A Spatial Rank-Ordered Probit Model with an Application to Travel Mode Choice

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ABSTRACT
Ranking data provide important additional information related to valuation because of the implied preference sequence among all alternatives, rather than just the top choice preference. This additional information from a preference ranking can be exploited to achieve a certain desired precision in choice model estimation with a much smaller sample size, making ranked data surveys much more cost-effective than first-choice surveys. In this paper, we propose a spatial rank-ordered probit (SROP) model that accommodates both spatial lag effects as well as spatial drift effects. To our knowledge, this is the first such formulation and application of an SROP model in the econometric and transportation literature. An application of the proposed model is demonstrated in a travel mode choice ranking experiment among seven alternatives, including autonomous vehicle (AV) private ride-hailing and AV pooled ride-hailing.

Keywords: Ranked data analysis, probit models, spatial econometrics, travel mode choice, autonomous vehicles.

1. INTRODUCTION

In consumer surveys involving choice situations, the first-choice ("the most preferred alternative") approach is the most common method to elicit preference information from respondents. However, more information regarding consumer preferences can be extracted if the individuals are asked to rank alternatives based on a preference ordering, instead of being asked to pick only the most preferred one. In particular, the additional information from a preference ranking can be exploited to achieve a certain desired precision in choice model estimation with a much smaller sample size, making ranked data surveys more cost-effective than first-choice surveys.

In spite of the promising theoretical efficiency of the rank-ordered model over first-choice models, the rank-based method continues to remain in the backseat as a tool for preference elicitation, especially when compared to the substantial methodological advancement and empirical implementation over the past three decades in the single-choice framework. This predilection toward first-choice models may be primarily attributed to the conception that ranked data are not very reliable because of the cognitive demands placed on respondents in ranking several alternatives. In particular, several studies in the past (for example, Chapman and Staelin, 1982, Foster and Mourato, 2002, and Hausman and Ruud, 1987) have raised concerns about the reliability of the rankings provided among the less-preferred alternatives, citing the inability of individuals to clearly distinguish between the utilities of alternatives lower down in the preference ordering. These studies illustrate that, when using the rank-ordered logit (ROL) model proposed by Beggs et al. (1981), which is the most commonly used structure for analyzing rank-ordered data, the coefficient estimates attenuate toward zero as one goes down the rank sequencing hierarchy (the decision among lower ranked alternatives is also referred to as the decision at higher "rank depth" or "explosion depth"). This attenuation has been ascribed to an increasing variance of the kernel extreme-value error term at higher rank depth, and has been taken as evidence that individuals are more precisely able to form their utilities for alternatives and translate those utilities into an equivalent choice at higher levels of rankings than at lower levels of ranking. Or, equivalently, that individual responses at lower ranking levels are not reliable, calling into question the veracity of using ranking data (relative to traditional first-choice data) as a means to collect individual responses (see, for example, Caparros et al., 2008 and Scarpa et al., 2011).

In contrast to the generally prevailing notion that ranking data are inherently unreliable, Yan and Yoo (2014) used simulations and analytical computations to show that the attenuation of parameter estimates at higher rank depth is specific to the ROL model, which explodes the rank-ordering decision framework into sequential pseudo-choice decisions. In doing so, the ROL model completely ignores the fact that individuals become decreasingly able, in a systematic sense, to discriminate among the alternatives at higher rank depth, incorrectly interpreting this systematic discrimination difficulty as a higher error variance (which then leads to the attenuation of coefficients toward zero at higher rank depths). But this attenuation-associated limitation is specific to the use of the type-1 extreme value error term for the utilities in the ROL model, and does not, in general, extend to models that use any other (even independent and identically distributed or IID) error distribution. In this context, as Nair et al. (2019) point out, it is by far much more appropriate to use normal error terms (even if only IID) for the utilities of alternatives in the analysis of ranked data, resulting in the rank-ordered probit (ROP) model.[1] Besides, the ROL maintains independence across the utilities of the ranked alternatives, which can be relaxed in the ROP model.

Even as the field is moving more toward embracing ranking data through the use of the ROP model, there continues to be a rapidly increasing interest in recognizing and incorporating spatial dependency among decision-makers. Such spatial inter-relationships engender a stochastic dependence in choice across individuals, and is inspired by Tobler's (1970) first law of geography that: "Everything is related to everything else, but near things are more related than distant things." (see also LeSage and Pace, 2009, Anselin, 2010, Arbia, 2014, Moscone and Tosetti, 2014, Bhat, 2015, Franzese et al., 2016, and Elhorst et al., 2016; Billé and Arbia, 2019 provide a good recent review). In this spatial econometrics field, the workhorse spatial formulation continues to be the spatial lag structure that allows spatial spillover effects due to both observed and unobserved factors. However, much of this focus on spatial/social dependency has been confined to traditional first-choice discrete models. In fact, we are unaware of a spatial/social model in the context of rank-ordered models. Also to be noted is that, even within the context of spatial first-choice discrete models, the use of normal kernel error terms (leading to a binary/multinomial probit framework) in the utility function is ubiquitous, because of the conceptual ease in allowing spatial dependence through a single autoregressive parameter and generating global spatial dependency effects through the use of the very desirable properties of a multivariate normal distribution.

In this paper, we propose a methodological framework for a spatial ROP model and illustrate its effectiveness in empirical implementation. The estimation of the proposed model is undertaken using the maximum approximate composite marginal likelihood (MACML) inference approach proposed by Bhat (2011, 2014). Relative to several other full-information methods in the literature to accommodate spatial dependency in the single-choice context, including the recursive importance sampling (RIS) estimator (see Beron and Vijverberg, 2004) and the Bayesian Markov Chain Monte Carlo (MCMC)-based estimator (see LeSage and Pace, 2009), the MACML approach involves a much lower dimensionality of integration (of the order of twice the number of alternatives minus one rather than the product of the number of observational units times the number of alternatives minus one). Also, unlike Generalized Method-of-Moment (GMM) type

[1] If the error terms of the utility of alternatives do not follow a type 1 extreme value distribution (even still assuming independently and identically distributed (IID) error terms across alternatives), the probability of a ranking pattern can no longer be written as the product (across rank depths) of the probabilities of choosing the most preferred alternative among the unranked alternatives at each rank depth. That is, for any generic distribution (even maintaining IID) of the error terms, except for the type 1 extreme value (as maintained in the ROL model), the probability of selecting an alternative at a rank level must be conditioned on the ordering of alternatives that have already been ranked. This immediately diminishes the coefficient instability pattern that is manifested across rank depths in the ROL. More specifically speaking, the difference between the ROP and ROL models for ranking data is not the same as the difference between a multinomial probit model and a multinomial logit model in the context of first-choice data analysis, but substantially more dramatic; conceptually speaking, the ROL model is an "impossible" structure for ranking data analysis, based on Luce and Suppes's (1965) impossibility theorem (Theorem 51, page 357). In other words, this is another way to state that the ROL is an "impossible" structure for ranking data and should be avoided (see also Schechter, 2010).

estimators (see Klier and McMillen, 2008, Carrion-Flores et al., 2018) that are applicable to low intensities of spatial dependency (see Franzese et al., 2016), the MACML approach is applicable for low as well as high levels of spatial dependency. The MACML approach is also easily able to accommodate unobserved heterogeneity in the sensitivity to exogenous variables (thereby also allowing spatial drift effects, in which unobserved factors affecting the sensitivity to a variable themselves get spatially correlated).[2] Finally, the MACML approach benefits from the use of recent analytic approximation methods (see Bhat, 2018) that allow the computation of the multivariate normal cumulative distribution (MVNCD) accurately and substantially faster compared to traditional simulation-based methods. As we demonstrate in the current paper, all of these advantages of the MACML inference approach in the context of single-choice models can be harnessed to accommodate spatial dependence within ranked data too.

To summarize, the focus of the current paper is to develop a methodological framework and formulation for the spatial rank-ordered probit (SROP) model and propose a MACML inference approach to estimate parameters. The rest of this paper is structured as follows. The next section presents the model framework and estimation procedure. Section 3 presents a simulation experiment to evaluate the performance of our proposed estimator. Section 4 presents the results from the application of the proposed model to a "future mobility" travel mode choice context. The final section summarizes important findings from the study and concludes the paper.

2. METHODOLOGY

2.1 Model Formulation

Consider a consumer q (q = 1, 2,…,Q) who ascribes a utility U_{qi} to alternative i (i=1, 2,…,I). For ease in presentation, we assume that all alternatives are available for ranking for each individual, and that individuals provide a full ranking of the alternatives. These assumptions are innocuous and help in presentation.[3] In our proposed methodology we employ a spatial lag type dependence.[4]

[2] See Bhat (2015) for a detailed treatment of spatial drift effects. As explained there, such drift effects are not only due to unobserved location-specific unobservables that may be correlated over space, but also motivated from the perspective of self-selection in the social interactions literature (see Moffitt, 2001 and Hartman et al., 2008). If not appropriately recognized and accommodated, the resulting self-selection can manifest itself incorrectly as social/spatial interactions. For example, in a travel mode choice context, households and individuals who intrinsically prefer walking or bicycling may be drawn toward neo-urbanist built environment residential locations with good land-use mixing and high built-up density. The net result is that individuals and households with similar mode use propensities and sensitivity to observed built environment attributes may be in close proximity, but this is not a result of social/spatial interactions after locating in a neighborhood or the causal influence of built environment attributes; rather, it may be attributable to self-selection of individuals regarding their residential location choices based on their mode use preferences (see Guo and Bhat, 2007 for a detailed discussion of such residential self-selection effects).

[3] The case of tied rankings provided by one of more consumers can be easily extended within this framework. This entails a simple modification to the contrast matrix as discussed in Nair et al. (2018).

[4] Interestingly, many spatial formulations in the literature have considered spatial interactions to be a "nuisance" issue, and have employed a spatial error structure, which cannot accommodate spatial spillover effects (that is, a change in a variable affecting the dependent outcome of one individual will not affect the dependent outcome of other individuals in proximal space). As indicated by Beck et al. (2006), McMillen (2010), and Bhat (2015), the spatial error structure necessitates the rather illogical position that space matters in the error process but not in the effects of exogenous variables. On the other hand, the spatial lag specification, in reduced form, allows symmetry in spatial dependence

In a typical spatial lag formulation, we write the spatially-dependent utility as follows:

$$U_{qi} = \delta\left(\sum_{q'} w_{qq'}\, U_{q'i}\right) + \boldsymbol{\beta}'_q \mathbf{x}_{qi} + \varepsilon_{qi};\ \boldsymbol{\beta}_q = \mathbf{b} + \tilde{\boldsymbol{\beta}}_q,\ \tilde{\boldsymbol{\beta}}_q \sim MVN_K(\mathbf{0}, \boldsymbol{\Omega}), \tag{1}$$

where $w_{qq'}$ is the usual distance-based spatial weight corresponding to units q and q' (with $w_{qq} = 0$ and $\sum_{q'} w_{qq'} = 1$) for each (and all) q, δ $(-1 < \delta < 1)$ is the spatial lag autoregressive parameter, $\mathbf{x}_{qi}$ is a $(K \times 1)$-column vector of exogenous attributes (including a constant for each alternative, except one of the alternatives), and $\boldsymbol{\beta}_q$ is an individual-specific $(K \times 1)$-column vector of corresponding coefficients that varies across individuals based on unobserved individual attributes. As indicated in Equation (1), $\boldsymbol{\beta}_q$ is assumed to be a realization from a K-variate multivariate normal distribution (denoted by $MVN_K(.,.)$ in Equation (1)), with a mean vector $\mathbf{b}$ and covariance matrix $\boldsymbol{\Omega} = \mathbf{LL}'$. We also assume that ε_{qi} is independent and identically normally distributed across q, but allow a general covariance structure across alternatives for individual q. Specifically, let $\boldsymbol{\varepsilon}_q = (\varepsilon_{q1}, \varepsilon_{q2}, ..., \varepsilon_{qI})'$ ($I \times 1$ vector). Then, we assume $\boldsymbol{\varepsilon}_q \sim MVN_I(0, \boldsymbol{\Lambda})$. Appropriate (and innocuous) scale and level normalization must be imposed on $\boldsymbol{\Lambda}$ for identifiability. This is because the utility of all the alternatives can be multiplied by a positive constant without changing the rank-ordering of the utilities (relating to the need for scale normalization); similarly, a constant can be added to all the utilities which will once again not alter the rank-ordering of the utilities (relating to the need for level normalization). Specifically, only utility differentials matter in ranking choice models, just as in traditional discrete choice models (see Alvo and Yu, 2014; page 171). Taking the utility differentials with respect to the first alternative, only the elements of the covariance matrix $\boldsymbol{\Lambda}_1$ of $\tilde{\varepsilon}_{qi1} = \varepsilon_{qi} - \varepsilon_{q1}$ $(i \neq 1)$ are estimable. However, the inference approach proposed here, like the traditional GHK simulator, takes the difference in utilities in a specific way that is a function of the observed ranking (as discussed later). Thus, if individual q is observed to choose ranking r_q, the covariance matrix $\boldsymbol{\Lambda}_{r_q}$ is desired for the individual. But, even though different differenced covariance matrices are used for different individuals, they must originate in the same matrix $\boldsymbol{\Lambda}$. To achieve this consistency, $\boldsymbol{\Lambda}$ is constructed from $\boldsymbol{\Lambda}_1$ by adding an additional row on top and an additional column to the left. All elements of this additional row and additional column are filled with values of zeros. An additional normalization needs to be imposed on $\boldsymbol{\Lambda}$ because the scale is also not identified. For this, we normalize the element of $\boldsymbol{\Lambda}$ in the second row and second column to the value of one. Note that these normalizations are innocuous and are needed for identification. The $\boldsymbol{\Lambda}$ matrix so constructed is fully general. Also, as in MNP models, identification is tenuous when only individual-specific covariates (that is, covariates such as age, gender, and household income that do not vary across alternatives) are used (see Keane, 1992 and Munkin and Trivedi, 2008). In particular, exclusion

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through both spatial spillover effects as well as spatial error correlation effects.

restrictions are needed in the form of at least one individual characteristic being excluded from each alternative's utility in addition to being excluded from a base alternative (but appearing in some other utilities). But these exclusion restrictions are not needed for covariates whose values vary across alternatives (in a travel mode context, such covariates may include mode-specific in-vehicle travel times, travel costs, and waiting times).

The model above may be written in a more compact form by specifying the $w_{qq'}$ terms as the elements of an exogenously defined distance-based spatial weight matrix $\mathbf{W}$ that is row-normalized by construction (note that $\sum_{q'} w_{qq'} = 1$), and defining the following vectors and matrices:

$\mathbf{U}_q = (U_{q1}, U_{q2}, ..., U_{qI})'$ ($I \times 1$ vector),

$\mathbf{U} = (\mathbf{U}_1', \mathbf{U}_2', ..., \mathbf{U}_Q')'$, $\boldsymbol{\varepsilon} = (\boldsymbol{\varepsilon}_1', \boldsymbol{\varepsilon}_2', ..., \boldsymbol{\varepsilon}_Q')'$ ($QI \times 1$ vectors),

$\mathbf{x}_q = (\mathbf{x}_{q1}, \mathbf{x}_{q2}, ..., \mathbf{x}_{qI})'$ ($I \times K$ matrix), $\mathbf{x} = (\mathbf{x}_1', \mathbf{x}_2', ..., \mathbf{x}_Q')'$ ($QI \times K$ matrix), $\tilde{\boldsymbol{\beta}} = \left(\tilde{\boldsymbol{\beta}}_1', \tilde{\boldsymbol{\beta}}_2', ..., \tilde{\boldsymbol{\beta}}_Q'\right)'$ ($QK \times 1$ vector), $\tilde{\boldsymbol{\Omega}} = IDEN_Q \otimes \boldsymbol{\Omega}$ ($QK \times QK$ matrix), $\tilde{\boldsymbol{\Lambda}} = IDEN_Q \otimes \boldsymbol{\Lambda}$ ($QK \times QK$ matrix)

$$\tilde{\mathbf{x}} = \begin{bmatrix} \mathbf{x}_1 & 0 & 0 & 0. \cdots & 0 \\ 0 & \mathbf{x}_2 & 0 & 0 \cdots & 0 \\ 0 & 0 & \mathbf{x}_3 & 0 \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \dots & \vdots \\ 0 & 0 & 0 & 0 \dots & \mathbf{x}_Q \end{bmatrix}_{(QI \times QK)}$$

$\mathbf{S} = \left[\mathbf{IDEN}_{QI} - \left(\delta \mathbf{W} \otimes \mathbf{IDEN}_I\right)\right]^{-1}$ ($QI \times QI$ matrix). $\mathbf{IDEN}_H$ refers to the identity matrix of H dimensions. Then, we can write Equation (1) in matrix notation as:

$$\mathbf{U} = \mathbf{S}\left[\mathbf{xb} + \tilde{\mathbf{x}}\tilde{\boldsymbol{\beta}} + \boldsymbol{\varepsilon}\right], \text{ so that } \mathbf{U} \sim MVN_{QI}(\mathbf{B}, \boldsymbol{\Xi}), \text{ with } \mathbf{B} = \mathbf{Sxb} \text{ and } \boldsymbol{\Xi} = \mathbf{S}\left(\tilde{\mathbf{x}}\tilde{\boldsymbol{\Omega}}\tilde{\mathbf{x}}' + \tilde{\boldsymbol{\Lambda}}\right)\mathbf{S}'. \quad (2)$$

Note that the above model includes the usual spatial lag effect through the component $\mathbf{S}[\mathbf{xb} + \boldsymbol{\varepsilon}]$, but also the spatial drift effect as captured by the component $\mathbf{S}\tilde{\mathbf{x}}\tilde{\boldsymbol{\beta}}$. That is, allowing for unobserved heterogeneity in the coefficients on one or more independent variables immediately also leads to a spatially-correlated nature of the sensitivity to the corresponding independent variables, as discussed earlier.

2.2 Model Formulation

To progress to estimation, define a contrast matrix for individual q based on the observed ranking r_q of alternatives for the individual. Specifically, let the first ranked alternative for individual q be r_q^1, the second r_q^2, and so on until the last-ranked alternative r_q^I. Then, the following $(I-1)$ inequalities should hold: $U_{qr_q^2} - U_{qr_q^1} < 0, U_{qr_q^3} - U_{qr_q^2} < 0, \dots U_{qr_q^I} - U_{qr_q^{I-1}} < 0.$ To write these inequalities in vector notation, define a contrast matrix $\mathbf{M}_q$ with (I-1) rows and I columns, each row representing one inequality and each column representing an alternative. Fill all the elements of the matrix with zeros. Then, in the first row, place an entry of '1' in the column corresponding to the second-ranked alternative, and a '-1' in the column corresponding to the first-ranked alternative (corresponding to the first inequality above). In the second row, place an entry of '1' in the column corresponding to the third-ranked alternative, and a '-1' in the column corresponding to the second-ranked alternative (corresponding to the second inequality above), and so on until placing an entry of '-1' in the column corresponding to the penultimate-ranked alternative, and a '1' in the column corresponding to the last-ranked alternative (corresponding to the last inequality above). Next develop a $Q(I-1)\times QI$ block-diagonal matrix $\mathbf{M}$, with each $(I-1)\times I$ block containing the matrix $\mathbf{M}_q$ for a specific individual. Thus, in the case of I=5 and Q=2, and if the first individual's ranking (from top choice to last choice) is 4>1>2>3>5, and the second individual's ranking is 2>5>3>4>1, the $\mathbf{M}$ matrix is as below:

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix}$$

Then, the likelihood of the observed sample (*i.e.*, individual 1 having the ranking r_1, individual 2 having the ranking r_2,…, individual Q having the ranking r_Q) may be written succinctly as $\text{Prob}[\mathbf{MU} < \mathbf{0}_{QI}]$, where $\mathbf{0}_{QI}$ is a column vector of zeros of length QI. The parameter vector to be estimated is $\boldsymbol{\theta} = (\mathbf{b}', \overline{\boldsymbol{\Omega}}', \overline{\boldsymbol{\Lambda}}', \delta)'$, where $\overline{\boldsymbol{\Omega}}$ is a column vector obtained by vertically stacking the upper triangle elements of the matrix $\boldsymbol{\Omega}$, and $\overline{\boldsymbol{\Lambda}}$ is another column vector obtained by vertically stacking the non-zero and non-normalized elements of the matrix $\boldsymbol{\Lambda}$. Defining $\mathbf{C} = \mathbf{MB}$ and $\boldsymbol{\Psi} = \mathbf{M\Xi M}'$, the likelihood function is:

$$L(\boldsymbol{\theta}) = \text{Prob}(r_1, r_2, ..., r_Q) = \Phi_{(Q-1)I}\left[\boldsymbol{\omega}_{\Psi}^{-1}(-\mathbf{C}), \boldsymbol{\Psi}^*\right], \tag{3}$$

where $\boldsymbol{\Psi}^* = \boldsymbol{\omega}_{\Psi}^{-1}\boldsymbol{\Psi}\boldsymbol{\omega}_{\Psi}^{-1}$, $\boldsymbol{\omega}_{\boldsymbol{\Psi}}$ is the diagonal matrix of standard deviations of $\boldsymbol{\Psi}$, and $\Phi_{(Q-1)I}[.,.]$ is the standard multivariate cumulative normal distribution function of dimension $Q(I-1)$.

The likelihood function above entails the evaluation of a $Q(I-1)$-dimensional integral, which is literally infeasible to compute accurately using traditional frequentist and Bayesian simulation techniques. In this paper, we use Bhat's (2011) maximum approximate composite marginal likelihood (MACML) approach, enhanced with new procedures to accurately evaluate the MVNCD function based on Bhat (2018).

2.3 The MACML Procedure

The MACML procedure (see Bhat, 2011 for details) constitutes an important and pivotal contribution in the field that combines the composite marginal likelihood (CML) approach with the use of accurate analytic approximations for the MVNCD function. The CML inference approach is based on maximizing a surrogate likelihood function that compounds much easier-to-compute, lower-dimensional, marginal likelihoods (see Varin et al., 2011 for an extensive review of CML methods; Lindsay et al., 2011 and Bhat, 2014 are also useful references). The CML approach, in a pairwise context, has been used earlier in spatial contexts by Bhat and colleagues (see Ferdous and Bhat, 2013, Bhat et al., 2017), and is particularly suited for spatial analysis because it is based on compounding (multiplying) pairwise probabilities of observation q choosing the ranked slate of alternatives r_q and observation q' choosing the ranked slate $r_{q'}$. The CML estimator (in this instance, the pairwise CML estimator) is then the one that maximizes the compounded probability of all pairwise events. The properties of the CML estimator may be derived using the theory of estimating equations (see Cox and Reid, 2004, Yi et al., 2011, Bhat, 2014). Specifically, under usual regularity assumptions (Xu and Reid, 2011), the CML estimator is consistent and asymptotically normal distributed (this is because of the unbiasedness of the CML score function, which is a linear combination of proper score functions associated with the marginal event probabilities forming the composite likelihood; for a formal proof, see Bhat, 2014).

The pairwise CML function for the proposed model may be written as follows:

$$\begin{aligned} L_{CML}(\boldsymbol{\theta}) &= \prod_{q=1}^{Q-1} \prod_{q'=q+1}^{Q} L_{qq'} = \prod_{q=1}^{Q-1} \prod_{q'=q+1}^{Q} \text{Prob}(r_q, r_{q'}) \\ &= \prod_{q=1}^{Q-1} \prod_{q'=q+1}^{Q} \Phi_{2\times(I-1)+1}(-\vec{\mathbf{C}}_{qq'}, \boldsymbol{\Theta}^*_{qq'}), \end{aligned} \tag{4}$$

where $\vec{\mathbf{C}}_{qq'} = \boldsymbol{\omega}^{-1}_{\boldsymbol{\Theta}_{qq'}} \left(\boldsymbol{\Delta}_{qq'}\mathbf{C}\right)$, $\boldsymbol{\Theta}^*_{qq'} = \boldsymbol{\omega}^{-1}_{\boldsymbol{\Theta}_{qq'}} \boldsymbol{\Theta}_{qq'} \boldsymbol{\omega}^{-1}_{\boldsymbol{\Theta}_{qq'}}, \boldsymbol{\Theta}_{qq'} = \boldsymbol{\Delta}_{qq'} \boldsymbol{\Psi} \boldsymbol{\Delta}'_{qq'}$, $\boldsymbol{\omega}_{\boldsymbol{\Theta}_{qq}}$ is the diagonal matrix of standard deviations of $\boldsymbol{\Theta}_{qq'}$, and $\boldsymbol{\Delta}_{qq'}$ is a $[2\times(I-1)]\times[Q(I-1)]$-selection matrix constructed as follows: (1) First fill the entire matrix with values of zero, (2) Place an identity matrix of size $(I-1)$ occupying the first through $(I-1)^{th}$ rows and the $\left[(q-1)\times(I-1)+1\right]^{th}$ through $\left[q\times(I-1)+1\right]^{th}$ columns, and (4) place another identity matrix of size $(I-1)$ occupying the last $(I-1)$ rows and the $\left[(q'-1)\times(I-1)+1\right]^{th}$ through $\left[q'\times(I-1)\right]^{th}$ columns.

The CML function in the equation above needs the evaluation of a $2\times(I-1)+1$-dimensional MVNCD function. For that, we use Bhat's (2018) matrix-based implementations to analytically approximate the MVNCD function, leading up to the MACML estimator. In this paper, we use Bhat's Two-Variate Bivariate Screening (TVBS) method for evaluating the

MVNCD function. The pairwise marginal likelihood function of Equation (4) comprises the evaluation of $Q(Q-1)/2$ MVNCD evaluations if the weight matrix is based on an inter-observation distance metric with no distance bound. This can itself become quite time consuming for large Q. However, previous studies (see, for example, Varin and Vidoni, 2008) have shown that spatial dependency drops quickly with inter-observation distance. In this situation, the pairs formed from the closest observations provide much more information than pairs that are very far away. In fact, as demonstrated by Bhat et al. (2010) and Varin and Czado (2008) in different empirical contexts, retaining all pairs may reduce estimator efficiency. The "optimal" distance (say $\tilde{d}_{thresh}$) for including pairings can be based on minimizing the trace of the asymptotic covariance matrix $V_{CML}(\hat{\boldsymbol{\theta}})$ (see later for the formula for $V_{CML}(\hat{\boldsymbol{\theta}})$). Once $\tilde{d}_{thresh}$ is determined, construct a $Q\times Q$ matrix $\tilde{\mathbf{R}}$ with its q^{th} column filled with a $Q\times 1$ vector of zeros and ones as follows: if the observational unit q' is not within the threshold distance $\tilde{d}_{thresh}$ of unit q, the q'^{th} row has a value of zero; otherwise, the q'^{th} row has a value of one. By construction, the q^{th} row of the q^{th} column has a value of one. Let $[\tilde{\mathbf{R}}]_{qq'}$ be the qq^{th} element of the matrix $\tilde{\mathbf{R}}$, and let $\tilde{W}=\sum_{q=1}^{Q-1}\sum_{q'=q+1}^{Q}[\tilde{\mathbf{R}}]_{qq'}$.
Define a set G_q of all individuals (observation units) that have a value of '1' in the vector $[\tilde{\mathbf{R}}]_q$, where $[\tilde{\mathbf{R}}]_q$ is the q^{th} column of the vector $\tilde{\mathbf{R}}$. Then, the CML function may be reduced to the following:

$$L_{CML}(\boldsymbol{\theta})=\prod_{q=1}^{Q-1}\prod_{\substack{q'=q+1\\ q'\in G_q}}^{Q} L_{qq'}\,. \tag{5}$$

Under usual regularity assumptions (Molenberghs and Verbeke, 2005, Xu and Reid, 2011, Bhat, 2014), the CML estimator of $\boldsymbol{\theta}$ is consistent and asymptotically normal distributed with asymptotic mean $\boldsymbol{\theta}$ and asymptotic covariance matrix given by the inverse of the Godambe's (1960) sandwich information matrix as $V_{CML}(\hat{\boldsymbol{\theta}})=\dfrac{\left[\hat{\boldsymbol{H}}^{-1}\right]\left[\hat{\boldsymbol{J}}\right]\left[\hat{\boldsymbol{H}}^{-1}\right]'}{\tilde{W}}$, where

$$\hat{\boldsymbol{H}}=-\frac{1}{\tilde{W}}\left[\sum_{q=1}^{Q-1}\sum_{\substack{q'=q+1\\ q''\in G_q}}^{Q}\frac{\partial^2 \log L_{qq'}}{\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}'}\right]_{\hat{\boldsymbol{\theta}}_{CML}}, \text{ or alternatively,}$$

$$\hat{\boldsymbol{H}}=\frac{1}{\tilde{W}}\sum_{q=1}^{Q-1}\sum_{\substack{q'=q+1\\ q'\in G_q}}^{Q}\left(\left[\frac{\partial \log L_{qq'}}{\partial\boldsymbol{\theta}}\right]\left[\frac{\partial \log L_{qq'}}{\partial\boldsymbol{\theta}'}\right]\right)_{\hat{\boldsymbol{\theta}}_{CML}} \tag{6}$$

The "vegetable" matrix $\boldsymbol{J}$ may be computed using a windows resampling procedure (see Heagerty and Lumley, 2000). To do so, overlay the spatial region under consideration with a square grid providing a total of $\tilde{Q}$ internal and external nodes. Then, select the observational unit closest to each of the $\widetilde{Q}$ grid nodes to obtain $\widetilde{Q}$ observational units from the original Q observational units ($\tilde{q}=1,\ 2,\ 3,\ldots,\widetilde{Q}$). Let $\widetilde{\boldsymbol{R}}_{\tilde{q}}$ be the $Q\times 1$ matrix representing the $\tilde{q}^{th}$ column vector of the matrix $\widetilde{\boldsymbol{R}}$, let $\widetilde{\mathbf{C}}_{\tilde{q}}$ be the set of all individuals (observation units) that have a value of '1' in the vector $\widetilde{\boldsymbol{R}}_{\tilde{q}}$, and let $\boldsymbol{y}_{\tilde{q}}$ be the sub-vector of $\boldsymbol{y}$ with values of '1' in the rows of $\widetilde{\boldsymbol{R}}_{\tilde{q}}$. Let $N_{\tilde{q}}$ be the sum (across rows) of the vector $\widetilde{\boldsymbol{R}}_{\tilde{q}}$ (that is $N_{\tilde{q}}$ the cardinality of $\widetilde{\mathbf{C}}_{\tilde{q}}$), so that the dimension of $\boldsymbol{y}_{\tilde{q}}$ is $N_{\tilde{q}}\times 1$. Let $l_{\tilde{q}}$ be the index of all elements in the vector $\boldsymbol{y}_{\tilde{q}}$, so that $l_{\tilde{q}}=1,2,\ldots,N_{\tilde{q}}$. Next, define $\ddot{C}_{\tilde{q}}=\left[N_{\tilde{q}}(N_{\tilde{q}}-1)\right]/2$. Then, the $\boldsymbol{J}$ matrix maybe empirically estimated as:

$$\hat{\boldsymbol{J}}=\frac{1}{\widetilde{Q}}\left[\sum_{\tilde{q}=1}^{\widetilde{Q}}\left[\frac{1}{\ddot{C}_{\tilde{q}}}\left(\left[\sum_{l_{\tilde{q}}=1}^{N_{\tilde{q}}-1}\sum_{l'=l_{\tilde{q}}+1}^{N_{\tilde{q}}}\frac{\partial\log L_{l_{\tilde{q}}l'}}{\partial\boldsymbol{\theta}}\right]\left[\sum_{l_{\tilde{q}}=1}^{N_{\tilde{q}}-1}\sum_{l'=l_{\tilde{q}}+1}^{N_{\tilde{q}}}\frac{\partial\log L_{l_{\tilde{q}}l'}}{\partial\boldsymbol{\theta}}\right]'\right)\right]\right]_{\hat{\theta}_{CML}}. \tag{7}$$

One additional issue regarding estimation. The analyst needs to ensure the positive definiteness of the covariance matrix $\mathbf{\Omega}$ and the correlation matrix $\mathbf{\Lambda}_1$. We achieve this using a Cholesky decomposition approach (by parameterizing the likelihood function in terms of the Cholesky-decomposed parameters). In addition, the spatial dependence parameter δ needs to be bounded between –1 and +1. For this, we parameterize δ as follows:

$$\delta=\frac{\exp\left(\ddot{\delta}\right)-1}{\exp\left(\ddot{\delta}\right)+1}.$$

A further simplification is also possible in case the number of alternatives being ranked (I) is large. In such a case, even the CML estimator proposed above can get unwieldly, since it requires the estimation of an MVNCD function of dimension 2*(I–1). In such an event, a rank-breaking approach within each individual followed by the CML is possible. Such an approach has been suggested for an aspatial rank-order utility maximizing model by Zhao and Xia (2018) (see also Soufiani et al., 2014 and Khetan and Oh, 2016). For symmetric distributions such as the normal distribution in the utilities, Zhao and Xia show that the following rank-breaking approach will yield a consistent and asymptotically normal (CAN) CML estimator: Use a uniform ranking and uniform, fully connected, and symmetric CML weighing approach. That is, all sub-rankings of length l from the larger ranking are all given the same weights regardless of the ranking positioning of specific sub-rankings, and all implied sub-rankings are considered with the same weight regardless of the ordering of alternatives in a sub-ranking. For example, if the full ranking has three positions (with three alternatives), then sub-rankings of length 2 may be used during estimation as long as the weights for the sub-ranking of the first two ranked alternatives and another sub-ranking of length 2 comprising the last two ranked alternatives are equally weighted; and all sub-rankings of alternatives of length 2 are considered (position 1 and position 2, position 1 and position 3, and position 2 and position 3) with equal weight regardless of which alternative is in which position of the sub-ranking. Essentially, the approach has the potential to further reduce the dimensionality of the MVNCD evaluation in the CML estimator to 2**l* instead of 2*(*I*–1). But, in most contexts in which the number of alternatives is in the order of 10 or less, the analytic approximation for the MVNCD function used in the current paper should enable implementation of the MACML estimator without the need for any prior rank-breaking (the estimation routine coded in the GAUSS matrix programming language is available at https://www.caee.utexas.edu/prof/bhat/CodeRepository/CODES/SpatialRank/Gauss_Codes_SROP.zip).

A final note on estimation. An additional simplification is possible in estimation by approximating the matrix inverse in the autoregressive multiplier term **S**. Defining $\tilde{\mathbf{W}} = \mathbf{W} \otimes \mathbf{IDEN}_I$, we may write:

$$\mathbf{S} = \left[\mathbf{IDEN}_{QI} - \left(\delta\tilde{\mathbf{W}}\right)\right]^{-1} = \mathbf{IDEN}_{QI} + \left(\delta\tilde{\mathbf{W}}\right) + \left(\delta^2\tilde{\mathbf{W}}^2\right) + \left(\delta^3\tilde{\mathbf{W}}^3\right) + ...$$

Since taking the inverse of a matrix is time-consuming, approximating this inverse by using a suitable order of terms on the right side of the above equation can provide substantial computational efficiency. Future research should investigate, using simulation, the optimal "computational-accuracy trade-off" of using different orders of terms.

3. SIMULATION

The simulation exercises undertaken in this section assess the ability of the proposed MACML estimator in recovering the parameters of the proposed spatial rank-ordered probit model from finite samples, as well as investigate the effects of ignoring the spatial drift and spatial lag effects. This is achieved by generating simulated data sets with known underlying model parameters.

3.1 Simulation Design

For the simulation study, a four-alternative choice situation is considered. A total of Q=600 observation units is assumed. For this simulation experiment, we do not consider constants in the utilities, and focus on the parameters on the independent variables, and the spatial parameter. Two independent variables are used in the utility equation (these are the elements of the $\mathbf{x}_{qi}$ vector). For the first independent variable, the values for the first two alternatives are generated from a standard univariate normal distribution, while the values for the second two alternatives are generated from a univariate normal distribution with a mean of 0.5 and standard deviation of 1. For the second independent variable, the values for the first two alternatives are generated from a univariate normal distribution with a mean of 0.5 and standard deviation of one, while the values for the second two alternatives are generated from a standard univariate normal distribution. Once generated, these independent variable values are held fixed in the entire rest of the simulation exercise. A random coefficient (across observation units) is assumed on the first variable, while a fixed coefficient is assumed on the second variable. The mean of the coefficient on the first independent variable and the fixed coefficient on the second independent variable are both assumed to be one (that is, $\mathbf{b} = (b_1, b_2)' = (1,1)'$). The variance on the random coefficient on the first variable is set at 1.0; thus, $\mathbf{\Omega}=\begin{bmatrix}\Omega_{11} & 0, \\ 0 & 0\end{bmatrix}=\begin{bmatrix}1 & 0, \\ 0 & 0\end{bmatrix}$, which is a degenerate matrix but is written in this form to show the correspondence with the notation in the earlier section. Of course, in estimation, fixed coefficients are handled in a straightforward manner because they are completely excluded from the $\mathbf{x}_q$ vector in the construction of the $\tilde{\mathbf{x}}$ matrix in Equation (2).

For the covariance matrix $\mathbf{\Lambda}$ of the kernel error terms of the four alternatives, we use a diagonal matrix to restrict the number of parameters to be estimated in our simulation, while also focusing on the spatial dependency and spatial drift effects of interest.

$$\mathbf{\Lambda}=\begin{bmatrix}0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \Lambda_{33} & 0 \\ 0 & 0 & 0 & \Lambda_{44}\end{bmatrix}=\begin{bmatrix}0 & 0 & 0 & 0 \\ 0 & 1 & 0.0 & 0.0 \\ 0 & 0.0 & 1.2 & 0.0 \\ 0 & 0.0 & 0.0 & 1.5\end{bmatrix}$$

To generate the spatial lag dependency, a rectangular grid of size 5,800 meters (3.604 miles) by 3,800 meters (2.361 miles) is considered, and partitioned by vertical and horizontal gridlines spaced 200 meters apart. Next, 600 observational units are located at the intersection points of the gridlines. The spatial weight matrix $\mathbf{W}$ (of size 600×600) is created using the inverse of the square of distance on the coordinate plane between observational units. This spatial matrix, once generated, is held fixed in all simulations. Finally, we consider two cases of the spatial lag parameter δ, 0.25 and 0.7, reflecting low spatial dependency and high spatial dependency values.

3.2 Data Generation and Performance Metrics

In total, the simulation design includes six parameters: $(b_1, b_2, \Omega_{11}, \Lambda_{33}, \Lambda_{44}, \delta)$. The simulation experiments entail assuming underlying "true" values for these parameters and generating data sets for estimation for each of the two cases of $\mathbf{\Omega}$ values. In particular, for a given combination, first compute the matrix $\mathbf{S}=\left[\mathbf{IDEN}_{QI}-\left(\delta\mathbf{W}\otimes\mathbf{IDEN}_I\right)\right]^{-1}$. Next compute $\mathbf{xb}$, and also draw a realization for $\tilde{\boldsymbol{\beta}}$ as Q independent draws from a normal distribution with mean 0 and variance Ω_{11}. Then, draw a realization for $\boldsymbol{\varepsilon}\sim MVN(\mathbf{0},\mathbf{\Omega})$ using the constructed covariance matrix. Finally, one can construct the utility vector $\mathbf{U}=\mathbf{S}\left[\mathbf{xb}+\tilde{\mathbf{x}}\tilde{\boldsymbol{\beta}}+\boldsymbol{\varepsilon}\right]$. Then, for each observation unit, the alternatives are ranked based on these utility values, i.e., the alternative with the highest utility value in ranked 1, followed by assigning a rank of 2 to the alternative with the second highest utility value and so on. The above procedure is repeated 250 times with different realizations of $\tilde{\boldsymbol{\beta}}$ and $\boldsymbol{\varepsilon}$ to generate 250 different data sets for each of the two cases of δ. For each dataset, the six model parameters are estimated using the MACML method. In addition, we also estimate restrictive versions of the proposed spatial random-coefficients ROP (or simply SROP) model on the 500 data sets, corresponding to (1) the "no spatial drift" model (that is, $\Omega_{11}=0$, but $\delta\neq 0$, which corresponds to a spatial fixed coefficients model) (2) the "no spatial lag" model (that is,

$\delta = 0$, but $\Omega_{11} \neq 0$, which corresponds to the aspatial random-coefficients ROP), and (3) $\delta = 0$, and $\Omega_{11} = 0$, which, of course, is the regular aspatial fixed-coefficients ROP. Across all these models and the proposed model in the current paper, a total of 2000 estimations are undertaken.

The results from the estimations are translated to measures of performance in terms of the ability to recover the "true" parameters and their standard errors. A comparison of the many restricted models with the proposed model is also undertaken. The procedure is as follows, for each of the two cases of δ:

(1) Estimate the parameters using each of the models for each data set s (of the 250 data sets generated). Estimate the standard errors.

(2) Compute the mean estimate for each model parameter across the data sets to obtain **a mean estimate**. Compute the **absolute percentage (finite sample) bias** (APB) of the estimator as:

$$APB(\%) = \left| \frac{\text{mean estimate - true value}}{\text{true value}} \right| \times 100. \tag{8}$$

(3) Compute the standard deviation for each model parameter across the data sets, and label this as the **finite sample standard deviation or FSSD** (essentially, this is the empirical standard error).

(4) Compute the median standard error for each model parameter across the data sets and label this as **the asymptotic standard error or ASE** (essentially, this is the standard error of the distribution of the estimator as the sample size increases).

(5) Next, to evaluate the accuracy of the asymptotic standard error formula, compute the APB associated with the ASE of the estimator as:

$$APBASE(\%) = \left| \frac{\text{ASE-FSSD}}{\text{FSSD}} \right| \times 100 \tag{9}$$

(6) Compare the three restrictive formulations discussed in the first paragraph of this section with the proposed model, based on the mean APB measure across all parameters and the adjusted composite log-likelihood ratio test (ADCLRT) value (see Pace et al., 2011 and Bhat, 2014 for more details on the ADCLRT statistic, which is the equivalent of the log-likelihood ratio test statistic when a composite marginal likelihood inference approach is used; this statistic has an approximate chi-squared asymptotic distribution). The ADCLRT statistic needs to be computed for each data set separately, and compared with the chi-squared table value with the appropriate degrees of freedom. Here we identify the number of times (out of the 250 estimations corresponding to the 250 data sets) that the ADCLRT value rejects the restrictive model in favor of the proposed general spatial model.

In addition, to examine the efficiency gains from using a rank-ordered elicitation mechanism rather than the more traditional first-choice elicitation mechanism, we also estimate a spatial multinomial probit (SMNP) model, which considers only the top choice preference, for each of the generated datasets. Then, we compute the trace value of the covariance matrix of parameters given by $tr[\mathbf{V}_{CML}(\hat{\boldsymbol{\theta}})]$ from the SROP and SMNP models for each dataset (the trace value provides the sum of the sampling variances across all model parameters). Finally, we compute the mean $tr[\mathbf{V}_{CML}(\hat{\boldsymbol{\theta}})]$ values for the SROP and SMNP models across all the 250 datasets (and for both the cases of the spatial lag parameter values). The model that provides a lower mean trace value is the preferred one.

3.3 Simulation Results

The simulation results are discussed in three parts. First, an analysis of the ability of the proposed MACML approach to recover the spatial rank-ordered probit model parameters is presented in the next section. Second, the implications of ignoring the spatial dependency and spatial heterogeneity effects are discussed in Section 3.3.2. This is followed by a brief discussion on the comparison of the efficiency between the SROP and the SMNP models

3.3.1 Parameter Recovery Ability of our Proposed Model

Table 1 provides the results for the ability of our proposed MACML approach to recover the parameters of the SROP model. The top panel of the table is for the low spatial autoregressive coefficient (δ=0.25), while the bottom panel is for the high spatial autoregressive lag coefficient (δ=0.70). In the low spatial lag case, the absolute percentage bias (APB) ranges from 1.20% to 3.10 %; the corresponding APB for the high spatial lag ranges from 1.43% to 7.93%. The recovery of the mean parameters is generally better than the covariance parameters, particularly in the high spatial lag case. This result is not surprising, because the covariance parameters enter the likelihood function in a more complex non-linear fashion, and the extent of this non-linearity is more pronounced in the high spatial dependency case (note that the **S** matrix gets applied in a non-linear fashion to the $\boldsymbol{\Omega}$ matrix during estimation; see Equation (2)). The spatial parameter is recovered with remarkable accuracy in both the cases with APB values of 1.20% and 1.43%. Overall, the average APB across all parameters (see the last row under the "Absolute Percentage Bias (APB)" column in each panel) is 2.07% for the low spatial coefficient case and 4.16% for the high spatial correlation case, which indicates that our proposed MACML approach is able to recover parameters with substantial accuracy.

The APBASE metric values, presented in the last column of Table 1, provide a sense of the accuracy of the asymptotic standard error of the estimates. The mean APBASE values (of 7.93 for low spatial dependency and 12.87 for high spatial dependency) may seem to be on the high end, but is simply a reflection of the very low values of FSSD in the first place. In terms of absolute difference, the ASE values are quite close to the FSSD values. Interestingly, of all the parameters, the spatial dependency parameter is the most precisely estimated indicating that our approach is very effective in accurately and precisely capturing spatial effects. The APBASE values for the covariance terms are generally higher than the mean parameters for both cases, again due to the non-linearity effect discussed earlier.

3.3.2. Effects of Ignoring Spatial Dependence and Drift Effects

In this section, we discuss the implications of ignoring the spatial dependence effect and the spatial drift effect. Table 2 provides the results of four models (the proposed SROP model, the "no spatial drift" model, the "no spatial lag" model, and the traditional (aspatial fixed coefficients) ROP model. In comparing with the "no spatial drift" model, the mean APB values for the SROP model is computed without considering the APB values for the Ω_{11} parameter, because the Ω_{11} parameter is implicitly fixed at zero in the "no spatial drift" model. For comparison with the "no spatial lag" model, the mean APB value for the SROP model is computed without considering the APB values for the δ parameter, which is fixed at zero for the "no spatial dependence model". And, finally, for the comparison with the traditional ROP, the APB values for both the Ω_{11} and δ are ignored.

The results indicate that the mean APB values are higher for all the three restricted models relative to the SROP model proposed in this paper. It is illustrative to note that the "no spatial lag" model performs substantially better than the "no spatial drift" model in the low spatial dependency case, while the opposite is the case for high spatial dependency; this is to be expected because, at low spatial dependency, ignoring the spatial lag effect should be of less consequence than ignoring the aspatial unobserved heterogeneity component. Perhaps more importantly, the results clearly indicate the substantially higher bias of all the three restricted models for the high spatial lag case relative to the low spatial lag case. Again, this is to be expected, because a high level of spatial lag translates into high levels of spatial dependency as well as high spatial drift effects (both effects contain the **S** variable in which the spatial autoregressive parameter δ is embedded). The last row of Table 2 indicates that our proposed SROP model rejects the three restricted specifications for each of the 250 data sets generated and the mean adjusted composite likelihood ratio test (ADCLRT) statistic value is substantially higher than the relevant table chi-squared statistic at any reasonable significance level.

3.3.3. Efficiency Gain in the Use of the Rank-Ordered Framework.

In this section, we discuss the efficiency gains offered by the rank-ordered elicitation mechanism relative to the most preferred (first-choice) elicitation mechanism. For the low spatial lag case (δ=0.25), the mean $tr[\mathbf{V}_{CML}(\hat{\boldsymbol{\theta}})]$ value of the covariance matrix for the SROP model is 0.041, while the corresponding value is 0.122 for the SMNP model. For the high spatial lag case (δ=0.70), the values are 0.060 and 0.167, respectively, for the SROP and SMNP models. In both the low and high spatial lag cases, the trace value for the SROP model is almost a third of that from the SMNP model, indicating that the SROP model is by far the preferred model from an efficiency standpoint.

3.3.4. Simulation Results Summary

Overall, the simulation results show that, irrespective of the magnitude of spatial dependence, the MACML estimator recovers the parameters of the SROP model very well. In addition, the results clearly highlight the bias in estimates if spatial dependence and/or spatial drift are ignored when one or both are actually present. We also find that at high spatial dependency, ignoring one or both of the spatial lag and spatial drift effects has severe consequences to the parameter estimates, underscoring the importance of capturing both effects in spatial models (because the level of the spatial autoregressive lag parameter will not, in general, be known in advance). Finally, the efficiency gain in using the SROP model is also highlighted by the simulation results.

4. EMPIRICAL APPLICATION

Travel mode choice studies abound in the literature, though there have been much fewer of these that consider spatial dependency (see for example, Bhat and Zhao, 2002, Dugundji and Walker, 2005, Goetzke 2008, and Sidharthan et al., 2011). Further, even the few that recognize spatial dependence do so in very restrictive and limited ways and within a first-choice context. In fact, to our knowledge, the current application is the first to use a rank-ordered model of travel mode choice even in an aspatial context. Of course, besides using rank-ordered data, we also consider both spatial lag and spatial drift effects within a single-comprehensive and integrated framework.

4.1 Data and Sample Formation

We demonstrate an application of our proposed model in the context of non-work/non-mandatory mode-choice behavior using a sample drawn from the 2019 multi-city Transformative Technologies in Transportation (T4) Survey. The T4 survey was conducted in Phoenix (Arizona), Atlanta (Georgia), Tampa (Florida) and Austin (Texas). In this study, we use the sample collected from the Austin (Texas) area. The distribution effort resulted in a convenience sample of 1,127 respondents (for the city of Austin), which was reduced to a final set of 928 individuals, after removing 199 individuals who either did not correctly respond to the ranking question used in this study or submitted an incomplete response form.

The survey elicited several user characteristics and choices associated with travel behavior, including through the use of a stated preference (SP) question that asked respondents to rank, in the context of a future autonomous world, their mode choice preferences (from most preferred to least preferred) for non-work/non-mandatory trips. The mode choice alternatives were as follows: private vehicle (human driven or autonomous), bicycle, public transport (bus/rail), human-driven private ride-hailing (ride-hailing alone with a human driver), human-driven pooled ride-hailing (ride-hailing with others with a human driver), autonomous vehicle (AV) private ride-hailing (same as private ride-hailing, except the vehicle will be autonomous), and AV pooled ride-hailing (same as pooled ride-hailing, except the vehicle will be autonomous).[5]

The SP experimental design was characterized by three trip attributes -- wait time, in-vehicle travel time, and total trip cost. A total of 36 scenarios were developed based on a random blocking approach and an orthogonal fractional factorial design, and each respondent was asked to rank the alternatives for one randomly selected scenario. Appropriate procedures were in place to ensure that every scenario was realistic (for example, the cost by pooled ride-hailing cannot be higher than that of private ride-hailing). Additionally, a non-mandatory trip purpose was also

[5] There was no distinction in the survey between the human-driven private vehicle travel mode and an AV private vehicle travel mode, because the thrust of the ranking exercise was to elicit information on the preferences for a private vehicle (whether AV or human-driven) versus other non-private vehicle modes.

randomly assigned to this question (which varied across individuals) by selecting one of the following four statements before presenting the choice experiment:

- *Suppose you are going to a mall to do some shopping (e.g., to purchase clothes, books, etc.).*
- *Suppose you are going to the airport.*
- *Suppose you are going out to spend some time with your friends (e.g., going to their house or to a bar).*
- *Suppose you are going out to get food (e.g., dinner at a restaurant or breakfast at a diner).*

We define these four trip purposes as “Shopping”, “Airport-access”, “Socializing” and “Eating-out”. Figure 1 shows a sample of the actual question presented to respondents.

Additionally, point location information of each individual (in terms of latitudes and longitudes) was imputed based on the residential information provided in the survey. The distance between residential locations was used as the spatial proximity measure to develop the weight matrix, and also provided the basis for the construction of a comprehensive set of built environment (BE) characteristics associated with an individual’s residence. Specifically, geocoded residential locations were overlaid on census block groups, and then the BE characteristics of the block group (as extracted from the U.S. EPA Smart Location Database; see Ramsey and Bell, 2014) were attributed to the individual’s residence.

4.2 Data Description

Table 3 provides a description of the ranking preferences of the respondents. For ease in presentation and for an overall understanding of user preferences, we present the ranking preferences in groups of “Top ranked”, “Within top 2 ranks”, “Within top 3 ranks”, “Within top 4 ranks” and “Within top 5 ranks”. Also, we will use the acronym “HD” for human driven and “RH” for ride-hailing. Thus, the seven modes will be referred to as private vehicle, bicycle, public transport, HD private RH, HD pooled RH, AV private RH, and AV pooled RH. From the table, we observe that individuals are overwhelmingly in favor of private vehicle use, with more than 70% assigning the top rank to this mode; the second most favored (as the top rank) mode; public transport; is substantially behind at close to 9%. However, beyond the first rank, at every other rank depth, HD private RH appears to be the second ranked mode after private vehicle. That is, a large fraction of those who indicate private vehicle use as their top choice also indicate HD private driven RH as their second ranked choice. In terms of overall ranking, bicycle appears to be the least favored mode with about 64% ranking it outside the top five. The ranking trend also suggests that private RH is generally preferred over pooled RH. Interestingly, in spite of the general buzz about the advent of AVs, HD RH services appear to be preferred over AV RH services.

4.3 Variable Specification and Estimation Process

To employ our proposed spatial model, the weight matrix needs to be pre-specified. Several weight matrix specifications were considered in our empirical analysis to characterize the spatial lag dependence. These included (1) the inverse of a continuous distance specification where the distance is measured as the Euclidean distance between each individual’s locations (2) the inverse of the square of the continuous distance specification, and (3) the inverse of the cube of the continuous distance specification and (4) the inverse of the exponential of the continuous distance specification. We also explored alternative distance thresholds to select the pairs of observations for inclusion in the composite marginal likelihood (CML) estimation. As indicated earlier, this threshold band determination may be based on minimizing the trace of the variance matrix of parameters given by $tr[\mathbf{V}_{CML}(\hat{\boldsymbol{\theta}})]$. We explored alternative distance thresholds of 0.5 miles, 1 mile, 2.5 miles, 5 miles, 7.5 miles and 10 miles. We did not include thresholds beyond the 10 miles range because the implied spatial dependence fades very rapidly and becomes negligible beyond 10 miles; also, the trace of the variance matrix started to increase beyond the five miles range. For each of the four different weight matrix specifications, the pattern of the trace values was identical as we moved from the lowest distance threshold (0.5) to the highest distance threshold (10.0). In particular, for each of the four weight matrix specifications, the trace value decreased from 0.5 miles until 2.5 miles, and then started increasing beyond 2.5 miles. That is, the best estimator efficiency was obtained at 2.5 miles. Further, the results indicated that the best spatial weight matrix specification was the inverse of the square of the continuous distance specification with the 2.5 miles distance band. This determination was based on the composite likelihood information criterion (CLIC) statistic, which may be used to compare the data fit of non-nested formulations (see Varin and Vidoni, 2005).

Several different variable specifications and functional forms of the variables were also simultaneously explored to arrive at the final specification. Our specifications included 4 sets of variables: (1) individual characteristics, (2) household characteristics, (3) built environment (BE) attributes and (4) Trip level attributes. We also attempted random coefficients on many variables, but the only one that turned out to be statistically significant was the one related to population density specific to the bicycle mode.

Overall, while our emphasis here is on examining spatial dynamics/drift effects, we investigated alternative empirical specifications based on statistical fit, intuitiveness, parsimony considerations, and the insights offered by the substantial earlier mode choice literature. Specifically, in terms of statistical fit, we used the adjusted composite likelihood ratio test (AD*CLRT*) statistic (see Pace et al., 2011 and Bhat, 2011) to compare nested models and the composite likelihood information criterion (CLIC) introduced by Varin and Vidoni (2005) to test non-nested models. The final specification includes some variables that are not statistically significant at the usual 5% level of significance. These are retained because the effects of these variables are intuitive and also because of the relatively small sample size in our spatial analysis.

4.4 Estimation Results

The estimation results are presented in Table 4 for the proposed SROP model, and are discussed below by variable category. These parameter estimates provide the directional effects of variables on the utility of the different alternatives.[6] We do not present the corresponding results for the

[6] Please note that these parameter estimates do not provide the magnitude of variable effects on the choice probability of the different alternatives, nor do they necessarily even provide the directional effects of variables on the choice

other more restrictive models, because the substantive directions of effects on the utility of alternatives were similar in all models. However, we present the data fits from the more restricted models in Section 4.5, as well as compare the implied elasticity results from the SROP model and one of the restricted models in Section 4.6. Also, to maintain attention on spatial issues, we will keep the discussion of the estimation results concise and brief. However, our results do also contribute to the mode choice literature in the context of emerging mobility options. Of particular note is that we include both a human-driven option as well as an autonomous vehicle option for the ride-hailing mode.

4.4.1 Parameter Estimates for Individual Characteristics

The results in Table 4 indicate that women have a higher propensity to use private vehicles and the bicycle mode, and a lower propensity to use all forms of ride-hailing services (for both human-driven and autonomous vehicles), relative to the base mode of public transit. These results are consistent with those of Asmussen et al. (2020), who observe a higher need for driving control among women. Besides, women have been known to be (a) more reluctant to use human-driven RH services due to security considerations (see, for example, Tirachini, 2020 and Kang et al., 2021) as well as (b) more disinclined to embrace AV RH services due to experiencing stronger (relative to men) feelings of nervousness and fear in anticipation of negative outcomes when adopting new technology (see Croson and Gneezy, 2009 and Lavieri and Bhat, 2019). Finally, the higher tendency of women to use active (walk and bicycle) modes may be attributed to a generally heightened health consciousness of women relative to men (see Ek, 2013) as well as a higher degree of environmental concern and social altruism of women (Desrochers et al., 2019).

The effects of age and possession of driving license in Table 4 suggests that older individuals over the age of 50 years and those with a driving license are strongly inclined toward the use of privately-owned vehicles. The former result is possibly a manifestation of the heightened need for mobility control among older individuals (Nikitas et al., 2018, Asmussen et al., 2020). The latter result is not surprising and may be a reflection of a general affective emotion of personal vehicle ownership and personal control of mobility (see, for example, Haustein, 2021).

Individuals with a graduate degree have a low preference for the bicycle mode relative to other modes and a marginally significant higher preference for AV pooled RH. The first result is rather surprising, given the generally higher health consciousness and higher green lifestyle associated with highly educated individuals (see McCright, 2010, Jaafar et al., 2017). However, in a fast-changing mobility landscape when AVs are also brought into modal consideration sets, Lavieri and Bhat (2019) observe that productive use of travel time, along with environmental consciousness, starts playing an important role in the mobility choice decisions of those highly educated.

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probability of the alternatives. This is because of the non-linear nature of the probability model, as well as because of the spatial autocorrelation dependence among individuals. A better characterization of the directionality and magnitude of variable effects may be obtained through the elasticity effects discussed in Section 4.5.1.

Finally, in the set of individual characteristics, individuals who are employed have a higher inclination (relative to their unemployed peers) to use privately owned vehicles and private RH services (by both human-driven and autonomous vehicles). This generic preference for private travel modes is supported by earlier literature in the ergonomics and occupational health fields (see Jansen et al., 2003, Mohren et al., 2010), which suggest that employed individuals experience elevated psychosomatic feelings of stress and life balance conflicts, and are engulfed by feelings of 'wanting to be left alone for a while after work'.

4.4.2 Parameter Estimates for Household Characteristics

Among the household demographic data available, we find the rather expected results that households with more than one vehicle and with three or more members have a distinct preference for the private vehicle. These results reflect the ability, convenience, and desire to travel as a family in private vehicles for non-mandatory leisure-oriented trips.

Household income, not surprisingly, has an important impact on mode choice decisions. In particular, individuals in high income households have a strong preference for "private" travel, either in their own vehicles or in RH vehicles. This is a reflection of the traditional influence of high income on private car use as well as its extension to other new/emerging mobility options that preserve the element of privacy in travel.

4.4.3 Parameter Estimates for Built Environment Attributes

Several built environment attributes were tested (in continuous and categorical forms); however, only population and employment density (in their continuous forms) of the residential location turned out to be statistically significant as explanatory variables. Both population density and employment density have a positive effect on the use of the bicycle mode, potentially because of better bicycle lane infrastructure in neo-urbanist high density regions and short distances of travel (Braun et al., 2019). Also, individuals residing in high population density locations tend to have a higher propensity for human-driven (HD) pooled RH, reinforcing a similar finding in Kang et al. (2021). Additionally, the results show evidence of heterogeneity in the effect of population density on bicycle utility (see the standard deviation on population density in the "Bicycle" column in Table 4). When compared with the mean effect of population density on bicycle utility, the implication is that a vast majority (97%) of individuals residing in higher population density neighborhoods have a higher probability of selecting the bicycle mode relative to those residing in lower population density neighbourhood, though the effect is reversed for 3% of individuals. As importantly, this unobserved heterogeneity, when combined with the spatial lag effect, leads to a spatial drift effect, as discussed in Section 1. In particular, individuals who intrinsically prefer walking locate themselves close to one another and cluster in high density neighborhoods, not because of social interaction after locating there or because of high density living, but because they are commonly attracted to live in such neighborhoods.

4.4.4 Parameter Estimates for Trip Level Attributes

Our results support the increasing need to disaggregate non-work purposes in mode choice modeling. Not surprisingly, the use of private vehicles is preferred over all other alternatives for shopping trips, a reflection of the convenience of private vehicles when carrying shopping bags especially when shop-hopping from one store to another. In fact, as observed by Gilibert et al. (2019), individuals do not even prefer private RH modes for shopping because of wait times and shop-hopping. As far as airport trips are concerned, private vehicles and private RH services are the most preferred. Such private modes of transportation provide a sense of time control and peace of mind, particularly as travel time reliability and reaching the airport in good time become paramount (see Tam et al., 2011). For social trips, the private vehicle continues to be a preferred mode of choice, attributable to social activities being undertaken in groups (either inter- or intra-household) (unfortunately, the stated preference survey did not include attributes related to party size and composition; these would be variables to consider in future efforts). As indicated by the negative coefficient on the social trip purpose variable for the bicycle mode, the bicycle mode is not the most convenient and appropriate for social activities, with clothes getting ruffled and weather-caused appearance and body feel considerations also at play. The impact of the social trip purpose on the ride-hailing services provides interesting insights; specifically, individuals have a clear and positive preference for AV-based RH services (both in private and pooled modes) relative to human-driven RH services, the bicycle mode, and the public transport mode. RH services, in general, provide a good option to private vehicle use, particularly to avoid drinking while driving.

The travel time and travel cost coefficients are negative and estimated to be -0.09 and -0.31 respectively (these coefficients did not show substantial variation by travel mode or by trip purpose, nor did we find statistically significant unobserved heterogeneity in these coefficients; we should also note here that we did not partition total travel time by in-vehicle and out-of-vehicle times in our stated ranking experiment). The results suggest a value of travel time (VTT) of about $17.42/hour. This value of travel time estimate is close to that observed in Alonso-González et al. (2020) and Kang et al. (2021) in the context of ride-hailing services.

4.4.5 Spatial Dependency (Autoregressive Lag) Parameter

The spatial lag parameter is estimated to be 0.649 and is statistically very significant, clear evidence of substantial spatial dependency among individuals in mode choice preferences. As we note in Section 4.5, ignoring this spatial dependency has severe consequences for both model fit as well as model coefficient estimates. In particular, ignoring the spatial parameter is tantamount to ignoring heteroscedasticity across individuals in the error term (since the spatial error specification also leads to error heteroscedasticity across individuals in addition to spatial autocorrelation), rendering parameter estimation in an aspatial model both inconsistent and inefficient. The extent of the inconsistency and inefficiency will vary depending on the empirical context; however, testing and accommodating for any spatial effects should always be an important consideration.

4.4.6 Constants

The estimated constants in the last row of Table 4 do not have any substantive interpretations, and simply represent adjustments in the utilities of alternatives after accommodating the other variables in the model.

4.4.7 Covariance/Correlation Terms

In the estimation spatial rank-ordered model, only the covariance matrix of the error differences is estimable (with the scale for one of the error differences normalized to one). This condition is the same as for the single-choice multinomial probit model. The differenced error covariance matrix is not strictly interpretable, because multiple (undifferenced) error covariance matrices can all be consistent with the same differenced covariance matrix. However, the structure of the error-differenced covariance matrix appeared to point to a high positive correlation across all the four ride-hailing alternatives.

4.5 Measures of Data Fit

To evaluate the goodness of fit, we compare our model with the three restricted rank-ordered model versions of "no spatial drift", “no spatial lag”, and “aspatial fixed coefficients”. To do so, we use the ADCLRT statistic identified in an earlier section. In addition, we compute an average probability of correct prediction for each of the models. For this, we use a procedure identical to that used in the simulation experiment design, except that the parameters used in determining the individual level utilities of the many alternatives are based off the estimated coefficients and covariance matrix estimates from the MACML estimation. Next, the procedure is applied 10000 times and the probability of the observed ranking for each individual is obtained as the fraction of times of the 10000 repetitions that the observed ranking turns out to be the predicted ranking. The average probability of correct prediction (APCP) of the rank-ordering is computed as the average across individuals of the probability of the observed ranking. Next, using the procedure just discussed, we also compute the average probability of first choice (APFC) across individuals. Further, in addition to the disaggregate ADCLRT, APCP, and APFC metrics, we also examine the performance of the models in an intuitive way at the aggregate level by computing the predicted aggregate number of individuals selecting each modal alternative as the top (single) choice from each of the rank-ordered models, and comparing these predictions with the actual numbers to compute the absolute percentage error (APE) statistic.

The results of this data fit assessment are provided in Table 5. The MACML values at convergence clearly favor the SROP model over the other models, and this is also reflected in the ADCLRT statistics that show that the SROP model rejects all the other three restrictive models at any reasonable level of significance. The APCP and APFC metrics of the SROP are also higher than their counterparts from the other models. At the aggregate level, the SROP is once again the winner, especially compared to the “no spatial lag” and aspatial ROP models.

Finally, to compare the efficiency gain from using ranked data, we compute the trace value of the variance matrix of parameters, $tr[\mathbf{V}_{CML}(\hat{\boldsymbol{\theta}})]$, similar to the simulation experiment, for the

SROP model as well as a spatial multinomial probit (SMNP) model for first-choice. The $tr[\mathbf{V}_{CML}(\hat{\boldsymbol{\theta}})]$ of the SROP model is 0.245, while that of the SMNP model turned out to be 0.339, reinforcing the advantage of the ranked data as compared to the more traditional first-choice elicitation mechanism in terms of estimation efficiency.

4.5.1 Elasticity Effects and Implications

The coefficients in Table 4 do not directly provide a sense of the magnitude and direction of effects of each variable on each of the alternatives; therefore, we compute aggregate-level "pseudo-elasticity effects" of the exogenous variables to characterize the impact of each variable. In the current analysis, we have two types of exogenous variables: categorical variables (they are gender, age category, possession of license, educational qualification, employment status, vehicle availability, household size, income category and trip purpose category) and continuous variables (they are population density, employment density, travel time and travel cost). Except for trip purpose (which is a nominal exogenous variable), all other categorical variables appear as binary variables in the final specification. For each binary (dummy) variable, we first predict the shares of each mode in the sample using the probability of first choice, as discussed in Section 4.5, after assigning the base value of "0" for the binary variable for each individual. This provides the "Base" shares for each dummy variable. Next, we switch the value of the dummy variable from zero to one for every individual and repeat the entire process to obtain the average shares for this "Treatment" case. The average "pseudo" elasticity effect is then reported to be the difference between the "Treatment" and the "Base" vectors as a percentage of the "Base" vector. A similar procedure is used for the trip purpose multinomial model, except the "Treatment" case refers to each of the shopping, airport-access, and social trip purposes, with the "eat-out" purpose being the base. For the continuous variables, we increase the value of the variable by 20% and express the percentage change with respect to the original shares (i.e., keeping the original values of the continuous variables as "Base").

Table 6 provides the pseudo-elasticity effects for the SROP model.[7] The numbers in the table may be interpreted as the percentage change in the shares of each mode due to a change in the exogenous variable. For example, the first numeric entry of -23.31% in the table indicates that the probability of a woman choosing public transit is about 23.3% less than that of a man; in other words, in a random sample of 100 women, 23 fewer individuals would select public transit

[7] We also computed the corresponding effects for the aspatial random-coefficients ROP model that ignores the spatial drift and spatial lag effects but accommodates unobserved heterogeneity (this is the aspatial counterpart to the SROP model). These elasticities for the aspatial model are not presented in Table 6 to avoid clutter, but are available in an online supplement at https://www.caee.utexas.edu/prof/bhat/ABSTRACTS/SpatialRank/OnlineSupp.pdf. Suffice it to say that the elasticity effects from the aspatial model were significantly lower than those from the SROP model, a consequence of the "spillover" effects in the SROP model that causes a spatial multiplier effect. Specifically, a change in the utility for individual A (due to a change in a specific exogenous variable) influences the utilities of alternatives of other individuals, which then have a "circular" influence back on the utilities of the alternatives for individual A. This "circular" influence is reinforcing because of the positive spatial lag parameter. By ignoring such a multiplier effect, the aspatial model underestimates the effects of policies directed toward behavioural modification changes or BE/transportation system changes.

compared to in a random sample of 100 men. Similar interpretations can be provided for continuous variables as well. For example, the last row of Table 6 indicates that when AV pooled RH cost is increased by 20%, the probability of choosing public transport increases by 1.22%. The directions of the elasticity effects of the model are pretty much consistent with the discussions in the previous section. The table suggests that the most important variables among the binary variables affecting mode-choice decisions are (1) gender (2) age (3) possession of driving license (4) education level, (5) household vehicles and (6) annual income. Although the continuous variables cannot be directly compared in terms of relative importance, the percentage share changes across the modes for an increase of 20% in all the BE and travel level-of-service variables are moderate in magnitude compared to the impact of the individual and household variables.

By way of implications, our proposed SROP mode choice model suggests substantial and significant spatial interaction among decision-makers in selecting their mode preferences. Ignoring these impacts will, in general, lead to biased coefficient estimates and potentially misinformed policy actions. From a substantive perspective, our model results suggest, very broadly speaking, that policies to nudge individuals to adopt more environmentally friendly and pooled travel modes would be more effective if directed toward behavioural change in specific demographic groups rather than on built environment or trip level changes. Also, it is clear that gender effects play a substantial role in mode choice decisions in the emerging new era of mobility options. In particular, the first row in Table 6 highlights the strong aversion of women toward ride-hailing services, especially those that are autonomous (see the last two columns of the first numeric row which indicate more than 50% reduction in shares between men and women). As highlighted by Pflugfelder (2018), Asmussen et al. (2020), and Dannemiller et al. (2021), women have elevated levels of safety concerns regarding ride-hailing as well as place less trust in AV technology. Any efforts to increase comfort levels with ride-hailing in general, and autonomous RH use in particular, among even a small set of women may help get many more women to eschew private car use and promote shared RH service use (in private or pooled forms), especially because of spatial spillover effects. Similarly, our results also underscore the importance of promoting the use of ride-hailing services (private and shared as well as AV or non-AV) among older individuals, perhaps by making RH smartphone apps simpler and less cluttered to reduce the cognitive burden for older individuals. In fact, it is well established in the gerontology and psychology literature that aging is typically associated with a decline in cognitive ability (such as memory, attention, and verbal and visual/spatial information retention; see Deary et al., 2009, Boot et al., 2013), suggesting the need for human-machine interfaces (HMI) to be simple, uncluttered, voice-activated, and with multi-modal audio/visual interfaces (see Morgan et al., 2017). Besides by eschewing private vehicles and embracing RH options, older individuals also may be able to expand their social networks and benefit from less social exclusion.

The built environment effects reflect the fact that individuals residing in high (employment or population) density neighborhoods are more likely (than those residing in low density neighborhoods) to choose public transit, bicycle, and the pooled RH modes, with population density particularly increasing bicycle and HD pooled RH choice, and employment density particularly elevating the AV pooled RH mode.

From a trip purpose standpoint, the private vehicle seems to be the most preferred mode of transportation for shopping, airport-access and social purposes (relative to the eat-out purpose). However, the AV RH service (in both private and pooled modes) also appears to be an attractive option for the social trip purpose. The predisposition toward the AV RH pooled services can be further promoted by potentially discouraging private vehicle use in socially active downtown or other high density areas (for example, through the reduction of parking facilities or cordon-based congestion pricing strategies), while allowing RH pooled vehicles full access to these areas. This will not only nudge the public to shift to a more sustainable mode, but can also help in reducing driving while intoxicated (DWI) infractions. At the same time, though, in the context of modal level of service effects, the pseudo-elasticity measures suggest that disincentivizing private vehicle use (for example, by increasing cost) would provide rather limited returns because users have a strong preference for the private vehicle mode (see the low magnitude of private vehicle elasticity effects in response to private vehicle travel time/cost changes in Table 6). This "stickiness" toward the private vehicle mode suggests that government policies need to go beyond private vehicle-use disincentive schemes if they are to have any substantive impact on non-work private vehicle mode use share. A multi-pronged effort that combines congestion pricing and high parking cost schemes with policies promoting high occupancy vehicle use (such as subsidized pooled RH rides and public transit rides, and high occupancy vehicle use lanes) is needed.

5. CONCLUSIONS

In this paper, we propose a spatial rank-ordered probit (SROP) model that accommodates both spatial lag effects as well as spatial drift effects. To our knowledge, this is the first such formulation and application of a SROP model in the econometric and transportation literature. The SROP model is estimated using the maximum approximate composite marginal likelihood (MACML) method, employing a recently-developed fast and accurate analytical approximation method to evaluate the multivariate normal cumulative distribution function. We evaluate the parameter recovery ability of the MACML estimator through a simulation experiment and also estimate three restricted versions of our proposed model. The simulation results indicate that our proposed MACML approach is able to recover the true parameters remarkably well with an average (across parameters) absolute percentage bias less than 5%. Furthermore, the severe bias that one would introduce if the spatial lag and/or spatial drift effects are ignored is also evidenced in the simulation experiment.

We demonstrate an application of our proposed model in a mode choice context using a sample drawn from the 2019 multi-city Transformative Technologies in Transportation (T4) Survey for the city of Austin. Our analysis results indicate a strong and significant spatial autoregressive lag parameter and, because of unobserved heterogeneity on the population density variable, also spatial spillover effects. Ignoring these dependencies and dynamics will, in general, lead to inconsistent and inefficient estimates of parameter effects. This is highlighted by computing the elasticity effects of variables, which indicates that the model that ignores spatial effects quite substantially underestimates variable elasticity effects. From a substantive point of view, the results underscore the importance of demographics and lifecycle factors in mode choice preferences, which appear to be orders of magnitude more important than built environment attributes and transportation system attributes in an era of new and emerging mobility services.

A limitation of our empirical analysis is that the “private vehicle” considered both the human-driven and autonomous variants together. Therefore, it was not possible to disentangle the sensitivity of the variables separately for “private human-driven” and “private autonomous” vehicles. Future survey efforts should strive to separate these two modes. Even so our empirical application does provide important insights in the context of relatively recent and future ride-hailing services. Besides, the focus of this paper was to propose and demonstrate the value of a spatial rank-ordered model. In this regard, we hope that researchers and practitioners will reconsider the use of ranking data in modeling choice behavior, rather than inappropriately and summarily dismissing this type of data collection as being unreliable. Also, in an era of emerging mobility options, diffusion effects play a key role in market adoption rates, and our spatial rank ordered probit (SROP) provides the ability to explicitly and effectively account for such effects in travel modeling.

ACKNOWLEDGMENTS
The authors are grateful to Lisa Macias for her help in formatting this document. The authors are also grateful to two anonymous reviewers who provided useful comments on an earlier version of the paper.

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Suppose you are going to a mall to do some shopping (e.g., to purchase clothes, books, etc.). You have the following seven options for your transportation. Rank the alternatives listed from most preferred (Rank 1) to least preferred (Rank 7). Please do not give the same rank to multiple alternatives.

Rank	Alternative	Wait time	In-vehicle travel time	Cost for entire trip
	Private vehicle: Use your own private vehicle (human-driven or AV)	No wait	10 min	$0.40
	Bicycle	No wait	35 min	$0.00
	Public transit: Use bus or rail	10 min	23 min	$1.25
	Private ride-hailing: Get a ride with a human-driven ride-hailing service (*e.g.*, Uber, Lyft)	3 min	10 min	$12.50
	Shared ride-hailing: Get a human-driven ride in a vehicle in which other passengers may be added.	6 min	20 min	$9.40
	AV private ride-hailing: Same as ride-hailing, except that the vehicle will be autonomous.	3 min	10 min	$8.75
	AV shared ride-hailing: Same as shared ride-hailing, except that the vehicle will be autonomous.	6 min	20 min	$6.60

Figure 1: Example of ranking question presented to users (this example corresponds to a shopping purpose trip; the trip-purpose scenarios were varied across respondents)

Table 1: Parameter recovery ability of the MACML approach

Parameter	True Value	Mean Estimate	Absolute Percentage Bias (APB)	Finite Sample Standard Deviation (FSSD)	Asymptotic Standard Error (ASE)	APBASE (%)
Low spatial coefficient case (δ=0.25)						
b_1	1.000	1.016	1.60	0.065	0.071	9.23
b_2	1.000	1.020	2.00	0.092	0.088	4.35
Ω_{11}	1.000	0.969	3.10	0.098	0.088	10.20
Λ_{33}	1.200	1.169	2.58	0.077	0.072	6.49
Λ_{44}	1.500	1.471	1.93	0.097	0.111	14.43
δ	0.250	0.253	1.20	0.035	0.034	2.86
Overall mean value across parameters			2.07			7.93
High spatial coefficient case (δ=0.70)						
Parameter	**True Value**	**Mean Estimate**	**Absolute Percentage Bias (APB)**	**Finite Sample Standard Deviation (FSSD)**	**Asymptotic Standard Error (ASE)**	**APBASE (%)**
b_1	1.000	1.022	2.20	0.073	0.082	12.33
b_2	1.000	1.030	3.00	0.104	0.093	10.58
Ω_{11}	1.000	1.053	5.30	0.108	0.122	12.96
Λ_{33}	1.200	1.261	5.08	0.082	0.097	18.29
Λ_{44}	1.500	1.619	7.93	0.101	0.119	17.82
δ	0.700	0.710	1.43	0.019	0.018	5.26
Overall mean value across parameters			4.16			12.87

Table 2: Ignoring the effects of spatial dependency

Evaluation Metric	δ = 0.25				δ = 0.70			
	SROP Model	**"No spatial drift" Model**	**"No spatial lag" Model**	**ROP Model**	**SROP Model**	**"No spatial drift" Model**	**"No spatial lag" Model**	**ROP Model**
Mean APB	2.07	24.88	6.82	33.01	4.16	28.86	34.26	54.67
Mean APBASE	7.93	18.86	13.61	24.11	12.87	22.02	26.39	27.18
Mean composite log-likelihood value at convergence	-9989.26	-10576.15	-10139.62	-10706.22	-10101.12	-10534.07	-11151.77	-11641.99
Number of times the adjusted composite likelihood ratio test (ADCLRT) statistic favors the SRC-ROP model	-	All 250 times when compared with $\chi_1^2 = 3.84$ value (mean ADCLRT statistic is 803.01)	All 250 times when compared with $\chi_1^2 = 3.84$ value (mean ADCLRT statistic is 172.61)	All 250 times when compared with $\chi_2^2 = 5.99$ value (mean ADCLRT statistic is 947.11)	-	All 250 times when compared with $\chi_1^2 = 3.84$ value (mean ADCLRT statistic is 602.73)	All 250 times when compared with $\chi_1^2 = 3.84$ value (mean ADCLRT statistic is 1324.26)	All 250 times when compared with $\chi_2^2 = 5.99$ value (mean ADCLRT statistic is 2761.39)

Table 3: Dependent variable (ranking) description

Mode	Top ranked	Within top 2 ranks	Within top 3 ranks	Within top 4 ranks	Within top 5 ranks
Private vehicle	70.4%	79.6%	86.1%	89.3%	93.3%
Bicycle	4.3%	13.7%	21.9%	28.9%	36.3%
Public Transport	8.7%	29.5%	43.0%	55.9%	63.3%
HD private RH*	5.2%	31.9%	52.8%	73.3%	86.5%
HD pooled RH	2.3%	12.9%	34.2%	56.9%	82.4%
AV private RH	6.4%	20.8%	38.7%	55.8%	73.8%
AV pooled RH	2.7%	11.6%	23.3%	39.9%	64.4%

*HD – Human Driven, RH-Ride hailing, AV= Autonomous Vehicle.

Table 4: Estimation results for the empirical application

Variables	Public Transport (base)	Private vehicle	Bicycle	HD private RH	HD pooled RH	AV private RH	AV pooled RH
Individual characteristics							
Female (Base: Male)		0.142 (3.10)	0.083 (2.66)	-0.047 (-2.78)	-0.033 (-2.32)	-0.08 (-5.12)	-0.071 (-4.29)
Age greater than 50 years (Base: Less than 50 years)		0.581 (1.76)	-	-	-	-	-
Possession of a driver's license (Base: No possession)		0.138 (3.76)	-	-	-	-	-
Completed graduate degree (Base: lower than graduate degree)		-	-0.297 (-3.48)	-	-	-	0.012 (1.41)
Employed (Base: Not employed)		0.127 (2.96)	-	0.011 (1.54)	-	0.021 (4.31)	-
Household characteristics							
Household has more than one vehicle (Base: Less than 1 vehicle)		0.159 (1.81)	-	-	-	-	-
Household size 3 or more (Base: Less than 3)		0.209 (3.18)	-	-	-	-	-
Household income (Base: <$50,000)							
Income ≥ $100,000		0.234 (3.01)	-	0.024 (3.66)	-	0.011 (1.51)	-
Built environmental attributes							
Population density (people/acre)		-	0.081 (4.69)	-	0.041 (6.21)	-	-
Population density (standard deviation)			0.043 (1.89)				
Employment density (people/acre)		-	0.090 (1.76)	-	-	-	-
Trip level attributes							
Trip purpose (Base: Eat-out)							
Shopping purpose		0.156 (1.77)	-	-	-	-	-
Airport-access purpose		0.109 (2.17)	-	0.011 (1.69)	-	0.011 (1.69)	-
Social purpose		0.153 (2.84)	-0.047 (-1.72)	-	-	0.093 (1.87)	0.093 (1.87)
Travel time (minutes)	-0.090 (-3.42)						
Travel cost ($)	-0.310 (-9.12)						
Spatial dependency (δ)	0.649 (9.98)						
Constant	--	-0.284 (-4.56)	-0.392 (-6.07)	0.093 (5.62)	0.048 (2.21)	0.061 (1.76)	0.020 (1.12)

Table 5: Data fit measures with respect to restricted rank-ordered versions

Evaluation Metric	Models			
	SROP Model	"No spatial drift" Model	"No spatial lag" Model	ROP Model
Mean composite log-likelihood value at convergence	-37,638.45	-37,978.89	-40,783.22	-41,246.89
ADCLRT		394.49 ($\chi^2_2 = 3.84$)	4088.36 ($\chi^2_2 = 3.84$)	5189.22 ($\chi^2_2 = 5.99$)
APCP	0.0023	0.0021	0.0018	0.0017
APFC	0.359	0.348	0.258	0.256

Predicted shares for the first choice

Mode	Observed first choice	SROP Model		"No spatial drift" Model		"No spatial lag" Model		ROP Model	
		Predicted	APE	Predicted	APE	Predicted	APE	Predicted	APE
Public Transport	81	66	18.50	68	16.05	69	14.81	67	17.28
Private vehicle	653	565	13.47	560	14.24	525	19.60	522	20.06
Bicycle	40	77	92.50	71	77.50	89	122.50	88	120.00
HD private RH	48	56	16.67	58	20.83	66	37.50	69	43.75
HD pooled RH	21	36	71.43	39	85.71	33	57.14	35	66.67
AV private RH	59	82	38.98	80	35.59	78	32.20	76	28.81
AV pooled RH	26	46	76.92	52	100.00	68	161.54	71	173.08
Weighted MAPE			**22.19**		**22.85**		**30.17**		**31.25**

Table 6: Pseudo-elasticity effects

Variable	Base	Treatment	Public Transit	Private vehicle	Bicycle	HD private RH	HD pooled RH	AV private RH	AV pooled RH
Sociodemographic Effects									
Gender	Male	Female	-23.31	17.36	3.55	-49.87	-30.32	-58.54	-52.49
Age	Less than 50	Greater than 50	-38.90	23.34	-29.48	-55.02	-50.74	-35.39	-36.79
License possession	No	Yes	-45.81	31.40	-36.12	-70.10	-57.60	-51.91	-45.15
Completed graduation	No	Yes	31.47	5.12	-71.35	13.84	9.38	7.66	32.15
Employed	No	Yes	-9.88	6.56	-3.56	4.35	-11.58	7.76	-13.65
Household vehicles	1 or less	More than 1	-20.58	15.75	-16.23	-32.71	-23.18	-25.04	-14.78
Household size	Less than 3	3 or more	-15.35	10.71	-12.02	-26.30	-23.94	-17.44	-22.03
Annual income	< $100,000	> $100,000	-21.56	8.89	-12.56	8.22	-41.23	9.82	-48.69
Built-Environment Effects									
Population density	20% increase		2.21	-0.09	5.04	-6.35	12.55	-2.22	1.24
Employment density	20% increase		3.79	0.22	0.62	-14.14	8.58	-20.31	19.88
Trip Level Attributes									
Trip purpose	Eat-out	Shopping	-19.58	11.13	-13.90	-29.20	-23.03	-15.41	-19.45
Trip purpose	Eat-out	Airport-access	-23.26	8.04	-15.29	1.02	-29.59	0.65	-22.65
Trip purpose	Eat-out	Social	-17.99	8.98	-35.25	-21.58	-18.65	3.21	4.56
Public Transit Travel Time	20% increase		-10.22	0.21	2.65	0.65	6.25	-0.25	4.25
Private Vehicle Travel Time	20% increase		4.81	-0.36	2.14	5.58	-3.27	-0.77	3.18
Bicycle Travel Time	20% increase		2.22	0.68	-15.22	0.56	3.65	1.25	2.15
HD private RH Travel Time	20% increase		1.66	0.54	0.48	-14.76	4.25	3.56	4.22
HD pooled RH Travel Time	20% increase		1.25	0.38	2.54	4.35	-12.36	-0.22	3.56
AV private RH Travel Time	20% increase		2.24	0.32	2.04	3.22	4.56	-11.25	2.31
AV pooled RH Travel Time	20% increase		1.11	0.45	1.25	2.54	4.23	1.25	-15.99
Public Transit Travel Cost	20% increase		-21.55	0.59	3.11	1.25	2.56	-0.15	3.22
Private Vehicle Travel Cost	20% increase		4.05	-0.81	4.25	1.30	3.17	1.31	2.35
HD private RH Travel Cost	20% increase		2.15	0.21	1.52	-22.65	3.89	2.25	4.35
HD pooled RH Travel Cost	20% increase		1.05	0.15	-0.22	3.25	-29.65	5.63	4.25
AV private RH Travel Cost	20% increase		3.21	0.22	1.10	5.55	4.34	-20.98	3.25
AV pooled RH Travel Cost	20% increase		1.22	0.19	0.46	4.12	5.66	3.07	-31.22