A Multiple Discrete-Continuous Extreme Value Model: Formulation and Application to Discretionary Time-Use Decisions

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ABSTRACT

Several consumer demand choices are characterized by the choice of multiple alternatives simultaneously. An example of such a choice situation in activity-travel analysis is the type of discretionary (or leisure) activity to participate in and the duration of time investment of the participation. In this context, within a given temporal period (say a day or a week), an individual may decide to participate in multiple types of activities (for example, in-home social activities, out-of-home social activities, in-home recreational activities, out-of-home recreational activities, and out-of-home non-maintenance shopping activities).

In this paper, we derive and formulate a utility theory-based model for discrete/continuous choice that assumes diminishing marginal utility as the level of consumption of any particular alternative increases (*i.e.*, satiation). This assumption yields a multiple discreteness model (*i.e.*, choice of multiple alternatives can occur simultaneously). This is in contrast to the standard discrete choice model that is based on assuming the absence of any diminishing marginal utility as the level of consumption of any alternative increases (*i.e.*, no satiation), leading to the case of strictly single discreteness. The econometric model formulated here, which we refer to as the Multiple Discrete-Continuous Extreme Value (MDCEV) model, has a surprisingly simple and elegant closed form expression for the discrete-continuous probability of not consuming certain alternatives and consuming given levels of the remaining alternatives. To our knowledge, we are the first to develop such a simple and powerful closed-form model for multiple discreteness in the literature. This formulation should constitute an important milestone in the area of multiple discreteness, just as the multinomial logit (MNL) represented an important milestone in the area of single discreteness. Further, the MDCEV model formulated here has the appealing property that it collapses to the familiar multinomial logit (MNL) choice model in the case of single discreteness. Finally, heteroscedasticity and/or correlation in unobserved characteristics affecting the demand of different alternatives can be easily incorporated within the MDCEV model framework using a mixing approach.

The MDCEV model and its mixed variant are applied to analyze time-use allocation decisions among a variety of discretionary activities on weekends using data from the 2000 San Francisco Bay Area survey.

1. INTRODUCTION

Several consumer demand choices related to travel decisions are characterized by the choice of multiple alternatives simultaneously. Examples of such choice situations include vehicle type holdings and usage, and activity type choice and duration of time investment of participation. In the former case, a household may hold a mix of different kinds of vehicle types (for example, a sedan, a minivan, and a pick-up) and use the vehicles in different ways based on the preferences of individual members, considerations of maintenance/running costs, and the need to satisfy different functional needs (such as being able to travel on weekend getaways as a family or to transport goods). In the case of activity type choice and duration, an individual may decide to participate in multiple kinds of recreational and social activities within a given time period (such as a day) to satisfy variety seeking desires. Of course, there are several other travel-related and other consumer demand situations characterized by the choice of multiple alternatives, including airline fleet mix and usage, carrier choice and transaction level, brand choice and purchase quantity for frequently purchased grocery items (such as cookies, ready-to-eat cereals, soft drinks, yoghurt, *etc.*), and stock selection and investment amounts.

Classical discrete and discrete-continuous models deal with situations where only one alternative is chosen from a set of mutually exclusive alternatives. Such models assume that the alternatives are perfectly substitutable for each other. On the other hand, the simultaneous demand for multiple alternatives discussed above corresponds to the situation where the alternatives are imperfect substitutes for one another. In this paper, we formulate a new econometric model for such multiple discreteness in demand that is based on utility maximization theory. Specifically, we assume a translated non-linear, but additive, form for the specification of the direct utility function, as proposed recently by Kim *et al.* (2002). The translated non-linear form allows for multiple discreteness as well diminishing marginal returns (*i.e.*, satiation) as the consumption of any particular alternative increases. This is in contrast to standard discrete and discrete-continuous choice models that allow only single discreteness and assume a linear utility structure (*i.e.*, no satiation effects). The econometric model formulated here, which we refer to as the Multiple Discrete-Continuous Extreme Value (MDCEV) model, is based on introducing a multiplicative log-extreme value error term into the utility function. The result of such a specification is a surprisingly simple closed form expression for the discrete-continuous probability of not consuming certain alternatives and consuming given levels of the remaining alternatives. To our knowledge, we are the first to develop such a simple and powerful model for multiple discreteness in the literature. Further, the MDCEV model has the appealing property that it collapses to the familiar multinomial logit (MNL) choice model in the case of single discreteness, and represents an extension of the single discrete-continuous models of Dubin and McFadden, 1984, Hanemann, 1984, Chiang, 1991, Chintagunta, 1993, and Arora *et al.*, 1998. Finally, heteroscedasticity and/or correlation in unobserved characteristics affecting the demand of different alternatives can be easily incorporated within the MDCEV model framework. Such an extension represents the multiple discrete-continuous equivalent of the mixed multinomial logit (MMNL) model (see Bhat, 2003 or Train, 2003 for detailed reviews of the MMNL model).

There have been several relatively recent studies in the marketing literature on the topic of multiple-discreteness. Hendel (1999) and Dube (2004) consider the purchase of multiple varieties within a particular product category as the result of a stream of expected (but unobserved to the analyst) future consumption decisions between successive shopping occasions (see also Walsh, 1995). Due to varying tastes across individual consumption occasions between the current shopping purchase and the next, consumers are observed to purchase a variety of goods at the current shopping occasion. The above studies use a linear utility function at each individual consumption occasion, with the utility parameters varying across consumption occasions. A Poisson distribution is assumed for the number of consumption occasions and a normal distribution is assumed regarding varying tastes to complete the model specification. Such a "vertical" variety-seeking model may be appropriate for frequently consumed grocery items such as carbonated soft drinks, cereals, and cookies. However, in many other choice occasions, such as time allocation to different types of discretionary activities, the true decision process may be better characterized as "horizontal" variety-seeking, where the consumer selects an assortment of alternatives due to diminishing marginal returns for each alternative. Kim *et al.* (2002) propose a utility structure for such "horizontal" variety-seeking with a non-linear utility function to accommodate satiation behavior. This is the overall structure maintained in the current paper. However, the econometric development and the estimation procedure are different between our paper and Kim *et al.*'s paper. The MDCEV model formulated here also represents a very simple and parsimonious model structure compared to the model proposed in Kim *et al.* (2002). To be sure, Kim *et al.*'s model is not practical for realistic applications, while the MDCEV model of this paper is very practical even for situations with a large number of discrete consumption alternatives. In fact, we submit that the MDCEV model structure is the MNL model-equivalent for multiple discrete-continuous choice analysis. Extensions of the MDCEV model to accommodate unobserved heteroscedasticity and error correlation among alternatives is straightforward and is similar to the movement from the MNL to the MMNL model in the standard discrete choice literature[1].

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[1] There have also been other formulations proposed to handle multiple-discreteness in the literature. These include the Mixed Multinomial-Poisson Approach of Terza and Wilson (1990) and the multivariate probit (logit) approaches

In the current paper, we develop the multiple discrete-continuous extreme value (MDCEV) model in the context of individual time use in different types of activity pursuits using data from the 2000 San Francisco Bay area. However, the formulation is applicable to any other multiple discrete-continuous choice situation.

The next section of the paper introduces the importance of time use analysis in travel demand modeling, and briefly reviews earlier literature in the area. Section 3 advances the econometric framework for the MDCEV model of time allocation. Section 4 discusses the data source and sample used in the empirical analysis. Section 5 presents empirical results. The final section provides a summary and identifies directions for future research.

2. OVERVIEW OF TIME-USE ANALYSIS

2.1 Time-Use Analysis in the Travel Demand Context

In the past several years, the activity-based approach to travel demand analysis has received much attention and seen considerable progress (see Bhat and Koppelman, 1999; Pendyala and Goulias, 2002, and Arentze and Timmermans, 2004). A fundamental difference between the commonly-used trip-based approach and the activity-based approach is the way time is conceptualized and represented in the two approaches (Pas, 1996; Ye *et al.*, 2004). In the trip-based approach, time is reduced to being simply a "cost" of making a trip. The activity-based approach, on the other hand, treats time as an all-encompassing continuous entity within which individuals make activity/travel participation decisions. Thus, the central basis of the activity-

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of Manchanda *et al.* (1999), Baltas (2004), Edwards and Allenby (2003), and Bhat and Srinivasan (2004). But all these formulations do not model the continuous component in a multiple discreteness setting. Further, the multiple discreteness is handled through statistical methods that generate correlation between univariate utility maximizing models of single discreteness rather than being fundamentally derived from a rigorous underlying utility maximization model for multiple discreteness. The resulting multiple discrete models also do not collapse to the standard discrete choice models when all individuals choose one and only one alternative at each choice occasion. However, these non-utility maximization models of multiple discreteness are also of value, and can be extended to include a continuous component in a flexible manner.

based approach is that individuals' travel patterns are a result of their time-use decisions. Individuals have 24 hours in a day (or multiples of 24 hours for longer periods of time) and decide how to use that time among activities (and with whom) subject to their schedule, socio-demographic, locational, and other contextual constraints. These decisions determine the generation and scheduling of trips (see Bhat *et al.*, 2004 for details of an implementation of an activity-based approach to travel demand modeling).

2.2 Earlier Studies Relevant to Current Research

The study of activity time use has received attention in several fields, including psychology, anthropology, sociology, urban planning, economics, and travel behavior analysis. Qualitative paradigms and frameworks of time use have emerged from all these fields, while most of the mathematical frameworks have been developed in the microeconomics and travel demand fields. The next two sections provide a very brief overview of the mathematical studies in the microeconomics and travel demand fields that are relevant to the current research effort. Section 2.3 positions the current research effort in the context of the earlier studies.

2.2.1 Microeconomic studies

The economic approach to time use is based on the assumption that individuals (and the households of which they are a part) use their time so that the total utility derived from all the activities is maximized. Each person in the household allocates time as well as money income to various activities - receiving income from time expended in the market place and receiving utility from spending this income on the consumption of goods and services (Gramm 1975, Gronau 1973, Becker 1981; 1965, Mincer 1962; 1963). Individuals "produce" non-market activities using “inputs” - their time and market goods and services. An individual's choice of work time and time in other non-market activities depends on market wages and prices of the “inputs” used to produce non-market activities. In particular, non-market time and consumer goods used in “production” of each non-market activity is chosen so as to maximize utility subject to constraints imposed by wages, prices of consumption goods, and time (Juster, 1990). Recently, Jara Diaz (2003) has focused on the technological relationship between time and goods consumption in more detail, and shed new light on the technological relations and constraints characterizing the utility maximization problem.

The economic studies of time use have been comprehensive in their frameworks, and have considered a variety of constraints under which individuals make their time-use decisions. However, most of these studies are rather theoretical in nature (but see Jara Diaz and Guevera, 2003 for a study that formulates and applies a microeconomic model).

2.2.2 Travel behavior analysis

Travel behavior researchers have turned to the examination of activity time use from a need to better understand and forecast travel. Some of these studies are based on frameworks that are not rigorously derived from utility theory (for example, see Allaman *et al.*, 1982; Damm and Lerman, 1981; Lu and Pas, 1999; Bhat, 1998), while others use frameworks with utility theory as the fundamental basis for time use (Munshi, 1993; Kitamura *et al.*, 1996; Yamamoto and Kitamura, 1999; Bhat and Misra, 1999; Meloni *et al.*, 2004). It is not at all clear that one type of studies is necessarily better than the other; in fact, both classes of studies have provided important insights into time use behavior. It should also be noted that utility-theoretic based models in the travel behavior research arena have generally considered time as being the only constraint in time allocation, and focused on discretionary activities.

2.2.3 The current research effort

The research in this paper is aligned with the utility-theoretic class of studies in the travel behavior area. As with earlier studies within this class, time is the only resource constraint considered and the empirical focus is on allocation among discretionary activities. The current research, however, generalizes earlier models by formulating a structure that is applicable to allocation among any number of activity categories (rather than just two categories). The underlying utility structure of the proposed structure is also very closely tied to preference and indifference curve theory, as discussed next.

3. UTILITY STRUCTURE

Kim *et al.*'s (2002) proposed utility structure remains at the core of the current research effort. However, for completeness and also because our model development procedure in Section 3.1 is different from Kim *et al.*'s, the discussion here begins from first principles. In Section 3.2, we specify an alternative error structure specification to the one used in Kim *et al.* and adopt a different way of writing the likelihood function, leading to a much simpler model formulation.

Let there be K different activity purposes that an individual can potentially allocate time to. Let t_j be the time spent in activity purpose j (j = 1, 2,…, K). We specify the utility accrued to an individual as the sum of the utilities obtained from investing time in each activity purpose. Specifically, we define utility over the K purposes as:

$$U = \sum_{j=1}^{K} \psi(x_j)(t_j + \gamma_j)^{\alpha_j}, \tag{1}$$

where $\psi(x_j)$ is the baseline utility for time invested in activity purpose *j*, and γ_j and α_j are parameters (note that ψ is a function of observed characteristics, x_j, associated with purpose *j*).

As discussed by Kim *et al.* (2002), the utility form in Equation (1) belongs to the family of translated utility functions, with γ_j determining the translation and α_j influencing the rate of diminishing marginal utility of investing time in activity purpose *j*. The function in Equation (1) is a valid utility function if $\psi(x_j) > 0$ and $0 < \alpha_j \leq 1$ for all *j*. Further, the term γ_j determines if corner solutions are allowed (*i.e.*, an individual does not participate in one or more activity purposes) or if only interior solutions are allowed (*i.e.*, an individual is constrained by formulation to participate in all activity purposes). To see this, consider the case where an individual has a total time *T* available to participate in one or both of two activity purposes and spends all the time *T* between these two activity purposes. Thus $t_1 + t_2 = T$, which serves as the constraint when maximizing utility. Figure 1 presents this two activity purpose case. The slope of the indifference curve at any point (t_1, t_2) in Figure 1, is given by:

$$\text{Slope}(t_1, t_2) = \frac{(\partial U / \partial t_1)}{(\partial U / \partial t_2)} = \frac{(t_2 + \gamma_2)^{1-\alpha_2}}{(t_1 + \gamma_1)^{1-\alpha_1}} \times \frac{\psi(x_1)}{\psi(x_2)} \times \frac{\alpha_1}{\alpha_2} \tag{2}$$

The indifference curve is shown in Figures 1a and 1b for the case where $\psi(x_2) = 2\psi(x_1)$ and α_1 = α_2 = 0.5. In Figure 1a, γ_1 = γ_2 = 0, which leads to the case where the slope of the indifference curve approaches infinity at the y-axis (*i.e.*, t_1 = 0 in Equation 2) and approaches zero at the x-axis (*i.e.*, t_2 = 0 in Equation 2). Thus, the indifference curve is tangential to both axes, and only interior solutions are possible. In the figure, the linear line indicates a time budget of 5 hours, and the optimal consumption point is 1 hour of activity purpose 1 and 4 hours of activity purpose 2. Figure 1b shows the case when γ_1 = 1.25, but γ_2 = 0. In this situation, the indifference curve has a finite slope at the y-axis (since the slope is non-zero and finite by Equation (2) when t_1 = 0 and $\gamma_1 \neq 0$). But the indifference curve remains tangential to the x-axis (since the slope is zero by Equation (2) when t_2 = 0 and γ_2 = 0). In such a case, it is possible that no amount of time is invested in activity purpose 1, as is the case in Figure 1b (note, however, that an interior solution is still possible in Figure 1b depending on the values of the α, γ, and ψ parameters in an individual's utility function; the values in Figure 1b have been set such that a corner solution results). Figure 1c shows the reverse case when $\psi(x_2) = 0.5\psi(x_1)$, α_1 = α_2 = 0.5, γ_1 = 0, and γ_2 = 1.25. Here, the indifference curve is tangential to the y-axis, but has a finite non-zero slope at the x-axis. The values of the parameters are such that a corner solution arises with no consumption of activity purpose 2.

The discussion above indicates that the utility form of Equation (1) is flexible enough to accommodate both internal and corner solutions. Specifically, if $\gamma_j \neq 0$, it is possible that individual q allocates no time to activity purpose j. On the other hand, if γ_j = 0, it implies that individual q allocates some non-zero amount of time to activity purpose j. In addition, the utility form is also able to accommodate a wide variety of time allocation situations based on the values of $\psi(x_j)$ and α_j (j = 1, 2,…, J). A high value of $\psi(x_j)$ for one activity purpose (relative to all other activity purposes), combined with a value of α_j close to 1, implies a high baseline preference and a very low rate of satiation for activity purpose j. This represents the situation when individual q allocates almost all her/his time to only activity purpose j (*i.e.*, a "homogeneity-seeking" individual). On the other hand, about equal values of $\psi(x_j)$ and small values of α_j across the various purposes *j* represents the situation where the individual invests time in almost all activity purposes (*i.e.*, a "variety-seeking" individual). More generally, the utility form allows a variety of situations characterizing an individual's underlying behavioral mechanism with respect to time allocation to activity purpose *j*, including (a) low baseline preference and high satiation (low ψ_j and low α_j), (b) high baseline preference and high satiation (high ψ_j and low α_j), (c) low baseline preference and low satiation (low ψ_j and high α_j), and (d) high baseline preference and low satiation (high ψ_j and high α_j).

3.1 Random Utility Model

We develop a statistical model from the utility structure of the previous section by adopting a random utility specification. Specifically, we introduce a multiplicative random element to the baseline utility as follows:

$$\psi(x_j, \varepsilon_j) = \psi(x_j) \cdot e^{\varepsilon_j}, \tag{3}$$

where ε_j captures idiosyncratic (unobserved) characteristics that impact the baseline utility for purpose *j*. The exponential form for the introduction of random utility guarantees the positivity of the baseline utility as long as $\psi(x_j) > 0$. To ensure this latter condition, we further parameterize $\psi(x_j)$ as $\exp(\beta' x_j)$, which then leads to the following form for the baseline random utility:

$$\psi(x_j, \varepsilon_j) = \exp(\beta' x_j + \varepsilon_j). \tag{4}$$

The x_j vector in the above equation includes a constant term reflecting the generic preference in the population toward purpose *j*. The overall random utility function then takes the following form:

$$\widetilde{\mathrm{U}} = \sum_{j} \left[\exp(\beta' x_j + \varepsilon_j)\right] \cdot (t_j + \gamma_j)^{\alpha_j} \tag{5}$$

From the analyst's perspective, the individual is maximizing random utility ($\widetilde{\mathrm{U}}$) subject to the time budget constraint that $\sum_{j=1}^{K} t_j = T$, where T is the time available for allocation among the K activity purposes. The analyst can then solve for the optimal time allocations by forming the Lagrangian and applying the Kuhn-Tucker conditions. The Lagrangian function for the problem is:

$$\text{‹} = \sum_{j} \left[\exp(\beta' x_j + \varepsilon_j)\right](t_j + \gamma_j)^{\alpha_j} - \lambda \left[\sum_{j=1}^{K} t_j - T\right], \tag{6}$$

where λ is the Lagrangian multiplier associated with the time constraint. The Kuhn-Tucker (K-T) first-order conditions for the optimal time allocations (the t_j^* values) are given by:

$$\left[\exp(\beta' x_j + \varepsilon_j)\right]\alpha_j (t_j^* + \gamma_j)^{\alpha_j - 1} - \lambda = 0, \text{ if } t_j^* > 0, j = 1, 2, \ldots, K \tag{7}$$

$$\left[\exp(\beta' x_j + \varepsilon_j)\right]\alpha_j (t_j^* + \gamma_j)^{\alpha_j - 1} - \lambda < 0, \text{ if } t_j^* = 0, j = 1, 2, \ldots, K$$

The above conditions have an intuitive interpretation. For all activity purposes to which time is allocated (*i.e.*, $t_j^* > 0$), the time investment is such that the marginal utilities are the same across purposes (and equal to λ) at the optimal time allocations (this is the first set of K-T conditions; note that the first term on the left side of the K-T conditions corresponds to marginal utility). Also, for an activity purpose *j* in which no time is invested, the marginal utility for that activity purpose at zero time investment is less than the marginal utility at the consumed times of other purposes (this is the second set of K-T conditions in Equation 7).

The optimal demand satisfies the conditions in Equation (7) plus the time budget constraint $\sum_{j=1}^{K} t_j^* = T$. The time budget constraint implies that only K-1 of the t_j^* values need to be estimated, since the time invested in any one purpose is automatically determined from the time invested in all the other purposes. To accommodate this constraint, designate activity purpose 1 as a purpose to which the individual allocates some non-zero amount of time (note that the individual should participate in at least one of the K purposes, given that $T > 0$). For the first activity purpose, the Kuhn-Tucker condition may then be written as:

$$\lambda = \left[\exp(\beta' x_1 + \varepsilon_1)\right] \alpha_1 (t_1^* + \gamma_1)^{\alpha_1 - 1} \tag{8}$$

Substituting for λ from above into Equation (7) for the other activity purposes (j = 2,…, K), and taking logarithms, we can rewrite the K-T conditions as:

$$\begin{aligned} &V_j + \varepsilon_j = V_1 + \varepsilon_1 \text{ if } t_j^* > 0 \;\; (j = 2, 3, \ldots, K) \\ &V_j + \varepsilon_j < V_1 + \varepsilon_1 \text{ if } t_j^* = 0 \;\; (j = 2, 3, \ldots, K), \text{ where} \\ &V_j = \beta' x_j + \ln \alpha_j + (\alpha_j - 1)\ln(t_j^* + \gamma_j) \;\; (j = 1, 2, 3, \ldots, K). \end{aligned} \tag{9}$$

The satiation parameter, α_j, needs to be bounded between 0 and 1, as discussed earlier. To enforce this condition, we parameterize α_j as $1/[1 + \exp(-\delta_j)]$. Further, to allow the satiation parameters to vary across individuals, we write $\delta_j = \theta_j' y_j$, where y_j is a vector of individual characteristics impacting satiation for the j^{th} alternative, and θ_j is a corresponding vector of parameter. Also, note that, in Equation (9), a constant cannot be identified in the $\beta' x_j$ term for one of the K alternatives (because only the difference in the V_j from V_1 matters). Similarly, individual-specific variables are introduced in the V_j 's for (J-1) alternatives, with the remaining alternative serving as the base (these identification conditions are similar to those in the standard discrete choice model).

3.2 Econometric Model

3.2.1 Basic structure

To complete the model structure, we specify a standard extreme value distribution for ε_j and assume that ε_j is independent of x_j $(j = 1, 2, \ldots, K)$. The ε_j's are also assumed to be independently distributed across alternatives. From Equation (9), the probability that the individual participates in M of the K activity purposes ($M \geq 2$), given ε_1, is:

$$P\left(t_2^*, t_3^*, \ldots, t_M^*, 0, 0, \ldots, 0\right) | \, \varepsilon_1 \tag{10}$$

$$= \left\{ \left(\prod_{i=2}^{M} g(V_1 - V_i + \varepsilon_1) \right) | \, J \, | \right\} \times \left\{ \prod_{s=M+1}^{K} G(V_1 - V_s + \varepsilon_1) \right\},$$

where g is the standard extreme value density function, G is the standard extreme value distribution, the first M activity purposes are taken to be the ones in which the individual participates for notational convenience, and J is the Jacobian whose elements are given by:

$$J_{ih} = \frac{\partial [V_1 - V_{i+1} + \varepsilon_1]}{\partial t_{h+1}^*}; \; i, h = 1, 2, \ldots, M - 1. \tag{11}$$

The term in the first parenthesis in Equation (10) is the continuous density component corresponding to the optimal time investment for the M purposes in which the individual participates (the first purpose does not appear in this component because it is always selected for participation and because the optimal time allocation for this purpose is implicitly determined by the time allocation to other purposes). To see that the term in the first parenthesis in Equation (10) corresponds to the continuous density component, note from the first-order conditions in

Equation (9) that the optimal time for activity purposes with non-zero time investment is governed by the nonlinear function given by $\varepsilon_i = V_1 - V_i + \varepsilon_1$ (i = 2, 3, …, M). A change-of-variable technique is used to obtain the density of $t^* = (t_2^*, t_3^*, ..., t_M^*)$ from ε $(= \varepsilon_2, \varepsilon_3, ..., \varepsilon_M)$, and results in the Jacobian term J. The term in the second parenthesis in Equation (10) is a discrete distribution component corresponding to the purposes in which the individual does not participate.

Substituting the extreme value density and distribution functions for g(.) and G(.), respectively, Equation (10) can be equivalently written as:

$$P\left(t_2^*, t_3^*, ..., t_M^*, 0, 0, ..., 0\right) | \, \varepsilon_1$$

$$= \left\{ \prod_{i=2}^{M} e^{-(V_1 - V_i + \varepsilon_1)} \cdot e^{-e^{-(V_1 - V_i + \varepsilon_1)}} \right\} \left\{ \prod_{s=M+1}^{K} e^{-e^{-(V_1 - V_s + \varepsilon_1)}} \right\} \cdot |J| \tag{12}$$

$$= \left\{ \left[\prod_{i=2}^{M} e^{-(V_1 - V_i)} \right] \left[\left(e^{-\varepsilon_1} \right)^{M-1} \right] \left[\prod_{i=2}^{M} e^{-e^{-(V_1 - V_i + \varepsilon_1)}} \right] \right\} \left\{ \prod_{s=M+1}^{K} e^{-e^{-(V_1 - V_i + \varepsilon_1)}} \right\} \cdot | \, J \, |$$

$$= \left\{ \left[\prod_{i=2}^{M} e^{-(V_1 - V_i)} \right] \left[\left(e^{-\varepsilon_1} \right)^{M-1} \right] \left[\prod_{j=2}^{K} e^{-e^{-(V_1 - V_j + \varepsilon_1)}} \right] \right\} \cdot | \, J \, |$$

$$= \left\{ \left[\prod_{i=2}^{M} e^{-(V_1 - V_i)} \right] \left[\left(e^{-\varepsilon_1} \right)^{M-1} \right] \left[e^{-\sum_{j=2}^{K} \left[e^{-(V_1 - V_j + \varepsilon_1)} \right]} \right] \right\} \cdot | \, J \, | .$$

Next, the determinant of the Jacobian term can be derived to be as follows (see Appendix A):

$$| \, J \, | = \left(\prod_{i=1}^{M} c_i \right) \left(\sum_{i=1}^{M} \frac{1}{c_i} \right), \text{where } c_i = \left(\frac{1 - \alpha_i}{t_i^* + \gamma_i} \right) \tag{13}$$

Finally, one can uncondition out ε_1 from Equation (12) to obtain the following unconditional probability expression:

$$P\left(t_2^*, t_3^*, \ldots, t_M^*, 0, 0, \ldots, 0\right)$$

$$= \left[\prod_{i=1}^{M} c_i\right]\left[\sum_{i=1}^{M} \frac{1}{c_i}\right]\left[\prod_{i=2}^{M} e^{-(V_1 - V_i)}\right] \int_{\varepsilon_1=-\infty}^{+\infty} \left\{ \left(e^{-\varepsilon_1}\right)^{M-1} \cdot e^{-\sum_{j=2}^{K}\left[e^{-(V_1 - V_j + \varepsilon_1)}\right]} \cdot e^{-\varepsilon_1} \cdot e^{-e^{-\varepsilon_1}} \right\} d\varepsilon_1 \quad (14)$$

$$= \left[\prod_{i=1}^{M} c_i\right]\left[\sum_{i=1}^{M} \frac{1}{c_i}\right]\left[\prod_{i=2}^{M} e^{-(V_1 - V_i)}\right] \int_{\varepsilon_1=-\infty}^{+\infty} \left\{ \left(e^{-\varepsilon_1}\right)^{M-1} \cdot e^{-\sum_{j=1}^{K}\left[e^{-(V_1 - V_j + \varepsilon_1)}\right]} \cdot e^{-\varepsilon_1} \right\} d\varepsilon_1$$

The integral above can be simplified as shown in Appendix B. The final result is a remarkably elegant and compact closed form structure:

$$P\left(t_2^*, t_3^*, \ldots, t_M^*, 0, 0, \ldots, 0\right)$$

$$= \left[\prod_{i=1}^{M} c_i\right]\left[\sum_{i=1}^{M} \frac{1}{c_i}\right]\left[\frac{\prod_{i=1}^{M} e^{V_i}}{\left(\sum_{j=1}^{K} e^{V_j}\right)^{M}}\right](M-1)! \quad (15)$$

In the case when M = 1 *(i.e.*, only one alternative is chosen), the model in Equation (15) collapses to the standard MNL model (if M = 1, the continuous component drops out, because the time invested in the chosen activity will be T). Also, note that the utilities are assumed to be linear in the standard MNL model (*i.e.*, α_j = 1 for all j). This results in the V_j 's in Equation (9) becoming linear. Thus, the model proposed in this paper is a multiple discrete-continuous extension of the standard MNL model. In addition, the model also represents a multiple discrete-continuous extension of the single discrete-continuous models of Dubin and McFadden, 1984, Hanemann, 1984, Chiang, 1991, Chintagunta, 1993, and Arora *et al.*, 1998. Specifically, the case of a single discrete-continuous model may be viewed as a two alternative case within the multiple discrete-continuous formulation, with one alternative (say the first) always being consumed. To see this, assume that the objective was to analyze if an individual participates in recreational activities during a certain time period and the time invested in recreational activities. Then, the first activity purpose can be labeled as "non-recreational" (essentially, an "outside good") and the second as "recreational". Since all individuals would invest some amount of time in "non-recreational" activities, $\gamma_1 = 0$. One can then use the multiple discrete-continuous formulation to model participation choice and duration of time in recreational activity, with *T* being the total amount of time within the period under consideration (for example, 24 hours if the time period is a day).

3.2.2 Accommodating heteroscedasticity and error correlations across alternative utilities

The previous section assumed that the ε_j terms are independently and identically distributed across alternatives, and are distributed standard Gumbel. However, these assumptions are needlessly restrictive. Incorporating heteroscedasticity and error correlation in the MDCEV model is straightforward, and leads to the Mixed MDCEV (or MMDCEV) model (this is similar to the movement from the MNL model to the mixed MNL model). Specifically, the error term ε_j may be partitioned into three independent components ζ_j, $\eta' w_j$, and $\mu' z_j$. The first component, ζ_j, is assumed to be independently and identically standard Gumbel distributed across alternatives. The second component, $\eta' w_j$, allows the estimation of distinct scale (variance) parameters for the error terms across alternatives. w_j is a column vector of dimension *K* with each row representing an alternative. The row corresponding to alternative *j* takes a value of 1 and all other rows take a value of 0. The vector η (of dimension *K*) is specified to have independent, normally distributed and mean-zero elements, each element having a variance of ω_j^2. Let ω be a vector of true parameter characterizing the variance-covariance matrix of the multivariate normal distribution of η. The third component in the error term, $\mu' z_j$, constitutes the mechanism to generate correlation across unobserved utility components of the alternatives. z_j is specified to be a column vector of dimension *H* with each row representing a group *h* (*h* = 1,2,...,*H*) of alternatives sharing common unobserved components. The row(s) corresponding to the group(s) of which *j* is a member take(s) a value of one and other rows take a value of zero. The vector μ (of dimension *H*) may be specified to have independent normally distributed elements, each element having a variance component σ_h^2. The result of this specification is a covariance of σ_h^2 among alternatives in group *h*. Let σ be a parameter vector characterizing the variance-covariance matrix of μ.

For given values of the vectors η and μ, one can follow the derivation of the earlier section and obtain the usual MDCEV probability that the individual participates in *M* of the *J* activity purposes ($M \geq 2$):

$$P\left(t_2^*, t_3^*, \ldots, t_M^*, 0, 0, \ldots, 0\right) \mid (\eta, \mu)$$

$$= \left[\prod_{i=1}^{M} c_i\right]\left[\sum_{i=1}^{M} \frac{1}{c_i}\right]\left[\frac{\prod_{i=1}^{M} e^{V_i + \eta' w_i + \mu' z_i}}{\left(\sum_{j=1}^{K} e^{V_j + \eta' w_j + \mu' z_j}\right)^M}\right](M-1)! \qquad (16)$$

The unconditional probability can then be computed as:

$$P\left(t_2^*, t_3^*, \ldots, t_M^*, 0, 0, \ldots, 0\right)$$

$$= \int_{\eta} \int_{\mu} \left[\prod_{i=1}^{M} c_i\right]\left[\sum_{i=1}^{M} \frac{1}{c_i}\right]\left[\frac{\prod_{i=1}^{M} e^{V_i + \eta' w_i + \mu' z_i}}{\left(\sum_{j=1}^{K} e^{V_j + \eta' w_j + \mu' z_j}\right)^M}\right](M-1)!\, dF(\mu \mid \sigma)\, dF(\eta \mid \omega), \qquad (17)$$

where F is the multivariate cumulative normal distribution. The reader will note that the dimensionality of the integration above is dependent on the number of elements in η and μ.

3.2.3 Estimation of the mixed MDCEV model

The parameters to be estimated in the MMDCEV model of Equation (17) include the β vector, the θ_j vectors and γ_j scalars for each alternative j (these are embedded in the V_j values), and the σ and ω vectors. Let θ be a column vector that stacks all the θ_j vectors vertically, and let γ be another column vector of the γ_j elements stacked vertically.

We use the maximum likelihood inference approach to estimate the parameters of the MMDCEV model. Introducing the index q for individuals, we can write the likelihood function as:

$$\text{‹}(\beta,\theta,\gamma,\sigma,\omega)=\sum_{q=1}^{Q}\log\left[\int_{\eta}\int_{\mu}\left[\prod_{i=1}^{M}c_i\right]\left[\sum_{i=1}^{M}\frac{1}{c_i}\right]\left[\frac{\prod_{i=1}^{M}e^{V_i+\eta' w_i+\mu' z_i}}{\left(\sum_{j=1}^{K}e^{V_j+\eta' w_j+\mu' z_j}\right)^{M}}\right](M-1)!\,dF(\mu\,|\,\sigma)\,dF(\eta\,|\,\omega)\right] \quad (18)$$

We apply quasi-Monte Carlo simulation techniques to approximate the integrals in the likelihood function and maximize the logarithm of the resulting simulated likelihood function across all individuals with respect to β, θ, γ, σ, and ω. Under rather weak regularity conditions, the maximum (log) simulated likelihood (MSL) estimator is consistent, asymptotically efficient, and asymptotically normal (see Hajivassiliou and Ruud, 1994; Lee, 1992; McFadden and Train, 2000).

In the current paper, we use a scrambled version of the Halton sequence to draw realizations for η and μ from their population normal distributions. Details of the Halton sequence and the procedure to generate this sequence are available in Bhat (2003).

4. DATA SOURCES AND SAMPLE FORMATION

4.1 Data Sources

The data source used for this analysis is the 2000 San Francisco Bay Area Travel Survey (BATS). This survey was designed and administered by MORPACE International Inc. for the Bay Area Metropolitan Transportation Commission. The survey collected information on all activity episodes undertaken by individuals from over 15,000 households in the Bay Area for a two-day period (see MORPACE International Inc., 2002 for details on survey, sampling, and administration procedures). The information collected on activity episodes included the type of activity (based on a 17-category classification system), start and end times of activity participation, and the whether the episode was pursued in-home or out-of-home. Furthermore, data on individual and household socio-demographics, individual employment-related characteristics, household auto ownership, and internet access and usage were also obtained.

In addition to the 2000 BATS data, we also obtained zonal-level land-use and demographics data for each of the Traffic Analysis Zones (TAZ) in the San Francisco Bay area. This data included: (1) area by land-use purpose, (2) number of housing units, (3) employment levels by sector, (4) zonal population, income and age distribution of the population, and (5) area type of the zone (core CBD, other CBD, urban, suburban, or rural). This information was used to study the impact of the characteristics of the residence zone.

4.2 Sample Formation

The process of generating the sample for analysis involved several steps. First, only individuals 16 years or older were considered to focus the analysis on the subgroup of the population who clearly exercise a choice in their time-use. Second, we selected only weekend day data from the original survey sample. This was done because individuals participate more frequently, and for longer durations, in discretionary activities over the weekends than during the weekdays (as indicated earlier in the paper, our empirical focus is on discretionary activities in this paper). Individuals also participate in more variety of types of discretionary activities on the weekends than weekdays (Bhat and Lockwood, 2004). Thus, time use analysis modeling for discretionary activities is particularly interesting over the weekends. Third, social activity episodes (including conversation and visiting family/friends) and recreational activity episodes, including such activities as hobbies, exercising, and watching TV, were selected from the larger file of all activity episodes. Fourth, the total time invested during the weekend day in each of the following four activity purpose categories was computed based on appropriate time aggregation across individual episodes within each category: (1) time spent in in-home social activities (IHS), (2) time spent in in-home recreational (IHR) activities, (3) time spent in out-of-home social (OHS) activities, and (4) time spent in out-of-home recreational (OHR) activities. Fifth, out-of-home shopping activity episodes were selected from the original survey file and those episodes unrelated to grocery shopping were selected. The total time invested over the weekend day across all out-of-home non-grocery shopping episodes was then computed to provide a fifth category of discretionary time-use: time spent in out-of-home non-maintenance shopping activities. For convenience, we will refer to this fifth category as “time spent in out-of-home shopping (OHSh) activities” in the rest of this paper. Sixth, data on individual, household, and residence zone characteristics were appropriately cleaned and added. Finally, several screening and consistency checks were performed and records with missing or inconsistent data were eliminated.

The final sample for analysis includes the weekend day time-use information of 1917 individuals. The analysis of interest is the participation and time invested in five types of discretionary activities over the weekend day: in-home social (IHS), in-home recreation (IHR), out-of-home social (OHS), out-of-home recreation (OHR), and out-of-home shopping (OHSh). Of the 1917 individuals, 1169 (61%) participated in only one activity type, 605 (31.5%) participated in two activity types, 126 (6.5%) participated in three activity types, and 17 (1%) participated in four activity types (no individual participated in all activity types). These statistics clearly indicate the problem of using standard discrete choice models, since 39% of individuals participate in more than one activity type.[2]

Table 1 provides descriptive details of participation in each type of discretionary activity. The second and third columns indicate the number (percentage) of individuals participating in each activity type and the mean duration of participation among those who participate, respectively. Several observations may be made from the statistics in these two columns. First, individuals participate most in OHSh activity over the weekend from among the various discretionary activity types. However, the duration of participation in OHSh activity is short compared to other activity types. This suggests an overall high baseline preference, but also a high level of satiation, for OHSh activity in the population. Second, there is also a high

[2] In the current analysis, we assume that the total time invested in discretionary activities is given for each individual, and examine the allocation of this total discretionary time to the five types of discretionary activities. A more general empirical model would be one that considers non-discretionary activities as an "outside good", and models not only the allocation to the five different types of discretionary activities, but also the time invested in discretionary activities and non-discretionary activities.

likelihood of participation in IHR activities, and the participation duration in such activities is also long. This suggests a high baseline preference and a low satiation for IHR activities. Third, there appears to be a relatively low baseline preference for IHS activities; the overall baseline preferences for the OHS and OHR activities are between those of the IHS activity type and the OHSh/OHR activity types. Fourth, the extent of satiation for IHS, OHS, and OHR activities is in the same general range, and these are between the satiation levels for OHSh and IHR activities based on the mean durations of participation. The last two columns in Table 1 indicate the split between solo participations (*i.e.*, individual participation in only one activity type or a corner solution) and multiple activity participations (*i.e.*, individual participation in multiple activity types or interior solutions) for each activity type. Thus, the number for the IHS activity type indicates that, of the 118 individuals participating in IHS activity, 33 (or 28%) participated only in IHS activity during the day and 85 (or 72%) participated in IHS activity along with participation in other activity types during the day. The results clearly indicate that individuals tend to participate in IHS activity more often in conjunction with participation in other activity types during the day. This may be because individual observed and unobserved factors that increase participation in IHS activity also increase participation in other activity types or because of a high satiation rate for IHS activity. The model in the paper accommodates both possibilities and can disentangle the two alternative effects. The results also show that IHR activity is more often participated in isolation than other activity types. Again, this may reinforce the notion of low satiation for the IHR activity type (as discussed earlier) or may reflect a strong preference for IHR activity by some individuals.

5. EMPIRICAL ANALYSIS

5.1 Variables Considered

Several types of variables were considered in the discretionary time-use model. These included household sociodemographics (household size, presence and number of children, number of household vehicles, number of bicycles in the household, household income, family structure, and dwelling type), household location attributes (discussed below), individual demographics and employment characteristics (age, license holding to drive, student status, employment status, number of days of work, internet use, and ethnicity), and day of week/season of year.

The household location variables included a land-use mix diversity variable, fractions of detached and non-detached dwelling units, area type variables classifying zones into one of 6 categories (core central business districts, central business districts, urban business, urban, suburban, and rural), and residential density and employment density variables. The first of these variables, the land-use mix diversity variable, is computed as a fraction between 0 and 1. Zones with a value closer to one have a richer land-use mix than zones with a value closer to zero. Three categories of land-uses are considered in the computation of the mix diversity variable: acres in residential use (r), acres in commercial/industrial use (c), and acres in other land-uses (o). The actual form of the land-use mix diversity variable is:

$$\text{Land-use mix diversity } = 1 - \left\{ \frac{\left|\frac{r}{L} - \frac{1}{3}\right| + \left|\frac{c}{L} - \frac{1}{3}\right| + \left|\frac{o}{L} - \frac{1}{3}\right|}{(4/3)} \right\}, \tag{19}$$

where $L = r + c + o$. The functional form assigns the value of zero to zones in which land-use is focused in only one category, and assigns a value of 1 to zones in which land-use is equally split among the three land-use categories.

Finally, the day of week/season variables were introduced to capture the day of weekend (Saturday or Sunday) and season of year effects (fall, winter, spring, or summer).

5.2 Empirical Results

5.2.1 Model specification and error-component specification

As discussed in Section 3, the utility form of Equation (1) includes the γ_j translation parameters to allow the possibility of corner solutions for each activity type (*i.e.*, zero consumption of each activity type). However, we found it difficult to identify the γ vector and the satiation vector α separately, because both these vectors determine the slope of the indifference curves at the corner points (see Equation 2). Thus, we fixed the elements of the γ vector to 1.0 (see also Kim *et al.*, 2002).

In our analysis, we considered several error component specifications to introduce unobserved heteroscedasticity and correlation in the utilities of the five activity types. The best statistical result included the following error components: (1) five error components, one for each alternative, to capture the variance of the baseline utility terms, (2) one error component to accommodate correlation between the two in-home activities (IHS and IHR), and (3) one error component to accommodate correlation between the OHS and OHSh activity types. In the first category of error components, corresponding to pure variance elements, the variances of the out-of-home activity types were constrained to be equal for identification and stability.

5.2.2 Variable effects

The final specification results of the leisure time-use model are presented in Table 2. In the following sections, we discuss the effect of variables by variable category. In instances where some alternatives do not appear for a variable, the excluded alternatives constitute the base category.

5.2.2.1 Household Sociodemographics Among the household sociodemographic variables, the effect of the number of adults indicates that individuals in households with several adults have a higher baseline preference for IHR activity compared to individuals in households with few adult members. This may be a reflection of the increased opportunity for joint in-home recreational participation in households, such as watching a movie or television at home with other adults (see Bhat and Misra, 1999 and Kitamura *et al.*, 1996 for a similar result using a Dutch dataset).

The presence of very young children (0 to 4 years of age) increases the baseline preference for out-of-home activity types (OHS, OHR, and OHSh), perhaps because of a stronger need of adults in such households to have a change from the activity of caring for children inside the home. The same higher baseline propensity to participate in out-of-home activities is also observed among adults in households with young children (5-15 years of age), though the out-of-home activity types tend to be recreational or shopping rather than social. The higher propensity of adults in households with young children to participate in recreational activity is perhaps a result of the outdoor recreational pursuits with young children (such as participation in youth soccer and baseball leagues, family walks, and bicycle trips; see Mallett and McGuckin, 2000 for a similar result).

The next household attribute is the number of bicycles in the household. As the number of bicycles increases in an individual's household, the individual is more likely to pursue OHR activity. This is quite reasonable. Households who own more bicycles may be more outdoor-oriented by nature, and owning bicycles also provides an additional means to participate in outdoor recreation.

Finally, among the set of household sociodemographics, the results indicate that individuals in low income households have a higher baseline preference for in-home recreation than those in high income households. Further, individuals in the middle income range (35,000 to 95,000 dollars per annum) are more likely to participate in out-of-home recreation than those in the low or high income ranges. This non-monotonic income effect on time investment in OHR activity deserves additional attention; it appears that increasing income does increase the ability to participate in out-of-home recreation, but other constraints set in at high incomes.

<u>5.2.2.2 Household Location Variables</u> Among the many household location variables considered in the analysis, the only ones having a marginally significant effect on time use were the area type variable and the land-use mix variables. The results indicate that individuals residing in CBD areas have a higher baseline preference for OHR activity compared to individuals residing in non-CBD areas. This is possibly because of the "pedestrian-oriented" urban forms associated with the high density of CBD areas. In addition, CBD areas are likely to be correlated with better accessibility to recreational activity centers. The effect of the land-use mix variable indicates a higher propensity to participate in shopping activities among individuals residing in areas with a diverse land-use mix, perhaps because of the increased ease of reaching shopping activity centers and combining shopping with other out-of-home activity participations.

5.2.2.3 Individual Sociodemographics and Employment Characteristics Several individual characteristics were tested in the model, but only those related to age, vehicle license holding, employment, whether or not the individual shops over the internet, gender, and ethnicity appeared in the final specification. The results indicate that older individuals are less likely to participate in OHR activity compared to younger individuals. Further, teenagers (16-17 years of age) are less likely to participate in shopping, and teenagers and young adults (18-29 years of age) are more likely to participate in out-of-home social (OHS) activity. Also, the elderly (greater than 65 years of age) are more likely to participate in IHR activity and less likely to pursue shopping activity relative to younger individuals, perhaps reflecting mobility constraints.

The availability of a license to drive has a positive effect on participation in all out-of-home activity types, which can be attributed to the greater mobility to reach out-of-home activity centers. Employed individuals have a higher propensity to participate in shopping activity over the weekend than do unemployed individuals, perhaps because of the inability to access shopping activity centers and pursue shopping during the course of the work week. The effect of the male variable indicates that men pursue more IHR activity than women, a reinforcement of the notion of men being "glued to the tube". Individuals who shop over the internet also pursue more OHSh activity, suggesting a complementary effect of internet use on OHSh activity. Alternatively, it may be that the same unobserved shopping orientation factors affect both use of the internet for shopping and out-of-home shopping. The race-related variables indicate that African-Americans are less likely to pursue out-of-home recreation (OHR) activity relative to other races. This finding is similar to those of previous works in the area of recreational activity participation (see Bhat and Gossen, 2004 and Mallett and McGuckin, 2000). Also, Hispanic

Americans participate more in OHS activity, while Asian Americans are less likely to pursue OHS activity, relative to other races.

5.2.2.4 Day of Week and Seasonal Effects The results for the day of week effects shows a higher level of preference for in-home activities (IHS and IHR) on Sundays relative to Saturdays. This is reasonable since Sundays serve as a transition day between the weekend and the work week, and many individuals use it as an in-home "rest" day.

The seasonal effects reflect a lower propensity to participate in OHR activity during the winter season and, to a lesser extent, the fall season compared to the spring and summer seasons. This is intuitive, since the spring and summer seasons provide more conducive weather conditions for outdoor recreation than the fall and winter seasons in the San Francisco Bay area.

5.2.2.5 Baseline Preference Constants The baseline preference constants do not have any substantive interpretations because of the presence of two continuous exogenous variables (age and land-use mix). But since almost all of the variables are dummy variables, the constants may be viewed informally as providing the baseline preferences for the "base" individual defined by the combination of the base dummy variable categories. From this perspective, the constants reinforce our discussion in Section 4.2. Specifically, the IHS activity type (the base activity type) is least preferred of all activity types at the point when no time has yet been invested in any activity type. On the other hand, the OHSh activity type clearly has the highest baseline preference of all the activity types.

5.2.3 Satiation parameters

The satiation parameter, α_j, for each activity type j is parameterized as 1/[1+exp(- δ_j)], where $\delta_j = \theta'_j y_j$ (see Section 3.1). This parameterization allows α_j to vary based on individual and day of week/seasonal characteristics and still be bounded between 0 and 1. In our empirical analysis, we did not find any statistically significant variation in the α_j parameters based on individual and day of week/season of year characteristics for the IHS, IHR, OHS, and OHR activity types. However, there was variation in the satiation parameter for the OHSh activity type based on the sex of the individual. After estimating the θ_j parameters, one can compute the δ_j parameters and then the α_j parameters.

Table 3 provides the estimated values of α_j and the t-statistic with respect to the null hypothesis of α_j = 1 (note that standard discrete choice models assume α_j = 1). Several important observations may be drawn from the table. First, all the satiation parameters are significantly different from 1, rejecting the linear utility structure employed in standard discrete choice models. Thus, there are clear satiation effects in discretionary time use decisions. Second, satiation effects are lower for the in-home activity types than for the out-of-home activity types. Between the two in-home activity types, there is lower satiation for IHR compared to IHS. Third, the highest satiation occurs in the OHSh category. This indicates that individuals are not willing to invest too much time on shopping activity. The satiation effect for shopping is higher for men relative to women (that is, women tend to shop longer than men).

5.2.4 Variance-covariance parameters

The error components introduced in the baseline preference function (see Section 5.2.1) generate heteroscedasticity and covariance in unobserved factors across activity types. From the estimated standard deviations of the error components, it is straightforward to compute the estimated variance-covariance matrix. This is presented in Table 4. The variance terms (*i.e.*, the diagonal elements) indicate higher variance due to unobserved factors for the out-of-home activity types relative to the in-home activity types. Further, the matrix is dominantly diagonal, indicating that there is not much covariance in unobserved factors between the various activity types after controlling for the observed factors. However, there is significant covariance between the two in-home activity types, reflecting individual-specific unobserved components (such as inertial tendencies and preference for privacy of the home) that predispose individuals to in-home activity pursuits. The implied correlation between the baseline preferences of the in-home activity types is 0.5. There is also a marginally significant covariance in the baseline preferences of the out-of-home social and out-of-home shopping activity types due to unobserved individual-specific factors. The implied correlation is, however, very low at 0.05.

5.3 Overall Likelihood-Based Measures of Fit

The log-likelihood value at convergence of the final mixed multiple discrete-continuous extreme value (MMDCEV) model is -10,053. The corresponding value for the MMDCEV model with only the constants in the baseline preference terms, the satiation parameters, and the variance-covariance terms is -10,142. The likelihood ratio test for testing the presence of exogenous variable effects is 178, which is substantially larger than the critical chi-square value with 25 degrees of freedom at any reasonable level of significance. This clearly indicates variations in the baseline preferences for the discretionary activity types based on household demographics/location variables, individual demographics/employment attributes, and day of week/seasonal effects. Further, the log-likelihood value at convergence of the MDCEV model that does not allow unobserved heteroscedasticity and correlation across the baseline preferences of the different activity types is -10,102. The likelihood ratio test for comparing the MDCEV model with the MMDCEV model is 98, which is again substantially larger than the critical chi-square value with 5 degrees of freedom (corresponding to the five parameters estimated to characterize the variance-covariance matrix). Thus, there is statistically significant unobserved variation across individuals in their baseline preferences, and statistically significant correlation between the IHS and IHR activity types and the OHS and OHSh activity types.

5.4 Prediction Procedure

The final end-objective of the discretionary time model is to be able to predict the time use of individuals in the several discretionary activity types. This prediction provides information on participation, as well as the level of participation in each discretionary activity type.

The MMDCEV model can be used in a rather straight forward manner for prediction purposes. Note that consumer q allocates time to the various activities based on maximizing $\tilde{U}_q$ in Equation (5) subject to the time budget constraint that $\sum_j t_{jq} = T_q$ and $t_{jq} \geq 0$ for all j (the index q for individual is introduced in the notation here). Thus, the consumer's time allocation is based on the following problem:

$$\text{Max } \tilde{U}_q = \sum_j \left\{ \left[\exp(\beta' x_{qj} + \zeta_{qj} + \eta_q' w_j + \mu_q' z_j) \right] \cdot (t_{qj} + \gamma_j)^{\alpha_{qj}} \right\} \tag{20}$$

subject to

$$\sum_j t_{jq} = T_q,\ t_{qj} \geq 0 \text{ for all } j\ (j = 1, 2, \ldots K).$$

Of course, the error components ζ_{qj}, $\eta_q' w_j$, and $\mu_q' z_j$ are not observed to the analyst, making $\widetilde{\mathrm{U}}_q$ random. Thus, the predictions for individual q may be obtained by solving the following optimization problem:

$$\text{Max } \widetilde{\mathrm{U}}_q = \int_{\eta_q=-\infty}^{\infty} \int_{\mu_q=-\infty}^{\infty} \int_{\zeta_{q1}=-\infty}^{\infty} \int_{\zeta_{q2}=-\infty}^{\infty} \cdots \int_{\zeta_{qK}=-\infty}^{\infty} \sum_j \left\{ \left[\exp(\beta' x_{qj} + \zeta_{qj} + \eta_q' w_j + \mu_q' z_j)\right] \cdot (t_{qj} + \gamma_j)^{\alpha_{qj}} \right\} \\ dG(\zeta_{q1}) dG(\zeta_{q2}) \ldots dG(\zeta_{qs}) dF(\mu_q \mid \sigma) dF(\eta_q \mid \omega) \tag{21}$$

subject to

$$\sum_j t_{jq} = T_q,\ t_{qj} \geq 0 \text{ for all } j,$$

where G is the standard cumulative Gumbel distribution and F is the multivariate normal distribution function. The objective function above can be evaluated using simulation techniques and the time allocations t_{jq} can be predicted using a constrained optimization routine. In the current paper, the optimization was achieved using the constrained optimization application of the GAUSS matrix programming language. The multidimensional integral in the objective function was evaluated using 5,000 random draws (we tested the sensitivity of the t_{jq} values to the number of draws for the first several observations and found little difference in the predicted t_{jq} values beyond 5,000 draws).

The prediction procedure discussed above can be used to assess the performance of the model by comparing the actual time allocations to the predicted time allocations in the estimation sample. Table 5 shows the actual and predicted time allocations for the first 5 individuals in the sample. The predicted time allocations in Table 5 are rounded to the closest minute. As can be noticed, the predictions satisfy the non-negativity constraint and the total time budget constraint because of the constrained optimization prediction process.

Summary disaggregate non-likelihood measures of fit can be computed in several ways based on a comparison of actual and predicted time allocations. Two measures of fit are presented here to reflect the discrete as well as continuous nature of the predictions from the MMDCEV model. The first measure evaluates the ability of the model to correctly predict participation in the various activity types (this is the discrete component of the model). This measure, which we label as the "hit rate" measure, indicates the percentage of correct predictions across all individuals and activity types regarding participation, and is computed to be 66%. The second measure evaluates the ability of the model to predict the duration of participation conditional on a correct prediction regarding participation (this is the continuous component of the model). This measure, computed as the mean absolute percentage error (MAPE) ratio, is 29%. Both the "hit rate" and MAPE ratio measures indicate reasonable prediction fits, but also suggest that we are perhaps missing individual-specific factors (such as, for example, the intrinsic attitude/lifestyle preference for each kind of discretionary activity) that impact participation in different kinds of discretionary activity pursuits. Such individual-specific factors can be accommodated if data on multiple weekend days are collected from the same individual.

6. CONCLUSIONS

Classical discrete and discrete-continuous models deal with situations with only one alternative chosen from a set of mutually exclusive alternatives. On the other hand, many consumer demand situations are characterized by the choice of multiple alternatives simultaneously. Until recently, there has been limited research on modeling such multiple discreteness situations in the literature. This paper formulates a new model for multiple discreteness in demand that is derived from utility maximization theory. Specifically, based on Kim *et al.* (2002), we assume a translated non-linear, but additive, form for the specification of the utility function, which allows for multiple discreteness as well diminishing marginal returns *(i.e.*, satiation) as the consumption of any particular alternative increases. The econometric model formulated here, which we refer to as the Multiple Discrete-Continuous Extreme Value (MDCEV) model, is derived by introducing a multiplicative log-extreme value error term into the utility function. The result of such a specification is a surprisingly simple closed form expression for the discrete-continuous probability of not consuming certain alternatives and consuming given levels of the remaining alternatives. The paper proposes a mixing distribution to accommodate heteroscedasticity and covariance in unobserved characteristics affecting the demand for different alternatives, leading to the Mixed MDCEV (or MMDCEV) model structure. Estimation of the MDCEV model is straightforward and easily achieved using a maximum likelihood inference procedure, while estimation of the MMDCEV model is accomplished using a simulated maximum likelihood procedure.

In the current paper, we demonstrate an application of the model to individual time use in different types of discretionary activity pursuits on weekend days using data from the 2000 San Francisco Bay area. The analysis included several different kinds of variables, including household demographics, household location variables, individual demographics and employment characteristics, and day of week and season of year. Important findings from the analysis include the following:

1. Individuals in households with several other adults and in households with low incomes have a high propensity to participate in in-home recreation over the weekend days; on the other

hand, individuals in households with children, with medium household incomes, and with bicycles prefer out-of-home leisure activities relative to in-home leisure activities.

2. Household location variables do not significantly impact time use in leisure activities. However, this finding may be the result of using a coarse spatial level for computing location characteristics in the current study.
3. Older individuals, men, and African-Americans are less likely to participate in out-of-home recreation than younger individuals, women, and non-African-Americans, respectively. Young adults (16-17 years), Hispanic Americans, and individuals with a motor vehicle driving license are more likely to participate in out-of-home social pursuits than adults over the age of 17 years, non-Hispanic Americans, and individuals without a driving license, respectively. Young adults are not very likely to participate in out-of-home shopping activities over the weekend, while employed individuals and those who shop on the internet have a high likelihood of participating in out-of-home shopping activities. Men prefer in-home recreation more so than women.
4. Individuals prefer to pursue in-home leisure activities on Sundays relative to Saturdays. Individuals participate less in out-of-home recreation during the winter and fall seasons.

The model can be used to assess the impacts of changing demographics and employment patterns on time-use patterns, using the prediction process described in Section 5.4. Such an analysis is important at a time when demographics and employment characteristics are changing rapidly. The predicted changes in time use patterns can then be used within an activity-based modeling framework to examine the implied travel changes (see Bhat *et al.*, 2004).

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Appendix A: Computation of the Determinant of the Jacobian in Probability Expression

From Equation (11), the elements of the Jacobian are given by:

$$J_{ih} = \frac{\partial[V_1 - V_{i+1} + \varepsilon_1]}{\partial t^*_{h+1}};\ i, h = 1, 2, \ldots, M-1. \tag{A.1}$$

Using Equation (9) in the text, we can write:

$$V_i = d_i + a_i \ln(t^*_i + \gamma_i),\ i = 1, 2, \ldots, M\ , \tag{A.2}$$

where $d_i = \beta' x_i + \ln \alpha_i$ and $a_i = (\alpha_i - 1)$.

Then the element *ih* of the Jacobian is:

$$\begin{aligned} J_{ih} &= \frac{\partial[d_1 + a_1 \ln(t^*_1 + \gamma_1) - d_{i+1} - a_{i+1} \ln(t^*_{i+1} + \gamma_{i+1}) + \varepsilon_1]}{\partial t^*_{h+1}} \\ &= \frac{\partial\left[d_1 + a_1 \ln\left(T - \sum_{r=2}^{K} t^*_r + \gamma_1\right) - d_{i+1} - a_{i+1} \ln(t^*_{i+1} + \gamma_{i+1}) + \varepsilon_1\right]}{\partial t^*_{h+1}} \\ &= -\frac{a_1}{(t^*_1 + \gamma_1)} - z_{ih} \frac{a_{i+1}}{(t^*_{i+1} + \gamma_{i+1})};\ i, h = 1, 2, \ldots, M\text{-}1, \text{ where } z_{ih} = 1 \text{ if } i = h \text{ and } z_{ih} = 0 \text{ if } i \neq h. \end{aligned} \tag{A.3}$$

To compute the determinant of the Jacobian, consider a case where an individual participates in 4 activity types. Then the Jacobian matrix is:

$$J = \begin{bmatrix} \left(-\frac{a_1}{t^*_1 + \gamma_1} - \frac{a_2}{t^*_2 + \gamma_2}\right) & -\frac{a_1}{t^*_1 + \gamma_1} & -\frac{a_1}{t^*_1 + \gamma_1} \\ -\frac{a_1}{t^*_1 + \gamma_1} & \left(-\frac{a_1}{t^*_1 + \gamma_1} - \frac{a_3}{t^*_3 + \gamma_3}\right) & -\frac{a_1}{t^*_1 + \gamma_1} \\ -\frac{a_1}{t^*_1 + \gamma_1} & -\frac{a_1}{t^*_1 + \gamma_1} & \left(-\frac{a_1}{t^*_1 + \gamma_1} - \frac{a_4}{t^*_4 + \gamma_4}\right) \end{bmatrix} \tag{A.4}$$

It is straightforward to see that, because of the structure of the Jacobian, the determinant of the Jacobian is given by:

$$|J| = \left[\prod_{i=1}^{4} \left(-\frac{a_i}{t_i^* + \gamma_i} \right) \right] \left[\sum_{i=1}^{4} \left(\frac{t_i^* + \gamma_i}{a_i} \right) \right]. \tag{A.5}$$

In the general case when the individual participates in *M* alternatives, the determinant is given by the following expression after substituting $a_i = \alpha_i - 1$:

$$|J| = \left[\prod_{i=1}^{M} \left(\frac{1-\alpha_i}{t_i^* + \gamma_i} \right) \right] \left[\sum_{i=1}^{M} \left(\frac{t_i^* + \gamma_i}{1-\alpha_i} \right) \right] \tag{A.6}$$

Appendix B: Derivation of the Structure of the Multiple Discrete Continuous Extreme Value (MDCEV) Model

From Equation (14) of the text.

$$P\left(t_i^* > 0 \text{ and } t_s^* = 0;\ i = 2,\ 3,\ldots, M \text{ and } s = M+1,\ldots, K\right)$$

$$= \left[\prod_{i=1}^{M} c_i\right]\left[\sum_{i=1}^{\mathrm{M}} \frac{1}{c_i}\right]\left[\prod_{i=2}^{M} e^{-(V_1 - V_i)}\right]\left[\int_{\varepsilon_1=-\infty}^{+\infty} \left(e^{-\varepsilon_1}\right)^{M-1} \cdot e^{-\sum_{j=1}^{K}\left[e^{-(V_1 - V_j + \varepsilon_1)}\right]} \cdot e^{-\varepsilon_1} \cdot d\varepsilon_1\right] \qquad \text{(B.1)}$$

Now, consider the last term with the integral in the expression above, and let $k = e^{-\varepsilon_1}$. Then $dk = -e^{-\varepsilon_1} \cdot d\varepsilon_1$, and we can write:

$$\int_{\varepsilon_1=-\infty}^{+\infty} \left(e^{-\varepsilon_1}\right)^{M-1} \cdot e^{-\sum_{j=1}^{K}\left[e^{-(V_1 - V_j + \varepsilon_1)}\right]} \cdot e^{-\varepsilon_1} \cdot d\varepsilon_1 = -\int_{k=+\infty}^{0} k^{M-1} \cdot e^{-\left[k \cdot \sum_{j=1}^{K} e^{-(V_1 - V_j)}\right]} \cdot dk \qquad \text{(B.2)}$$

Next, let $b = -ak$, where $a = \left[\sum_{j=1}^{K} e^{-(V_1 - V_j)}\right]$. Then, $db = -adk$, and the integral in (B.2) can be rewritten as:

$$-\int_{b=-\infty}^{b=0} \left(-\frac{b}{a}\right)^{M-1} \cdot e^{b} \cdot \left(-\frac{1}{a}\right) \cdot db = -\left(-\frac{1}{a}\right)^{M} \cdot \int_{-\infty}^{0} b^{M-1} \cdot e^{b} \cdot db = (-1)^{M+1} \cdot \frac{1}{a^M} \int_{-\infty}^{0} b^{M-1} \cdot e^{b} \cdot db \qquad \text{(B.3)}$$

To evaluate the final integral, one can use the following recursive formula:

$$\int b^{M-1} \cdot e^{b} \cdot db = b^{M-1} \cdot e^{b} - (M-1)\int b^{M-2} \cdot e^{b} \cdot db \qquad \text{(B.4)}$$

This results in the following:

$$\int_{-\infty}^{0} b^{M-1}\cdot e^{b}\cdot db = \left[\begin{array}{l} b^{M-1}e^{b} - (M-1)b^{M-2}\cdot e^{b} + (M-1)(M-2)b^{M-3}\cdot e^{b} - \\ (M-1)(M-2)(M-3)b^{M-4}\cdot e^{b} + \ldots \\ (M-1)(M-2)(M-3)\ldots 1\cdot e^{b} \end{array}\right]_{b=-\infty}^{b=0}$$

$$= (-1)^{M-1}\cdot (M-1)! \qquad \text{(B.5)}$$

Putting the values of the integral back in (B.3), we get:

$$\int_{\varepsilon_1=-\infty}^{+\infty} \left(e^{-\varepsilon_1}\right)^{M-1}\cdot e^{-\sum_{j=1}^{J}\left[e^{-(V_1-V_j+\varepsilon_1)}\right]}\cdot e^{-\varepsilon_1}\cdot d\varepsilon_1 = (-1)^{2M}\cdot \frac{(M-1)!}{\left[\sum_{j=1}^{K} e^{-(V_1-V_j)}\right]^{M}} = \frac{(M-1)!}{\left[\sum_{j=1}^{K} e^{-(V_1-V_j)}\right]^{M}} \qquad \text{(B.6)}$$

Finally, we can re-write Equation (B.1) as:

$$P\left(t_i^* > 0 \text{ and } t_s^* = 0;\ i = 2,\ 3,\ldots, M \text{ and } s = M+1,\ldots, K\right) \qquad \text{(B.7)}$$

$$= \left[\prod_{i=1}^{M} c_i\right]\left[\sum_{i=1}^{\mathrm{M}} \frac{1}{c_i}\right]\left[\prod_{i=2}^{M} e^{-(V_1-V_i)}\right]\left[\frac{1}{\left[\sum_{j=1}^{K} e^{-(V_1-V_j)}\right]^{M}}\right](\mathrm{M}-1)!$$

$$= \left[\prod_{i=1}^{M} c_i\right]\left[\sum_{i=1}^{M} \frac{1}{c_i}\right]\left[\frac{\prod_{i=1}^{M} e^{V_i}}{\left(\sum_{j=1}^{K} e^{V_j}\right)^{M}}\right](M-1)! \qquad \text{(B.8)}$$

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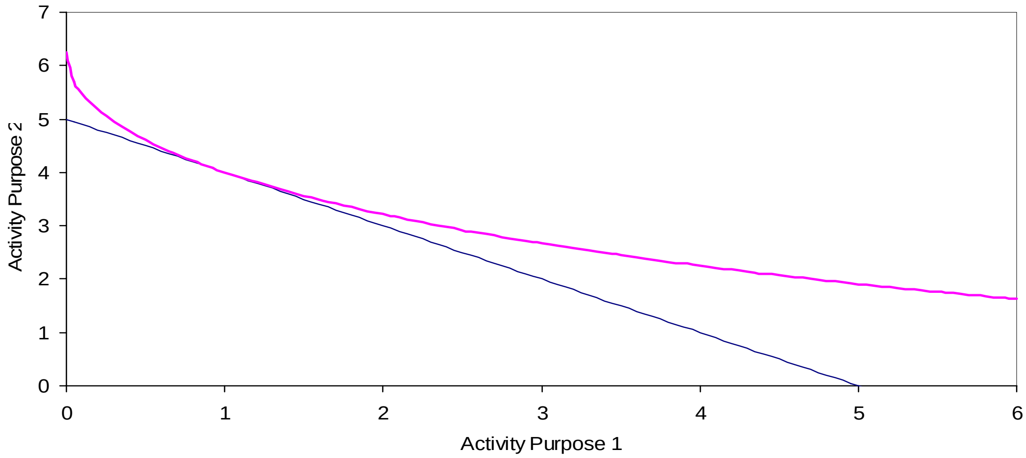

Figure 1a: $\gamma_1 = \gamma_2 = 0;\ \ \alpha_1 = \alpha_2 = 0.5;$ and $\psi(x_2) = 2\psi(x_1)$

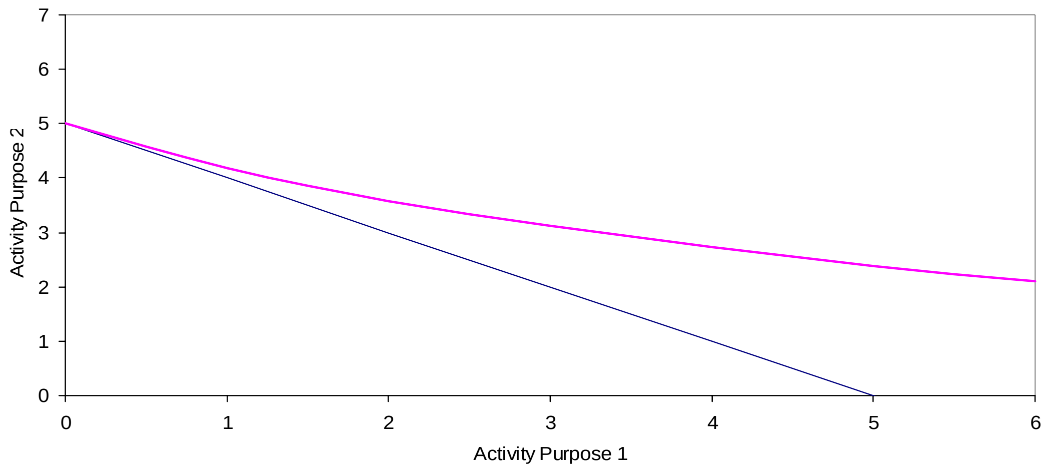

Figure 1b: $\gamma_1 = 1.25, \gamma_2 = 0;\ \ \alpha_1 = \alpha_2 = 0.5;$ and $\psi(x_2) = 2\psi(x_1)$

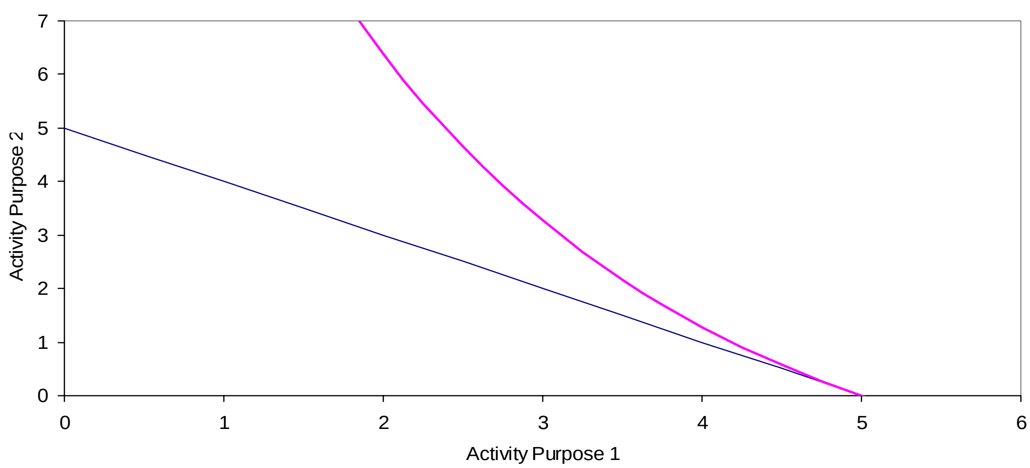

Figure 1c: $\gamma_1 = 0, \gamma_2 = 1.25;\ \ \alpha_1 = \alpha_2 = 0.5;$ and $\psi(x_2) = 0.5\psi(x_1)$

Figure 1: Indifference Curves and Optimal Consumption Points

Table 1. Descriptive Statistics of Activity Type Participation and Duration Over the Weekend Day

Activity Type	Total number (%) of individuals participating[a]	Mean duration of participation (mins)	Number of individuals (% of total number participating) who participate….[b]	
			Only in activity type	In the activity type and other activity types
In-home social (IHS)	118 (6.2)	197	33 (28%)	85 (72%)
In-home recreational (IHR)	738 (38.5)	275	355 (48%)	383 (52%)
Out-of-home social (OHS)	496 (25.9)	181	180 (36%)	316 (64%)
Out-of-home recreational (OHR)	632 (33.0)	188	262 (41%)	370 (59%)
Out-of-home shopping (OHSh)	841 (43.9)	81	339 (40%)	502 (60%)

[a] Percentages across rows in the column do not sum to 100% because some individuals participate in more than one activity type.
[b] Percentages sum to 100% for each row across the two columns, since the percentages are with respect to the total number of individuals participating in each activity type (the second column in the table).

Table 2. Effect of Exogenous Variables on Baseline Preference to Participate in Each Activity Type

Explanatory Variables	Parameter	t-statistic
Household sociodemographics		
Number of adults		
In-home recreation	0.2738	2.51
Presence of very young children (0 to 4 years of age)		
Out-of-home social, out-of-home recreation, and out-of-home shopping	0.5141	1.77
Presence of young children (5-15 years of age)		
Out-of-home recreation	0.9801	2.93
Out-of-home shopping	0.7069	2.60
Number of bicycles		
Out-of-home recreation	0.1142	1.45
Low annual household income (<35,000 dollars)		
In-home recreation	0.9323	3.64
Medium annual household income (35,000-90,000 dollars)		
Out-of-home recreation	0.5596	2.21
Household location variables		
Central business district		
Out-of-home recreation	0.6771	1.03
Diversity in land use-mix		
Out-of-home shopping	0.4846	0.92
Individual demographics and employment characteristics		
Age		
Out-of-home recreation	-2.1541	-2.65
Age 16 or 17 years		
Out-of-home social	1.1916	1.75
Out-of-home shopping	-1.7242	-2.54
Age 18-29 years		
Out-of-home social	0.5489	1.617
Age >65 years		
In-home recreation	0.3701	1.213
Out-of-home shopping	-0.7306	-1.913
Driver's license		
Out-of-home social, out-of-home recreation, and out-of-home shopping	1.4007	3.22
Employed		
Out-of-home shopping	0.3359	1.33
Male		
In-home recreation	0.5915	3.49
Shopping on the internet		
Out-of-home shopping	0.7958	1.81
African-American		
Out-of-home recreation	-2.3893	-1.92
Hispanic-American		
Out-of-home social	1.2894	2.40
Asian-American		
Out-of-home social	-0.7621	-2.26
Day of the week and seasonal effects		
Sunday		
In-home recreation and in-home social	1.0757	5.14
Winter		
Out-of-home recreation	-0.9564	-2.23
Fall		
Out-of-home recreation	-0.5128	-1.96
Baseline preference constants		
In-home recreation	1.9300	3.08
Out-of-home social	0.8161	1.07
Out-of-home recreation	2.2227	2.66
Out-of-home shopping	3.0191	3.75

Table 3. Satiation Parameters

Activity Type	Parameter	t-statistic[1]
In-home social (IHS)	0.8794	3.09
In-home recreational (IHR)	0.9556	3.47
Out-of-home social (OHS)	0.7660	6.34
Out-of-home recreational (OHR)	0.7822	6.39
Out-of-home shopping (OHSh)		
Women	0.4586	7.60
Men	0.4028	7.50

[1] The t-statistic is computed for the null hypothesis that the satiation parameter is equal to 1. Equivalently, the t-statistic is for the test that there are no satiation effects or that the utility structure is linear.

Table 4. Variance-Covariance Matrix

Activity Type	Activity Type				
	In-home social	In-home recreational	Out-of-home social	Out-of-home recreational	Out-of-home shopping
In-home social (IHS)	7.87 (2.98)	3.04 (2.50)	0	0	0
In-home recreational (IHR)		4.74 (3.85)	0	0	0
Out-of-home social (OHS)			11.88 (4.26)	0	0.64 (1.21)
Out-of-home recreational (OHR)				11.24 (4.11)	0
Out-of-home shopping (OHSh)					11.88 (4.26)

Table 5. Predicted and Actual Discretionary Time Allocations for the First Five Individuals in the Sample

Individual Number	Predicted (Actual) time use in…				
	In-home social activity	In-home recreational activity	Out-of-home social activity	Out-of-home recreational activity	Out-of-home shopping activity
1	0 (0)	116 (120)	0 (0)	0 (0)	4 (0)
2	0 (0)	238 (240)	0 (0)	0 (0)	2 (0)
3	0 (0)	7 (0)	0 (0)	6 (0)	2 (15)
4	0 (0)	84 (0)	9 (0)	0 (0)	27 (120)
5	0 (0)	330 (240)	0 (0)	0 (90)	0 (0)