MODEL BATHYMETRY FOR SINUOUS, DENDRITIC RESERVOIRS

BRIDGET WADZUK BEN R. HODGES

Department of Civil Engineering
University of Texas
Austin, Texas USA

ABSTRACT

A method for straightening bathymetries is proposed to facilitate a simplified form of three-dimensional (3D) modeling in sinuous, dendritic reservoirs. This method neglects the sinuosity of the reservoir, while preserving the 3D topology and the general relationship between water volume in littoral and main basin regions. This approach allows a model to capture the critical 3D effects of shallow littoral regions, while retaining simple Cartesian grids and methods.

INTRODUCTION

Reservoirs formed by drowned river valleys are often approximated as two-dimensional (2D), laterally-averaged hydraulic systems due to their long and narrow character. This approximation neglects both the sinuous and dendritic natures of many reservoirs, allowing 2D models to provide seasonal and annual simulations of reservoir thermal structure (i.e. the CE-QUAL-W2 model, Bales and Giorgino 1998, Garvey *et al* 1998). From a hydraulic perspective, this neglect is reasonable and effective as long as there are (1) low flow rates through the reservoir and (2) small volumes in the dendritic coves and sidearms. In contrast, water quality is strongly influenced by the spatial characteristics, transport and thermal gradients of the littoral region, which places importance on the shallow dendritic volumes. However, there is no evidence to suggest significant impact on water quality from sinuosity, so our hypothesis is the next step of improvement from 2D to 3D modeling requires capturing the exchanges between littoral regions and the main basin while continuing to neglect the curvature of the system. Developing a suitable "straightened" bathymetry for a 3D model that neglects curvature is the subject of this paper.

THE CASE FOR MODELING STRAIGHTENED RESERVOIRS

Numerical models based on Cartesian grids are generally the easiest to apply to any system. However, such models require square grid cells for a sinuous reservoir, resulting in an impractically large number of grid cells. As an example, Lake Travis (Figure 1), a reservoir formed by the Mansfield dam on the (Texas) Colorado River near Austin, Texas, USA, has a maximum depth of 156m, with a surface area of 18,929 acres. If we use a square Cartesian grid of 530×530×2 meters, 110000 grid cells are required to represent the interior volume of the reservoir. This would be too large for most practical applications using present desktop computers. As the reservoir is long and narrow, the number of grid cells could be reduced by

To appear in proceedings of the 6th International Workshop on Physical Processes in Natural Waters, June 27-29, Girona, Catalonia,Spain.

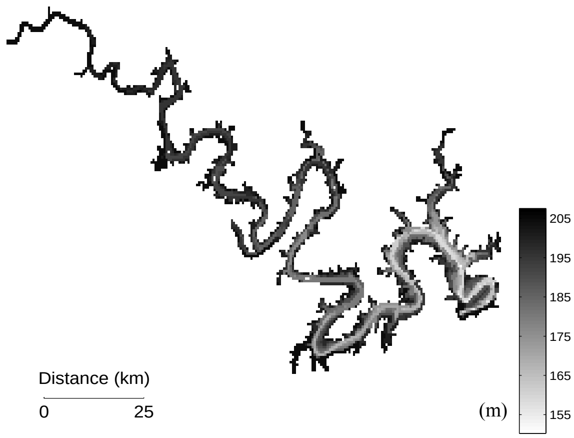

Figure 1 – Bathymetry of Lake Travis

using rectangular grid cells that are aligned with the channel centerline. Effectively, the reservoir needs to be "straightened," as shown in Figure 2. This approach was successfully used in a model of Lake Burragorang, Australia (Hodges *et al* 2000).

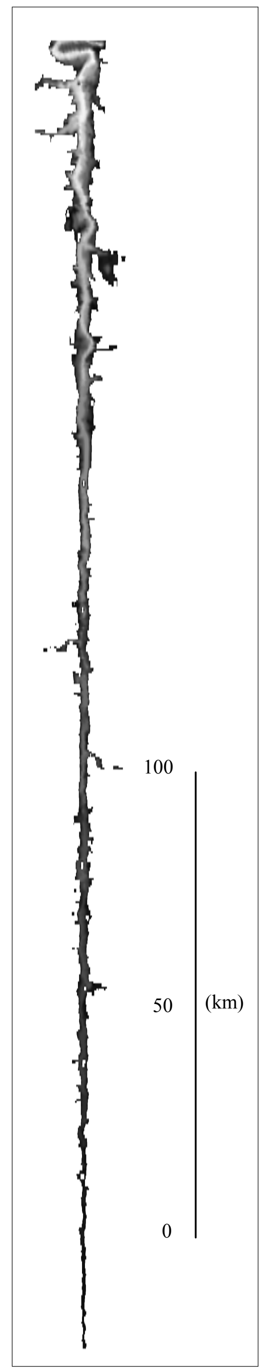

Figure 2 – Straightened bathymetry

MATHEMATICAL FOUNDATION FOR STRAIGHTENING

A straightened reservoir (Figure 2) has a 1:1 correspondence between radial and arc-length coordinates (r,s) of the straightened representation and the true physical space coordinates (x,y). The straightened grid can be displayed in physical space by using mappings of $r(x,y)$ and $s(x,y)$ as shown in Figure 3. Straightening is identical to a curvilinear transformation where the "straightened" representation is taken to be the "computational space" typical of curvilinear methods (e.g. Thompson *et al.* 1985). While curvilinear numerical methods are readily available, they add an extra level of complexity to modeling that may not be warranted given the truncation error associated with the coarse grid resolution used in many reservoir models. It can be shown (Hodges and Imberger 2000) that a system with a radius of curvature (R_0) and channel width (δ) has curvilinear transformation terms which are of order δ / R_0 or smaller, which is significantly less than unity for sinuous systems. Thus, the Cartesian equations of motion on a straightened grid are an

Figure 3 – Straightened bathymetry in physical space

approximation that neglects these small curvilinear terms. However, in addition to neglecting curvature, the straightened representation distorts the volume of individual grid cells. A simplified method for accounting for this effect in a Cartesian model is presently under investigation.

AN APPROACH TO STRAIGHTENING

Straightening a reservoir can be done in a methodical manner to preserve the topology and approximate the volumes. We have developed a Matlab® code to allow interactive straightening of a reservoir. The methodology consists of six steps:

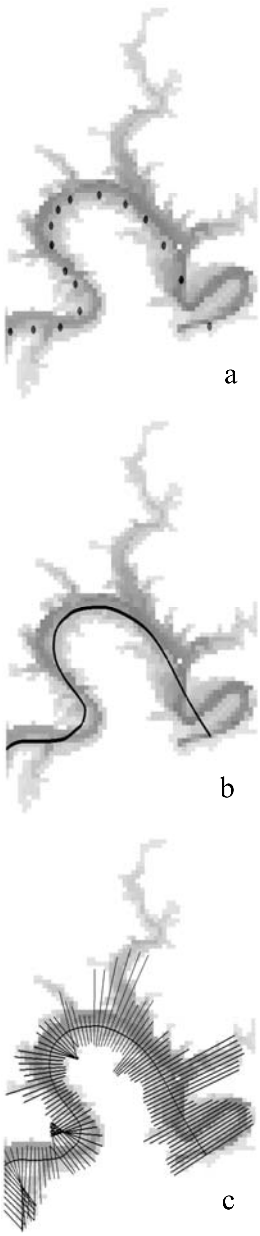

Figure 4 – (a) Centerline points, (b) smooth curve, (c) radial lines.

1) Select points to represent the approximate channel centerline – Figure 4a,
2) Spline the points to obtain a smooth curve – Figure 4b,
3) Compute radial lines, which are normal lines to the spline at selected intervals – Figure 4c,
4) Check for singularities where radial lines cross - Figure 5a, b,
5) Repeat steps 1 through 4 until there are no singularities,
6) Use selected intervals along radial lines to obtain bathymetry at grid points.

To begin this process, a smoothly curved line is needed to use as the "centerline" for straightening. This line, with the radial coordinate $r = 0$, is not necessarily the true geometric center of the channel or the line following the greatest depth (i.e. the thalweg), which may have sharper curvatures than the main body of the reservoir. Instead, the straightening centerline is a smoothly splined line with normal (i.e. radial) lines that do not cross within the bathymetry. Thus, for the bend of Lake Travis shown in Figure 5a, the selected points of the centerline are unacceptable, while a more acceptable set of points is shown in Figure 5b.

The radial lines are extended to a predetermined maximum value, which results in lines that may cross the bathymetry multiple times. This leads to a straightened bathymetry with spurious values. We use interactive graphics to allow the user to 'box' the spurious points in the straightened bathymetry, selecting them for deletion. This allows easy removal of more the 90% of the spurious points. To get points that are more difficult to identify, we allow interactive "clipping" of the radial lines. This procedure is more time consuming, but allows greater accuracy.

The user selects an arc-length grid scale, Δs, (i.e. along the spline), and a radial grid spacing, Δr. The bathymetry at each position (0,s) is obtained for positions separated by Δs along the centerline spline. The $r(x,y)$, $s(x,y)$ location of all the intersections for other grid lines in the straightened bathymetry is determined by simply stepping out along each of the radial lines using a radial grid separation, Δr. Using this approach, the radial distances are identical to physical

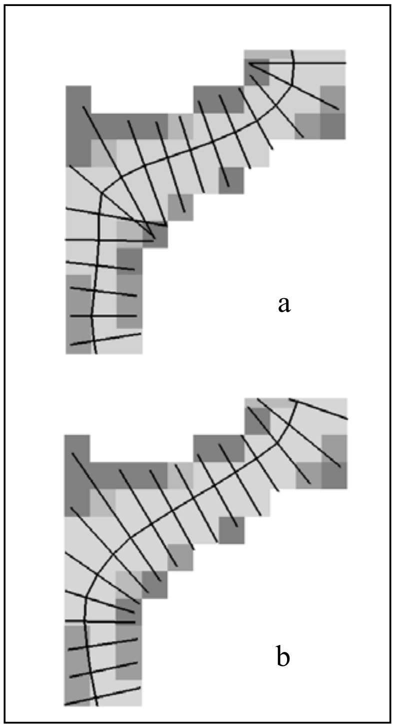

Figure 5 – (a) Unacceptable crossing radial lines, (b) acceptable radial lines.

distances. Thus, points P_1 and P_2 with corresponding straightened and physical coordinates $P_2(r_1,s)$, $P_1(r_2,s)$ and $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ have physical and radial distances related by

$$r_2 - r_1 \equiv \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2} \qquad (2)$$

However, the arc-length coordinates are only physical distances along the centerline, r = 0. At any other r location, the physical distance between two points, s_1 and s_2 along the arc length is related to the physical distance by the ratio of the radius of curvature $R(s,r)$ for the points and the radius of curvature along the centerline, R_0.

$$\frac{R(s,r)}{R_0}\left(s_2 - s_1\right) = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2} \qquad (3)$$

The volume distortion caused by the straightening can be approximated by the curvature ratios.

CONCLUSION

The curvilinear approach of Hodges and Imberger (2000) can be used as a rigorous basis for developing "straightened" bathymetries for modeling sinuous reservoirs. The proposed procedure for straightening reservoirs uses an interactive graphics program to set the centerline for straightening and identify spurious bathymetry additions that arise from radial point definitions. We are presently investigating methods to automate this methodology and allow bathymetry straightening in a completely mechanistic manner using GIS tools.

ACKNOWLEDGEMENTS

The bathymetric data used in this analysis was supplied by the Lower Colorado River Authority of the state of Texas (http://www.lcra.org). Funding for this research has been provided by the Vice President for Research, the Vice President and Dean of Graduate Studies, and the College of Engineering at the University of Texas, Austin.

REFERENCES

Bales, J.D. and M.J. Giorgino, (1998). "Dynamic modeling of water-supply reservoir physical and chemical processes," *Proceedings of the First Federal Interagency Hydrologic Modeling Conference*, April 19-23, 1998, Las Vegas, NV: Subcommittee on Hydrology of the Interagency Advisory Committee on Water Data, p. 2-61 to 2-67.

Garvey, E., J.E. Tobiason, M. Hayes, E. Wolfram, D.A. Reckhow, and J.W. Male (1998). "Coliform Transport in a Pristine Reservoir: Modeling and Field Studies," *Water Science Technology*, Vol. 37, No. 2, p137-144.

Hodges, B.R., J. Imberger, B. Laval, and J. Appt, (2000). "Modeling the Hydrodynamics of Stratified Lakes," *Hydroinformatics 2000*, Iowa Institute of Hydraulic Research, 23-27 July 2000.

Hodges, B.R. and J. Imberger (2000), "A Simplified Curvilinear Approach for Modeling Sinuous Systems," (submitted to *J. Hyd. Engrg*.).

Thompson, Joe F., Z.U.A. Warsi, and C. Wayne Mastin, (1985) *Numerical Grid Generation*, North Holland.