mathrm{Btu/lbm}_a \cdot$ °F or $\mathrm{kJ/kg}_a \cdot$ °C.
c_{pa}	Specific heat of dry air at constant pressure, $\mathrm{Btu/lbm}_a \cdot$ °F or $\mathrm{kJ/kg}_a \cdot$ °C.
c_{pw}	Specific heat of water vapor at constant pressure, $\mathrm{Btu/lbm}_a \cdot$ °F or $\mathrm{kJ/kg}_a \cdot$ °C.
H	Enthalpy, Btu or kJ; H_a for dry air; H_w for water or water vapor.
h	Specific enthalpy of moist air, $\mathrm{Btu/lbm}_a$ or $\mathrm{kJ/kg}_a$; h_a for dry-air; h_s for saturated moist air at t, $h_{as} = h_s - h_a$.
h_f	Specific enthalpy of liquid water, $\mathrm{Btu/lbm}_w$ or $\mathrm{kJ/kg}_w$.
h_g	Specific enthalpy of saturated water vapor, $\mathrm{Btu/lbm}_w$ or $\mathrm{kJ/kg}_w$.
$\bar{h}_g$	Average enthalpy of water vapor defined by Eq. (7.44), $\mathrm{Btu/lbm}_w$ or $\mathrm{kJ/kg}_w$.
h_{fg}^*	$h_g^* - h_f^*$, $\mathrm{Btu/lbm}_w$ or $\mathrm{kJ/kg}_w$.
h_{g0}	Enthalpy of saturated water vapor at 0 °F or 0 °C, $\mathrm{Btu/lbm}_w$ or $\mathrm{kJ/kg}_w$.
Δh_L	Latent change of specific enthalpy of moist air for a process, $\mathrm{Btu/lbm}_a$ or $\mathrm{kJ/kg}_a$.
Δh_S	Sensible change of specific enthalpy of moist air for a process, $\mathrm{Btu/lbm}_a$ or $\mathrm{kJ/kg}_a$.
h_w	Enthalpy of water added to moist air, $\mathrm{Btu/lbm}_w$ or $\mathrm{kJ/kg}_w$; $h_w = h_g$ for low-pressure water vapor.