Reducing Numerical Diffusion Effects with Pycnocline Filter

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Abstract: Numerical or artificial diffusion is the unintentional smoothing of gradients associated with the discretization of the transport equations. In lakes and reservoirs where through-flow is small, the effects of numerical diffusion of mass are cumulative, leading to a progressive weakening of vertical density stratification. This density field misrepresentation precludes accurate, long-term, three-dimensional (3D), hydrodynamic simulations on fixed grids in closed basins with an active thermocline. An ad hoc technique to limit the destratifying effects of numerical diffusion of mass is presented and tested for a 3D, hydrostatic, Z-coordinate numerical model. The technique quantifies the domain-integrated numerical diffusion by assessing the change in the background potential energy E_b. At each time step, the change in E_b associated with numerical diffusion is calculated, then removed using a sharpening filter applied to each water column. In idealized test cases, the filtering technique is effective in maintaining density stratification over one year while undergoing periodic, large-amplitude forcing by internal waves. Forty-day simulations of Lake Kinneret compared to field measurements demonstrate improved representation of density stratification using the filtering technique.

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Introduction

Conservative discrete advection schemes produce numerical diffusion, leading to unphysical gradient smoothing. This irreversible grid-scale averaging of scalar gradients occurs as transported water parcels mix their concentrations. In mass transport for a density-stratified system, the cumulative effect is a slow reduction of the background vertical stratification.

Density stratification in typical lakes is determined by the cumulative competition between stratification by solar heating, wind mixing, and night-time cooling, and destratification by internal mixing processes [see Imberger and Patterson (1990) for a review of lake dynamics]. During the summer, strong density stratification inhibits vertical transport flux (Imberger and Patterson 1990; Wüest et al. 1996, 2000; Saggio and Imberger 2001), with basin-scale internal waves playing an important role in horizontal and vertical transport (Antenucci et al. 2000). There is feedback between internal waves and stratification: Internal waves energize mixing, thereby altering the stratification, which modifies the internal wave field and changes the mixing characteristics (Imberger 1994). Thus, in lake simulations, the misrepresentation of internal waves and mixing causes further degradation of the simulated stratification and internal waves.

In numerical simulations, wind forcing stimulates basin-wide internal seiches which sweep the strong density stratification through the numerical grid resulting in numerical diffusion. Hodges et al. (2000) demonstrated that the duration of simulations of a highly stratified, strongly wind-forced lake, is limited by the accumulation of numerical error in the vertical stratification. It is clear this effect was caused by strong internal wave motions captured by the simulation, leading to enhanced numerical diffusion. There have been papers in the literature demonstrating seasonal simulation of stratified lakes using three-dimensional (3D) models (e.g., Ahsan and Blumberg 1999; Beletsky and Schwab 2001) that when appropriately calibrated, captured the thermal budget and the temperature stratification. However, to our knowledge, none of the existing work has demonstrated a simultaneous representation of large-amplitude internal waves (which cause numerical diffusion) and the seasonal evolution of the thermocline.

In simulations, both vertical and horizontal advection contribute to smearing the pycnocline. The physics of a particular simulation will determine whether vertical or horizontal advection contributes most to overall numerical diffusion. Simulations of linear seiching with simple up-down advection of isopycnals will likely suffer most from numerical diffusion due to vertical advection. In contrast, internal waves may degenerate into steep fronts which propagate across the basin (Farmer 1978; Horn et al. 2001). A consequence of these steep fronts is the advection of horizontal gradients which are grossly underresolved by the coarse horizontal grid size typically used in geophysical modeling and, hence, numerical diffusion due to horizontal advection may dominate. After a wind event, the internal wave field in a lake will contain a superposition of basin-scale, linear, and nonlinear waves with a range of horizontal and vertical mode waves (Saggio and Imberger 1998). Lake simulations are therefore likely to suffer from both vertical and horizontal numerical diffusion, and it is difficult to determine a priori which will dominate. Our goal here is to show that numerical diffusion in lake simulations is inherently multidimensional, but that the end result is a thickening of

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