Accommodating Flexible Substitution Patterns in Multi-Dimensional Choice Modeling: Formulation and Application to Travel Mode and Departure Time Choice

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Abstract

The nested logit model has been used extensively to model multi-dimensional choice situations. A drawback of the nested logit model is that it does not allow choice alternatives to share common unobserved attributes along all the dimensions characterizing the multi-dimensional choice context. This paper formulates a mixed multinomial logit structure that accommodates unobserved correlation across both dimensions in a two-dimensional choice context. The mixed multinomial logit structure is parsimonious in the number of parameters to be estimated and is also relatively easy to estimate using simulation methods. The mixed multinomial logit model is applied to an analysis of travel mode and departure time choice for home-based social-recreational trips using data drawn from the 1990 San Francisco Bay Area household survey. The empirical results underscore the need to capture unobserved attributes along both the mode and departure time dimensions, both for improved data fit as well as for more realistic policy evaluations of transportation control measures.

Keywords: Nested logit model, error-components logit, mixed multinomial logit, simulation estimation technique, nonwork trip modeling, travel mode choice modeling, departure time analysis.

1. Introduction

Many discrete choice contexts are characterized by alternatives which represent a combination of two or more underlying choice dimensions. Examples of such multi-dimensional choice situations include (to list a few) purchase incidence and brand choice in marketing (Bucklin and Gupta, 1992), auto ownership and work travel mode choice in transportation (Train, 1980, and Koppelman and Pas, 1986), residential location and workplace choice in geography (Waddell, 1993 and Evers, 1990), and dwelling type and residential location choice in urban economics (Fischer and Aufhauser, 1988).

The need to jointly analyze the dimensions characterizing a multi-dimensional choice situation arises from three considerations. First, the feasible choice set for a decision-making agent may be determined by a combination of the underlying choice dimensions. Second, there may be important observed determinants of choice which are associated with the combination of the underlying dimensions. Third, some of the joint choice alternatives may share unobserved attributes, leading to different patterns of substitution among different pairs of alternatives.

The model structure used to analyze multi-dimensional choice depends, to a large extent, on the assumptions made regarding shared unobserved attributes. The multinomial logit (MNL) structure assumes the absence of any common unobserved attributes among the utilities of the joint choice alternatives. This assumption results in the Independence from Irrelevant Alternatives (IIA) property, which is untenable in most multidimensional choice contexts (and, at the least, should be empirically tested; see Stopher *et al.*, 1981).

The nested logit (NL) model (Daly and Zachary, 1979; McFadden, 1978) generalizes the MNL model. It has a closed-form mathematical structure and is relatively easy to estimate. The drawback of the NL model is that it imposes the rather unrealistic restriction that shared unobserved attributes can be associated with only one or the other (as opposed to both) choice dimensions (in the rest of this paper, we will restrict the presentation and discussion to a two-dimensional choice setting since most applications of multi-dimensional discrete choice models are confined to two dimensions). For example, in a joint travel mode and departure time context, it is likely that there will be unobserved factors (such as comfort and privacy) common to joint choice alternatives sharing the same travel mode as well as unobserved factors (such as personal scheduling preferences) common to joint choice alternatives sharing the same departure time. However, the NL model would allow shared unobserved attributes along the mode dimension only or along the departure time dimension only.

The multinomial probit (MNP) structure allows a flexible structure for the covariance among the unobserved attributes of the alternatives. Consequently, it allows a flexible substitution pattern among the joint choice alternatives. Unfortunately, in most choice contexts, the increase in flexibility of the MNP structure comes at the expense of evaluating very high dimensional multivariate normal integrals for the choice probabilities. Methodological developments in the past few years suggest approximating the high-dimensional integral with smooth, unbiased and efficient simulators which provide strictly positive values for the choice probabilities. A simulator that satisfies all these properties is one that combines accept/reject simulation techniques with a logit kernel (Ben-Akiva and Bolduc, 1996). The traditional accept/reject simulator is constructed for the MNP structure (or for any random utility structure) by (a) drawing values for the random terms from the multivariate normal distribution (or from the appropriate distribution for any random utility model); (b) calculating the resulting utility of each alternative and identifying the one with the highest utility; (c) repeating steps (a) and (b) several times, and (d) computing a simulated probability for each alternative as the proportion of draws for which that alternative had the highest utility. Such a simulator is unbiased, but not smooth (with respect to model parameters) and does not guarantee strictly positive choice probabilities for a finite number of draws (*i.e.*, the simulated probability for an alternative may be zero if it is never the one with the highest utility in any of the draws). The use of a logit kernel replaces the deterministic 0-1 assignment for each alternative in each draw by a logit formula that takes the utility values as its arguments. The resulting choice probability simulator is smooth and strictly positive. The use of the logit kernel accept/reject simulator is equivalent to the addition of a Gumbel error term (distributed identically and independently across alternatives) to the utility of each alternative. Of course, the addition of such an error term changes the structure of the model. But, as indicated formally by McFadden and Train (1997) and more intuitively by Train (1997a), the addition of such a gumbel term is innocuous and does not change utility relationships. In fact, the model with the additional gumbel error term can be made to approximate the model without the gumbel term to any desired degree of closeness.

Another impediment to the use of a MNP is the large number of parameters to be estimated if one allows a completely free covariance matrix (subject to certain identification considerations). The large number of covariance parameters generates a number of conceptual, statistical and practical problems, including difficulty in interpretation, highly non-intuitive model behavior, and low precision of covariance parameter estimates (see Horowitz, 1991 for a discussion). An approach to alleviate the situation is to impose constraints on the covariance matrix, so the number of parameters is reduced while still providing sufficiently realistic substitution patterns among the alternatives for the choice situation under consideration. There are two ways of imposing such constraints. One approach is to impose them directly on the MNP covariance matrix. The second is to use an error-components approach that induces the required correlations over alternatives along with the inclusion of an IID gumbel term. In the first approach, the dimensionality of the integral in the choice probabilities is on the order of J-1, J being the number of alternatives. In the second approach, the dimensionality is equal to the number of error components. When the number of alternatives is large, as is the case in many choice contexts, the number of error components will be smaller than the number of alternatives. Thus, there can be substantial gains in simulator efficiency and estimation cost if one uses the error-components approach (see also Brownstone and Train, 1996 for an application that shows that the error-components approach with the gumbel IID error term can approximate MNP probabilities considerably more accurately than direct MNP simulators, given a specified amount of computer time).

In the error-components approach with the IID gumbel term, the choice probabilities of the alternatives conditional on the error-components takes the familiar multinomial logit form. The unconditional choice probabilities are obtained by integrating the multinomial logit form over the distribution of the random parameters in the error-components. For this reason, the error-components approach can be viewed as a "mixed multinomial logit" (MMNL) structure where the (unconditional) choice probability is a mixture of a logit form with the specified distribution of the random parameters. It is instructive to note here that a random-coefficients specification within a multinomial logit formulation also leads to the mixed multinomial logit structure (Revelt and Train, 1998; Train, 1997b; Bhat, 1996a, 1997a; and Mehndiratti, 1996 for random-coefficients logit applications). Thus, a mixed MNL structure may be generated from an intrinsic motivation to allow flexible substitution patterns across alternatives (error-components) or from a need to accommodate unobserved heterogeneity across individuals in their sensitivity to observed exogenous variables (random-coefficients).

In this paper, we will use the term "mixed logit" recognizing that this structure is motivated by a need to allow a flexible substitution pattern across alternatives. Specifically, we present a mixed

MNL model formulation that accommodates shared unobserved attributes along both dimensions in a two-dimensional choice situation. The particular form of the error-components structure used in this study has not been applied previously in the mixed logit literature and should be useful in several other choice contexts in addition to the one in the current paper.

The rest of this paper is organized as follows. The next section develops the model formulation. Section 3 discusses the estimation procedure. Section 4 presents the empirical results obtained from applying the model to travel mode and departure time choice for home-based social-recreational trips. The final section provides a summary of the research findings and identifies possible extensions of the current research.

2. Model Formulation

We will develop the model formulation in the context of a joint model of travel mode and departure time choice. For simplicity in presentation, we will assume that all individuals have all alternatives available to them. Extension of the formulation to the case where some individuals have only a subset of alternatives available is straightforward.

Assume *M* travel modes and *T* departure time periods in the choice set (the departure time choice is represented by several temporally contiguous discrete time periods which collectively span the entire day). Let the utility U_{mt} that an individual associates with the mode-departure time alternative $\{m,t\}$ be the sum of a deterministic component V_{mt} (that depends on observed attributes of the alternative and the individual) and a random component $\zeta_{mt} = \mu' z_m + \eta' w_t + \varepsilon_{mt}$ (we develop the model structure at the individual level and so do not use an index for individuals). z_m is a row vector of dummy variables (of dimension *M*), each row being associated with a travel mode m' $(m' = 1,2,...,m,...M)$. If $m = m'$, the corresponding row has a value of one; otherwise, the value is 0. μ is a random row vector (of dimension *M*) with zero mean, whose elements are assumed to be independent of each other and normally distributed. The variance matrix of $\mu(=\Sigma)$ is diagonal with the parameter $\sigma^2_{m'}$ in the m'th row. η and w_t are row vectors of dimension *T* defined in a manner similar to μ and z_m, respectively, but in the context of departure times. The variance matrix of $\eta(=\Omega)$ is diagonal with the parameter $\delta^2_{t'}$ in the t'th row. The random vectors μ and η are assumed to be mutually independent, and independent of ε_{mt}. The ε_{mt} terms are assumed to be independent and identically standard Gumbel distributed (across alternatives).

The error components specification adopted for the composite error term ζ_{mt} generates a covariance across alternatives with the same mode $(\mathrm{Cov}\{U_{mt}, U_{mt^*}\} = \sigma_m^2;\ t \neq t^*)$ and also a covariance across alternatives with the same departure time $(\mathrm{Cov}\{U_{mt}, U_{m^*t}\} = \delta_t^2;\ m \neq m^*)$. If we impose the restriction that $\sigma_m^2 = 0$ for all *m* and $\delta_t^2 = 0$ for all *t*, we obtain the MNL structure. If we impose the restriction that $\sigma_m^2 = 0$ for all *m*, but $\delta_t^2 \neq 0$ for all *t*, we obtain an analog to the nested NL model with shared unobserved attributes along the departure time dimension. On the other hand, if we impose the restriction that $\sigma_m^2 \neq 0$ for all *m*, but $\delta_t^2 = 0$ for all *t*, we obtain an analog of the NL model that allows shared unobserved attributes along only the mode dimension.

For given values of μ and η in the MMNL model, we get the familiar MNL form for the probability of choosing alternative *mt* (McFadden, 1973):

$$P_{mt} \mid (\mu, \eta) = \frac{\exp(V_{mt} + \mu' z_m + \eta' w_t)}{\sum_{(m^*, t^*)} \exp(V_{m^*t^*} + \mu' z_{m^*} + \eta' w_{t^*})} \tag{1}$$

The unconditional probability of choosing alternative *mt* can now be obtained by integrating the conditional multinomial choice probabilities in equation (1) with respect to the assumed normal (and independent) distributions for the vectors μ and η.:

$$P_{mt} = \int_{\mu=-\infty}^{+\infty} \int_{\eta=-\infty}^{+\infty} \left\{ \frac{\exp(V_{mt} + \mu' z_m + \eta' w_t)}{\sum_{(m^*, d^*)} \exp(V_{m^*t^*} + \mu' z_{m^*} + \eta' w_{t^*})} \right\} f(\mu)\, f(\eta)\, \mathrm{d}\mu\, \mathrm{d}\eta \tag{2}$$

Note that μ and η are vectors with *M* and *T* (independent and identically distributed) elements, respectively. Thus, the expression in equation (2) involves a (*M*+*T*)-dimensional integral.

3. Model Estimation

We assume a linear-in-parameters specification for the systematic utility of each joint choice alternative given by $V_{qmt} = \beta' X_{qmt}$ for the individual q and alternative mt (we introduce the index for individuals in the following presentation since the purpose of the estimation is to obtain the model parameters by maximizing the likelihood function over all individuals in the sample). The composite random error term for the qth individual is given by $\zeta_{qmt} = \mu_q' z_m + \eta_q' w_t + \varepsilon_{qmt}$. We assume that μ_q, η_q, and ε_{qmt} are each independently and identically distributed across individuals: $\mu_q \sim N(0, \Sigma)$, $\eta_q \sim N(0, \Omega)$, and $\varepsilon_{qmt} \sim G(0,1)$. The parameters to be estimated in the MMNL model are the parameter vector β, the diagonal variance matrix Σ (that is, the parameters σ_m for each m), and the diagonal variance matrix Ω (that is, the parameters δ_t for each t). Now define s_q and u_q as standard-normal row vectors of dimension M and T, respectively, so that $s_q = \Sigma^{-1/2}\mu_q$ and $u_q = \Omega^{-1/2}\eta_q$. Also, let $[\Sigma^{1/2}]$ and $[\Omega^{1/2}]$ represent row vectors obtained by picking off the diagonal entries of $\Sigma^{1/2}$ and $\Omega^{1/2}$, respectively. Then, the log-likelihood function for a given value of the parameter vector $\theta = (\beta', [\Sigma^{1/2}]', [\Omega^{1/2}]')'$ takes the form shown below:

$$
\begin{aligned}
\ell(\theta) &= \sum_q \sum_{m,t \varepsilon C_q} y_{qmt} \log P_{qmt}(\theta) \\
&= \sum_q \sum_{m,t \varepsilon C_q} y_{qmt} \log \left[\int_{s_q=-\infty}^{+\infty} \int_{u_q=-\infty}^{+\infty} \left\{ \frac{\exp(\beta' X_{qmt} + \Sigma^{1/2} s_q + \Omega^{1/2} u_q)}{\sum_{m^*, t^* \varepsilon C_q} \exp(\beta' X_{qm^*t^*} + \Sigma^{1/2} s_q + \Omega^{1/2} u_q)} \right\} \cdot \phi(s_q)\phi(u_q) \mathrm{d}s_q \mathrm{d}u_q \right] \quad (3)
\end{aligned}
$$

where C_q is the choice set of alternatives available to the qth individual, $\phi(.)$ represents the standard normal density function, and

$$
y_{qmt} = \begin{cases} 1 & \text{if the } q \text{th individual chooses alternative } mt, \\ 0 & \text{otherwise.} \end{cases} \quad (4)
$$

The log-likelihood function for the estimation of the parameters (equation 3) involves a M+T-dimensional integral, which cannot be evaluated analytically since it does not have a closed-form solution. Further, conventional quadrature techniques cannot be used to compute the integrals with sufficient precision and speed for estimation via maximum likelihood, since the dimension of integration exceeds two (Revelt and Train, 1998; Hajivassiliou and Ruud, 1994).

We apply simulation techniques to approximate the choice probabilities in the log-likelihood function of equation (3) and maximize the resulting simulated log-likelihood function. The use of simulation techniques to evaluate multi-dimensional integrals has received substantial attention in recent years. Our implementation of simulation methods to estimate the mixed multinomial logit model takes the same form as the procedure adopted by Revelt and Train (1998) and Bhat (1996a, 1997a). The simulation technique approximates the choice probabilities by computing the integrand in equation (3) at randomly chosen values for each s_q vector and for each u_q vector. Since the elements within the vectors s_q and u_q are independent of each other and independent across individuals, and also because the vectors s_q and u_q are themselves independent (by assumption), we generate a matrix w of standard normal random numbers with $Q^*(M+T)$ elements (one element for each individual-travel mode combination and one element for each individual-departure time combination) and compute the corresponding choice probabilities for a given value of the parameter vector θ. We then repeat this process R times for the given value of the parameter vector θ. Let $\widetilde{P}^r_{qmt}(\theta)$ be the realization of the choice probability in the rth draw (r = 1,2,..., R). The choice probabilities are then approximated by averaging over the $\widetilde{P}^r_{qmt}(\theta)$ values:

$$\widetilde{P}_{qmt}(\theta) = \frac{1}{R} \sum_{r=1}^{R} \widetilde{P}^r_{qmt}(\theta) , \tag{5}$$

where $\widetilde{P}_{qmt}(\theta)$ is the simulated choice probability of the qth individual choosing alternative mt given the parameter vector θ. $\widetilde{P}_{qmt}(\theta)$ is an unbiased estimator of the actual probability $P_{qmt}(\theta)$. Its variance decreases as R increases. It also has the appealing properties of being smooth (*i.e.*, twice differentiable) and being strictly positive for any realization of the finite R draws. The former property is important since it implies that conventional gradient-based optimization methods can be used in the maximization of the simulated log-likelihood function. The latter property ensures that the simulated log-likelihood function (which involves the logarithm of the choice probabilities) is always defined.

The simulated log-likelihood function is constructed as:

$$S\text{‹}(\theta) = \sum_{q=1}^{Q} \sum_{m,t \in C_q} y_{qmt} \log\left[\widetilde{P}_{qmt}(\theta)\right] \tag{6}$$

The parameter vector θ is estimated as the vector value that maximizes the above simulated function. Under rather weak regularity conditions, the maximum simulated log-likelihood (MSL) estimator is consistent, asymptotically efficient, and asymptotically normal (see Hajivassiliou and Ruud, 1994 and Lee, 1992). However, the MSL estimator will generally be a biased simulation of the maximum log-likelihood (ML) estimator because of the logarithmic transformation of the choice probabilities in the log-likelihood function. The bias of the MSL estimator decreases with the variance of the probability simulator; that is, it decreases as the number of repetitions increase. Brownstone and Train (1996) have shown the bias to be rather negligible with 250 repetitions in the context of the MMNL model. In the current paper, we use 500 repetitions for accurate simulations of the choice probabilities and to reduce simulation variance of the MSL estimator.

All estimations and computations were carried out using the GAUSS programming language on a pentium personal computer. Gradients of the simulated log-likelihood function with respect to the parameters were coded.

4. Application to Travel Mode and Departure Time Choice

4.1. Background

Mode choice and departure time choice are important components of a traveler's decision regarding trip-making. At an aggregate level, these choices determine the number and temporal pattern of vehicle trips on urban roadways. From a policy standpoint, the recent Intermodal Surface Transportation Efficiency Act (ISTEA) and the Clean Air Act Legislations (CAAAs) require that travel demand models be able to evaluate a variety of transportation control measures (TCMs) such as peak-period pricing, congestion-pricing, restrictions on use of single occupancy vehicle (SOV) during certain time periods, and ride-sharing and transit-use incentives (Stopher, 1993; Weiner and Ducca, 1996). These TCMs may have an impact on travel mode, or departure time choice, or both. Consequently, understanding the factors that affect travelers' mode and departure time is a necessary prerequisite to examining the potential effectiveness of policy measures aimed at alleviating traffic congestion and reducing mobile source emissions.

Previous research on trip mode choice, and the very limited research on trip departure time choice, has primarily focused on the work trip (see Bhat, 1997a; Horowitz, 1993; Ben-Akiva and Lerman, 1985; Swait and Ben-Akiva, 1987; and Train, 1980 for work mode choice modeling and Abkowitz, 1981; Mannering, 1989; Chin, 1990; Hendrickson and Plank, 1984; Mahmassani and Jou, 1996; and Small, 1982 for work departure time choice modeling). However, nonwork travel accounts for about three-fourths of the total trips in urban areas and projections suggest that this proportion is only likely to increase as suburbanization and lifestyle changes impact individuals' travel behavior (for a detailed discussion, see Lockwood and Demetsky, 1994). Further, in the context of departure time choice, nonwork trips offer a more interesting challenge than work trips. Most individuals do not have much flexibility in changing their work departure time because of the relatively rigid nature of work schedules; on the other hand, individuals are likely to have considerably more departure time flexibility for nonwork trips. The above reasons motivate our focus on nonwork trips; specifically, we direct our attention to home-based social-recreational (HBSR) trips in this paper.

4.2. Data Source and Model Specification

The data for the study are drawn from the San Francisco Bay Area Household Travel Survey conducted by the Metropolitan Transportation Commission (MTC) in the Spring and Fall of 1990 (see White and Company, Inc., 1991 for details of survey sampling and administration procedures). This survey included a single-weekday travel diary of households, and it is this single-day sample that is used here. In addition to the travel diary, detailed individual and household socio-demographic information was also collected in the survey.

The modal alternatives include drive alone, shared-ride, and transit. The departure time choice is represented by six time-periods: early morning (12:01-7 a.m.), a.m. peak (7:01-9 a.m.), a.m. offpeak (9:01 a.m.-12 noon), p.m. offpeak (12:01-3 p.m.), p.m. peak (3:01-6 p.m.), and evening (6:01 p.m.-12 midnight). For some individual trips, modal availability is a function of time-of-day (for example, transit mode may be available only during the a.m. and p.m. peak periods). Such temporal variations in modal availability are accommodated by defining the feasible set of joint choice alternatives for each individual trip.

Level of service data were generated for each zonal pair in the study area and by five time periods: early morning, a.m. peak, mid-day, p.m. peak, and evening.[1] The mid-day impedances were applied to both the a.m. offpeak and p.m. offpeak periods. The impedance data were appropriately appended to the home-based trips based on the origin-destination of trips.

The sample used in this paper comprises 3000 home-based social-recreational person-trips obtained from the overall single-day travel diary sample. The mode choice shares in the sample are as follows: drive alone (45.7%), shared-ride (51.9%) and transit (2.4%). The departure time distribution of home-based social-recreational trips in the sample is as follows: Early morning (4.6%), a.m. peak (5.5%), a.m. offpeak (10.3%), p.m. offpeak (17.2%), p.m. peak (16.1%), and evening (46.3%).

Four sets of variables were used in the model specification: (a) alternative specific constants, (b) individual/household socio-demographics; (c) trip destination attributes, and (d) level-of-service variables. We arrived at the final specification based on a systematic process of eliminating variables found to be statistically insignificant in previous specifications and combining variables found not to have statistically different effects on the joint mode-departure time utilities. The sociodemographic variables influencing mode/departure time choice in the final specification included employment status (whether employed or not), age, an elderly flag indicator (whether above 65 yr or not), sex, a flag variable indicating presence of children (less than 16 yr) in the individual's household, income of individual's household, and the ratio of the number of vehicles to adults in the individual's household. The trip destination attributes included a San Francisco downtown destination indicator that identified whether a trip terminated in the San Francisco downtown area, and a Central Business District (CBD) destination flag that indicated whether a trip terminated in a CBD.[2] Three level-of-service variables were used in the current analysis: travel cost, total travel time, and out-of-vehicle travel time over trip distance.

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[1] The Metropolitan Transportation Commission in Oakland provided zone-to-zone level-of-service data by travel mode for two time periods of the day: AM peak and mid-day. We applied mode-specific factors to the AM peak and mid-day level-of-service data to obtain the level-of-service measures for the other time periods of the day. The factors were developed based on information extracted from the household travel survey. For a detailed discussion of the procedure, see Bhat (1997b).

[2] The CBD districts include the San Francisco superdistricts (except the downtown superdistrict which has an extremely high employment density and is identified separately) and the superdistricts of San Jose and Oakland. The superdistrict classification is based on a 34 system categorization developed by the Metropolitan Transportation Commission.

4.3. Empirical Results

We estimated four different models of mode-departure time choice: (1) the (MNL) model; (2) the MMNL model which accommodates shared unobserved random utility attributes along the departure time dimension only (we will refer to this model as the MMNL-T model); (3) the MMNL model which accommodates shared unobserved random utility attributes along the mode dimension only (we will refer to this model as the MMNL-M model); and (4) the proposed MMNL model which accommodates shared unobserved attributes along both the dimensions of mode and departure time (we will refer to this model as the MMNL-MT model). In the MMNL models, we allowed the sensitivity among joint choice alternatives sharing the same mode (departure time) to vary across modes (departure times). It is useful to note that such a specification generates heteroscedasticity in the random error terms across the joint choice alternatives. In the MMNL-T and MMNL-MT models, we found statistically insignificant shared unobserved components specific to the morning departure times (*i.e.*, early morning, a.m. peak, and a.m. offpeak periods). Consequently, the MMNL-T and MMNL-MT model results presented here restrict these components to zero.

The level-of-service parameter estimates, implied money values of travel time, data fit measures, and the variance parameters in $[\Sigma]$ and $[\Omega]$ from the different models are presented in Table 1 (other parameter estimates are presented in section 4.5). The signs of the level-of-service parameters are consistent with *a priori* expectations in all the models. Also, as expected, travelers are more sensitive to out-of-vehicle travel time than in-vehicle travel time. A comparison of the magnitudes of the level-of-service parameter estimates across the four specifications reveals a progressively increasing magnitude as we move from the MNL model to the MMNL-MT model (this is an expected result since the variance before scaling is larger in the MNL model compared to the mixture models, and in the MMNL-M and MMNL-T models compared to the MMNL-MT model; see Revelt and Train, 1997 for a similar result). The implied money values of in-vehicle and out-of-vehicle travel times are lesser in the MMNL models relative to the MNL model.

The four alternative models in Table 1 can be evaluated formally using conventional likelihood ratio tests. A statistical comparison of the MNL model with any of the mixture models leads to the rejection of the MNL. Further likelihood ratio tests among the MMNL-M, MMNL-T, and MMNL-MT models result in the clear rejection of the hypothesis that there are shared unobserved attributes along only one dimension; that is, the tests indicate the presence of statistically significant shared unobserved components along both the mode and departure time dimensions (the likelihood ratio test statistic in the comparison of the MMNL-T model with the MMNL-MT model is 14.2; the corresponding value in the comparison of the MMNL-M model with the MMNL-MT model is 23.8; both these values are larger than the chi-squared distribution with three degrees of freedom at any reasonable level of significance). Thus, the MNL, MMNL-T, and MMNL-M models are mis-specified.

The variance parameters provide important insights regarding the sensitivity of joint choice alternatives sharing the same mode and departure time. The variance parameters specific to departure times (in the MMNL-T and MMNL-MT models) show statistically significant shared unobserved attributes associated with the afternoon/evening departure periods. However, as indicated earlier, we did not find statistically significant shared unobserved components specific to the morning departure times (*i.e.*, early morning, a.m. peak, and a.m. offpeak periods). The implication is that home-based social-recreational trips pursued in the morning are more flexible and more easily moved to other times of the day than trips pursued later in the day. Social-recreational activities pursued later in the day may be more rigid because of scheduling considerations among household members and/or because of the inherent temporal "fixity" of late-evening activities (such as attending a concert or a social dinner). The magnitude of the departure time variance parameters reveal that late evening activities are most rigid, followed by activities pursued during the p.m. offpeak hours. The p.m. peak social-recreational activities are more flexible relative to the p.m. offpeak and late-evening activities. The variance parameters specific to the travel modes (in the MMNL-M and MMNL-MT models) confirm the presence of common unobserved attributes among joint choice alternatives that share the same mode; thus, individuals tend to maintain their current travel mode when confronted with transportation control measures such as ridesharing incentives and auto-use disincentives. This is particularly so for individuals who rideshare, as can be observed from the higher variance associated with the shared-ride mode relative to the other two modes. In the context of home-based social-recreational trips, most ridesharing arrangements correspond to travel with children and/or other family members; it is unlikely that these ridesharing arrangements will be terminated after implementation of transportation control measures such as transit-use incentives.

The different variance structures among the four models imply different patterns of inter-alternative competition. To demonstrate the differences, Table 2 presents the disaggregate self- and cross-elasticities (for a person-trip in the sample with close-to-average modal level-of-service values) in response to peak period pricing implemented in the p.m. peak (*i.e.*, a cost increase in the "drive alone-p.m. peak" alternative). We group all morning time periods together in the Table since the cross-elasticities for these time periods are the same for each mode (due to the absence of shared unobserved attributes specific to the morning time periods).

The MNL model exhibits the familiar Independence from Irrelevant Alternatives (IIA) property (that is, all cross-elasticities are equal). The MMNL-T model shows equal cross-elasticities for each time period across modes, a reflection of not allowing shared unobserved attributes along the modal dimension. However, there are differences across time periods for each mode. First, the shift to the shared ride-p.m. peak and transit-p.m. peak is more than to the other non-p.m. peak joint choice alternatives. This is, of course, because of the increased sensitivity among p.m.-peak joint choice alternatives generated by the error variance term specific to the p.m. peak period. Second, the shift to the evening-period alternatives are lower compared to the shift to the p.m. offpeak period alternatives for each mode. This result is related to the heteroscedasticity in the shared unobserved random components across time periods. The variance parameter in Table 1 associated with the evening period is higher than that associated with the p.m. offpeak period; consequently, there is less shift to the evening alternatives (see Bhat, 1995 for a detailed discussion of the inverse relationship between cross-elasticities and the variance of alternatives). The MMNL-M model shows, as expected, a heightened sensitivity of drive alone alternatives (relative to the shared-ride and transit alternatives) in response to a cost increase in the DA-p.m. peak alternative. The higher variance of the unobserved attributes specific to shared-ride (relative to transit; see Table 1) results in the lower cross-elasticity of the shared-ride alternatives compared to the transit alternatives. The MMNL-MT model shows higher cross-elasticities for the drive alone alternatives as well as for the non-drive alone p.m. peak period alternatives since it allows shared-unobserved attributes along both the mode and time dimensions.

The drive-alone p.m. peak period self-elasticities in Table 2 are also quite different across the models. The self-elasticity is lower in the MMNL-T model relative to the MNL mode. The MMNL-T model recognizes the presence of temporal rigidity in social-recreational activities pursued in the p.m. peak. This is reflected in the lower self-elasticity effect of the MMNL-T model. The self-elasticity value from the MMNL-M model is larger than that from the MMNL-T model. This is because individuals are likely to maintain their current travel mode (even if it means shifting departure times) in the face of transportation control measures. But the MMNL-T model accommodates only the rigidity effect in departure time, not in travel mode. As a consequence, the rigidity in mode choice is manifested (inappropriately) in the MMNL-T model as a low drive alone p.m.-peak self-elasticity effect. Finally, the self-elasticity value from the MMNL-MT model is lower than the value from the MMNL-M models. The MMNL-M model ignores the rigidity in departure time; when we include this effect in the MMNL-MT model, the result is a depressed self-elasticity effect.

4.4. Substantive Policy Implications

The substitution structures among the four models imply different patterns of competition among the joint mode-departure time alternatives. We now turn to the aggregate self- and cross-elasticities to examine the substantive implications of the different competition structures for the level-of-service variables. To limit the discussion, we focus only on the travel cost elasticities for the drive alone and transit joint choice alternatives in response to a congestion pricing policy implemented in the p.m. peak.

Table 3 provides the cost elasticities obtained from the various models. The aggregate cost elasticities reflect the same general pattern as the disaggregate elasticities discussed earlier. Some important policy-relevant observations that can be made from the aggregate elasticities are as follows. The DA-p.m. peak self-elasticities show that the MNL and MMNL-T models under-estimate the decrease in peak period congestion due to peak-period pricing, while the MMNL-M model over-estimates the decrease. Thus, using the DA-p.m. peak cost self-elasticities from the MNL and MNL-T models will make a policy analyst much more conservative than (s)he should be in pursuing peak-period pricing strategies. On the other hand, using the DA-p.m. peak cost self-elasticity from the MMNL-M model provides an overly-optimistic projection of the congestion alleviation due to peak period pricing. From a transit standpoint, the MNL and MMNL-T under-estimate the increase in transit share across all time periods due to p.m. peak period pricing. Thus, using these models will result in lower projections of the increase in transit ridership and transit revenue due to a peak period pricing policy. The MMNL-M model under-estimates the projected increase in transit share in all the non-evening time periods, and over-estimates the increase in transit share for the evening time period. Thus, the MNL, MMNL-T, and MMNL-M models are likely to lead to inappropriate conclusions regarding the necessary changes in transit provision to complement peak-period pricing strategies.

4.5. Detailed MMNL-MT Model Results

In this section, we present and discuss the parameter estimation results from the MMNL-MT model (see Table 4). We do not present the alternative-specific constant values due to space constraints. We also do not discuss the effect of level-of-service variables or the correlation parameters, since these have been presented earlier in Table 1.

Among the socio-demographic variables, we observe that employed individuals tend to participate in home-based social-recreational (HBSR) activities primarily during the evening period. Employed individuals are particularly unlikely to pursue HBSR activities during the a.m. offpeak and p.m. offpeak periods since they would be at work during these times. The effect of employment on mode choice indicates that employed individuals are more likely to use the drive alone mode for HBSR trips than unemployed individuals. Age has a negative effect on making HBSR trips in the evening; in addition, individuals over 65 yr (the “elderly”) are most likely to pursue HBSR activities during the mid-day. These results suggest that older individuals tend to stay away from pursuing HBSR activities in the early and late parts of the day and also from the peak hours (possibly due to perceived safety/security considerations). The effect of sex on mode use suggests that women are more predisposed toward ridesharing arrangements than men. Individuals in households with young children have to work around the biological needs and sleeping schedules of the children, which make it difficult for them to pursue out-of-home activities in the early and late parts of the day (see Bhat and Koppelman, 1993). The negative effect of presence of children on early morning and evening departures from our analysis appears to confirm this. The strong positive effect of presence of children on use of the shared-ride mode is simply a reflection of adults traveling with their children to participate in social-recreational activities. The effects of income and the ratio of vehicles to adults in the household on departure time choice and mode choice, respectively, are also quite reasonable.

The impact of the trip destination attributes on mode choice indicates that individuals who travel to the San Francisco downtown area or other CBDs are very likely to use the transit mode. This is to be expected because of the high traffic congestion in these areas and also since these high land-use density corridors are likely to be well served by transit.

5. Conclusions and Direction for Future Research

This paper proposes a mixed multinomial logit (MMNL) structure that is able to capture shared unobserved attributes along both dimensions in a two-dimensional choice context. In concept, the MMNL model generalizes the nested logit model which allows shared unobserved attributes along one or the other dimension (but not both).

The MMNL model is applied to the estimation of mode-departure time choice for home-based social-recreational trips using data drawn from the 1990 Bay area household travel survey. We estimated four alternative models: the MMNL model allowing unobserved attributes along both the mode and departure time dimension (MMNL-MT model), the MMNL model allowing unobserved attributes along the time dimension only (MMNL-T model), the MMNL model allowing unobserved attributes along the mode dimension only (MMNL-M model), and the commonly used multinomial logit (MNL) model. The results indicate that the MMNL-MT model outperforms the other models in terms of data fit. We also find that failure to accommodate shared unobserved attributes along both the mode and departure time dimensions leads to incorrect conclusions regarding the (disaggregate-level and aggregate-level) elasticity effects of level-of-service variables. In summary, failure to accommodate shared unobserved attributes along both the mode and departure time dimensions can lead to inappropriate evaluations of transportation control measures and, consequently, mis-informed policy actions.

Several methodological/empirical extensions of the MMNL model proposed here may be considered. First, the extent of covariance (or sensitivity) among alternatives that share the same mode and/or same departure time may be specified to be a function of observed (to the analyst) individual characteristics (see Brownstone and Train, 1996 and Bhat, 1996b for related models in the context of uni-dimensional choice situations). Second, the level-of-service response parameters may be parameterized to be functions of observed individual characteristics, while ensuring at the same time that the sign on the level-of-service parameters are always in the appropriate direction. Third, the level-of-service parameters may be specified to be functions of observed as well as unobserved individual characteristics (see Bhat, 1996a for a model that accommodates the second and third extensions in a uni-dimensional context). Fourth, the model may be extended to analyze destination choice along with mode and departure time choice. The current effort considers mode and departure time choice as decisions conditional on destination choice. However, it is likely that all three decisions are made jointly. For example, the spatial non-uniformity in the implementation of policy actions such as congestion pricing can lead to changes in choice of destination.

The extensions identified above are conceptually straight-forward. However, they lead to added dimensions of integration for the choice probabilities. Consequently, the increase in computation time necessary to achieve a desired level of accuracy can become quite substantial and may lead to unacceptably large convergence times in the simulated maximum likelihood estimation. It is, therefore, important to explore methods that can increase the accuracy of the logit simulator for a given number of simulation replications.

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List of Tables

Table 1. Level of Service Parameters, Implied Money Values of Travel Time, Data Fit Measures, and Error Variance Parameters

Attributes/data fit measures	MNL model	MMNL-T model	MMNL-M model	MMNL-MT model
Level of service[1]				
Travel cost (in cents)	-0.0031 (-3.13)	-0.0036 (-3.02)	-0.0044 (-2.88)	-0.0045 (-2.83)
Total travel time (in mins.)	-0.0319 (-3.15)	-0.0336 (-2.87)	-0.0382 (-3.22)	-0.0408 (-3.33)
Out-of-vehicle time/distance	-0.2363 (-3.42)	-0.2429 (-4.82)	-0.2508 (-4.19)	-0.2589 (-4.26)
Implied money values of time ($/hr)				
In-vehicle travel time	6.17	5.60	5.21	5.44
Out-of-vehicle travel time[2]	13.66	12.23	10.80	11.09
LL at Convergence[3]	-6393.6	-6382.9	-6387.7	-6375.8
Error variance parameters				
$\delta_{pm\ offpeak}$	-	0.8911 (2.76)	-	0.9715 (2.96)
$\delta_{pm\ peak}$	-	0.7418 (2.83)	-	0.3944 (1.88)
$\delta_{evening}$	-	1.9771 (2.70)	-	1.6421 (3.02)
$\sigma_{drive\ alone}$	-		0.6352 (1.91)	0.5891 (1.98)
$\sigma_{shared\ ride}$	-		1.9464 (3.06)	1.9581 (3.20)
$\sigma_{transit}$	-		0.7657 (1.73)	0.7926 (2.07)

[1] The entries in the different columns correspond to the parameter values and their t-statistics (in parenthesis).

[2] Money value of out-of-vehicle time is computed at the mean travel distance of 6.11 miles.

[3] The LL (Log-Likelihood) at equal shares is -8601.24 and the LL with only alternative specific constants and an IID error covariance matrix is -6812.07

Table 2. Disaggregate Travel Cost Elasticities in Response to a Cost Increase in the Drive Alone (DA) Mode during PM Peak

Effect on Joint Choice Alternative	MNL model	MMNL-T model	MMNL-M model	MMNL-MT model
DA-morning periods[1]	0.0072	0.0085	0.0141	0.0165
DA-PM offpeak	0.0072	0.0060	0.0141	0.0131
DA-PM peak	-0.1112	-0.0993	-0.1555	-0.1423
DA-evening	0.0072	0.0042	0.0141	0.0099
SR-morning periods[1]	0.0072	0.0085	0.0059	0.0072
SR-PM offpeak	0.0072	0.0060	0.0059	0.0055
SR-PM peak	0.0072	0.0120	0.0059	0.0079
SR-evening	0.0072	0.0042	0.0059	0.0045
TR-morning periods[1]	0.0072	0.0085	0.0119	0.0131
TR-PM offpeak	0.0072	0.0060	0.0119	0.0106
TR-PM peak	0.0072	0.0120	0.0119	0.0150
TR-evening	0.0072	0.0042	0.0119	0.0082

[1] The morning periods include early morning, AM peak, and AM off-peak. The cross-elasticities for the morning periods within each mode with respect to a PM peak cost increase in the drive alone mode are the same in the mixture logit models because of the absence of shared unobserved attributes specific to the morning time periods.

Table 3. Aggregate Travel Cost Elasticities in Response to a Cost Increase in the Drive Alone (DA) Mode during PM Peak

Effect on Joint Choice Alternative	MNL model	MMNL-T model	MMNL-M model	MMNL-MT model
Drive alone (DA) alternatives				
Early morning	0.0146	0.0202	0.0290	0.0392
AM peak	0.0125	0.0166	0.0259	0.0334
AM offpeak	0.0121	0.0155	0.0250	0.0317
PM offpeak	0.0123	0.0136	0.0254	0.0265
PM peak	-0.1733	-0.1536	-0.2355	-0.2192
Evening	0.0146	0.0088	0.0293	0.0204
Transit (TR) alternatives				
Early morning	0.0197	0.0260	0.0280	0.0371
AM peak	0.0188	0.0237	0.0283	0.0358
AM offpeak	0.0163	0.0195	0.0236	0.0291
PM offpeak	0.0168	0.0175	0.0246	0.0251
PM peak	0.0218	0.0393	0.0333	0.0485
Evening	0.0205	0.0120	0.0299	0.0203

Table 4. MMNL-MT Model Results

Variables	Departure Choice Sub-Model		Mode Choice Sub-Model	
	Parameter	t-stat.	Parameter	t-stat.
Socio-demographic Attributes				
Employment status				
Early morning	-0.4792	-0.99	-	-
AM peak	-1.3739	-2.91	-	-
AM offpeak	-1.8546	-4.04	-	-
PM offpeak	-1.7450	-3.50	-	-
PM peak	-0.6484	-1.45	-	-
Shared ride	-	-	-0.5776	-3.12
Transit	-	-	-1.0630	-2.48
Age (specific to evening departure time)	-0.0233	-2.40	-	-
Elderly (specific to AM/PM offpeak)	0.2345	1.57	-	-
Female (specific to shared ride)	-	-	0.6201	2.43
Presence of children				
Early morning	-0.6204	-2.85	-	-
Evening	-0.7569	-3.12	-	-
Shared ride	-	-	1.5016	2.76
Income (specific to evening departure time)	0.0043	1.87	-	-
No. of vehicles/No. of adults (specific to transit)	-	-	-1.1158	-2.91
Trip Destination Attributes				
San Francisco downtown (specific to transit)	-	-	3.2144	3.83
Other CBDs (specific to transit)	-	-	1.9372	4.11