**Online supplement to**

**“New Matrix-Based Methods for the Analytic Evaluation of the Multivariate Cumulative Normal Distribution Function”**

By Chandra R. Bhat

Performances in each of four sets of cases corresponding to (a) low correlations, high MVNCD values, (b) low correlations, low MVNCD values, (c) high correlations, high MVNCD values, and (d) high correlations, low MVNCD values.

**Table 1. MVNCD evaluations results: low correlation, high MVNCD value**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Dim.** | **Measure** | **GHK** | **SSJ** | **Univariate truncated methods** | **Bivariate truncated methods** |
| **500 rep.** | **10,000 rep.** | **1 perm.** | **10 perm.** | **TGME** | **ME** | **OVUS** | **OVBS** | **TGBME** | **BME** | **TVBS** |
| *H=5* | MAE | 0.00034 | 0.00004 | 0.00068 | 0.00037 | 0.00937 | 0.00045 | 0.00026 | 0.00014 | 0.00660 | 0.00028 | 0.00018 |
| MAPE | 0.10 | 0.01 | 0.22 | 0.12 | 2.37 | 0.12 | 0.07 | 0.03 | 1.69 | 0.07 | 0.05 |
| %MAE>0.005 | 0.0 | 0.0 | 0.0 | 0.0 | 66.8 | 0.4 | 0.0 | 0.0 | 49.2 | 0.0 | 0.0 |
| %MAPE>2 | 0.0 | 0.0 | 0.4 | 0.0 | 49.2 | 0.4 | 0.0 | 0.0 | 29.2 | 0.0 | 0.0 |
| Time (s) | 0.018 | 0.247 | 0.009 | 0.056 | 0.008 | 0.016 | 0.018 | 0.018 | 0.009 | 0.013 | 0.017 |
| *H=7* | MAE | 0.00052 | 0.00006 | 0.00085 | 0.00045 | 0.01193 | 0.00036 | 0.00029 | 0.00020 | 0.00940 | 0.00029 | 0.00022 |
| MAPE | 0.19 | 0.02 | 0.35 | 0.19 | 3.75 | 0.13 | 0.11 | 0.07 | 2.97 | 0.11 | 0.08 |
| %MAE>0.005 | 0.0 | 0.0 | 0.1 | 0.0 | 21.2 | 0.0 | 0.0 | 0.0 | 18.0 | 0.0 | 0.0 |
| %MAPE>2 | 0.0 | 0.0 | 0.3 | 0.0 | 20.8 | 0.0 | 0.0 | 0.0 | 17.6 | 0.0 | 0.0 |
| Time (s) | 0.026 | 0.418 | 0.014 | 0.097 | 0.012 | 0.028 | 0.032 | 0.035 | 0.014 | 0.019 | 0.031 |
| *H=10* | MAE | 0.00091 | 0.00009 | 0.00093 | 0.00045 | 0.01312 | 0.00039 | 0.00032 | 0.00023 | 0.01091 | 0.00032 | 0.00025 |
| MAPE | 0.39 | 0.04 | 0.49 | 0.26 | 5.15 | 0.21 | 0.17 | 0.12 | 4.31 | 0.17 | 0.13 |
| %MAE>0.005 | 0.1 | 0.0 | 0.2 | 0.0 | 21.0 | 0.0 | 0.0 | 0.0 | 19.6 | 0.0 | 0.0 |
| %MAPE>2 | 0.1 | 0.0 | 0.9 | 0.2 | 22.8 | 0.0 | 0.0 | 0.0 | 22.2 | 0.0 | 0.0 |
| Time (s) | 0.043 | 0.757 | 0.028 | 0.198 | 0.017 | 0.053 | 0.058 | 0.068 | 0.020 | 0.031 | 0.053 |
| *H=12* | MAE | 0.00098 | 0.00010 | 0.00094 | 0.00047 | 0.01342 | 0.00036 | 0.00029 | 0.00022 | 0.01129 | 0.00028 | 0.00023 |
| MAPE | 0.53 | 0.05 | 0.60 | 0.32 | 6.45 | 0.22 | 0.18 | 0.14 | 5.45 | 0.17 | 0.13 |
| %MAE>0.005 | 0.1 | 0.0 | 0.2 | 0.0 | 21.4 | 0.0 | 0.0 | 0.0 | 19.5 | 0.0 | 0.0 |
| %MAPE>2 | 0.4 | 0.0 | 1.1 | 0.3 | 23.6 | 0.1 | 0.0 | 0.0 | 22.7 | 0.1 | 0.0 |
| Time (s) | 0.055 | 1.163 | 0.038 | 0.286 | 0.021 | 0.074 | 0.082 | 0.096 | 0.024 | 0.047 | 0.074 |
| *H=15* | MAE | 0.00132 | 0.00012 | 0.00109 | 0.00047 | 0.01269 | 0.00033 | 0.00029 | 0.00023 | 0.01094 | 0.00026 | 0.00022 |
| MAPE | 0.86 | 0.08 | 0.85 | 0.36 | 7.42 | 0.24 | 0.22 | 0.17 | 6.49 | 0.20 | 0.17 |
| %MAE>0.005 | 0.3 | 0.0 | 0.3 | 0.0 | 21.2 | 0.0 | 0.0 | 0.0 | 20.6 | 0.0 | 0.0 |
| %MAPE>2 | 2.0 | 0.0 | 3.0 | 0.1 | 24.2 | 0.0 | 0.0 | 0.0 | 23.8 | 0.0 | 0.0 |
| Time (s) | 0.073 | 2.205 | 0.067 | 0.439 | 0.029 | 0.140 | 0.143 | 0.146 | 0.034 | 0.066 | 0.106 |
| *H=18* | MAE | 0.00140 | 0.00015 | 0.00080 | 0.00042 | 0.01195 | 0.00028 | 0.00024 | 0.00020 | 0.01027 | 0.00022 | 0.00020 |
| MAPE | 1.06 | 0.11 | 0.75 | 0.42 | 7.87 | 0.27 | 0.24 | 0.20 | 6.96 | 0.22 | 0.19 |
| %MAE>0.005 | 0.5 | 0.0 | 0.0 | 0.0 | 19.9 | 0.0 | 0.0 | 0.0 | 19.2 | 0.0 | 0.0 |
| %MAPE>2 | 3.7 | 0.0 | 2.3 | 0.4 | 24.5 | 0.2 | 0.1 | 0.1 | 24.2 | 0.1 | 0.1 |
| Time (s) | 0.100 | 2.313 | 0.099 | 0.663 | 0.034 | 0.169 | 0.173 | 0.211 | 0.041 | 0.092 | 0.143 |
| *H=20* | MAE | 0.00148 | 0.00015 | 0.00080 | 0.00044 | 0.01139 | 0.00026 | 0.00024 | 0.00020 | 0.01000 | 0.00021 | 0.00019 |
| MAPE | 1.24 | 0.13 | 0.81 | 0.45 | 8.86 | 0.28 | 0.25 | 0.21 | 7.80 | 0.23 | 0.20 |
| %MAE>0.005 | 1.0 | 0.0 | 0.1 | 0.0 | 18.7 | 0.0 | 0.0 | 0.0 | 18.3 | 0.0 | 0.0 |
| %MAPE>2 | 4.6 | 0.0 | 2.2 | 0.6 | 24.2 | 0.0 | 0.0 | 0.0 | 24.0 | 0.0 | 0.0 |
| Time (s) | 0.114 | 2.848 | 0.106 | 0.773 | 0.039 | 0.200 | 0.220 | 0.247 | 0.049 | 0.109 | 0.163 |

**Table 2. MVNCD evaluation results: low correlation, low MVNCD value**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Dim.** | **Measure** | **GHK** | **SSJ** | **Univariate truncated methods** | **Bivariate truncated methods** |
| **500 rep.** | **10,000 rep.** | **1 perm.** | **10 perm.** | **TGME** | **ME** | **OVUS** | **OVBS** | **TGBME** | **BME** | **TVBS** |
| *H=5* | MAE | 0.00013 | 0.00002 | 0.00070 | 0.00035 | 0.00220 | 0.00013 | 0.00009 | 0.00005 | 0.00163 | 0.00010 | 0.00005 |
| MAPE | 0.18 | 0.02 | 2.53 | 1.36 | 2.06 | 0.17 | 0.12 | 0.06 | 1.52 | 0.12 | 0.07 |
| %MAE>0.005 | 0.0 | 0.0 | 0.4 | 0.0 | 13.6 | 0.0 | 0.0 | 0.0 | 8.8 | 0.0 | 0.0 |
| %MAPE>2 | 0.0 | 0.0 | 29.6 | 13.2 | 33.6 | 0.0 | 0.0 | 0.0 | 19.6 | 0.0 | 0.0 |
| Time (s) | 0.018 | 0.247 | 0.009 | 0.056 | 0.008 | 0.016 | 0.018 | 0.018 | 0.009 | 0.013 | 0.017 |
| *H=7* | MAE | 0.00013 | 0.00002 | 0.00060 | 0.00024 | 0.00198 | 0.00009 | 0.00009 | 0.00005 | 0.00160 | 0.00007 | 0.00006 |
| MAPE | 0.39 | 0.05 | 7.63 | 5.90 | 3.64 | 0.40 | 0.37 | 0.27 | 3.01 | 0.30 | 0.26 |
| %MAE>0.005 | 0.0 | 0.0 | 0.0 | 0.0 | 10.0 | 0.0 | 0.0 | 0.0 | 8.0 | 0.0 | 0.0 |
| %MAPE>2 | 0.8 | 0.0 | 55.6 | 34.4 | 62.8 | 2.8 | 2.0 | 2.0 | 50.0 | 1.6 | 1.6 |
| Time (s) | 0.026 | 0.418 | 0.014 | 0.097 | 0.012 | 0.028 | 0.032 | 0.035 | 0.014 | 0.019 | 0.031 |
| *H=10* | MAE | 0.00009 | 0.00001 | 0.00031 | 0.00017 | 0.00077 | 0.00002 | 0.00002 | 0.00001 | 0.00062 | 0.00002 | 0.00001 |
| MAPE | 1.29 | 0.16 | 16.17 | 10.38 | 6.43 | 1.11 | 1.00 | 0.72 | 5.19 | 0.74 | 0.62 |
| %MAE>0.005 | 0.0 | 0.0 | 0.0 | 0.0 | 2.8 | 0.0 | 0.0 | 0.0 | 1.6 | 0.0 | 0.0 |
| %MAPE>2 | 18.8 | 0.0 | 74.8 | 62.0 | 76.8 | 8.4 | 7.6 | 4.8 | 74.0 | 6.4 | 5.2 |
| Time (s) | 0.043 | 0.757 | 0.028 | 0.198 | 0.017 | 0.053 | 0.058 | 0.068 | 0.020 | 0.031 | 0.053 |
| *H=12* | MAE | 0.00006 | 0.00001 | 0.00017 | 0.00008 | 0.00030 | 0.00002 | 0.00001 | 0.00001 | 0.00024 | 0.00001 | 0.00001 |
| MAPE | 2.43 | 0.31 | 26.38 | 18.04 | 8.26 | 1.94 | 1.86 | 1.45 | 7.14 | 1.82 | 1.42 |
| %MAE>0.005 | 0.0 | 0.0 | 0.4 | 0.0 | 1.2 | 0.0 | 0.0 | 0.0 | 0.8 | 0.0 | 0.0 |
| %MAPE>2 | 37.6 | 0.8 | 77.6 | 72.0 | 83.2 | 14.8 | 14.4 | 11.2 | 81.6 | 10.0 | 8.8 |
| Time (s) | 0.055 | 1.163 | 0.038 | 0.286 | 0.021 | 0.074 | 0.082 | 0.096 | 0.024 | 0.047 | 0.074 |
| *H=15* | MAE | 0.00003 | 0.00000 | 0.00005 | 0.00003 | 0.00011 | 0.00000 | 0.00000 | 0.00000 | 0.00010 | 0.00000 | 0.00000 |
| MAPE | 6.24 | 1.29 | 26.65 | 23.75 | 11.29 | 3.24 | 2.94 | 2.65 | 9.90 | 2.19 | 1.93 |
| %MAE>0.005 | 0.0 | 0.0 | 0.0 | 0.0 | 0.8 | 0.0 | 0.0 | 0.0 | 0.8 | 0.0 | 0.0 |
| %MAPE>2 | 62.0 | 11.2 | 83.6 | 78.0 | 86.8 | 22.8 | 20.8 | 19.2 | 84.4 | 18.0 | 17.2 |
| Time (s) | 0.073 | 2.205 | 0.067 | 0.439 | 0.029 | 0.140 | 0.143 | 0.146 | 0.034 | 0.066 | 0.106 |
| *H=18* | MAE | 0.00001 | 0.00000 | 0.00002 | 0.00001 | 0.00004 | 0.00000 | 0.00000 | 0.00000 | 0.00004 | 0.00000 | 0.00000 |
| MAPE | 10.64 | 2.94 | 35.36 | 24.03 | 14.07 | 4.90 | 4.48 | 4.25 | 11.86 | 3.02 | 2.73 |
| %MAE>0.005 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| %MAPE>2 | 75.6 | 32.0 | 86.4 | 82.0 | 82.4 | 31.2 | 28.8 | 28.8 | 86.4 | 22.8 | 22.4 |
| Time (s) | 0.100 | 2.313 | 0.099 | 0.663 | 0.034 | 0.169 | 0.173 | 0.211 | 0.041 | 0.092 | 0.143 |
| *H=20* | MAE | 0.00001 | 0.00000 | 0.00001 | 0.00001 | 0.00004 | 0.00000 | 0.00000 | 0.00000 | 0.00004 | 0.00000 | 0.00000 |
| MAPE | 15.49 | 4.52 | 38.38 | 32.65 | 17.04 | 6.17 | 5.81 | 5.36 | 14.93 | 4.44 | 4.83 |
| %MAE>0.005 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| %MAPE>2 | 87.2 | 46.4 | 87.2 | 85.2 | 86.4 | 33.2 | 32.8 | 31.6 | 84.0 | 29.6 | 29.2 |
| Time (s) | 0.114 | 2.848 | 0.106 | 0.773 | 0.039 | 0.200 | 0.220 | 0.247 | 0.049 | 0.109 | 0.163 |

**Table 3. MVNCD evaluation results: high correlation, high MVNCD value**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Dim.** | **Measure** | **GHK** | **SSJ** | **Univariate truncated methods** | **Bivariate truncated methods** |
| **500 rep.** | **10,000 rep.** | **1 perm.** | **10 perm.** | **TGME** | **ME** | **OVUS** | **OVBS** | **TGBME** | **BME** | **TVBS** |
| *H=5* | MAE | 0.00141 | 0.00024 | 0.00348 | 0.00223 | 0.03741 | 0.00334 | 0.00215 | 0.00122 | 0.02587 | 0.00231 | 0.00144 |
| MAPE | 0.43 | 0.08 | 1.34 | 0.90 | 10.59 | 1.21 | 0.79 | 0.44 | 7.27 | 0.86 | 0.51 |
| %MAE>0.005 | 2.8 | 0.8 | 22.4 | 7.6 | 93.2 | 20.8 | 11.2 | 3.6 | 88.8 | 10.0 | 4.0 |
| %MAPE>2 | 2.8 | 0.8 | 17.6 | 10.8 | 90.8 | 17.6 | 10.8 | 3.6 | 88.8 | 9.6 | 5.2 |
| Time (s) | 0.018 | 0.247 | 0.009 | 0.056 | 0.008 | 0.016 | 0.018 | 0.018 | 0.009 | 0.013 | 0.017 |
| *H=7* | MAE | 0.00186 | 0.00023 | 0.00271 | 0.00171 | 0.03293 | 0.00239 | 0.00188 | 0.00127 | 0.02575 | 0.00176 | 0.00131 |
| MAPE | 0.71 | 0.09 | 1.53 | 1.09 | 12.24 | 1.38 | 1.14 | 0.78 | 9.76 | 0.97 | 0.69 |
| %MAE>0.005 | 5.2 | 0.4 | 18.0 | 4.4 | 94.0 | 11.2 | 6.8 | 3.2 | 92.8 | 6.0 | 3.2 |
| %MAPE>2 | 6.0 | 0.4 | 19.2 | 12.8 | 94.8 | 18.0 | 11.6 | 6.8 | 96.0 | 10.4 | 6.8 |
| Time (s) | 0.026 | 0.418 | 0.014 | 0.097 | 0.012 | 0.028 | 0.032 | 0.035 | 0.014 | 0.019 | 0.031 |
| *H=10* | MAE | 0.00259 | 0.00031 | 0.00228 | 0.00133 | 0.03179 | 0.00153 | 0.00128 | 0.00096 | 0.02589 | 0.00116 | 0.00095 |
| MAPE | 1.13 | 0.13 | 1.22 | 0.73 | 12.87 | 0.86 | 0.71 | 0.51 | 10.57 | 0.64 | 0.48 |
| %MAE>0.005 | 14.0 | 0.0 | 8.4 | 2.0 | 94.8 | 4.4 | 3.2 | 2.0 | 94.0 | 2.4 | 1.2 |
| %MAPE>2 | 16.0 | 0.0 | 18.4 | 8.4 | 97.2 | 10.8 | 8.0 | 4.4 | 96.8 | 6.8 | 4.4 |
| Time (s) | 0.043 | 0.757 | 0.028 | 0.198 | 0.017 | 0.053 | 0.058 | 0.068 | 0.020 | 0.031 | 0.053 |
| *H=12* | MAE | 0.00283 | 0.00034 | 0.00197 | 0.00126 | 0.02854 | 0.00113 | 0.00096 | 0.00077 | 0.02366 | 0.00095 | 0.00076 |
| MAPE | 1.49 | 0.17 | 1.37 | 0.92 | 13.53 | 0.82 | 0.69 | 0.54 | 11.17 | 0.70 | 0.56 |
| %MAE>0.005 | 17.6 | 0.0 | 8.4 | 1.6 | 95.2 | 0.8 | 0.4 | 0.0 | 93.2 | 0.4 | 0.4 |
| %MAPE>2 | 23.2 | 0.0 | 17.6 | 9.2 | 98.8 | 9.6 | 8.0 | 4.4 | 98.0 | 7.6 | 5.6 |
| Time (s) | 0.055 | 1.163 | 0.038 | 0.286 | 0.021 | 0.074 | 0.082 | 0.096 | 0.024 | 0.047 | 0.074 |
| *H=15* | MAE | 0.00302 | 0.00043 | 0.00179 | 0.00095 | 0.02388 | 0.00082 | 0.00072 | 0.00057 | 0.02029 | 0.00068 | 0.00056 |
| MAPE | 1.77 | 0.23 | 1.32 | 0.71 | 12.63 | 0.59 | 0.51 | 0.40 | 10.68 | 0.48 | 0.39 |
| %MAE>0.005 | 16.0 | 0.0 | 4.4 | 0.0 | 94.4 | 0.8 | 0.4 | 0.4 | 95.2 | 0.4 | 0.4 |
| %MAPE>2 | 32.8 | 0.0 | 18.8 | 6.4 | 98.8 | 6.8 | 5.6 | 3.6 | 99.2 | 6.0 | 2.8 |
| Time (s) | 0.073 | 2.205 | 0.067 | 0.439 | 0.029 | 0.140 | 0.143 | 0.146 | 0.034 | 0.066 | 0.106 |
| *H=18* | MAE | 0.00310 | 0.00043 | 0.00150 | 0.00092 | 0.02216 | 0.00066 | 0.00058 | 0.00044 | 0.01913 | 0.00054 | 0.00045 |
| MAPE | 2.24 | 0.31 | 1.27 | 0.80 | 13.69 | 0.58 | 0.50 | 0.39 | 11.91 | 0.45 | 0.38 |
| %MAE>0.005 | 19.6 | 0.0 | 1.2 | 0.0 | 95.6 | 0.0 | 0.0 | 0.0 | 92.4 | 0.0 | 0.0 |
| %MAPE>2 | 41.2 | 1.2 | 20.4 | 8.8 | 98.8 | 4.4 | 4.0 | 3.6 | 98.8 | 3.2 | 2.4 |
| Time (s) | 0.100 | 2.313 | 0.099 | 0.663 | 0.034 | 0.169 | 0.173 | 0.211 | 0.041 | 0.092 | 0.143 |
| *H=20* | MAE | 0.00345 | 0.00049 | 0.00148 | 0.00083 | 0.02012 | 0.00057 | 0.00049 | 0.00041 | 0.01718 | 0.00047 | 0.00042 |
| MAPE | 2.82 | 0.45 | 1.70 | 0.99 | 15.00 | 0.65 | 0.58 | 0.49 | 13.07 | 0.56 | 0.49 |
| %MAE>0.005 | 23.6 | 0.0 | 2.8 | 0.0 | 86.0 | 0.0 | 0.0 | 0.0 | 86.4 | 0.0 | 0.0 |
| %MAPE>2 | 50.8 | 2.0 | 24.8 | 11.6 | 98.0 | 6.8 | 4.4 | 3.6 | 99.2 | 4.0 | 3.2 |
| Time (s) | 0.114 | 2.848 | 0.106 | 0.773 | 0.039 | 0.200 | 0.220 | 0.247 | 0.049 | 0.109 | 0.163 |

**Table 4. MVNCD evaluation results: high correlation, low MVNCD value**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Dim.** | **Measure** | **GHK** | **SSJ** | **Univariate truncated methods** | **Bivariate truncated methods** |
| **500 rep.** | **10,000 rep.** | **1 perm.** | **10 perm.** | **TGME** | **ME** | **OVUS** | **OVBS** | **TGBME** | **BME** | **TVBS** |
| *H=5* | MAE | 0.00055 | 0.00006 | 0.00243 | 0.00168 | 0.01159 | 0.00105 | 0.00064 | 0.00040 | 0.00849 | 0.00064 | 0.00038 |
| MAPE | 2.97 | 0.46 | 11.37 | 9.40 | 21.28 | 5.73 | 5.15 | 3.38 | 16.40 | 4.24 | 2.65 |
| %MAE>0.005 | 0.8 | 0.0 | 13.2 | 7.6 | 50.4 | 4.0 | 0.4 | 0.4 | 42.4 | 0.8 | 0.4 |
| %MAPE>2 | 14.4 | 2.8 | 62.8 | 55.2 | 86.4 | 35.6 | 26.4 | 16.8 | 80.4 | 22.4 | 14.4 |
| Time (s) | 0.018 | 0.247 | 0.009 | 0.056 | 0.008 | 0.016 | 0.018 | 0.018 | 0.009 | 0.013 | 0.017 |
| *H=7* | MAE | 0.00050 | 0.00005 | 0.00176 | 0.00100 | 0.00522 | 0.00040 | 0.00029 | 0.00022 | 0.00406 | 0.00029 | 0.00022 |
| MAPE | 5.81 | 1.57 | 21.41 | 15.02 | 25.79 | 6.05 | 5.89 | 6.32 | 19.40 | 5.19 | 4.87 |
| %MAE>0.005 | 0.4 | 0.0 | 8.4 | 1.6 | 26.8 | 1.2 | 0.4 | 0.4 | 20.8 | 0.8 | 0.4 |
| %MAPE>2 | 38.8 | 8.4 | 76.4 | 68.4 | 93.6 | 36.4 | 34.0 | 26.8 | 86.8 | 28.8 | 23.2 |
| Time (s) | 0.026 | 0.418 | 0.014 | 0.097 | 0.012 | 0.028 | 0.032 | 0.035 | 0.014 | 0.019 | 0.031 |
| *H=10* | MAE | 0.00024 | 0.00004 | 0.00056 | 0.00033 | 0.00193 | 0.00008 | 0.00007 | 0.00006 | 0.00160 | 0.00008 | 0.00007 |
| MAPE | 18.50 | 7.40 | 26.20 | 24.07 | 34.76 | 9.91 | 10.28 | 10.05 | 31.16 | 11.74 | 12.20 |
| %MAE>0.005 | 0.0 | 0.0 | 1.6 | 0.4 | 9.6 | 0.0 | 0.0 | 0.0 | 8.8 | 0.0 | 0.0 |
| %MAPE>2 | 69.2 | 34.4 | 84.4 | 82.0 | 93.6 | 47.6 | 45.2 | 41.2 | 92.0 | 42.4 | 41.2 |
| Time (s) | 0.043 | 0.757 | 0.028 | 0.198 | 0.017 | 0.053 | 0.058 | 0.068 | 0.020 | 0.031 | 0.053 |
| *H=12* | MAE | 0.00016 | 0.00003 | 0.00030 | 0.00015 | 0.00088 | 0.00003 | 0.00003 | 0.00002 | 0.00073 | 0.00002 | 0.00002 |
| MAPE | 23.69 | 8.17 | 38.90 | 30.90 | 30.26 | 10.60 | 9.50 | 8.97 | 26.34 | 8.28 | 7.72 |
| %MAE>0.005 | 0.0 | 0.0 | 0.4 | 0.0 | 5.2 | 0.0 | 0.0 | 0.0 | 4.4 | 0.0 | 0.0 |
| %MAPE>2 | 79.6 | 42.8 | 91.2 | 87.2 | 94.4 | 48.4 | 45.6 | 42.8 | 95.2 | 42.4 | 39.6 |
| Time (s) | 0.055 | 1.163 | 0.038 | 0.286 | 0.021 | 0.074 | 0.082 | 0.096 | 0.024 | 0.047 | 0.074 |
| *H=15* | MAE | 0.00008 | 0.00002 | 0.00013 | 0.00006 | 0.00041 | 0.00001 | 0.00001 | 0.00001 | 0.00037 | 0.00001 | 0.00001 |
| MAPE | 34.86 | 17.84 | 36.00 | 28.07 | 33.52 | 13.10 | 11.98 | 12.01 | 31.41 | 12.29 | 12.68 |
| %MAE>0.005 | 0.0 | 0.0 | 0.0 | 0.0 | 1.6 | 0.0 | 0.0 | 0.0 | 1.6 | 0.0 | 0.0 |
| %MAPE>2 | 85.6 | 61.6 | 91.6 | 90.0 | 90.8 | 55.6 | 54.4 | 48.8 | 93.6 | 46.8 | 46.8 |
| Time (s) | 0.073 | 2.205 | 0.067 | 0.439 | 0.029 | 0.140 | 0.143 | 0.146 | 0.034 | 0.066 | 0.106 |
| *H=18* | MAE | 0.00002 | 0.00000 | 0.00002 | 0.00001 | 0.00003 | 0.00000 | 0.00000 | 0.00000 | 0.00003 | 0.00000 | 0.00000 |
| MAPE | 48.76 | 24.69 | 39.31 | 33.11 | 36.07 | 16.47 | 14.63 | 13.75 | 33.07 | 13.04 | 11.76 |
| %MAE>0.005 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| %MAPE>2 | 95.2 | 78.8 | 94.8 | 89.2 | 94.0 | 62.0 | 61.2 | 59.6 | 93.2 | 57.2 | 56.4 |
| Time (s) | 0.100 | 2.313 | 0.099 | 0.663 | 0.034 | 0.169 | 0.173 | 0.211 | 0.041 | 0.092 | 0.143 |
| *H=20* | MAE | 0.00001 | 0.00000 | 0.00001 | 0.00001 | 0.00001 | 0.00000 | 0.00000 | 0.00000 | 0.00001 | 0.00000 | 0.00000 |
| MAPE | 55.30 | 38.15 | 40.19 | 33.88 | 39.39 | 15.18 | 14.52 | 15.06 | 35.83 | 14.91 | 15.39 |
| %MAE>0.005 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| %MAPE>2 | 96.8 | 88.0 | 94.0 | 90.0 | 97.6 | 64.0 | 62.4 | 63.6 | 96.4 | 61.6 | 63.6 |
| Time (s) | 0.114 | 2.848 | 0.106 | 0.773 | 0.039 | 0.200 | 0.220 | 0.247 | 0.049 | 0.109 | 0.163 |