Recent Methodological Advances Relevant to Activity and Travel Behavior Analysis

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ABSTRACT

This paper presents an overview of the considerable progress in modeling methodology that has been made in recent years and that is directly relevant to improved transportation policy analysis and travel demand forecasting. The overview is organized under three broad classes of models: discrete choice models, hazard-based duration models, and limited-dependent variable models. Because of the objective of the paper, the focus here is on the methodological aspects of various studies rather than on the empirical findings from the studies. Some important methodological topics have necessarily been omitted from this review because of space considerations. We have tried to specifically identify these topics at the beginning of the paper and provide references to recent reviews on these topics.

1. INTRODUCTION

This paper reviews recent methodological advances relevant to modeling activity and travel behavior. The overview of the state-of-the-art in modeling is motivated by two considerations. First, discrete-choice models that have been well established in the past (such as the multinomial logit and nested logit models) have been generalized in several ways to make them more realistic in their representations of travel choice behavior. Second, the increasing realization of the need to model travel as part of a larger (and holistic) activity-travel pattern has led to the analysis of activity attributes (such as activity participation, activity duration, home-stay duration, *etc.*) either in isolation or jointly with one another. This has led to the adoption of relatively non-traditional (in the travel analysis field) methodologies such as duration analysis, limited-dependent variable models, and computational process models.

Several points should be noted before proceeding to the remainder of the paper. First, this paper is not intended to provide detailed information regarding the structures or estimation procedures for the models reviewed. Such a task would not be feasible within the space constraints of a single paper. Second, the paper does not touch on methodological developments in survey data collection techniques or analytic imputation techniques for missing data. These issues have been addressed in papers presented at a recent conference in Stockholm (see Stopher, 1996 and Brownstone, 1996). Third, the paper does not address methodological advances in joint estimation from revealed preference (RP) as well as stated preference (SP) data. Instead, the paper focuses on estimation from revealed preference data only. The reader is referred to Hensher (1994a) for an overview of the methods of RP-SP estimation. Fourth, this paper does not review computational process models since comprehensive reviews of these models have been conducted not very long ago by pioneers of the approach who are more knowledgeable regarding the approach than the current author (see Gärling *et al.*, 1994; Golledge *et al.*, 1994; Kitamura and Fujii, 1996; and Kurani and Kitamura, 1996). An additional reason for not focusing on computation process models (CPMs) here is that while there has been considerable advancement in these methods, some basic issues related to statistical estimation and calibration of CPMs are yet to be defined and resolved (Golledge *et al*., 1994). Fifth, we focus on methods for cross-sectional data rather than longitudinal (or panel) data in the current paper. In concept, the methods discussed here for cross-sectional analysis can be extended to panel analysis after accommodating the additional econometric issues introduced by panel data. Finally, the paper does not review advances in methods related to demand-supply interaction analysis or demand-supply equilibration.

The paper is organized as follows. The next section focuses on discrete choice models. Section 3 presents models for time duration. Section 4 discusses limited-dependent variable models in activity-travel behavior analysis. Section 5 concludes the paper.

2. DISCRETE CHOICE MODELS

The multinomial logit (MNL) model has been the most widely used structure for modeling discrete choices in travel behavior analysis. The random components of the utilities of the different alternatives in the MNL model are assumed to be independent and identically distributed (IID) with a type I extreme-value (or Gumbel) distribution (McFadden, 1973). The MNL model also maintains an assumption of homogeneity in responsiveness to attributes of alternatives across individuals (*i.e.*, an assumption of response homogeneity). For example, in a mode choice model, the MNL maintains the same utility parameters on the level-of-service attributes across individuals. Finally, the MNL model also maintains an assumption that the error variance-covariance structure of the alternatives is identical across individuals (*i.e.*, an assumption of error variance-covariance homogeneity). The three assumptions together lead to the simple and elegant closed-form mathematical structure of the MNL. However, these assumptions also leave the MNL model saddled with the "independence of irrelevant alternatives" (IIA) property at the individual level (Ben-Akiva and Lerman, 1985).[1] In the next three sections, we will discuss generalizations of the MNL structure along each of the three dimensions mentioned above: a) Relaxation of the IID (across alternatives) error structure, b) Relaxation of response homogeneity, and c) Relaxation of the error variance-covariance structure homogeneity. While we discuss each of the dimensions separately, one can combine extensions across different dimensions to formulate several more generalized and richer structures.

2.1. Relaxation of the IID (across alternatives) error structure

The rigid inter-alternative substitution pattern of the multinomial logit model can be relaxed by removing, fully or partially, the IID assumption on the random components of the utilities of the different alternatives. The IID assumption can be relaxed in one of three ways: a) allowing the random components to be correlated while maintaining the assumption that they are identically distributed (identical, but non-independent random components), b) allowing the random components to be non-identically distributed (different variances), but maintaining the independence assumption (non-identical, but independent random components), and c) allowing the random components to be non-identical and non-independent (non-identical, non-independent random components). We discuss each of these alternatives below.

[1] Travel demand literature, in general, attributes the IIA property to the IID assumption of the error covariance structure and does not discuss the assumptions of response homogeneity and error variance-covariance homogeneity.

2.1.1. Identical, non-independent random components

The distribution of the random components in models which use identical, non-independent random components can be specified to be either normal or type I extreme value. Discrete choice literature has mostly used the type I extreme value distribution since it nests the multinomial logit and results in closed-form expressions for the choice probabilities.

The models with the type I extreme value error distribution belong to the Generalized Extreme Value (GEV) class of random utility-maximizing models. Five model structures have been formulated and applied within the GEV class. These are: the Nested Logit (NL) model, the Paired Combinatorial Logit (PCL) model, the cross-nested logit (CNL) model, the Ordered GEV (OGEV) model, and the Multinomial Logit-Ordered GEV (MNL-OGEV) model.

The nested logit (NL) model permits covariance in random components among subsets (or nests) of alternatives (each alternative can be assigned to one and only one nest). Alternatives in a nest exhibit an identical degree of increased sensitivity relative to alternatives not in the nest (Williams, 1977 and Daly and Zachary, 1978). Each nest in the NL structure has associated with it a dissimilarity (or logsum) parameter that determines the correlation in unobserved components among alternatives in that nest (see Daganzo and Kusnic, 1993). The range of this dissimilarity parameter should be between 0 and 1 for all nests if the NL model is to remain globally consistent with the random utility maximizing principle. A problem with the NL model is that it requires *a priori* specification of the nesting structure. This requirement has at least two drawbacks. First, the number of different structures to estimate in a search for the best structure increases rapidly as the number of alternatives increases. Second, the actual competition structure among alternatives may be a continuum that cannot be accurately represented by partitioning the alternatives into mutually exclusive subsets. The NL model has been applied to multidimensional choice contexts (for example, see Waddell 1993 and Evers, 1990) and unidimensional contexts where subsets of the available alternatives share common unobserved components of utility (for example, see Forinash and Koppelman, 1993 and Brownstone and Small, 1989).

The paired combinatorial logit (PCL) model initially proposed by Chu (1989) and recently examined in detail by Koppelman and Wen (1997) generalizes, in concept, the nested logit model by allowing differential correlation between each pair of alternatives (the nested logit model, however, is not nested within the PCL structure). Each pair of alternatives in the PCL model has associated with it a dissimilarity parameter (subject to certain identification considerations that Koppelman and Wen are currently studying) that is inversely related to the correlation between the pair of alternatives. All dissimilarity parameters have to lie in the range of 0 to 1 for consistency with random utility maximization. In the intercity mode choice empirical analysis of Koppelman and Wen, the PCL model which allows correlation between air and car modes as well as between train and car modes performed better than the nested logit models which nests air and car only or train and car only. Koppelman and Wen derive the expressions for the self- and cross-elasticities in the PCL model and show empirically that the policy impacts suggested by the restrictive MNL and nested logit models can be quite different from those suggested by the statistically superior (in their empirical context) PCL model.

Another generalization of the nested logit model is the cross-nested logit (CNL) model of Vovsha (1996). In this model, an alternative need not be exclusively assigned to one nest as in the nested logit structure. Instead, an alternative can appear in different nests with different probabilities based on what Vovsha refers to as allocation parameters. A single dissimilarity parameter is estimated across all nests in the CNL structure. Unlike in the PCL model, the nested logit model can be obtained as a special case of the CNL model when each alternative is unambiguously allocated to one particular nest.[2] Vovsha proposes a heuristic procedure for estimation of the CNL model. This procedure appears to be rather cumbersome and its heuristic nature makes it difficult to establish the statistical properties of the resulting estimates.

The ordered GEV model was developed by Small (1987) to accommodate correlation among the unobserved random utility components of alternatives close together along a natural ordering implied by the choice variable (examples of such ordered choice variables might include car ownership, departure time of trips, *etc.*). The simplest version of the OGEV model (which Small refers to as the standard OGEV model) accommodates correlation in unobserved components between the utilities of each pair of adjacent alternatives on the natural ordering; that is, each alternative is correlated with the alternatives on either side of it along the natural ordering.[3] The standard OGEV model has a dissimilarity parameter that is inversely related to the correlation between adjacent alternatives (this relationship does not have a closed form, but the correlation implied by the dissimilarity parameter can be obtained numerically). The dissimilarity parameter has to lie in the range of 0 to 1 for consistency with random utility maximization. Empirical applications of the OGEV model have not been very successful thus far (that is, the OGEV model was not significantly better than the MNL or the dissimilarity

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[2] A related model is the cross-correlated logit (CCL) model of Williams (1977). The CCL model allows correlation among alternatives across both dimensions in a two-dimensional choice model by specifying the error covariance matrix to include variance terms specific to each dimension (the error terms specific to each dimension and to the combination of dimensions are assumed to be gumbel distributed). Vovsha's CNL model, on the other hand, enables a flexible correlation structure by allowing the same alternative to appear in multiple nests. The CCL model is not consistent with random utility maximization, while the CNL model is.

[3] The reader will note that the nested logit model cannot accommodate such a correlation structure because it requires alternatives to be grouped into mutually exclusive nests.

parameter exceeded one). However, it is important to note that only two such attempted applications have been documented to date, both by Small (1987).

The MNL-OGEV model formulated by Bhat (1997a) generalizes the nested logit model by allowing adjacent alternatives within a nest to be correlated in their unobserved components. This structure is best illustrated with an example. Consider the case of a multi-dimensional model of travel mode and departure time for nonwork trips. Let the departure time choice alternatives be represented by several temporally contiguous discrete time periods in a day such as AM peak (6AM-9AM), AM mid-day (9AM-12Noon), PM mid-day (12Noon-3PM), PM peak (3PM-6PM), and other (6PM-6AM). An appropriate nested logit structure for the joint mode-departure time choice model may allow the joint choice alternatives to share unobserved attributes in the mode choice dimension, resulting in an increased sensitivity among time-of-day alternatives of the same mode relative to the time-of-day alternatives across modes. However, in addition to the uniform correlation in departure time alternatives sharing the same mode, there is likely to be increased correlation in the unobserved random utility components of each pair of adjacent departure time alternatives due to the natural ordering among the departure time alternatives along the time dimension. Accommodating such a correlation generates an increased degree of sensitivity between adjacent departure time alternatives (over and above the sensitivity among non-adjacent alternatives) sharing the same mode. A structure that accommodates the correlation patterns just discussed can be formulated by using the multinomial logit (MNL) formulation for the higher-level mode choice decision and the standard ordered generalized extreme-value (OGEV) formulation (see Small, 1987) for the lower-level departure time choice decision (*i.e.*, the MNL-OGEV model). The MNL-OGEV structure, in the context of the mode-departure time example, has two dissimilarity parameters: one is associated with the correlation among joint alternatives sharing the same mode, and the other is associated with the increased correlation between adjacent departure time alternatives of the same mode. For consistency with random utility maximization, both these parameters should be less than 1 and the latter dissimilarity parameter should be smaller than the former dissimilarity parameter.

The advantage of all the GEV models discussed above is that they allow partial relaxations of the independence assumption among alternative error terms while maintaining closed-form expressions for the choice probabilities. The problem with these models is that they are consistent with utility maximization only under rather strict (and often empirically violated) restrictions on the dissimilarity parameters. The origin of these restrictions can be traced back to the requirement that the variance of the joint alternatives be identical in the GEV models.

2.1.2. Non-identical, independently distributed random components

The concept that heteroscedasticity in alternative error terms (*i.e.*, independent, but not identically distributed error terms) relaxes the IIA assumption is not new (see Daganzo, 1979), but has received little (if any) attention in travel demand modeling and other fields. In fact, the IIA property has become virtually synonymous with the assumption of lack of similarity (or independence of random components) among the choice alternatives in travel demand literature. Four models have been proposed which allow non-identical random components. The first is the negative exponential model of Daganzo (1979), the second is the heteroscedastic multinomial logit (HMNL) model of Swait and Stacey (1996), the third is the oddball alternative model of Recker (1996) and the fourth is the heteroscedastic extreme-value (HEV) model of Bhat (1995).

Daganzo (1979) used independent negative exponential distributions with different variances for the random error components to develop a closed-form discrete choice model that does not have the IIA property. His model has not seen much application since it requires that the perceived utility of any alternative not exceed an upper bound (this arises because the negative exponential distribution does not have a full range).[4] Daganzo's model does not nest the multinomial logit model.

Swait and Stacey (1996) allowed heteroscedasticity by specifying the variance of the alternative error terms to be functions of observed alternative characteristics. The error terms themselves are assumed to be type I extreme-value. The scale parameter θ_i characterizing the variance of each alternative i is written as $\theta_i = \exp(\beta' z_i)$, where z_i is a vector of attributes associated with alternative i and β is a corresponding vector of parameters to be estimated. The resulting model has a closed-form structure, though it also places the restriction that the differing variances of the alternatives can be attributed solely to observed alternative characteristics. Swait and Stacey applied the model to brand choice modeling using scanner panel data.

Recker (1996) proposed the oddball alternative model that permits the random utility variance of one "oddball" alternative to be larger than the random utility variances of other alternatives. This situation might occur because of attributes that define the utility of the oddball alternative, but are undefined for other alternatives. Then, random variation in the attributes that are defined only for the oddball alternative will generate increased variance in the overall random component of the oddball alternative relative to others. For example, operating schedule and fare structure define the utility of the transit alternative, but are not defined for other modal alternatives in a mode choice model. Consequently, measurement error in schedule and fare structure will contribute to the increased variance of transit relative to other alternatives. The

[4] It is useful only in instances where there is a clear bound to the perceived attractiveness of an alternative, such as "in route choice models where it may not be unreasonable to assume that the perceived attractiveness of a route cannot be positive, since perceived travel time cannot be reasonably expected to be negative" (Daganzo, 1979; p16).

model has a closed-form structure for the choice probabilities based on convenient distributional assumptions on the random components. However, the model is quite restrictive in requiring that all alternatives except one have identical variance.

Bhat (1995) formulated the heteroscedastic extreme-value (HEV) model which assumes that the alternative error terms are distributed with a type I extreme value distribution. The variance of the alternative error terms is allowed to be different across all alternatives (with the normalization that the error terms of one of the alternatives has a scale parameter of one for identification). Consequently, the HEV model can be viewed as a generalization of Recker's oddball alternative model. The HEV model is applied to an intercity mode choice context. The motivation is that unequal error variances are likely to occur when the variance of an unobserved variable that affects choice is different for different alternatives. For example, if comfort is an unobserved variable whose values vary considerably for the train mode (based on, say, the degree of crowding on different train routes) but little for the automobile mode, then the random components for the automobile and train modes will have different variances (Horowitz, 1981). The HEV model does not have a closed-form solution for the choice probabilities, but involves only a one-dimensional integration regardless of the number of alternatives in the choice set. Bhat develops an efficient Gauss-Laguerre quadrature technique to approximate the one-dimensional integral. The HEV model can be modified to accommodate variations in the scale parameter because of observed alternative attributes, as done by Swait and Stacey (1996).[5]

The advantage of the heteroscedastic class of models discussed above is that they allow a flexible cross-elasticity structure among alternatives than many of the GEV models discussed

[5] The reader is referred to Hensher (1996a; 1996b) and Hensher *et al*. (1996) for applications of the HEV model to estimation from revealed and stated preference data. The HEV model has also been applied in a marketing context by Allenby and Ginter (1995).

earlier. Specifically, the models (except the oddball model) permit differential cross-elasticities among all pairs of alternatives. The limitation (relative to the GEV models) is that the choice probabilities do not have a closed-form analytical expression in the HEV model.

2.1.3. Non-identical, non-independent random components

Models with non-identical, non-independent random components use one of two general structures: the first is an error-components structure and the second is the general multinomial probit (MNP) structure.

The error-components structure partitions the overall error into two components: one component which allows the random components to be non-identical and non-independent, and the other component which is specified to be independent and identically distributed across alternatives. In particular, consider the following utility function for alternative *i*:

$$\begin{aligned} U_i &= V_i + \zeta_i \\ &= V_i + \mu' z_i + \varepsilon_i \end{aligned} \tag{1}$$

where V_i and ζ_i are the systematic and random components of utility, and ζ_i is further partitioned into two components, $\mu' z_i$ and ε_i. z_i is a vector of observed data associated with alternative *i*, μ is a random vector with zero mean and density $g(\mu \mid \Sigma)$, Σ is the variance-covariance matrix of the vector μ, and ε_i is independently and identically standard distributed across alternatives with density function f(.). The component $\mu' z_i$ induces heteroscedasticity and correlation across unobserved utility components of the alternatives (see Train, 1995). While different distributional assumptions might be made regarding *f*(.) and *g*(.), it is typical to assume a standard type I extreme value for *f*(.), and a normal distribution for *g*(.). This results in an error-components model with a logit kernel. On the other hand, if a standard normal distribution is used for $f(.)$, the result is a error-components probit model. Both these structures will involve integrals in the choice probability expressions which do not have a closed-form solution. The estimation of these models is achieved using logit simulators (in the first case) or probit simulators (in the second case). Different and very general patterns of heteroscedasticity and correlation in unobserved components among alternatives can be generated by appropriate specification of the μ and z_i vectors. For example, z_i may be specified to be a row vector of dimension M with each row representing a group m of alternatives sharing common unobserved components. The row(s) corresponding to the group(s) of which i is a member take(s) a value of one and other rows take a value of zero. The vector μ (of dimension M) may be specified to have independent elements, each element having a variance component σ_m^2. The result of this specification is a covariance of σ_m^2 among alternatives in group m and heteroscedasticity among the groups of alternatives. This structure is less restrictive than the nested logit structure in that an alternative can belong to more than one group. Also, by structure, the variance of the alternatives is different (see Bhat, 1997b for application of this structure to a multi-dimensional choice context).[6] More general structures for $\mu' z_i$ in equation (1) are presented by Ben-Akiva and Bolduc (1996) and Brownstone and Train (1996).

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[6] Appropriate identification conditions will have to be imposed in this structure. In the most general case, each group can represent a pair of alternatives. If there are I alternatives, the number of pairs of alternatives is $I(I\text{-}1)/2$. However, we cannot accommodate a covariance term for each pair; one of the pairs should be normalized to have a covariance of zero, so the covariance of the other pairs is relative to that of the base pair. Unfortunately, the covariances are generated by variance terms and so are pre-specified to be positive. Thus, the normalization of which pair to select as the base is not innocuous; the base pair should be the one with least covariance, which of course we do not know *a priori* (see also Ben-Akiva and Bolduc, 1996 for a related discussion). Thus, in general, we have to impose a restrictive structure for the covariance patterns based on *a priori* theoretical considerations.

The general multinomial probit (MNP) structure does not partition the error terms, and estimates (subject to certain identification considerations) the variance-covariance matrix of the overall random components among the different alternatives (see Bunch and Kitamura, 1990; Lam, 1991; Lam and Mahmassani, 1991; and Chintagunta and Honore, 1996). The advantage of the MNP model is that the structure is more general than the error-components models (the error components structure essentially parameterizes the variance-covariance matrix in an MNP model using an *a priori* structure). However, McFadden and Train (1996) have shown that the error-components formulation can approximate a multinomial probit formulation as closely as one needs it to. Also, the MNP model introduces several additional parameters in the covariance matrix which generates a number of conceptual, statistical and practical problems, including difficulty in interpretation, highly non-intuitive model behavior, low precision of covariance parameter estimates, and increased difficulty in transferring models from one space-time sampling frame to another (see Horowitz, 1991 and Currim, 1982). Further, the error-components models can be estimated using simulators which are conceptually simple and easy to program. These simulators involve simultaneous draws from the appropriate density function with unrestricted ranges for all alternatives. Consequently, they are inherently faster than simulators for the MNP model where the range for the random draw of one alternative is dependent on the value of the earlier draws for other alternatives (see Brownstone and Train, 1996).

2.2. Relaxation of response homogeneity

The standard multinomial logit, and other models which relax the IID assumption across alternatives, typically assume that the parameters determining the sensitivity to attributes of the alternatives are the same across individuals in the population. Ideally, we should obtain individual-specific parameters for the subjective evaluations of alternative attributes. However, the data used for travel behavior modeling is usually cross-sectional. This precludes estimation at the individual level and constrains the modeler to pool the data across individuals. In such pooled estimations, the analyst should accommodate differences in responsiveness to alternative attributes (response heterogeneity) across individuals. In particular, if the assumption of homogeneity is imposed when there is heterogeneity, the result is biased and inconsistent parameter and choice probability estimates (see Chamberlain, 1980).

Response homogeneity may be relaxed in one of two ways. In the first approach (which we will refer to as the varying coefficients approach), the coefficients on alternative attributes are allowed to vary across individuals while maintaining a single utility function. In the second approach (which we will refer to as the segmentation approach), individuals are assigned to segments based on their personal/trip characteristics and a separate utility function is estimated for each segment. Each of these approaches is discussed in the subsequent two sections.

2.2.1. Varying coefficients approach

Consider the utility U_{qi} that an individual q associates with an alternative i and write it as:

$$U_{qi} = \alpha_i + \delta_i' z_q + \varepsilon_{qi} + \eta_q' x_{qi} \tag{2}$$

where α_i is an individual-invariant bias constant, z_q is a vector of observed individual characteristics, δ_i is a vector of parameters to be estimated, ε_{qi} is a random term representing idiosyncrasies in preferences, and η_q is a vector representing the responsiveness of individual q to a corresponding vector of alternative-associated variables x_{qi}. The ε_{qi} terms may be specified to have any of the structures discussed in Section 2. Conditional on η_q and the assumption regarding the ε_{qi} terms, the form of the conditional choice probabilities can be developed. The unconditional choice probabilities corresponding to the conditional choice probabilities will depend on the response heterogeneity specification adopted for the η_q vector. Three specifications are possible, as discussed next.

The <u>first</u> specification allows for systematic response heterogeneity by writing each element η_{qk} of the vector η_q as a function of a vector w_{qk} of relevant observed individual characteristics: $\eta_{qk} = \gamma_k + \beta_k f(w_{qk})$ (η_{qk} represents the response coefficient of the *qth* individual to the *kth* alternative attribute. If we pre-specify a functional form for $f(w_{qk})$, then the unconditional choice probabilities take the same structure as the conditional (on η_q) choice probabilities. A problem with the specification of the form $\eta_{qk} = \gamma_k + \beta_k f(w_{qk})$, however, is that it does not guarantee the correct sign of the response coefficient η_{qk} for all individuals. For example, in a mode choice context, we expect the effect of the travel time and cost variables to be negative for all individuals, which is not guaranteed by writing $\eta_{qk} = \gamma_k + \beta_k f(w_{qk})$. An alternative method that accommodates systematic response heterogeneity and, at the same time, ensures the appropriate sign on the response coefficients is to specify $\eta_{qk} = \pm\exp(\gamma_k + \beta_k' w_{qk})$. The '+' sign is applied for a non-negative response coefficient and the '-' sign is applied for a non-positive response coefficient. The unconditional choice probabilities with this exponential specification are the same as the conditional probabilities after replacing η_{qk} with $\pm\exp(\gamma_k + \beta_k' w_{qk})$. The resulting model, however, now has a non-linear-in-parameters utility function. It is important to emphasize that incorporating systematic heterogeneity may contribute to a more realistic representation of response sensitivity, but does not relax the IIA property (if the IID error assumption across alternatives of the MNL is maintained).

The second specification for the response coefficients allows random variation is sensitivity, but does not accommodate differences in sensitivity due to observed individual attributes. One form for the random variation may be $\eta_{qk} = \gamma_k + v_{qk}$, where γ_k is the mean response sensitivity across all individuals in the population and v_{qk} is a term representing random taste variation of individual *q* from the mean. Alternatively, if the response coefficient needs to be of a particular sign, then one can use an alternative form: $\eta_{qk} = \pm\exp(\gamma_k + v_{qk})$. v_{qk} is typically assumed to be normally distributed, which implies that the response coefficients are normally distributed if one uses the first form and log-normally distributed if one uses the second form. Applications of random response heterogeneity include Fisher and Nagin, 1985, Revelt and Train, 1996, Train, 1996, Ben-Akiva *et al.*, 1993, Gonül and Srinivasan, 1993, and Mehndiratta, 1996. The v_{qk} terms in the random response heterogeneity specification represent the random tastes of person *q* and are common to the utility of all alternatives *i*. Therefore, variance in the v_{qk} terms across individuals induces a correlation among the utility of different alternatives (see McFadden and Train, 1996). As a result, the random response specification does not exhibit the restrictive independence from irrelevant alternatives (IIA) property even if the IID error assumption across alternatives of the MNL is maintained.

The third (and most general) specification for the response coefficients superimposes random response heterogeneity over the systematic response heterogeneity: $\eta_{qk} = \pm\exp(\gamma_k + \beta_k' w_{qk} + v_{qk})$, where v_{qk} is a term representing random taste variations across individuals with the same observed characteristics w_{qk}. Bhat (1996a) adopts such a specification in an intercity mode choice context.

2.2.2. Segmentation approaches

Two segmentation approaches may be identified depending on whether the assignment of individuals to segments is exogenous (deterministic) or endogenous (probabilistic).

The exogenous segmentation approach to capturing heterogeneity assumes the existence of a fixed, finite number of mutually-exclusive market segments (each individual can belong to one and only one segment). The segmentation is based on key socio-demographic variables (sex, income, *etc*.). Within each segment, all individuals are assumed to have *identical* preferences and identical sensitivities to level-of-service variables (*i.e.*, the same utility function). In the exogenous segmentation approach, the assignment of individuals to segments is deterministic and is implicit in the definition of the segments. A choice model is estimated subsequently for each segment. The total number of segments is a function of the number of segmentation variables and the number of segments defined for each segmentation variable. Ideally, the analyst would consider all socio-demographic and trip-related variables available in the data for segmentation (we will refer to such a segmentation scheme as full-dimensional exogenous market segmentation). However, the full-dimensional segmentation approach has a practical limitation; the total number of segments grows very fast with the number of segmentation variables, creating both interpretational and estimation problems due to inadequate observations in each segment (with typical sample sizes used for mode choice analysis). To overcome this limitation, researchers have resorted to a Limited-Dimensional Exogenous Market Segmentation method, which uses only a subset of the demographic and trip variables (typically one or two) for segmentation. It is not uncommon for the subset of variables to be decided *a priori* based on judgment, though one could estimate models with different subsets and then select the preferred subset for segmentation. The advantage of the limited-dimensional approach is that it is practical (the parameters can be efficiently estimated with data sizes generally available for mode choice analysis) and is easy to implement. The disadvantage is that its practicality comes at the expense of suppressing potentially higher-order interaction effects of the segmentation variables on response to alternative attributes. In addition, an intrinsic problem with the exogenous market segmentation methods is that the threshold values of the continuous segmentation variables (for example, income) which define segments have to be established in a rather *ad hoc* fashion. Also, the exogenous approach does not relax the IIA property if the IID (across alternatives) assumption on the random components is maintained.

The endogenous market segmentation approach attempts to accommodate heterogeneity in a practical manner not by suppressing higher-order interaction effects of segmentation variables (on response to alternative attributes), but by reducing the dimensionality of the segment-space. Each segment, however, is allowed to be characterized by a large number of segmentation variables. The appropriate number of segments representing the reduced segment-space is determined statistically by successively adding an additional segment till a point is reached where an additional segment does not result in a significant improvement in fit. Individuals are assigned to segments in a probabilistic fashion based on the segmentation variables. The approach jointly determines the number of segments, the assignment of individuals to segments, and segment-specific choice model parameters. Since this approach identifies segments without requiring a multi-way partition of data as in the full-dimensional exogenous market segmentation method, it allows the use of many segmentation variables in practice and, therefore, facilitates incorporation of the full order of interaction effects of the segmentation variables on preference and sensitivity to alternative attributes. The method also obviates the need to (arbitrarily) establish the threshold values defining segments for continuous segmentation variables. The approach does not exhibit the individual-level independence from irrelevant alternatives (IIA) property of the exogenous segmentation approach even if a multinomial logit structure is maintained within each segment. A potential disadvantage is that the model estimation can be unstable. However, Bhat (1997c) has recently proposed a stable and effective hybrid estimation approach for the endogenous segmentation model that combines an Expectation-Maximization (EM) algorithm with standard likelihood maximization routines. Other applications of the endogenous segmentation approach may be found in Gopinath and Ben-Akiva (1995), Swait (1994), Gupta and Chintagunta (1994), Dayton and Macready (1988), and Swait and Sweeney (1996).[7]

2.3. Relaxation of error variance-covariance structure homogeneity

The assumption of error variance-covariance structure homogeneity across individuals can be relaxed either by a) allowing the variance components to vary across individuals (variance relaxation), b) allowing the covariance components to vary across individuals (covariance relaxation), and c) allowing both variance and covariance components to vary across individuals (variance-covariance relaxation).

[7] In concept, the endogenous segmentation approach is equivalent to a random-coefficients approach with non-parametric discrete probability distributions for the heterogeneity specification (see Jain *et al*., 1994 and Chintagunta and Honore, 1996).

2.3.1. Variance relaxation

Swait and Adamowicz (1996) formulate a heteroscedastic multinomial logit (HMNL) model that allows the variance of alternatives to vary across individuals based on attributes characterizing the individual and her/his environment (the variance, however, does not vary across alternatives). The motivation for such a model is that individuals with the same deterministic utility for an alternative may have different abilities to accurately perceive the overall utility offered by the alternative. The HMNL model has exactly the same structure as the heteroscedastic model described in Section 2.1.2, though the motivations for their development are different. Swait and Adamowicz apply their model to analyze market structure in a consumer behavior study and find evidence of varying variance components across individuals. McMillen (1995) also proposes a heteroscedastic model in the context of spatial choice. Both the above studies specify the variance of alternatives to be a deterministic function of individual-related characteristics and do not relax the IIA property if the IID (across alternatives) structure on the random components is maintained. Steckel and Vanhonacker (1988), on the other hand, develop a heteroscedastic logit model that treats the heteroscedasticity across individuals in the variance of alternatives as a random variable. This random variable is assumed to take an exponential distribution, and appears as a parameter in a generalized type I extreme value distribution for the random components of utility. The resulting mixing distribution for the random components of utility provides a closed-form expression for choice probabilities. Steckel and Vanhonacker show that their model is not saddled with the IIA property.

2.3.2. Covariance relaxation

Bhat (1997d) develops a nested logit model that allows heterogeneity across individuals in the magnitude of covariance among alternatives in a nest. The heterogeneity is incorporated by specifying the logsum (dissimilarity) parameter(s) in the nested logit model to be a deterministic function of individual-related characteristics. The model is applied to intercity mode choice analysis, where such heterogeneity may be likely to occur. For example, consider a nested model with car and train grouped as surface modes and air treated as a non-nested alternative. The degree of (increased) sensitivity (or cross-elasticity) between the two surface transportation modes relative to the air mode may differ based on characteristics of the traveler such as income (lower income may imply greater sensitivity between the surface modes) and attributes of the traveler's trip such as trip distance (shorter trip distances may lead to greater sensitivity between surface modes). Kamakura *et al*. (1996) adopt a different approach to accommodating covariance heterogeneity across individuals in their joint product form type-brand choice marketing analysis of peanut butter purchase behavior (there are two major product forms; creamy and crunchy; and several major brands such as Peter Pan and Skippy). They specify two nesting structures based on whether product form type (brand choice) is at the top (bottom) level of bottom (top) level and then assign individuals to each nesting structures probabilistically.

The author is not aware of any study that allows both variance and covariance components to vary across individuals (variance-covariance relaxation), though in concept the extension involves just a combination of the variance and covariance relaxations discussed earlier.

3. HAZARD DURATION MODELS

Hazard-based duration models are ideally suited to modeling duration data. Such models focus on an end-of-duration occurrence (such as end of shopping activity participation) given that the duration has lasted to some specified time (Kiefer, 1988; Hensher and Mannering, 1994). This concept of conditional probability of "failure" or termination of activity duration recognizes the dynamics of duration; that is, it recognizes that the likelihood of ending a shopping activity participation depends on the length of elapsed time since start of the activity.

Hazard-based duration models, which had their roots in biometrics and industrial engineering, are being increasingly used to model duration time in the fields of economics, transportation, and marketing (see Kiefer, 1988, Hensher and Mannering, 1994, and Jain and Vilcassim, 1991 for a review of the applications of duration models in economics, transportation, and marketing, respectively). To include an examination of covariates which affect duration time, most studies use a proportional hazard model which operates on the assumption that covariates act multiplicatively on some underlying or baseline hazard.

Two important specification issues in the proportional hazard model are a) the distributional assumptions regarding duration (equivalently, the distributional assumptions regarding the baseline hazard) and b) the assumptions about unobserved heterogeneity (*i.e.*, unobserved differences in duration across people). We discuss each of these issues in Sections 3.1 and 3.2, respectively. The extension of the simple univariate duration model to include multiple duration processes, multiple spells from the same individual, and related issues is the focus of Section 3.3.

3.1. Baseline hazard distribution

The distribution of the hazard may be assumed to be one of many parametric forms or may be assumed to be nonparametric. Common parametric forms include the exponential, Weibull, log-logistic, gamma, and log-normal distributions. Different parametric forms imply different assumptions regarding duration dependence. For example, the *exponential* distribution implies no duration dependence; that is, the time to "failure" is not related to the time elapsed. The *Weibull* distribution generalizes the exponential distribution and allows for monotonically increasing or decreasing duration dependence. The form of the duration dependence is based on a parameter that indicates whether there is positive duration dependence (implying that the longer the time has elapsed since start of the duration, the more likely it is to exit the duration soon), negative duration dependence (implying that the longer the time has elapsed since start of the duration, the less likely it is to exit the duration soon), or no duration dependence (which is the exponential case). The *log-logistic* distribution allows a non-monotonic hazard function. The choice of the distributional form for the hazard function may be made on theoretical grounds. However, a serious problem with the parametric approach is that it inconsistently estimates the baseline hazard and the covariate effects when the assumed parametric form is incorrect (Meyer, 1990). Sometimes, there may be little theoretical support for any particular parametric shape. In such cases, one might consider using a nonparametric baseline hazard. The advantage of using a nonparametric form is that even when a particular parametric form is appropriate, the resulting estimates are consistent and the loss of efficiency (resulting from disregarding information about the hazard's distribution) may not be substantial (Meyer, 1987).

Within the nonparametric approach, one may use the partial likelihood framework suggested by Cox (1972) which estimates the covariate effects but not the baseline hazard, or the approach suggested by Han and Hausman (1990) which estimates both the covariate effects and the baseline hazard parameters (also sometimes referred to as the incidental or nuisance parameters) simultaneously (the Han and Hausman approach is an alternative formulation of the approach originally proposed by Prentice and Gloeckler, 1978 and extended by Meyer, 1987). Between the Cox and Han and Hausman (HH) approaches, the HH approach has many advantages. First, in many studies, the dynamics of duration is itself of direct interest; the Cox approach, however, conditions out the nuisance parameters. Second, the Cox approach becomes cumbersome in the presence of many tied failure times (Kalbfleisch and Prentice, 198, page 101). Third, unobservable heterogeneity (which we discuss in the next section) cannot be accommodated within the Cox partial likelihood framework without the presence of multiple integrals of the same order as the number of observations in the risk set at each time period. Estimation in the presence of such large orders of integration is impractical even with recent advances in the computation of multidimensional integrals. In addition, the HH approach is the only appropriate method when duration models are to be estimated from interval-level data arising from the grouping of underlying continuous duration times. The parametric and Cox approaches use density function terms in their respective likelihood functions which are appropriate only for estimation from continuous duration data. If they are used to model grouped (or interval-level) duration data, the resulting estimates would generally be inconsistent (Prentice and Gloeckler, 1978).

Most studies of duration to date have made an *a priori* assumption of a parametric hazard. The most relevant duration studies for activity-travel modeling include a) the homestay duration models for commuters (*i.e.*, the time between coming home from work and leaving home for another out-of-home activity participation) of Mannering *et al.* (1992) and Hamed and

Mannering (1993), b) the sex-differentiated shopping duration models of Niemeier and Morita (1996), c) the shopping activity duration during the evening work-to-home commute of Bhat (1996b), and d) the delay duration model for border crossings by Paselk and Mannering (1992). These studies have been reviewed in greater detail by Pas (1996). Of these studies, Bhat (1996b) uses a nonparametric baseline hazard specification, while others use a parametric baseline hazard specification. Some studies in the marketing literature have used general parametric forms which nest the more frequently used Weibull, exponential and Gompertz distributions. Examples include Jain and Vilcassim (1991) and Vilcassim and Jain (1991).

3.2. Unobserved heterogeneity

Unobserved heterogeneity arises when unobserved factors (*i.e.*, those not captured by the covariate effects) influence durations. It is well established now that failure to control for unobserved heterogeneity can produce severe bias in the nature of duration dependence and the estimates of the covariate effects (Heckman and Singer, 1984; Lancaster, 1985; Sharma, 1987).

The standard procedure used to control for unobserved heterogeneity is the random effects estimator (see Flinn and Heckman, 1982). This involves specification of a distribution for the unobserved heterogeneity (across individuals) in the population. Two general approaches may be used to specify the distribution of unobserved heterogeneity. One approach is to use a parametric distribution such as a gamma distribution or a normal distribution (most earlier research has used a gamma distribution). The problem with the parametric approach is that there is seldom any justification for choosing a particular distribution; further, the consequence of a choice of an incorrect distribution on the consistency of the model estimates can be severe (see Heckman and Singer, 1984). A second approach to specifying the distribution of unobserved heterogeneity is to use a nonparametric representation for the distribution and to estimate the distribution empirically from the data. This is achieved by approximating the underlying unknown heterogeneity distribution by a finite number of support points and estimating the location and associated probability masses of these support points. The nonparametric approach enables consistent estimation since it does not impose a prior probability distribution.

Application of duration models in the transportation field have, for the most part, ignored unobserved heterogeneity (but see Bhat, 1996b and Hensher, 1994b). Researchers in the marketing and economics fields have paid more attention to unobserved heterogeneity. However, even in these fields, most applications have employed a parametric heterogeneity specification (see Gupta, 1991, Manston *et al.*, 1986, Meyer, 1990, Han and Hausman, 1990, all of whom use a gamma distribution). Very few studies have adopted a nonparametric heterogeneity distribution (see Jain and Vilcassim, 1991 and Vilcassim and Jain, 1991).

Among the duration studies mentioned above, Bhat (1996b) uses a nonparametric baseline hazard (based on the Han and Hausman approach) and a nonparametric unobserved heterogeneity specification (based on the Heckman and Singer approach). By allowing a nonparametric distribution for both the baseline hazard and unobserved heterogeneity, this paper sheds light on the importance of allowing a nonparametric specification for the baseline hazard, for unobserved heterogeneity, and for both of these. The finding from the study indicates that, at least in the context of the empirical analysis of the paper, the nonparametric baseline-nonparametric unobserved heterogeneity specification is preferable to other parametric specifications for the baseline or for heterogeneity or both. This result is important. It is contrary to the commonly held view that the choice of the mixing distribution may not be important if the baseline hazard is nonparametrically specified (see Meyer, 1990; Han and Hausman, 1990; Manston *et al.*, 1986).

3.3. Multiple duration processes

The discussion thus far has focused on the case where durations end as a result of a single event. For example, the length of unemployment ends when an individual gains employment (see Meyer, 1990) or home stay duration ends when an individual leaves home to participate in an activity (Mannering *et al.* 1992). A limited number of studies have been directed toward modeling the more interesting and realistic situation of multiple duration-ending outcomes. For example, failure in the context of unemployment duration (*i.e.*, exit from the unemployment spell) can occur either because of a new job, recall to the old job, or withdrawal from the labor force. Similarly, home stay duration may be terminated because of participation in out-of-home shopping activity, social activity, or personal business.

Previous research on multiple duration-ending outcomes (*i.e.*, competing risks) has extended the univariate proportional hazard model to the case of two competing risks in one of three ways. The <u>first method</u> assumes independence between the two risks (see Katz, 1986 and Gilbert, 1992). Under such an assumption, estimation proceeds by estimating a separate univariate hazard model for each risk. Unfortunately, the assumption of independence is untenable in most situations and, at the least, should be tested. The <u>second method</u> generates a dependence between the two risks by specifying a bivariate parametric distribution for the underlying durations directly. For example, Diamond and Hausman (1985) specify a log bivariate-normal distribution for the durations. This method has the result of placing very strong (and non-testable) parametric restrictions on the form of the baseline cause-specific hazard functions. The <u>third method</u> accommodates interdependence between the competing risks by allowing the unobserved components affecting the underlying durations to be correlated. Cox and Oakes (1984, page 159-161) develop a model which generates a positive dependence between the underlying durations based on common dependence on an observed random variable. More recently, Han and Hausman (1991) propose a model which allows unrestricted correlation in random unobserved components affecting the competing risks. This model permits nonparametric baseline hazard estimation, enables estimation from interval-level data of the type commonly found in econometrics and other fields, and retains an interpretation as an incompletely observed continuous-time hazard model.

A shortcoming of all the competing risk methods discussed above is that they tie the exit state of duration very tightly with the length of duration. The exit state of duration is not explicitly modeled in these methods; it is characterized implicitly by the minimum competing duration spell. Such a specification is restrictive, since it assumes that the exit state of duration is unaffected by variables other than those influencing the duration spells and implicitly determines the effects of exogenous variables on exit state status from the coefficients in the duration hazard models (this situation is analogous to the difference between a general endogenous switching regression equation system and the more restrictive disequilibrium market model of demand and supply; see Maddala, 1983, page 308).

Bhat (1996c) considers a generalization of the Han and Hausman competing risk specification where the exit state is modeled explicitly and jointly with duration models for each potential exit state. The resulting formulation follows strictly from the proportional hazard specification for the duration spells. This is in contrast to the Han and Hausman specification which uses an approximation to the proportional hazard specification. The model also extends the Han and Hausman framework to multivariate competing risks.[8] Bhat's formulation does not require placing parametric restrictions on the shapes of hazards within discrete time intervals, as required in the specifications of Han and Hausman, 1991 and Sueyoshi, 1992 (Han and Hausman and Sueyoshi maintain an assumption of a constant hazard within each discrete time-interval in deriving the competing-risk model specification). Another desirable characteristic of the model is that it is a generalized multiple durations model where the durations can be characterized either by multiple entrance states or by multiple exit states or by a combination of entrance and exit states. The focus of econometric literature has been on multiple durations due to multiple exit states (*i.e.*, the competing risk model). However, in many applications, multiple durations may arise because of multiple entrance states. Examples of multiple entrance states include layoffs, being fired, or first-time labor force entry for unemployment duration, activity-type participation choice (shopping, recreation, visiting, *etc.*) for activity duration, and type of initial acquaintance (in college, though personal advertisement, *etc.*) for marriage durations. Ignoring the entrance state when there are common unobserved factors affecting entrance status and spell duration will lead to biased and inconsistent hazard model parameters due to classic sample selection problems. In this context, information on the absence of a duration spell itself may be valuable; that is, it may be important to consider the "no-entry" state (for example, the "employed" state in unemployment duration modeling, the "home" state in activity duration modeling, or the "unmarried" state in marriage duration modeling) as an explicit entrance state in modeling durations for other entrance states.

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[8] Sueyoshi (1992) has also extended the Han and Hausman framework to the multivariate case. However, like all earlier competing risk models, he characterizes the exit state implicitly based on the duration spells. Further, the Sueyoshi approach becomes cumbersome when dealing with multivariate competing risks since it requires computation of multivariate integrals. In contrast, Bhat's approach requires only the computation of bivariate integrals independent of the number of competing risks.

Most multiple-duration hazard formulations do not accommodate unobserved heterogeneity because it makes the estimation difficult. However, with the computing capabilities available today, this should not be an excuse for ignoring unobserved heterogeneity. There has also been only limited work in accommodating dependence in the effect of unobserved variables across multiple spells from the same individual. Hensher (1994b) and Mealli and Pudney (1996) have formulated and estimated a competing risks model that captures both unobserved heterogeneity specific to each spell as well as unobserved "fixed" dependence across multiple spells from the same individual. These papers also serve as exhaustive reviews of recent competing risk formulations.[9] Other issues in the context of hazard models not discussed here include incorporating the time-invariant effect of time varying covariates or allowing for time-varying effects of time-invariant covariates. For recent work in these areas, the reader is referred to Hensher (1994b), McCall (1994) and Wedel *et al*. (1995).

4. LIMITED-DEPENDENT VARIABLE MODELS

Limited-dependent variable models encompass a wide variety of structures which may be classified in one of two broad categories. The first category recognizes the discontinuous nature of a variable (such as the ordinal nature of number of activity stops or several zero values for out-of-home activity duration because of non-participation in out-of-home activity). The second category accommodates the interdependence between a discrete choice variable and another related continuous or grouped or ordinal variable (such as the interdependence between mode

[9] Ettema *et al*., 1995 also formulate a competing risk model to model activity duration with the termination states being any one of several activity types such as in-home leisure, work/education, shopping, *etc*. They use an accelerated lifetime model to include the effect of covariates so that the covariates rescale time directly. Unfortunately, with such a specification, they are unable to capture unobserved heterogeneity and also they have to impose the assumption of independence among risks.

choice to work and the number of activity stops during the evening commute). In general, the structures for the second category subsume those of the first category as special cases. Thus, modeling out-of-home activity participation (a discrete choice) and out-of-home activity duration (a continuous choice) using a discrete/continuous framework is more general than considering duration as being a discontinuous variable with bunching of values at zero for individuals who do not participate in out-of-home activity. In this paper, we will focus on the more general second category of inter-related discrete and non-discrete variable systems. The non-discrete variable can take several forms. However, the three most interesting cases in the context of travel and activity modeling are the continuous, ordinal, and grouped forms. Further, the structure for the discrete/ordinal and discrete/grouped variable systems are very similar; so we will examine limited-dependent variable systems under two headings: discrete/continuous and discrete/ordinal models.

4.1. Discrete/continuous models

The methods developed for, and applications of, discrete/continuous choices can be broadly classified under two categories based on the number of alternatives involved in the discrete choice decision. The first category is the dichotomous alternative case and the second is the polychotomous alternative case. By far, most of the attention to date has focused on the dichotomous case (see Heckman, 1976 and Lee, 1981 for estimation methods and Willis and Rosen, 1980 and Sakamoto and Chen, 1991 for applications; Amemiya, 1985 and Maddala, 1983 provide a review of economic applications, while Winship and Mare, 1992 provide a review of sociological applications). In contrast to the dichotomous case, the polychotomous case has received much lesser attention (see Hay 1980, Dubin and McFadden 1983, Hanemann 1984, and

Lee 1983 for estimation methods and Hensher *et al.* 1992, Bhat 1996, Barnard and Hensher 1992, Hamed and Mannering 1993, and Mannering and Winston 1985 for applications; Mannering and Hensher, 1987 provide a review of transportation-related applications). Almost all applications of the dichotomous case have used either a logit or probit approach to model the discrete choice in the discrete/continuous model. Applications of the polychotomous case have generally used a multinomial logit-based approach to model the discrete choice due to the resulting simplicity in structure.

Hamed and Mannering (1993) use the discrete/continuous model framework to model activity type choice, travel time duration to the activity, and activity duration. They use a limited-information (two-stage) maximum likelihood method in their estimation where all variables specific to (or determined by) the activity type in which the traveler participated and appearing in the continuous travel time/activity duration equations are replaced by their expected values as obtained from the discrete activity type choice model. A similar limited-information procedure is used by Damm (1981) in his study of activity participation choice and activity duration, as well as by Hensher *et al.* (1992), Train (1986), and Mannering and Winston (1985) in their automobile brand type/automobile use models.

Barnard and Hensher (1992) estimate a discrete/continuous model of shopping destination choice and retail expenditure. They use Lee's (1983) transformation method for polychotomous choice situations with non-normal error distributions in the choice model. Bhat (1996d) has also used Lee's method for discrete/continuous models, but extends the method to jointly estimate a polychotomous discrete choice and two continuous choices (rather than a single continuous choice). Lee's method has two advantages over the other polychotomous (two-stage) methods discussed earlier. First, Lee's method enables full-information maximum-likelihood estimation, while the other methods are two-stage methods in which the discrete choice model is estimated first and then the continuous choice model is estimated using one of several methods to account for selectivity bias (see Dubin 1985, page 158). Thus Lee's method facilitates asymptotically more efficient estimates in the discrete/continuous choice model. Second, the expressions for the asymptotic covariance matrices of the two-stage estimates are very complicated, while the asymptotic covariance matrix in Lee's method can be obtained directly from the maximum likelihood estimation. Lee's method is also very flexible and can accommodate any model formulation for the discrete choice decision with little change in the methodology.

4.2. Discrete/ordinal models

There has been relatively little empirical work in the area of joint discrete/ordinal variable systems compared to the joint discrete/continuous systems reviewed in the previous section. As in the case of discrete/continuous systems, discrete/ordinal models can also be classified under the dichotomous and polychotomous categories based on the number of alternatives involved in the discrete choice decision. The next two paragraphs review studies in each of the two categories.

Bhat and Koppelman (1993) estimate a joint model of employment status (represented by a binary flag indicating whether or not an individual is employed) and annual income earnings. Observed income earnings in their data is in grouped form (*i.e.*, observed only in grouped categories such as < 20,000, 20,000-39,999, 40,000-59,999, *etc.*). Since it is likely that people who are employed are also likely to be the people who can earn higher incomes, the two variables are modeled jointly. Bhat (1996) subsequently has used a similar structure to impute a continuous income value from missing and grouped income observations in a data set. The motivation for this work is two fold. First, while income is measured in grouped form, it is the continuous measure of income that frequently appears as an explanatory variable in labor supply models, market research models and travel demand models (Killingsworth 1983, Koppelman *et al.* 1993, Golob 1989). Second, there might be systematic differences in unobserved characteristics affecting income between respondent and non-respondent households (or individuals). For example, it seems at least possible that households with above-average income, other things being equal, will be more reluctant than other households to provide information on income (see Lilliard *et al.*, 1986).

Bhat (1997e) has recently developed a joint model of polychotomous work mode choice and number of non-work activity stops during the work commute (*i.e.*, the total number of non-work stops made during the morning home-to-work commute and evening work-to-home commute). The joint model provides an improved basis to evaluate the effect on peak-period traffic congestion of conventional policy measures such as ridesharing improvements and solo-auto use dis-incentives. Traditional mode choice models address the question “What is the effect of a change in, say, solo-auto in-vehicle travel time (for example, due to conversion of an existing general-purpose lane to a high-occupancy lane) on work mode choice?”. If commute trips were the sole contributors to peak period congestion, then the shifts in work mode choice provide a direct indication of the potential impact on congestion. A more pertinent question to address today, however, is “What is the effect of a change in, say again, solo-auto in-vehicle time on work mode choice and number of non-work stops?”. This question is prompted by the recognition that vehicle trips due to non-work stops also add to peak period congestion. Thus, understanding the effect of a policy action on work mode choice and number of non-work stops allows us to evaluate the effect on peak-period congestion through the impact on both direct commute vehicle-trips and additional vehicle-trips due to non-work stops.

5. SUMMARY AND CONCLUSIONS

This paper has reviewed methodological developments in the econometric field of direct relevance to activity and travel behavior modeling. Clearly, there has been substantial progress in the development and practical applicability of the methodologies in the recent past. This progress can be traced to at least four factors: a) The need for realistic representations of the behavioral decision processes underlying activity-travel decisions, b) The ability to provide micro-level demographic inputs required by activity-travel models, c) Better tools for data storage and processing, and d) The advent of simulation techniques to approximate multi-dimensional integrals.

5.1. Need for realistic representation of behavioral decision processes

The travel demand models used widely today were developed in the late sixties and have seen little change over the years. These models were developed primarily to evaluate alternative major capital improvements. While this continues to remain an important objective of travel demand models, there is a shift in emphasis from evaluating the long-term investment-based strategies to understanding travel behavior responses to shorter term congestion management policies such as alternate work schedules, telecommuting, and congestion-pricing. The traditional travel demand models are not suited to such a task because, due to their many simplifying assumptions and narrow “individual-trip” perspective, they are unable to examine the potentially complex behavioral responses to demand management actions (Spear, 1994). For example, a change in work schedule to an early arrival home may lead to increased trip-making at the evening because of the additional time available to participate in out-of-home activities. If some of this travel is undertaken during the same time as the PM peak-period travel, the extent of congestion alleviation projected by traditional models will not be realized (see Jones *et al.*, 1990). Similarly displacements of travel (and its associated consequences) to other times of day due to a change in activity patterns caused by adoption of work telecommuting strategies cannot be examined by traditional models (see Mokhtarian, 1993). Also, traditional models do not incorporate adequate richness in the substitution pattern among alternatives or the different sensitivities of individuals to changes in the transportation system. This can lead to inappropriate evaluations of travel demand management policies (Stopher, 1993). Finally, from a transportation and regional planning perspective, reasonably accurate forecasts of travel demand are needed to be better prepared for the future and to endeavor to avoid serious conflicts between transportation supply and demand. Inasmuch as the travel needs of the population is changing rapidly (due to changes in lifestyle, changes in activity needs of particular subgroups such as the elderly, changes in household structure and social environment, changes in urban structure, changes in technology, *etc.*), it is obvious that models with a sound behavioral casual linkage between travel patterns and the travel environment will be critical to good design and planning of future transportation infrastructure.

5.2. Ability to provide micro-level inputs for activity-travel models

The need for realistic representations of activity and travel decisions requires modeling of these choices at the individual (or household) level. Once the individual-level models are estimated from a sample, they can be used to examine the impact of various policies (in the short-term) or forecast activity-travel patterns (in the long term). In either case, detailed disaggregate-level inputs of the characteristics of the decision-making entities and other attributes (such as options available and constraints encountered) of the choice context are required. Oftentimes, such information is not readily available. For example, consider a destination choice model which has been estimated on a sample and is to be applied to study the policy impact of imposing congestion-pricing on selected spatial corridors. The destination choice model might include household and individual level characteristics as exogenous determinants (for example, older individuals might prefer destinations which are close by or higher income earning individuals may be willing to travel greater distances, *etc*.). It is quite possible, however, that we will not have observations of individuals in the sample making trips between certain zonal pairs. In such a case, we cannot study the impact of the congestion-pricing policy on trip-making between those zonal pairs. Similarly, in a forecasting context, there will be changes in the characteristics of the population and the currently available sample may become unrepresentative of the future population. In both the cases mentioned above, there is a need for a mechanism to generate the appropriate disaggregate-level inputs. This issue has been at the core of the debate on the practical usefulness of disaggregate-level models. Miller (1996) summarizes it well as follows "I believe a strong case can be made that a primary reason for the relatively slow diffusion of disaggregate modeling methods into travel demand forecasting practice is due to the difficulty practitioners have in generating the disaggregate forecast inputs required by these methods". However, with the development of micro-simulation techniques to generate the required disaggregate-level inputs either through updating of the current sample over time or by “synthesizing” a representative sample from other supplementary aggregate-level information such as census data, it is now possible to apply models which are more realistic in their representations of behavior to policy analysis and forecasting. The reader is referred to the comprehensive review by Miller (1996) on techniques and research issues associated with micro-simulation.

5.3. Better tools for data storage and processing

The tools available for data storage and processing have seen dramatic improvement over the past few years. Desktop and even notebook computers are able to store data of large sizes and are remarkably fast in the retrieval and processing of such data. This has made possible the estimation of models deemed earlier to be impractical. The improved computer processing capabilities has also spurred the development of new and behaviorally rich model formulations. Another area that has developed quite considerably is Geographic Information Systems (GIS) technology. Fotheringham and Rogerson (1993) discuss the potential of integrating travel analysis methods with GIS technology. A specific application of GIS technology to activity-travel analysis is the development of a measure of spatial accessibility for use in the modeling of multistop and multi-purpose travel (see Arentze *et al*., 1994a,b,c; Lee, 1996). Golledge *et al.* (1994) and Kwan (1994) have used GIS to calibrate a production system model of activity scheduling behavior. Caliper Corporation's TransCAD GIS software represents an important bridge in linking GIS developments with travel demand modeling practice. Specifically, TransCAD attempts to package advanced econometric modeling techniques within an interface that is user-friendly, enables spatial representation of the transportation network and geographic database management, and allows an intuitive spatial display of the results from the travel demand models.

5.4. Advent of simulation techniques to approximate multi-dimensional integrals

Recent advances in the field of Monte Carlo simulation methods to evaluate multi-dimensional integrals have contributed considerably to the feasibility in estimating complex discrete-choice and other limited-dependent variable models. Two types of simulators that are of particular interest in the activity-travel area are the probit-based and the logit-based simulators. The former is suitable for discrete-choice structures that use a normal distribution for the random components and the latter is appropriate for various extensions of the multinomial logit structure (see Chib and Greenberg, 1996, Hajivassiliou *et al.*, 1996, and Brownstone and Train, 1996 for reviews of such simulation techniques). The underlying concept in such methods is to approximate the integration by computing the integrand at various values drawn from the appropriate multi-variate distribution of the variable vector over which the integration is being carried out and taking the mean across the computed integrand values. Several issues arise during the actual implementation of the approach, which we do not discuss here. The application of probit-simulators in the travel behavior field can be found in the work of Mahmassani and his colleagues who have used the multinomial probit structure to examine the day-to-day dynamics in departure time and route choice of commuters (see Lam and Mahmassani, 1991; Mahmassani and Jou, 1996; Mahmassani, 1997). Logit-based simulators have been used in the travel demand field by Brownstone and Train (1996), Bhat (1996a, 1997b), and Ben-Akiva and Bolduc (1996).

The formulation and estimation of behaviorally rich models has been greatly facilitated by the developments discussed above. However, the fields of micro-simulation, Geographic Information Systems, and simulation of integrals are continually evolving and by all accounts there is still considerable progress to be made. As these fields develop, and as practitioners and researchers in the activity-travel behavior field become familiar with them, there is bound to be more empirical applications using these tools. This, along with the need for improved policy analysis and accurate demand forecasting, should contribute further toward the implementation of improved methodologies in the area of activity and travel behavior research.

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