UNDERSTANDING ACTIVITY ENGAGEMENT ACROSS WEEKDAYS AND WEEKEND DAYS: A MULTIVARIATE MULTIPLE DISCRETE-CONTINUOUS MODELING APPROACH

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ABSTRACT
This paper is motivated by the increasing recognition that modeling activity-travel demand for a single day of the week, as is done in virtually all travel forecasting models, may be inadequate in capturing underlying processes that govern activity-travel scheduling behavior. The considerable variability in daily travel suggests that there are important complementary relationships and competing tradeoffs involved in scheduling and allocating time to various activities across days of the week. Both limited survey data availability and methodological challenges in modeling week-long activity-travel schedules have precluded the development of multi-day activity-travel demand models. With passive and technology-based data collection methods increasingly in vogue, the collection of multi-day travel data may become increasingly commonplace in the years ahead. This paper addresses the methodological challenge associated with modeling multi-day activity-travel demand by formulating a multivariate multiple discrete-continuous probit (MDCP) model system. The comprehensive framework ties together two MDCP model components, one corresponding to weekday time allocation and the other to weekend activity-time allocation. By tying the two MDCP components together, the model system also captures relationships in activity-time allocation between weekdays on the one hand and weekend days on the other. Model estimation on a week-long travel diary data set from the United Kingdom shows that there are significant inter-relationships between weekdays and weekend days in activity-travel scheduling behavior. The model system presented in this paper may serve as a higher-level multi-day activity scheduler in conjunction with existing daily activity-based travel models.

Keywords: activity-travel demand, modeling weekly activity schedules, time allocation, multiple discrete-continuous probit model, weekday and weekend travel.

1. INTRODUCTION

Activity-travel demand model systems in use around the world simulate or predict activity-travel patterns and choices for a single day, typically a "representative" weekday (e.g., Bradley et al., 2010; Yagi and Mohammadian, 2010), although the notion of what constitutes a "representative" weekday may be debated. One-day travel diary data is typically used to estimate and calibrate model systems (Frazis and Stewart, 2012). More recently, there has been a growing interest in understanding broader, longer-term time use and activity patterns that span multiple days or weeks (see Jara-Díaz and Rosales-Salas, 2015). Activity and time use patterns not only vary among persons, but they also vary within persons between different days (Pas and Sundar, 1995; Zhou and Golledge, 2003), with estimates of within-person variability as high as 60 percent of the total variability in travel. A number of studies have made a case for why one-day data is insufficient to model activity travel patterns, and why variability analysis that examines activity time allocation and travel over a multi-day period is more appropriate to explain and forecast travel demand (Bhat and Koppelman, 1993; Liu et al., 2015; Jara-Díaz and Rosales-Salas, 2015). The use of one-day data to understand and model activity-travel patterns implicitly assumes that each day is independent of the other, and that people's activity-travel scheduling process is generally based on a one-day horizon. However, it is not possible to fully capture weekly cycles in activity-travel engagement by looking at single day activity-travel data. For example, a worker may focus on pursuing work activities during the weekdays and leave shopping/leisure pursuits for the weekend days.

From a system performance standpoint, the variation in individual activity-travel rhythms between weekdays and weekend days manifests itself in network traffic congestion patterns which, in turn, affect activity-travel rhythms in a cyclical fashion. For example, peak traffic congestion on weekdays often corresponds to morning and evening commute periods, while peak traffic on weekend days corresponds to the mid-day periods (Agarwal, 2004; Bhat and Gossen, 2004). Consistent with these variations across days of the week, weekend and weekday public transport options and services often differ. This further deepens the distinction between the weekday and weekend travel options, modal accessibility, and constraints. The differences in activity-travel patterns between different days of the week (especially weekdays and weekend days), coupled with rhythms of behavior depicting a multi-day periodicity, implies an inter-dependency in activity-travel patterns across days of the week that is likely engendered by activity-travel scheduling horizons that exceed a 24 hour period. The possible existence of a longer planning horizon for personal and household activity schedules has been recognized in the literature (Doherty, 2005), leading to notable data collection activities aimed at gathering information about people's activity-travel schedules over a period of a week or longer (Muthyalagari et al., 2001; Axhausen et al., 2002).

One of the challenges associated with developing and deploying multi-day activity-travel models is that there is very limited data on multi-day activity-travel behavior. While there are small scale data sets (Zhou and Golledge, 2003) that include information on activity-travel schedules over multiple days, the vast majority of regular household travel surveys (upon which activity-travel models are estimated) are limited to one-day activity-travel diary data. In an era of limited attention spans and declining survey participation and response rates, it has proven to be an immense challenge to collect information for multiple days due to respondent burden and reluctance (Singer and Presser, 2008). Even with new travel survey data collection methods that employ smart phones, GPS technologies, or passive tracking mechanisms, the collection of large sample data for multiple travel days remains a challenge due to cost considerations, technology limitations, or privacy concerns (National Research Council, 2007; Nitsche et al., 2014).

Even if the data collection challenges can be overcome, methodological challenges remain. In the activity-travel literature, a few studies have examined time use patterns over the course of an entire week to reflect longer term needs and rhythms (e.g., Lee and McNally, 2003). However, these studies do not explicitly capture weekday-weekend interactions in their modeling framework. The complementary relationships and competing trade-offs in activity scheduling and time use across days of the week are poorly understood, and methods to effectively model activity-travel scheduling processes over multiple days have proven to be elusive. This paper aims to address this gap by presenting a holistic and comprehensive model capable of modeling activity scheduling and time allocation behavior across days of the week, explicitly accounting for inherent complementary relationships and competing trade-offs. The model takes the form of a multivariate multiple-discrete continuous probit (MDCP) model system, in which two distinct MDCP components – corresponding to weekday and weekend activity-time allocation respectively – are stitched together in a multivariate framework. By tying the two MDCP components together in a joint modeling framework, the model is able to explicitly account for relationships in activity engagement between weekdays on the one hand and weekend days on the other. The model system presents a methodological basis for modeling week-long activity-travel schedules at a macro-level, but does not operate at a day-level to explicitly model activity-time allocation for each day of the week (this remains a future effort). Rather, the model in this paper is capable of depicting how the entire time budget encompassing all weekdays may be allocated for various activity pursuits; and similarly for the entire weekend time budget. The model system is estimated and its efficacy demonstrated through an application to a week-long travel diary data set collected in the United Kingdom in 2015. The data set is a typical large sample travel survey data set and ideally suited to study and model multi-day activity-travel engagement and time allocation behavior.

The remainder of this paper is organized as follows. The next section presents a brief discussion of considerations in modeling activity-travel behavior over a week-long time horizon. The third section presents an overview of the data and survey sample, the fourth section presents the modeling methodology, and fifth section offers details of the model estimation results. The sixth section compares and contrasts the prediction results of the joint model system with that of independent model systems (in which relationships in activity engagement between weekdays and weekend days are note accounted for). Discussion and concluding thoughts are offered in the seventh and final section.

2. CONSIDERING ACTIVITY-TRAVEL BEHAVIOR OVER A WEEK

Interest in understanding the variation in activity-travel patterns across multiple days has motivated considerable research in this domain. Jones and Clarke (1988) document the analytical as well as policy advantages of using multi-day data to understand the variability in travel behavior, and illustrate various graphical and numerical methods to measure such variability. Simma and Axhausen (2001) used the German Mobi*drive* (six-week travel diary) data to study relationships in travel behavior patterns between successive days of a week. They find that the activity-travel pattern of one day significantly impacts the activity-travel pattern of the subsequent day; they also identify key differences in travel behavior between weekdays and weekend days. Tarigan and Kitamura (2009) also used the Mobi*drive* data to analyze week-to-week variability in leisure trip making, and find that individuals with a higher frequency of leisure trips per week exhibit greater variability in such trips over different weeks.

Using data from a one-week GPS travel study conducted by the US Department of Transportation, Zhou and Golledge (2003) explored variability in weekday and weekend travel patterns. In addition to confirming that considerable differences in travel behavior exist between weekdays and weekends, they also report variability in travel patterns across weekdays (which are generally expected to have more stable travel patterns). Aguiléra et al. (2009) argue that the difference in activity-travel patterns between working and non-working days has increased over the past 20 years. They show that non-work trips have declined on working days and significantly increased on non-working days, with growing work-related constraints contributing to difficulty in allocating time for non-work trips on working days. Ziems et al. (2010) report that participation in out-of-home discretionary activities on weekend days provides greater utility than on weekdays; such differences could naturally lead to or be manifestations of differential activity engagement among days of the week. Lee and McNally (2003) developed a computerized survey instrument that records the evolution of activity schedules of a household from plans and intentions to actual outcomes, over the period of a week. Their work shows that activities of longer duration are likely to be planned and scheduled multiple days in advance, while activities of short duration are inserted opportunistically at short notice. Other studies provide ample evidence that weekend activity-travel patterns are substantially different than weekday patterns (Yamamoto and Kitamura, 1999; Lockwood et al., 2005; Craig and Mullan, 2010).

Despite considerable progress in the development of activity-based microsimulation models of travel behavior, it is clear that research efforts have largely focused on examining either weekday or weekend travel behavior without accounting for deep inter-relationships that exist between the two types of days. For example, Garikapati et al. (2014) estimate a multiple discrete-continuous model of activity participation and time allocation for weekday tours, while Born et al. (2014) apply a similar approach to model weekend discretionary activity participation and duration. Arentze and Timmermans (2009) present a heuristic multi-person, multi-day activity generation model that is capable of simulating household activity agendas for a multi-day period (say a week) and allocating tasks among members of the household. Ho and Mulley (2013) developed separate weekday and weekend models for joint household travel arrangements and mode choices. While their models are comprehensive in that they are able to account for differences in households interactions between weekdays and weekends (separately), they do not account for variability of travel patterns within a week and the inter-dependencies in activity-travel patterns between weekdays and weekend days. This research effort aims to fill a major gap in understanding and modeling weekly activity-time allocation behavior by presenting a joint multivariate multiple discrete-continuous probit (MDCP) model that captures relationships in the allocation of activities and time between weekdays and weekend days.

3. DATA AND SAMPLE DESCRIPTION

The data used in this study is derived from the 2015 United Kingdom (UK) National Travel Survey (UK Department for Transport, 2016). This survey is part of a continuous data collection effort that began in July 1988, and is designed to provide data on trends in personal travel behavior. The survey gathered information from 7,564 households (18,071 individuals) in England (Scotland and Wales were not included) through interviews and a seven-day travel diary. Interviews were conducted face-to-face with all household members to collect data on person and household characteristics as well as vehicles used. The data set includes information such as household size, household income, household vehicles, person work status, and residential location area type. Household members then recorded all of their trips over a seven-day period in the travel diary.

Only personal travel, i.e., travel for private purposes, work, or education, within Great Britain is included in the survey data. Travel for leisure purposes is included, but trips which are themselves recreational are not included in the data. Distance traveled, stage of the trip, trip mode(s), and trip purpose were recorded for each trip. Individuals were asked to designate a single activity purpose at the destination end of each trip, based on the purpose for which most time was spent.

An extensive data cleaning and filtering process was applied to derive the analysis data set for this study. Person level activity scheduling and time allocation patterns were chosen as the unit of analysis. Choosing person-level (as opposed to household-level) time allocation patterns provides a way to estimate and interpret the model results in an intuitive fashion. For example, aggregating work time allocation over workers and non-workers in the household and taking that as the weekly work time allocation for the households would seem unintuitive and incoherent for model estimation and interpretation. The analysis in this study is restricted to understanding multiday travel behavior and activity allocation for working adults 18 years and over, in recognition of the structural differences that may exist between workers and non-workers in how they allocate activities and time across weekdays and weekend days. While the focus of this paper is on capturing work-leisure cycles of adult workers, future research on this topic should focus on developing similar models for non-workers and student segments, so that the time allocation of patterns of all the members of a household can be captured.[1] All individuals less than 18 years of age and all non-workers regardless of age were removed from the analysis data set, yielding a sample of 8,334 adults. Further filters were applied to eliminate individuals who indicated that they telecommute. Including telecommuters in the analysis could lead to possible under estimation of time allocation to work, and hence this segment was excluded from analysis.[2] Eliminating telecommuters from the dataset resulted in a reduction in the usable sample to 6,323 adults. After further cleaning to eliminate records with incomplete or erroneous diaries or missing data for variables of interest used in the model specification of this study (i.e., socio demographic information and trip related information), the final analysis sample comprised 4,543 adult workers. For the final analysis sample, necessary checks were carried out to ensure continuity in trip/activity reporting. Weekdays were defined as beginning on Monday at 12:00 AM and ending at 11:59 PM on Friday. Weekend days were defined as beginning on Saturday at 12:00 AM and ending at 11:59 PM on Sunday. Trip records were transformed into activity episodes to derive time allocation

[1] It is important to account for intra-household interactions when modeling activity scheduling and time allocation decisions of individuals. This topic is beyond the scope of the current research, but constitutes an excellent direction to extend the research presented in this paper.

[2] The average work duration of individuals from the analysis sample is 32.39 hours on weekdays and 8.68 hours on weekends. For the telecommuter sample, these numbers are 22.61 hours on the weekdays and 3.15 hours on the weekends (owing to reporting of work duration as 'in-home activity' when these individuals work from home).

across weekdays and weekend days for various activity (trip) purposes.[3] All time was allocated into one of seven activity purposes:[4]

- Work
- Education (go to school)
- Escort (pick-up/drop-off passenger)
- Shopping (buying groceries or other goods, and all visits to shops, even if there was no intention to purchase anything)
- Personal business (visits to services, hair dressers, laundry and dry cleaning services, banks, estate agents, libraries, churches, medical consultation or treatment, eat/drink alone)
- Social-recreation (visit friends, sports, social activities, volunteer work, eat/drink with friends, entertainment)
- In-home (cannot include work-at-home because only individuals who indicated that they never work at home were retained in the final analysis sample)

In the above purpose definitions, we include work as an activity purpose. This recognizes that, in a long-term (weekly planning) framework, individuals pre-decide on how much work they would like to perform on weekdays (owing to constraints or personal obligations) and what part they want to leave out for the weekend.

Table 1 presents an overview of the sample. The sample is about evenly split between females and males. About 10 percent of the sample is 60 years or over, with the remainder of the sample rather uniformly distributed across the other younger age groups. Given that this is an exclusively adult worker sample, it is not surprising that only 10.2 percent of the sample reports being a student.

[3] Travel time is excluded from the time budget based on the activity-based travel demand modeling perspective (used on almost all activity time-use studies) that individuals determine their activity needs and time-use in activities, and then travel features during the step of scheduling the activities to fulfill the desired activity agenda. Also to be noted is that we undertook a descriptive analysis on the average participation rates and time use patterns of the analysis sample for each day of the week. Results of this analysis are documented in an online supplement to this paper (Astroza et al., 2018; see http://www.caee.utexas.edu/prof/bhat/ABSTRACTS/MultivariateMDCP/OnlineSupplement.pdf). The descriptive analysis (presented in Tables A.1 and A.2 in the supplement) do suggest that while considerable differences exist in time allocation patterns between weekdays and weekend days, there are no major variations (at least at an aggregate level) for within weekdays and within weekend days activity participation and time allocation patterns. This is, of course, a characteristic of this specific dataset and cannot be generalized. A system that models activity-time allocation for each day of the week, while accounting for interdependencies across various days remains a future effort.

[4] The activity categories were chosen such that they provide sufficient amount of detail and variety in defining activities, while ensuring adequate sample sizes in each of the categories for model estimation. Also, the activity purpose classification adopted in this paper has been used in many other contexts (Bhat et al., 2016; Jara-Díaz et al., 2016; Astroza et al., 2017). Using a similar activity classification would help compare and contrast the results of this analysis with existing work on the topic of activity time allocation patterns.

TABLE 1 Description of Survey Sample Used for Analysis

Person Characteristics (N = 4,543 Adult Workers)		**Household Characteristics** (N = 3,104 Hhlds)	
Variable	*Value*	*Variable*	*Value*
Gender		Household Income	
Female	51.0%	Less than £25K	22.2%
Age		£25K to <£50K	38.0%
18-29 years	21.1%	£50K or over	39.8%
30-39 years	21.3%	Presence of Children	
40-49 years	23.8%	No child	62.4%
50-59 years	23.4%	Average Household Size	3.03
60 years and over	10.4%	Average Household Vehicles	1.61
Student Status			
Student	10.2%		
Educational Attainment			
Not a college graduate	73.5%		
College graduate or higher	26.5%		
Work Status			
Full-time worker	73.1%		
Work Place Location			
Same location everyday	83.0%		
Frequency of Bus Use			
At least once a week	20.4%		
Less than once a week	79.6%		
Frequency of Bicycle Use			
At least once a week	13.0%		
Less than once a week	87.0%		
Frequency of Walking			
At least once a week	67.3%		
Less than once a week	32.7%		

Nearly three-quarters of the sample are full-time workers and 83 percent of the sample reports working at the same location every day. In general, a high percent of the sample (67.3 percent) report walking at least once a week. About one-in-five individuals is a frequent bus user; just about 13 percent are regular bicycle users. The household income distribution shows that nearly 40 percent of the sample is in the higher income bracket of £50,000 or over. The average household size is 3.03, with 62.4 percent of the sample reporting no children in the household. The average number of vehicles per household is 1.61. To ensure that eliminating erroneous records did not alter the overall composition of the survey sample, a descriptive analysis was conducted for the sample excluded on grounds of erroneous data or missing information. The descriptive statistics of the sample with missing/erroneous information is documented in Table A.3 of the online supplement (Astroza et al., 2018). From the analysis, it was observed that the socio-demographic characteristics of individuals who were removed from the analysis are quite similar to those of the sample of individuals used in the analysis.

Table 2 presents the weekday-weekend activity and time allocation pattern for the sample. The table shows the percent of individuals participating in each activity separately on weekdays and weekend days. For each of the two day-types, the table also shows the mean duration (in hours) that those who participated in the activity dedicated to each activity purpose. Finally, from among those who participated in a specific activity purpose on weekdays (weekend days), the table shows the percent who also participated in that specific activity purpose on weekend days (weekdays).[5]

TABLE 2 Activity Participation and Time Allocation Pattern of the Analysis Sample

Activity Purpose	Weekdays			Weekend		
	Participation (%)	Mean duration (hrs) among those participating	Among those participating, % that also participate on weekend	Participation (%)	Mean duration (hrs) among those participating	Among those participating, % that also participate on weekdays
Work	82.6	32.39	26.6	28.6	8.68	76.8
Education	1.8	13.15	47.0	1.2	7.92	70.9
Escort	25.7	2.02	40.8	15.8	1.43	66.3
Shopping	59.8	2.28	37.8	33.0	1.69	68.5
Personal business	26.3	2.21	22.0	13.2	1.72	44.0
Recreation	65.9	9.15	51.5	44.0	5.81	77.1
In-home	100.0	64.45	100.0	100.0	26.65	100.0

Consistent with the fact that this is a worker sample, 82.6 percent reported participating in work during weekdays and 28.6 percent report participating in work on weekend days. Nearly 60 percent report participating in shopping on at least one weekday; the corresponding percentage for weekend days is 33 percent. About 66 percent of the sample engaged in a social-recreational activity on at least one weekday; the corresponding percentage for weekend days is 44 percent. Over the course of the five weekdays, those who reported at least one work activity spent, on average, 32.4 hours for work. Only 1.8 percent participated in education activities during the weekdays; the few who participated spent, on average, 13.2 hours on this activity. Other activities get an allocation of a little over two hours over the course of five weekdays, although social-recreational activities get a higher time allocation at 9.2 hours on average. A substantial percent of individuals who participated in an activity purpose on weekdays also participated in the same activity on weekend days. As expected, the corresponding percentages are higher (in the last column) for weekend days, consistent with the fact that weekends account for two days of the week but weekdays account for five days of the week. Overall, the sample depicts characteristics that are consistent with expectations with no anomalies, rendering the data set suitable for weekly activity-time use analysis.

[5] To check if activities/trips are under-reported as the survey progressed (due to survey fatigue), a descriptive analysis of the average number of activities reported per day from the first day to the seventh day of the travel diary was undertaken. Results of the analysis are documented in Table A.4 of the online supplement (Astroza et al., 2018). From that table, it may be observed that the number of activities reported per day do not show any major variations from day 1 to day 7 of the survey data, suggesting no fatigue in the estimation sample.

4. MODELING METHODOLOGY

To understand the time allocation among weekdays and weekend days, a multivariate multiple discrete-continuous probit (MDCP) modeling approach is adopted in this study. This section provides a brief overview of the methodology; a more detailed description of the modeling framework, identification considerations, and the estimation method may be found in Astroza et al. (2015).

In the proposed model, there are two dependent variables of the multiple discrete-continuous (MDC) type (time allocation of weekdays and time allocation of the weekend) and g is the index for these variables ($g=1$ for weekdays time allocation and $g=2$ for weekend time allocation). For each MDC variable, there are seven alternatives corresponding to the different activity purposes (work, education, escort, shopping, personal business, recreation, in-home) and k is the index for activity purpose. The order of alternatives does not matter, except that for presentation purposes, the "in-home" activity is placed at the end (the seventh alternative). Individuals are assumed to maximize their time utility subject to a binding time budget constraint:

$$\max U_g(\boldsymbol{t}_g) = \sum_{k=1}^{6} \frac{\tau_{gk}}{\alpha_{gk}} \psi_{gk} \left(\left(\frac{t_{gk}}{\tau_{gk}} + 1 \right)^{\alpha_{gk}} - 1 \right) + \frac{1}{\alpha_{g7}} \psi_{g7} (t_{g7})^{\alpha_{g7}} \tag{1}$$

$$s.t. \sum_{k=1}^{7} t_{gk} = T_g \,,$$

where the utility function $U_g(\boldsymbol{t}_g)$ is quasi-concave, increasing and continuously differentiable, $\boldsymbol{t}_g$ is the time investment vector of dimension 7×1 with elements t_{gk} $(t_{gk} \geq 0)$, τ_{gk}, α_{gk}, and ψ_{gk} are parameters associated with activity purpose k, and T_g represents the time budget to be allocated among the seven activity purposes. T_1 is 120 hours (the total hours of the 5 weekdays) minus the travel time and T_2 is 48 hours (the total hours of the weekend) minus the travel time. Travel time is excluded from the time budget because the analysis focuses exclusively on activity time allocations across weekdays and weekend days. It should be noted here that excluding travel times from the weekday and weekend time budgets will lead to varying time budgets for different individuals in the dataset (as opposed to a fixed time budget of 120 hours and 48 hours respectively for weekdays and weekends for all individuals in the dataset). While varying time budgets do not impose any additional constraints in the model estimation process, additional steps need to be incorporated while applying the model in forecasting mode. If the budget varies from one unit to the other (in this case, individuals), separate budget models need to be estimated which can be provided as input to the forecasting process. For example, You et al. (2014) have estimated a household mileage budget prediction model that is provided as an input to a vehicle fleet composition model system (developed as an MDCEV model). Pinjari et al. (2016) proposed a stochastic frontier approach to relax the 'fixed budget' assumption in the MDCEV model system.

The utility function form in Equation (1) allows corner solutions (*i.e.*, zero consumptions) for the first six out-of-home activity purposes through the parameters τ_{gk}, which allow corner solutions for these alternatives while also serving the role of satiation parameters ($\tau_{gk} > 0$: $k = 1, 2, ..., 6$). On the other hand, the functional form for the final activity purpose ensures that some time is invested for the "in-home" activity; this is usually referred to as an *essential outside good* in the microeconomics literature (see Bhat, 2008). The role of α_{gk} is to capture satiation effects, with a smaller value of α_{gk} implying higher satiation for activity purpose k. ψ_{gk} represents the stochastic baseline marginal utility; that is, it is the marginal utility at the point of zero time investment for alternative k. τ_{gk} and α_{gk} influence satiation, but in different ways: τ_{gk} controls satiation by translating consumption quantity, while α_{gk} controls satiation by exponentiation of the consumption quantity. Since it is difficult to empirically disentangle the effects of τ_{gk} and α_{gk} separately, Bhat (2008) recommends estimating a τ-profile (in which $\alpha_k \to 0$ for all alternatives, and the τ_k terms are estimated) and an α-profile (in which the τ_k terms are normalized to the value of one for all alternatives, and the α_k terms are estimated), and choosing the profile that provides a better statistical fit.

Stochasticity is introduced through the baseline marginal utility function, ψ_{gk}, as:

$$\psi_{gk} = \exp(\boldsymbol{\beta}'_g \boldsymbol{z}_{gk} + \xi_{gk}) \text{ or } \overline{\psi}_{gk} = \ln(\psi_{gk}) = \boldsymbol{\beta}'_g \boldsymbol{z}_{gk} + \xi_{gk}, \tag{2}$$

where $\boldsymbol{z}_{gk}$ is a A_g-dimensional vector of attributes that characterizes alternative k (including a constant for each alternative except one, to capture intrinsic preferences for each alternative relative to a base alternative); $\boldsymbol{\beta}_g$ is a consumer-specific vector of coefficients (of dimension $A_g \times 1$) and ξ_{gk} captures the idiosyncratic (unobserved) characteristics that impact the baseline utility of alternative k. Initially, $\boldsymbol{\beta}_g$ was considered to be a vector of random parameters to account for potential taste heterogeneity. However, this taste heterogeneity was tested and found insignificant for variables in the model specifications of this study (mainly because all $\boldsymbol{z}_{gk}$ variables are socio-demographics and hence individual-specific; it is difficult to estimate random coefficients on such individual-specific variables and, in fact, can lead to identification problems; and taste heterogeneity is typically considered only for variables that vary across alternatives).

The error term $\boldsymbol{\xi}_g [= (\xi_{g1}, \xi_{g2}, ..., \xi_{g7})']$ is distributed multivariate normal. That is, $\boldsymbol{\xi}_g \sim \text{MVN}_7(\mathbf{0}_7, \boldsymbol{\Lambda}_g)$, where $\text{MVN}_K(\mathbf{0}_K, \boldsymbol{\Lambda})$ indicates a K-variate normal distribution with a mean vector of zeros denoted by $\mathbf{0}_K$ and a covariance matrix $\boldsymbol{\Lambda}$. Bhat (2008) shows that only differences in the logarithm of the baseline utilities matter, not the actual logarithm of the baseline utility values. Thus, it is possible to work with the logarithm of the baseline utilities of the first six alternatives, and normalize the logarithm of the baseline utility for the last alternative to zero. Then, only the covariance matrix, say $\breve{\boldsymbol{\Lambda}}_g$ of the error differences $\breve{\xi}_{gk} = (\xi_{gk} - \xi_{g7})$ is estimable, and not the covariance matrix $\boldsymbol{\Lambda}_g$ of the original error terms. In addition, a scale normalization is needed and the element of $\breve{\boldsymbol{\Lambda}}_g$ in the first row and first column is fixed to the value of one. To facilitate easy interpretation of the covariance matrix $\breve{\boldsymbol{\Lambda}}_g$, it is assumed that the error term of the "outside" alternative ("in-home" activity), ξ_{g7}, is independent of the error terms of the "inside" alternatives, ξ_{gk} $(k = 1, 2, ..., 6)$. With this assumption, each covariance matrix element of $\breve{\boldsymbol{\Lambda}}_g$ can then immediately be interpreted as an indicator of the extent of variance and covariance in the utilities of the inside alternatives.

To estimate both time allocations jointly (weekdays and weekend days), jointness is generated in the unobserved portion of the utility of different MDC variables; define $\breve{\boldsymbol{\xi}}_g = (\breve{\xi}_{g1}, \breve{\xi}_{g2}, ..., \breve{\xi}_{g6})'$ and $\breve{\boldsymbol{\xi}} = (\breve{\xi}_1', \breve{\xi}_2')'$ of size 12×1. Then the distribution of the vector $\breve{\boldsymbol{\xi}}$ can be written as:

$$\breve{\boldsymbol{\Lambda}} = \begin{bmatrix} \breve{\boldsymbol{\Lambda}}_1 & \breve{\boldsymbol{\Lambda}}_{12} \\ \breve{\boldsymbol{\Lambda}}_{12}' & \breve{\boldsymbol{\Lambda}}_2 \end{bmatrix} \tag{3}$$

where $\breve{\boldsymbol{\Lambda}}_1$ captures the covariance between errors of the weekdays time allocation, $\breve{\boldsymbol{\Lambda}}_2$ captures the covariance between errors of the weekend time allocation, and $\breve{\boldsymbol{\Lambda}}_{12}$ captures the dependence between the errors of weekdays time allocation and weekend time allocation. The proposed model can be extended to incorporate a more disaggregate day of the week categorization (Monday, Tuesday…, Sunday) in a straightforward way by including a separate MDC model for each day of the week, with the error covariance matrix extended to capture variations across different days of the week. The model is estimated using the maximum approximated composite marginal likelihood (MACML) approach (Bhat, 2011).

It is worth noting here that the model proposed in this paper focuses exclusively on identifying patterns and tradeoffs involved in scheduling and allocating time to various activities across weekdays and weekend days. Therefore, only a temporal constraint (total time minus the travel time to various activities) is considered in the modeling framework. However, it could be argued that participation in some activities can be constrained not only by temporal considerations but also monetary resources. For example, some recreational episodes would not be possible if individuals are not willing to spend the money to participate in that activity (movie ticket, restaurant check, museum entrance, etc.). When consumption (or monetary) constraints are not considered in a time allocation model, the different effects that monetary constraints could have on people’s time allocation decisions simply become a part of the unobserved effects. This has important implications when the model is applied in prediction mode, in that the model might forecast activity-time allocation profiles that might not be feasible for specific individuals (i.e., the profile violates the constraints that were not explicitly considered).

Addressing this issue, many studies in the time use domain have discussed the importance of considering multiple constraints (associated with time, money, and capacity) in the understanding time allocation decisions. Some studies have recognized that utility is derived both from allocation of time to activities, and consumption of goods (Konduri et al., 2011; Jara-Díaz and Astroza, 2013; Astroza et al., 2017) and others have posited that both time and monetary constraints govern activity participation and goods consumption decisions (Castro et al., 2012; Jara-Díaz et al., 2016). Although the model framework presented in this paper could be extended to allow multiple variable types (such as a combination of two MDC variables, one for time allocation and another for goods consumption), the possible interactions between goods consumption and time allocation (and between monetary and temporal constraints) would entail considerable reformulations to the model and are outside the scope of this paper. Such an extension to the model framework in the context of weekday and weekend joint time allocation models is a fruitful direction for future research.

While this solution for jointly modeling weekday and weekend time allocation patterns presented in this paper (stitching two MDCP models together) is one of the ways to approach this problem, it is certainly not the only way. Another way to approach the joint modeling of weekday and weekend time allocation decisions is using the generalized heterogeneous data model

(GHDM) proposed by Bhat et al. (2016). The GHDM allows for joint estimation of multiple MDC variables along with continuous, count, ordinal, and nominal variables. However, the GHDM model structure requires assumptions about the latent constructs, and can be computationally intensive. Since the current context does not include multiple types of dependent variables, the joint MDCP model framework was adopted to gain efficiencies in model run times.

5. MODEL ESTIMATION RESULTS

This section presents a detailed overview of model estimation results. A number of model specifications were tested, and variables were carefully introduced or excluded based on considerations of behavioral intuition and statistical significance. The τ -profile was found to offer a better statistical fit in this study. Hence, only the τ_{gk} parameters, $\boldsymbol{\beta}_g$ parameter vector (baseline utility parameters), and Vech($\breve{\boldsymbol{\Lambda}}$) are estimated. Vech($\breve{\boldsymbol{\Lambda}}$) represents the vector of upper triangle elements of the non-zero and non-fixed elements of $\breve{\boldsymbol{\Lambda}}$. Multivariate MDCP model estimation results for weekday time allocation are presented in Table 3, while model results for weekend day time allocation are presented in Table 4. It should be noted that the weekday and weekend model components comprise a single model system that is estimated jointly to reflect trade-offs and complementary relationships that may be at play in weekly activity-time use allocation processes. The log-likelihood at convergence for the model system is -31,891.01; the log-likelihood corresponding to a constants-only model is -50,745.23, yielding an adjusted ρ^2(c) of 0.3645. In addition to estimating a joint weekday-weekend model, an independent model that ignored error covariances across days was also estimated. This model exhibited a log-likelihood value at convergence equal to -32,168.37. Comparing the joint and independent models yields a log-likelihood ratio χ^2 statistic of 92.72 with 10 degrees of freedom, which is statistically significant at any level of confidence. This suggests that the estimation of the joint model is warranted in the context of explaining weekly activity-time use allocation behavior (on the part of adult workers).

Model estimation results presented in Table 3 are largely intuitive and consistent with expectations.[6] As expected, individuals 60 years and over allocate less time to work on weekdays than other age groups, with all other age groups showing a perfectly intuitive and increasing progression in coefficient values. Students dedicate less time to work, as expected, consistent with their school obligations. They do allocate more time to education activity.

[6] The estimation results in Table 3 (except for the last row labeled “Satiation parameters”) correspond to the baseline utility parameters. In the MDC model, a positive coefficient on a variable for a specific purpose *k* (relative to the base purpose) implies that the variable increases the utility of purpose *k* (relative to the base purpose) for discrete participation as well as increases the utility corresponding to the continuous participation (for a given satiation parameter). In terms of the satiation parameters, as discussed earlier, a smaller magnitude for purpose *k* (for a given baseline utility) implies higher satiation (lower duration) for activity *k*.

TABLE 3 Estimation Results of the Multivariate MDCP Model – Weekday Time Allocation

Variables	Work		Education		Escort		Shopping		Personal Business		Recreation	
	Coeff.	t-stat	Coeff.	t-stat	Coeff.	t-stat	Coeff.	t-stat	Coeff.	t-stat	Coeff.	t-stat
Baseline utility parameters												
Constants	-1.982	-17.23	-7.215	-22.39	-2.891	-15.37	-2.005	-16.99	-2.803	-20.19	-2.104	-22.11
Individual characteristics												
Gender (base: male)												
Female	--	--	--	--	0.257	1.99	0.893	3.27	--	--	-0.563	-3.19
Age range (base: 18-29 years old)												
30-39 years old	0.101	2.36	--	--	0.154	2.43	0.204	2.63	--	--	-0.103	-2.88
40-49 years old	0.257	2.99	--	--	0.269	3.22	0.100	1.98	--	--	-0.221	-3.11
50-59 years old	0.301	3.75	-0.276	-3.22	0.203	1.98	-0.145	-2.00	0.104	2.24	-0.325	-3.17
60 or older	-0.152	-2.64	-0.651	-2.01	--	--	-0.264	-2.35	0.175	2.33	-0.378	-2.90
Student status (base: non-student)												
Student	-0.275	-3.86	1.031	9.27	--	--	--	--	--	--	0.107	2.25
Educational attainment (base: no degree)												
Has a college or graduate degree	0.651	4.20	0.164	3.20	--	--	--	--	0.320	3.94	0.355	2.10
Work status (base: full-time worker)												
Part-time worker	-0.265	-3.16	--	--	0.274	2.11	0.274	2.44	--	--	--	--
Household demographics												
Household income (base: high income)												
Low income	0.239	4.09	--	--	-0.264	-3.22	-0.196	-3.64	-0.175	-2.31	-0.388	-2.54
Medium income	0.104	3.65	--	--	-0.189	-3.73	-0.115	-2.96	-0.096	-1.97	-0.174	-3.22
Presence of children (base: no children)												
At least one child in the household	-0.186	-2.64	--	--	0.671	6.55	0.363	3.26	--	--	0.264	4.87
Household size	0.364	3.15	--	--	0.244	4.32	0.231	4.11	--	--	0.438	3.85
Number of vehicles	--	--	--	--	0.305	3.27	--	--	--	--	0.324	2.13
Travel behavior												
Frequency of bus use (base: infrequent user)												
Frequent user	--	--	--	--	-0.103	-2.00	--	--	--	--	--	--
Frequency of bicycle use (base: infrequent user)												
Frequent user	--	--	--	--	-0.205	-2.66	--	--	--	--	0.163	2.74
Frequency of walking (base: frequent pedestrian)												
Infrequent pedestrian	--	--	--	--	0.124	3.21	0.239	3.28	--	--	-0.254	-3.16
Satiation parameters*	12.644	8.99	1.033	9.15	1.754	6.55	1.677	5.63	1.864	7.33	4.276	9.47

--: Not significant.

*: Since all individuals in the sample participate in the in-home activity, no satiation parameter is estimated for that activity purpose.

Females are found to dedicate more time to escort and shopping activities, but less time to recreational activities, consistent with traditional gender roles in which women assume a greater share of household maintenance (Mensah, 1995; Garikapati et al., 2014). Younger individuals are less inclined to allocate time to escort activities as they are not likely to have household members who need chauffeuring; on the other hand, they dedicate more time to social-recreation activities. Older individuals dedicate less time to recreation, presumably due to household obligations and greater levels of work commitment. Education is positively associated with time allocation to work and school, and part-time workers dedicate less time to work – but more time to fulfilling other household obligations such as escort and shopping. Those in low income households dedicate more time to work on weekdays, presumably because they have to work long hours to make sustainable income. Individuals in higher income households dedicate more time to shopping and discretionary activities than those in lower income households, consistent with their monetary resource availability (Garikapati et al., 2014). Vehicle availability positively influences time allocation to escort and recreation activities, consistent with the notion that the availability of a vehicle facilitates chauffeuring individuals and accessing recreational opportunities to a greater degree. Individuals who use alternative modes of transportation spend less time on weekdays for escorting activities; as expected, those who frequently bicycle and walk are more likely to dedicate time for recreational activities (consistent with their active lifestyle). Satiation parameters show values consistent with the time allocation patterns depicted in Table 2.

Results of the weekend activity-time allocation model are presented in Table 4. An interesting difference between the weekday and weekend models is that part-time workers dedicate more time to work on weekend days compared to full-time workers, suggesting that part-time workers are likely to have alternative work schedules that involve working on weekends. Also, those who have different work locations are found to dedicate more time to work on weekend days; again, it is likely that workers with multiple work locations have alternative work schedules that involve greater work obligations on weekends. Females exhibit a similar pattern on weekends as weekdays (more escort and shopping, and less recreation). Younger individuals are more likely to dedicate time to shopping and recreation than their older counterparts, presumably because of fewer household constraints. Lower income individuals dedicate less time to all maintenance and discretionary activities when compared with high income individuals, which is consistent with their need to work longer hours and the monetary constraints they face. The presence of children is associated with a greater degree of time allocation to non-work activities on weekend days. Frequent bicycle and walk mode users are more likely to dedicate time to recreational activities on weekend days (similar to weekdays and once again consistent with their more active lifestyle).

In the weekend model, a few weekday activity participation variables were found to be statistically significant (note that the model continues to be a joint model of weekday and weekend activity participation and time-use; these weekday effects on the weekend time-use are after controlling for error correlations between the weekday and weekend time allocations). These endogenous variable effects illustrate the trade-offs and complementary relationships embedded in weekly activity-time allocation behavior, thus supporting the need to move towards multiday activity-travel demand modeling to better replicate what happens on any given day of the week.

TABLE 4 Estimation Results of the Multivariate MDCP Model – Weekend Time Allocation

Variables	Work		Education		Escort		Shopping		Personal Business		Recreation	
	Coeff.	t-stat	Coeff.	t-stat	Coeff.	t-stat	Coeff.	t-stat	Coeff.	t-stat	Coeff.	t-stat
Baseline utility parameters												
Constants	-1.274	-22.75	-4.673	-19.25	-1.755	-12.00	-1.650	-12.11	-1.910	-17.22	-1.622	-15.06
Individual characteristics												
Gender (base: male)												
Female	--	--	--	--	0.136	2.01	0.318	2.18	--	--	-0.105	-2.74
Age range (base: 18-29 years old)												
30-39 years old	0.125	3.16	--	--	0.092	2.40	0.121	3.10	--	--	--	--
40-49 years old	0.187	2.75	--	--	0.187	3.76	0.096	2.04	0.175	3.11	--	--
50-59 years old	0.227	2.86	-0.154	-2.78	0.226	2.10	-0.100	-2.11	0.206	3.75	-0.145	-2.88
60 or older	-0.086	-1.97	-0.398	-2.35	--	--	-0.187	-2.20	0.287	2.98	-0.186	-3.76
Student status (base: non-student)												
Student	--	--	1.255	8.11	--	--	--	--	--	--	--	--
Educational attainment (base: no degree)												
Has a college or graduate degree	--	--	--	--	--	--	--	--	0.111	4.85	0.186	3.27
Work status (base: full-time worker)												
Part-time worker	0.367	4.28	--	--	--	--	--	--	--	--	--	--
Work place location (base: same location everyday)												
Different location	0.461	2.86	--	--	--	--	--	--	--	--	--	--
Household demographics												
Household income (base: high income)												
Low income	0.239	4.09	--	--	-0.305	-2.86	-0.181	-3.96	-0.287	-3.40	-0.578	-3.89
Medium income	0.104	3.65	--	--	-0.275	-4.21	-0.107	-3.22	-0.123	-2.87	-0.386	-4.16
Presence of children (base: no children)												
At least one child in the household	-0.186	-2.64	--	--	0.723	5.87	0.210	3.61	--	--	0.411	2.65
Household size	0.364	3.15	--	--	0.266	5.11	0.126	3.12	--	--	0.127	2.64
Number of vehicles	--	--	--	--	0.426	4.98	--	--	--	--	0.186	2.06
Travel behavior												
Frequency of bus use (base: infrequent user)												
Frequent user	--	--	--	--	-0.218	-2.45	--	--	--	--	--	--
Frequency of bicycle use (base: infrequent user)												
Frequent user	--	--	--	--	--	--	--	--	--	--	0.210	3.11
Frequency of walking (base: frequent pedestrian)												
Infrequent pedestrian	--	--	--	--	0.205	2.75	--	--	--	--	-0.109	-2.86
Weekdays time allocation												
Participates in work during weekdays	-0.764	-4.86	--	--	--	--	--	--	--	--	--	--
Participates in shopping during weekdays	--	--	--	--	--	--	-0.185	-3.26	--	--	--	--
Participates in personal business during weekdays	--	--	--	--	--	--	--	--	-0.186	-2.76	--	--
Satiation parameters*	4.234	10.11	3.087	10.90	1.327	4.18	1.154	6.97	1.344	5.86	2.996	11.97

--: Not significant; *: Since all the individuals in the sample participate in the in-home activity, no satiation parameter is estimated for that activity purpose.

In this joint model, weekday work time allocation negatively impacts weekend work activity time allocation; weekday shopping time allocation negatively impacts weekend shopping time allocation; and similarly, weekday personal business activity time allocation negatively impacts weekend personal business activity time allocation. These three trade-offs between weekday and weekend time allocation are found to be significant. No other relationships were found statistically significant within the context of this data set. Also, the model structure where weekday time allocation variables entered the weekend time allocation model component provided a better fit than the model where weekend time allocation variables were entered into the weekday time allocation model (only recursive effects are identified in joint limited dependent variable models; see Bhat, 2015).

The error covariance matrix is presented in Table 5. The elements in the table are statistically significant at the 5% level of significance, unless otherwise stated (with a * sign). Interestingly, the diagonal blocks of the matrix (corresponding to the covariance within alternatives for each of the weekday and weekend MDC models) were not significantly different from diagonal values of one and covariances of 0.5. Thus, these are fixed in our estimation. The implication is that the original error terms $\xi_{g1},\xi_{g2},...,\xi_{g7}$ of alternatives within each MDC model are identically distributed and independently distributed (IID) of each other (for each of g=1 corresponding to weekdays time allocation and g=2 corresponding to weekend time allocation). The reader will note that this is so because the covariance matrix in Table 5 corresponds to the difference in error terms $\breve{\xi}_{g1},\breve{\xi}_{g2},...,\breve{\xi}_{g6}$, where $\breve{\xi}_{gk}=(\xi_{gk}-\xi_{g7})$ for k=1,2,…6. And for identification, we have made the innocuous assumption that the variance of ξ_{g7}=0.5. Then, if all the error terms $\xi_{g1},\xi_{g2},...,\xi_{g7}$ are IID, the result is the pattern obtained along the diagonal blocks of Table 5. This is a result that just happens to be in our case, and we would not know if this is the case or not unless we estimated a more general model that allows the error terms within each MDC model to be non-identical and correlated. From the results presented in Table 5, it can also be observed that there are a number of significant error correlations across overall weekday time-use and overall weekend day time-use (see the off-diagonal block of the matrix). Focusing on the error covariances across day types, several interesting relationships emerge. For example, the error covariance between weekend work and weekday work is negative, suggesting that common unobserved attributes that increase work time allocation on weekends are likely to decrease work time allocation on weekdays. An individual who seeks a flexible schedule or has unobserved obligations or constraints on weekdays may seek to dedicate more time for work on weekends rather than weekdays. Alternatively, another segment of workers who would like to stick to the "work week" schedule might prefer to work more on weekdays and avoid working (or work less) on weekend days. In case of education, a positive error covariance is seen between unobserved attributes of weekend and weekday time allocation, suggesting that individuals who spend time on education related activities (such as completing homework, working on projects etc.,) over the weekend, might also allocate a time to education on weekdays (attending schools and colleges) or vice versa.

The error covariance between weekend shopping and weekday recreation is negative, suggesting that shopaholics may save their monetary resources for weekend shopping activities and skimp on weekday recreational activities. On the other hand, it is also possible that individuals who prefer to use the weekdays for recreation refrain from shopping on weekends. Unobserved attributes that increase recreational activity time allocation on weekends are also found to increase shopping time allocation on weekdays. People with active lifestyles may seek to undertake recreational activities on the weekend; and would naturally allocate more time to shopping on weekdays to support the active weekend lifestyle. Conversely, individuals who allocate time for recreation on weekdays, might not have any time left for shopping on weekdays and hence end up allocating more time to shopping activity on the weekends. A positive error covariance was found between unobserved attributes of recreational activity time allocation across weekends and weekdays. This indicates that individuals who are naturally gregarious and outgoing might allocate time for recreational activities on weekdays as well as weekends. Overall, the presence of significant error correlations (or unobserved effects) corroborates the need for a joint modeling framework that closely ties the weekday and weekend time allocation behaviors together.

One additional important note here is that we did consider built environment variables such as residential density, distance from home location to the nearest hospital, grocery store, shopping center, bus stop, and railway station. But, surprisingly, none of these turned out to be statistically significant. This may be a result of not using micro-level built environment variables (which were not available in the data), or could also be indicative of the fact that activity time-use patterns are not too impacted by our built environment. There have been studies in the literature that suggest this latter result (see, for example, Grigolon et al., 2013 and Sreela and Anjaneyulu, 2017, who report that lifestyles, and demographics are the main drivers of activity participation and time-use, while built environment/level of service have little impact). Suffice it to say that considering more detailed built-environment factors remains a fruitful direction for future research.

TABLE 5 Estimated Covariance Matrix of Errors in the Joint Weekdays-Weekend Model

	Weekdays						Weekend					
	Work	Education	Escort	Shopping	Personal Business	Recreation	Work	Education	Escort	Shopping	Personal Business	Recreation
Weekdays												
Work	1.000*											
Education	0.500*	1.000*										
Escort	0.500*	0.500*	1.000*									
Shopping	0.500*	0.500*	0.500*	1.000*								
Personal Business	0.500*	0.500*	0.500*	0.500*	1.000*							
Recreation	0.500*	0.500*	0.500*	0.500*	0.500*	1.000*						
Weekend												
Work	-0.322	0.000*	0.000*	0.000*	0.000*	0.218	1.000*					
Education	0.000*	0.175	0.000*	0.000*	0.000*	0.000*	0.500*	1.000*				
Escort	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.500*	0.500*	1.000*			
Shopping	0.000*	0.000*	0.000*	0.253	-0.241	-0.419	0.500*	0.500*	0.500*	1.000*		
Personal Business	0.000*	0.000*	0.000*	0.105	0.305	0.000*	0.500*	0.500*	0.500*	0.500*	1.000*	
Recreation	0.000*	0.000*	0.000*	0.207	0.000*	0.120	0.500*	0.500*	0.500*	0.500*	0.500*	1.000*

Note: * sign refers to fixed values. All the other elements in the table are significant at 5% level of significance.

6. COMPARISON WITH ALTERNATIVE MODELS

To the knowledge of the authors, this is the first attempt at developing a joint model of activity scheduling and time allocation behavior across weekdays and weekend days, while explicitly accounting for relationships in activity engagement between different day types. Since no prior research exists in this domain to compare and contrast the joint model estimation results, the efficacy of the modeling framework presented in this paper is demonstrated by applying the joint multivariate MDCP model to the estimation sample to examine the degree to which it can replicate observed weekly activity-time allocation patterns. The prediction methodology presented in Pinjari and Bhat (2014) is used to derive estimates of activity-time allocation. The prediction performance of the model system is compared with that of three other models. First, an independent multivariate MDCP model that ignores error covariances is estimated. Then, for each sample individual, a random weekday and a random weekend day is drawn. A multivariate MDCP model is estimated for the one random weekday and one random weekend day. Two versions of this model are estimated, one with error covariances (joint) and one without error covariances (independent). The predictions from these models are then converted to equivalent weekly activity-time allocation by multiplying the weekday patterns by five weekdays and multiplying the weekend patterns by two weekend days. The predicted time allocation patterns are tabulated, and the mean absolute percentage error (MAPE) is calculated for each model. The prediction results for each of the models are shown in Table 6. The table shows comparisons of participation rates and average weekly duration for the seven aggregate activity categories considered in this analysis. From the table, it can be observed that the predictions of the joint models (both the participation rates and the average durations) are the closest to the observed values. In comparing the mean absolute percentage error (MAPE) values across the four models presented, it can be observed that "joint weekly" model has a lower MAPE (14.3 for participation rates, and 15.7 for average durations) than the "independent weekly" model (MAPE of 17.3 for participation rates, and 17.1 for average durations) in which error covariances are not considered. The same pattern is observed in comparing MAPE values of the joint and independent models for the "one-day" model. To further reinforce the efficacy of the joint model system, the model predictions of time allocations and participation rates were obtained using joint and independent "weekly models" for various socio-demographic segments. Then, the MAPE values are compared across both these models (similar to the values presented in the last row of Table 6). The results of this analysis have been documented in Table A.5 of the online supplement (Astroza et al., 2018). From the additional analysis, it was observed that the MAPE values for the joint model are less than that of the independent model for each and every demographic segment. In addition to the overall MAPE values, Astroza et al. (2018) also provides the detailed prediction comparison tables for a couple of demographic segments (females, and individuals from the high-income households) in the survey sample (see Tables A.6 and A.7 of the online supplement). Based on all of the forecasting results presented, it can be concluded that the proposed joint multi-weekday and multi-weekend day multivariate MDCP model system provides a superior fit relative to the independent model system, and that this improved fit is not simply an artifact of overfitting.

TABLE 6 Predictions of Participation Shares and Average Duration (Among Those Participating in the Corresponding Activity)

Activity purpose	Weekly model (the entire diary)				"One-day" model (one random weekday * 5 plus one random weekend day * 2)				Sample	
	Joint		Independent		Joint		Independent			
	Participation (%)	Average duration (hours)	Participation (%)	Average duration (hours)	Participation (%)	Average duration (hours)	Participation (%)	Average duration (hours)	Participation (%)	Average duration (hours)
Weekdays										
Work	86.4	34.14	88.2	35.6	76.0	27.6	74.4	27.0	82.6	32.39
Education	2.1	8.70	2.2	8.7	1.0	17.1	1.1	17.3	1.8	13.15
Escort	20.6	2.97	19.2	3.01	15.3	3.13	17.5	3.15	25.7	2.02
Shopping	61.0	2.77	61.6	2.77	55.4	2.36	57.8	2.40	59.8	2.28
Personal Business	22.1	2.55	20.5	2.58	22.3	1.78	25.6	1.73	26.3	2.21
Recreation	67.1	10.10	67.2	10.23	59.3	7.89	60.1	7.88	65.9	9.15
In-home	--	60.36	--	61.0	--	66.3	--	65.9	100.0	64.45
Weekend										
Work	31.4	10.23	32.1	10.56	34.6	7.31	32.7	7.22	28.6	8.68
Education	1.9	7.50	2.0	7.23	0.9	8.56	0.9	8.57	1.2	7.92
Escort	14.7	1.68	15.6	1.69	9.8	2.45	9.9	2.50	15.8	1.43
Shopping	35.0	1.75	36.1	1.78	32.0	1.82	31.5	1.79	33.0	1.69
Personal Business	15.7	1.48	16.3	1.45	10.2	2.03	9.5	2.06	13.2	1.72
Recreation	48.5	6.70	48.6	6.72	38.6	7.78	3.84	8.01	44.0	5.81
In-home	--	28.45	--	29.00	--	30.10	--	30.15	100.0	26.65
Mean absolute percentage error	**14.3**	**15.7**	**17.1**	**17.3**	**20.6**	**21.9**	**24.7**	**23.1**		

7. DISCUSSION AND CONCLUSIONS

There are many reasons to examine and model travel behavior over multiple days. Activity-travel patterns show considerable variation from day to day and across day types (weekdays versus weekend days), people plan and schedule (some) activities over a multi-day horizon, and there are trade-offs and complementary relationships in activity-time allocation across days of the week and between weekdays and weekend days. Despite these motivations for capturing and modeling multi-day activity-travel behavior, travel demand forecasting models in research and practice continue to model daily travel demand for a single day, essentially treating each of the day of the week as an independent entity.

This paper aims to address this serious shortcoming in travel demand modeling practice by offering a methodology and framework for representing the relationships embedded in multi-day activity-travel engagement patterns. In this study, a multivariate multiple discrete-continuous probit (MDCP) modeling framework is employed to model activity-time allocation within weekdays, within weekend days, and between weekday and weekend days. One week travel diary data derived from the 2015 United Kingdom National Travel Survey is used for the analysis. The model is estimated for a sample of more than 4,500 adult workers who reported that they do not telecommute. The multivariate MDCP framework essentially stitches together two MDCP models, one model for weekday activity-time allocation and one model for weekend activity-time allocation. Each MDCP model component is able to reflect the allocation of available time budget to various activities over the course of multiple days (as opposed to a single day) – five days for the weekday MDCP and two days for the weekend MDCP. The multivariate MDCP ties these two MDCP models together so that trade-offs and complementary relationships in activity-time allocation between the two day types can be fully captured. More importantly, the model framework accounts for the presence of error covariances that may arise from common unobserved attributes simultaneously affecting activity-time allocation on weekdays and weekend days.

Advances in technology and passive data collection methods should increasingly enable the collection of multi-day travel behavior data, and the estimation and implementation of model systems such as that presented in this paper. The model system presented here may be used in conjunction with activity-based travel forecasting models to first predict broad activity-time allocation patterns for an entire week, focusing on the relationships between weekdays and weekend days. The weekday and weekend activity-time allocation predictions can then be further disaggregated to derive activity patterns for any individual day of the week. Weekday and weekend day predictions of activity-time allocation obtained through such a multi-day week-long approach are likely to more accurately reflect observed patterns because they are produced with an explicit recognition of inter-day activity-time allocation relationships. Future research efforts should focus on model implementation and testing efforts, together with enhancing the specification to also account for trade-offs and complementary relationships between in-home and out-of-home activities. The proposed model may be extended to include a more disaggregate categorization of days of the week (a separate MDC model for each day of the week), to further explore time use and activity scheduling tradeoffs within different weekdays (Monday-Friday) and weekend days (Saturday-Sunday). If adequate data is available, a more disaggregate classification of the activity categories could be included in the model structure to better understand the nuances in trade-offs across various activity types. Future research on this topic could adopt the proposed framework to develop similar models for the non-worker adult and student adult segments. Further, examining multi-day activity-travel patterns of children would be another direction of investigation, especially because most current studies of children's activity pattern focus on a single day (see, for example, Copperman and Bhat, 2007). Such multi-day modeling of the activity scheduling and time allocation patterns of all members of a household would pave the way to estimate multi-day household-level models that account for intra-household interactions in activity scheduling and time allocation decisions (such as parents picking up the children on some days but not all, planning a weekend recreational event together, etc.). Finally, advancing the modeling methodology to incorporate monetary budgets (in addition to time budgets) would further enhance the ability of the model to accurately predict multi-day activity-travel patterns.

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