SIMPLE CURVILINEAR METHOD FOR NUMERICAL METHODS OF OPEN CHANNELS

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ABSTRACT: Three-dimensional finite-difference or finite-volume models of sinuous open channels (e.g., narrow rivers, estuaries, and reservoirs) generally require boundary-fitted grids and curvilinear flow solution. Cartesian models with square grid cells are simpler to apply, but require a larger number of cells, as the cell size is determined by cross-stream resolution. This paper presents a simplified curvilinear approach suitable for systems where the along-stream length scale is larger than the cross-stream scale. The curvilinear Navier-Stokes equations are manipulated so the left-hand side is identical to the Cartesian momentum equations. The right-hand side then consists of grid-stretching curvature terms. These terms are written as functions of a perturbation parameter, so the first-order curvilinear effects are obtained with the lowest-order perturbation terms. As the Cartesian equations' form is preserved, we can readily adapt a Cartesian model to this perturbation curvilinear approach by adding the small curvilinear terms as explicit momentum sources.

INTRODUCTION

Rivers, estuaries, and reservoirs are generally characterized by large aspect ratios (i.e., length/width or width/depth), allowing effective two-dimensional (2D) modeling of large-scale flow dynamics. However, the three-dimensional (3D) heterogeneity of biota and nutrient fluxes (especially near boundaries) is critical to modeling the evolution of water quality; the cross-channel variability and vertical concentration profiles are as important as the downstream flux. As a result, efficient models of 3D transport are needed for seasonal or annual simulations of flow physics and water quality. The 3D methods presently available are: (1) finite-element methods on triangular or prismatic meshes; (2) finite-difference/volume methods on Cartesian (square) meshes; and (3) finite-difference/volume methods on boundary-fitted curvilinear or unstructured meshes. Each approach has its advantages and drawbacks, the relative importance of which is a subject of dispute in the numerical modeling community and cannot be fully addressed without a treatise. In this paper, we add a new approach that complements the methods already available.

Our new ''perturbation curvilinear'' method is appropriate for narrow sinuous systems and can be seen as a compromise between modeling on a simple square Cartesian mesh or applying a complicated curvilinear grid. The primary drawback of modeling with a Cartesian mesh on a sinuous channel is that the grid is not aligned with the streamwise and cross-stream axes, so a square mesh is required (Fig. 1). For a channel with a width of 200 m, a mesh size of $\sim 10 \times 10$ m is required to obtain reasonable resolution of cross-stream variability. If merely five grid cells are used in the vertical (marginal resolution) and the Cartesian model stores only the ''wet'' cells, the 3D Cartesian representation will require 10^4 grid points for every kilometer of channel. Furthermore, for a downstream velocity of O (0.1) ms^{-1}, a transport algorithm limited by the CFL condition will set the maximum time step ~ 100 s for the 10×10 m grid cells. Thus, modeling a 20 km stretch of channel for a year requires 2×10^5 grid cells and 3×10^5 time steps—a task that is presently only practical on a supercomputer. Some Cartesian models require extra storage and trivial computations for the ''dry'' cells in the system (which increases cell counts in sinuous systems by an order of magnitude); however, cell counts in this paper are based on Cartesian methods that store and compute only the ''wet'' cells.

In contrast to the Cartesian approach, boundary-fitted curvilinear methods (Fig. 2) align the mesh with streamwise and cross-stream flow, allowing use of grid cells with large aspect ratios. The channel width is a reasonable length scale for streamwise flow physics, so in the example above we could apply horizontal curvilinear mesh spacing of 200×10 m. Again considering five grid cells in the vertical, the curvilinear 3D model requires only 500 grid cells per kilometer of channel. Furthermore, the time step could be raised to ~ 30 min (assuming that cross-channel flow rates are small), making the ratio of real time to computational time nominally two orders of magnitude faster than solution on a square Cartesian mesh. The primary drawbacks of this approach are: (1) the construction of a boundary-fitted curvilinear mesh is a difficult and time-consuming process for irregular domains; (2) implementation of curvilinear equations is significantly more complex

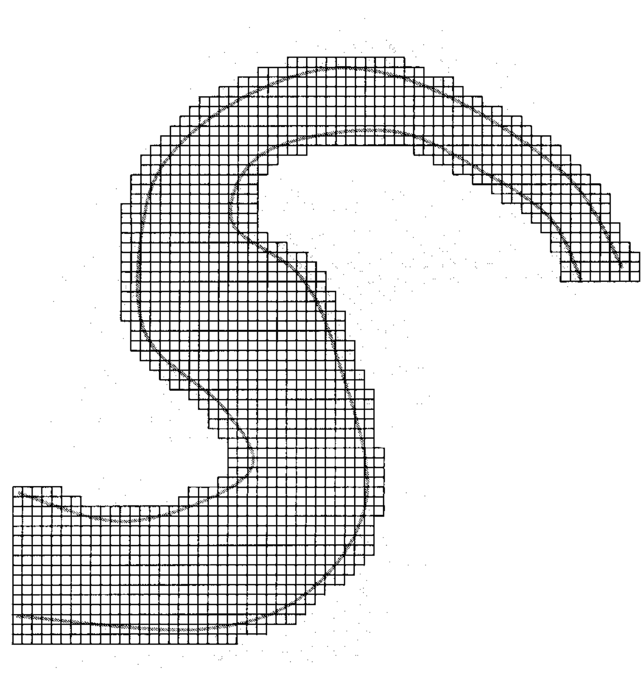

FIG. 1. Channel Bend with Square Cartesian Mesh

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