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IMPROVED TECHNIQUES FOR GRAVITY CURRENT MODELING

Paula S. Kulis and Ben R. Hodges
University of Texas at Austin
Department of Civil, Architectural
and Environmental Engineering
Austin, Texas 78712, U.S.A.
(512) 471-3131
paulakulis@mail.utexas.edu

ABSTRACT

Boundary gravity currents play a key role in circulation for estuaries, bays, lakes and the coastal ocean. Three-dimensional models of large basins using z-level coordinates tend to overestimate entrainment of boundary gravity currents as the stair-step topography leads to artificial mixing. Boundary following models using sigma-coordinates avoid the stair-step problem, but introduce difficulties for the 3D processes in the basin interior and allow artificial entrainment based on changes of the grid cell height where the density current thickness is poorly resolved. Previous researchers have proposed isolating the gravity current physics into a separate depth-averaged model that is coupled to the interior of the 3D z-level model. However, existing methods for coupling the models have adhoc features that are undesirable and, in some cases, may result in non-physical solutions. Approaches to accurately coupling gravity current and three dimensional models are explored, the problems are investigated, and possible solutions are proposed.

INTRODUCTION

Gravity current flows play a major role in circulation of lakes and estuaries. They can be caused by horizontal gradients in density due to salinity, suspended particles, or temperature; a density gradient of only a few percent is required to initiate an underflow [1]. The inflow to a lake or estuary from a river is rarely of the same density of the fluid in the water body itself. This often results in a positively or negatively buoyant plume. These currents also occur when waste is discharged into a water body from any source, and when daily heating and cooling patterns occur unevenly over a lake.

Gravity currents in the environment can introduce pollutants into a water body [2], and tracking their fate is essential to water quality protection. Gravity currents in coastal embayments, lakes and estuaries are a source of density stratification, which increases the energy required to fully mix the system [3], potentially decreasing the rate of replenishment for bottom waters. Conversely, shear-induced entrainment of ambient fluid into the underflow can cause enough mixing to have little impact on the stratification in the water body.

The issue of bottom water replenishment is key to shallow coastal areas like Corpus Christi Bay in Texas, where bottom waters may be episodically hypoxic during the summer in a basin only 3-5 m deep [4]. The disposal of 25 mgd (1.09 m^3/s) of desalination brine into Corpus Christi Bay has been proposed [5], and the brine fate is of interest as there is a question as to whether it will enhance stratification and increase hypoxia duration or extent. We are investigating the proposed discharge impacts on energy requirements for vertical mixing. The present work is based on representative vertical scales of Corpus Christi Bay to provide insight and guidance for future quantitative investigations.

Modeling environmental water bodies requires taking into account many forcings of varied scales. Because of competing requirements for computational power, providing sufficient resolution for a density current of an unknown scale becomes difficult. The stair-step topography in z-coordinate models has been shown to lead to artificial entrainment (e.g. [6,7]). While sigma coordinate grids do not pose this problem, the effects of insufficient vertical resolution in a sigma coordinate model have not yet been quantified. As we show in this paper, sigma coordinate models require fine grid resolution in order to accurately represent underflows.

The z-coordinate problems have been addressed by coupling a 2D bottom boundary layer model to a 3D hydrodynamic model (e.g. [8-10]). The goal of the present work is to establish a need for coupling an underflow model with a sigma-coordinate hydrodynamic model, the Environmental Fluid Dynamics Computer Code (EFDC) of Hamrick [11]. EFDC solves the free surface, hydrostatic, Reynolds averaged equations of motion for a variable density fluid using a Cartesian or curvilinear orthogonal horizontal grid and a sigma-coordinate vertical grid. In this paper, we first explore the grid resolution required in EFDC to capture gravity current motions in an idealized basin with scales based on Corpus Christi Bay. We then discuss consequences on mixing and energetics calculations, and finally propose solutions.

NOMENCLATURE

The following variables and notations are used in this paper:

H	total water depth
g	gravitational acceleration
?PE	change in potential energy
?z	vertical grid height
$?z_o$	grid height at source
??	density difference
?	density
$?_o$	reference density
$?_m$	mixed density

DESCRIPTION OF TEST BASIN

Corpus Christi Bay is shallow, with an average depth of 3.6 meters. We use an idealized basin (Fig. 1) that is uniform in the y-direction to examine the 2D down slope behavior. The basin is 6 km wide in the y-direction, and 12 km in the x-direction. At x=0, the basin is 2 m deep. A linearly sloping bottom extends for 6 km, where the maximum depth of 6 m is reached. From 6 km to 12 km, the basin has a flat bottom. At 0 m in the x-direction there is a dense fluid source from a depth of 1m to 2 m for all values of y, and a simple open boundary is located at the basin's opposing end.

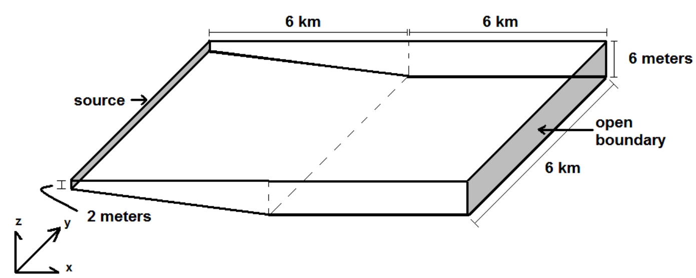

Figure 1: Test Basin.

The estimated flow rate of desalination brine to be discharged into the bay is 25 mgd (1.09 m^3/s), with a salinity of 60‰. If the flow entering our basin has already entrained enough bay water (at 30‰) to double in volume, the flow rate is increased to 50 mgd (2.18 m^3/s) and the salinity of the inflow is reduced to 45‰. This prior entrainment would likely also involve some degree of transverse plume spreading, so we take this flow rate and spread it over 29.2 meters in the y-direction. This lower flow rate is made uniform in the y-direction so that additional transverse spreading effects of the plume can be neglected.

Five vertical grid resolutions were tested. For all tests, horizontal resolution is 400 meters in both x and y-directions, and test duration is 120 hours of constant flow. Vertical resolution is 4, 6, 10, 20 and 30 uniformly-spaced layers. We will refer to these resolutions by their dimensionless ratio ($h/?z_o$) of initial underflow height, h, to layer thickness at the source, $?z_o$. These ratios are 2, 3, 5, 10, and 15, respectively. The minimum eddy viscosity and diffusivity are set at molecular levels of 10^{-6} and 10^{-9} m^2/s, respectively. Horizontal momentum and mass diffusivity was set to zero. At the end of each test, the curtain of salinity and x-direction velocity were taken at 8 km in the y-direction for all values of depth and length (Fig. 2).

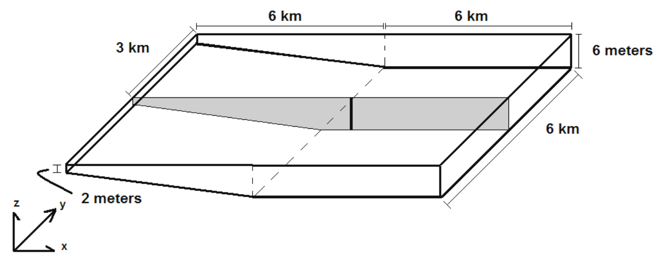

Figure 2: The gray curtain in the basin is the cross-section of data taken from EFDC for analysis. The black vertical line shows the location where the vertical profile was taken for the energy analysis.

EFDC TEST RESULTS

The model results are shown in Fig. 3. Contours of buoyancy, defined as $??/?_o$, where $?_o$ is the initial density of the basin and ?? is the density difference between $?_o$ and the density of a cell. From visual inspection we see that entrainment decreases as h/?z is increased from 2 to 15. This result suggests that the entrainment seen in Fig. 3a, 3b and 3c is a result of numerical error. Relative to the differences between Fig. 3a and 3d, the difference between Fig. 3d and 3e is small, indicating that the resolution provided in Fig. 3d (at $h/?z_o$ of 10) relative to the underflow size is representative of reasonable resolution of the bulk dynamics of the underflow. Furthermore, the underflow thickness in the flat portion of the basin is approximately 1.5 meters in depth in Figure 3d, and therefore 5 vertical layers resolve the underflow. Dallimore et al. [10] also presented the idea that 5 vertical layers was enough to resolve an underflow in a z-coordinate grid.

ENERGY ANALYSIS

The impacts of vertical grid resolution are made apparent by an analysis of mixing energetics. If we want to know how much external energy is needed to mix the water column, we must examine the potential energy required for "lifting" the dense fluid and mixing it with the ambient fluid. The ability of a water body to meet this energy requirement gives us information on the fate of the fluid in the underflow.

The ideal minimum mixing energy required per unit area in the x and y directions is the difference in potential energy before and after complete mixing:

$$\Delta PE = \frac{1}{2} g ?_m H^2 - \sum_{i=1}^{N} \frac{1}{2} g ?_i \left[z_{t,i}^{\ 2} - z_{b,i}^{\ 2} \right] \qquad (1)$$

where $?_i$ is the density in kg/m^3, z_t and z_b are the elevations at the top and bottom of each cell layer, respectively, N is the total

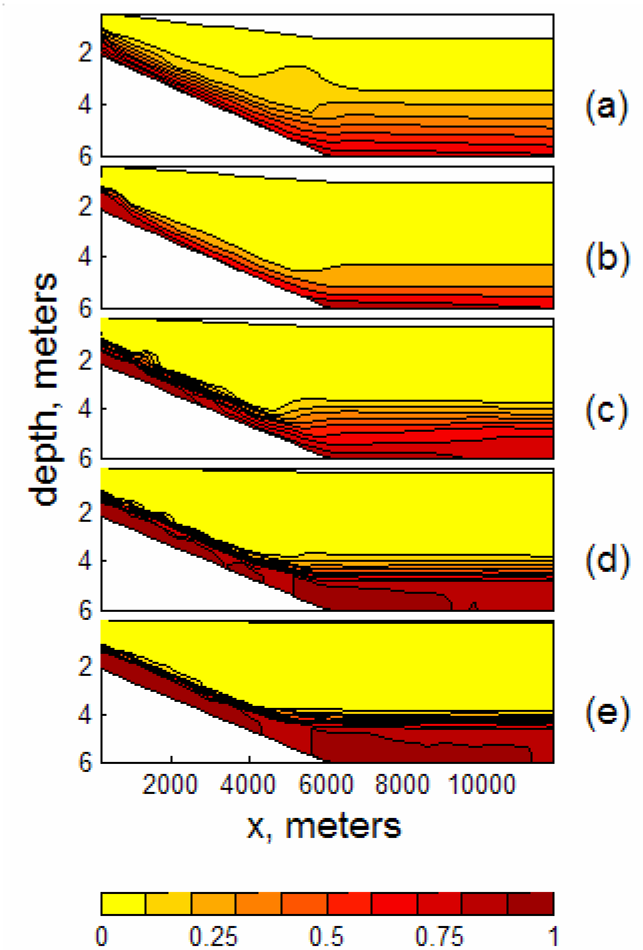

Figure 3: Buoyancy contours at a cross section in x-z plane after 120 hours for h/?z values of (a) 2, (b) 3, (c) 5, (d) 10 and (e) 15.

number of layers, ?$_m$ is the depth-averaged density, and H is the total water depth. This change in potential energy must come from turbulent kinetic energy (tke). Only a fraction of the energy from an external source is converted to tke, and only a fraction of the tke produced actually performs lifting, or conversion to PE.

We assume that the initial kinetic energy of the water column available for mixing is negligible. To prove that this assumption is valid, we calculate mixing energy assuming that all kinetic energy in the water column is used for mixing as a "worst-case scenario". This assumption results in an energy difference of less than 2% from the calculation not taking kinetic energy into account. Because our calculation of energy requirements makes many simplifications, a 2% error is not considered significant.

The results of this analysis are presented in Fig. 4. Energies are presented as ?PE/? E_{ref}, where ? E_{ref} is the energy needed to achieve the potential energy difference between a column of entirely ambient fluid at 30‰ and a column of entirely underflow fluid at 45‰. We see that the difference in energy required by the h/? z_o=15 and h/? z_o=10 test cases is only 1.4%. As stated previously, an error of 1.4% is not significant relative to other simplifications made in this analysis. For example, we do not calculate losses of tke in the lifting process or losses of energy in the production of tke. Therefore, a finer grid resolution will not significantly improve our ability to calculate mixing energy requirements. Therefore, for the sake of comparison in this paper, we refer to h/? z_o =10 as the fully resolved case. A 16% increase in the energy required for mixing was seen from h/? z_o of 2 to 10. This difference suggests that erroneous modeling of gravity currents can lead to further erroneous nutrient or pollutant fate and transport calculations.

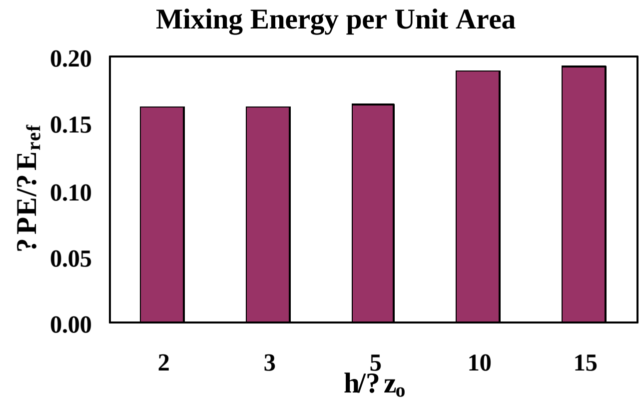

Figure 4: Energy requirements for mixing the water column at different grid resolutions.

CONCLUSIONS AND FUTURE WORK

While sigma coordinate models have been shown in the past to more accurately represent dense underflows than z-coordinate models, we have shown through an examination of EFDC that instances exist in which sigma coordinates have short-comings in capturing gravity current physics. The resolution required to capture an underflow is approximately 5 vertical layers within the height of the underflow, which requires a sigma coordinate model to utilize a very fine resolution for capturing thin underflows. Poorly resolved underflow model results can cause inaccurate calculations of mixing energy. According to these results, a model testing the fate of desalination brine in Corpus Christi Bay with a low grid resolution could inadvertently predict full mixing of the plume given a moderate wind.

A test of modeling such a wind forcing on a steady state gravity current at varied grid resolutions would be interesting, and would also validate the approximations that we have made in this paper. The energy calculations presented in the present work are "back of the envelope" and make many assumptions, such as neglecting kinetic energy in the water column. However, these preliminary calculations give estimates of scales for Corpus Christi Bay and indicate the need for further investigation into this topic.

We propose a more in depth energy analysis relating mixing energy to wind forcing, and examining slope dependency on grid resolution implications for results. In addition, the results from the present work encourage the concept of coupling a 2D submodel based on others developed for sigma coordinates with sigma coordinate models. Coupling a depth-integrated, separate model with a sigma coordinate hydrodynamic model would make underflow resolution independent of vertical grid size.

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