Supplement to CE319F Lecture on Bernoulli Equation

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Bernoulli Equation

$$-\frac{\partial}{\partial s}(p+\gamma z)=\rho a_s=\rho V\frac{\partial V}{\partial s}=\frac{\partial}{\partial s}\left(\rho\frac{V^2}{2}\right)$$

$$\frac{\partial}{\partial s}\left(p+\gamma z+\rho\frac{V^2}{2}\right)=0$$

$$p+\gamma z+\rho\frac{V^2}{2}=\text{Constant (along streamline)}$$

or (after dividing by γ)

$$\frac{p}{\gamma}+z+\frac{V^2}{2g}=\text{Constant}$$

So, for points 1 & 2 on the same streamline:

$$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}$$

Assumptions:

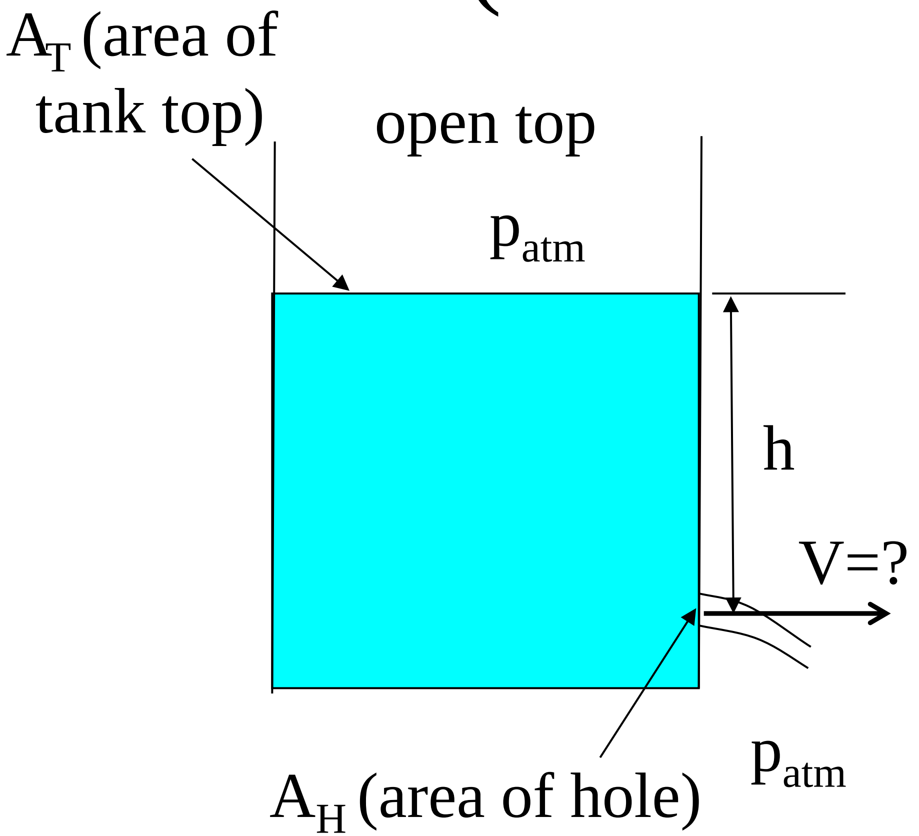
- ***Along streamline, s***
- ***Steady flow***
- ***Incompressible fluid***
- ***Inviscid flow***

$$\frac{p}{\gamma}+z=\textit{Piezometric head}$$

$$\frac{V^2}{2g}=\textit{Velocity (dynamic) head}$$

Note for $V_1=V_2=0$ we recover law of hydrostatics for incompressible fluid.

Velocity of fluid out of hole in tank (same as Example 4.4)



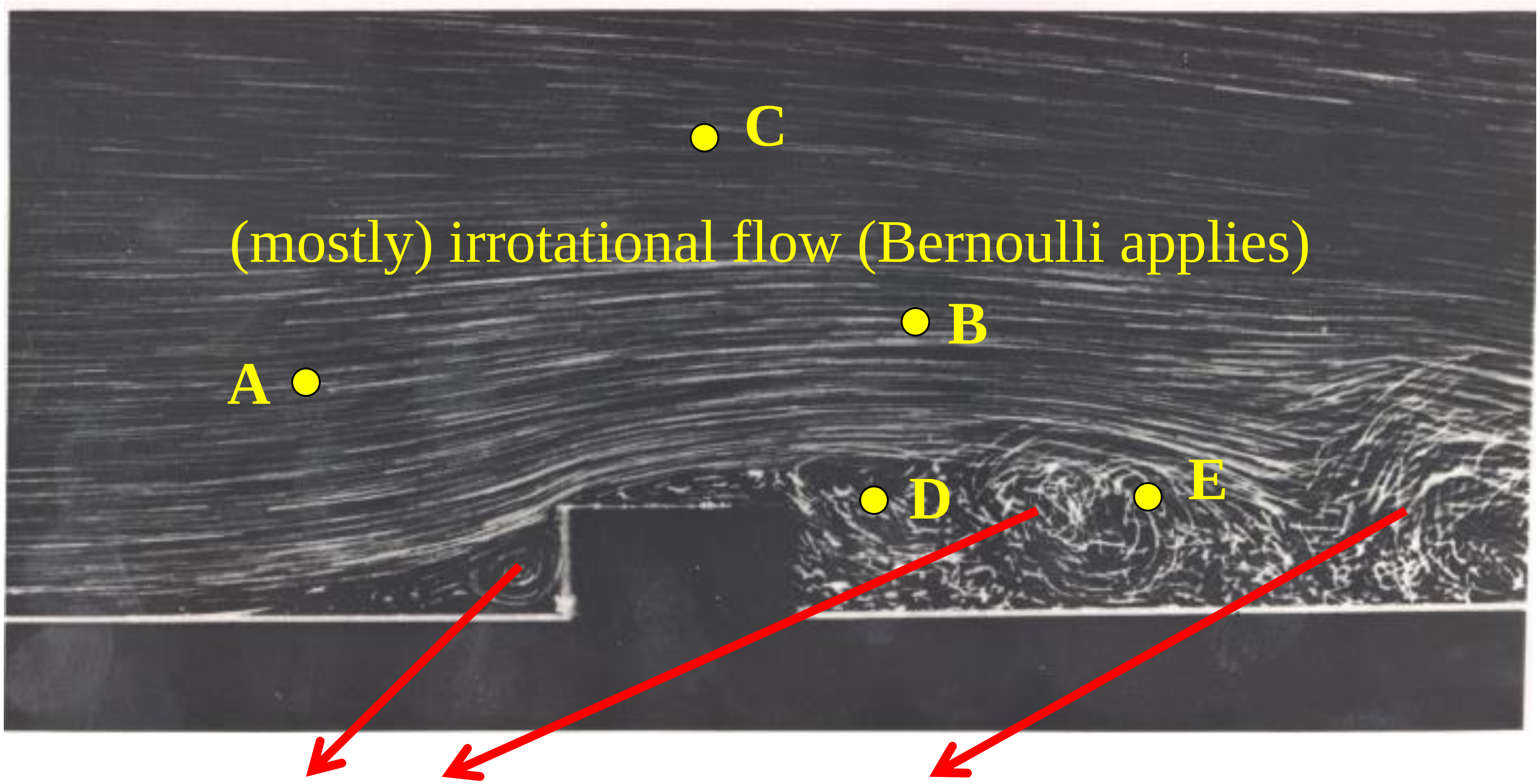
Q1) V=? If tank top open

Q2) V=? If tank top closed

Answers to be presented in class and in the lab!

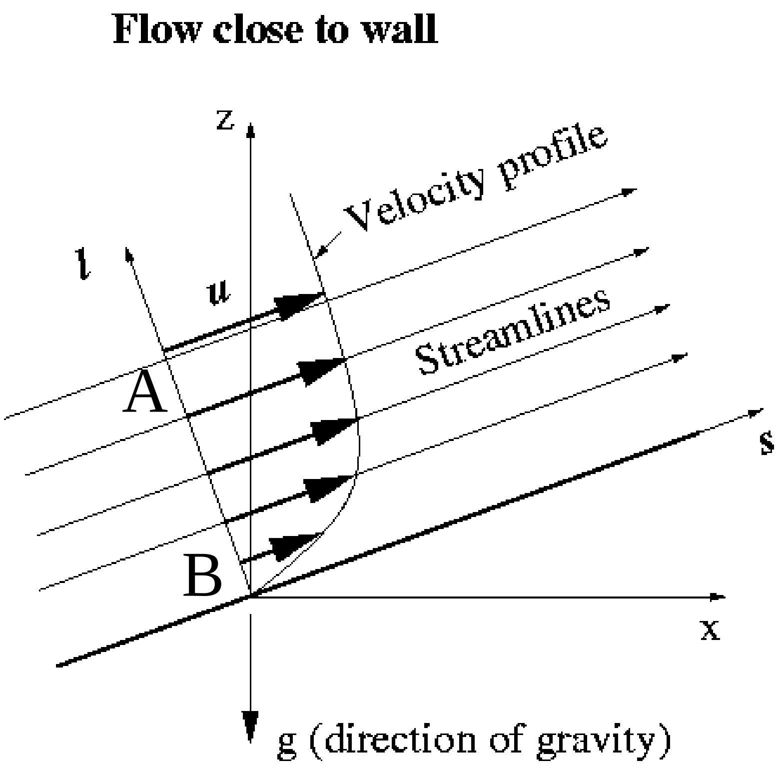
Q1: Can Bernoulli equation apply between points which do NOT belong to the same streamline?

- Bernoulli applies among points A, B, C, where flow is IRROTATIONAL
- Bernoulli does NOT apply between points D & E, where flow is ROTATIONAL



Highly **rotational** flow (where eddies, swirl, vorticity are formed)

Q2: How pressure varies along lines normal to the direction of parallel flow?



Derivation to be presented in class!

- **Bernoulli does NOT apply between points A & B (flow is rotational and viscous)**
- **Instead the hydrostatic law applies between points A & B:**

$$p_A + \gamma z_A = p_B + \gamma z_B$$

Stagnation Tube (p. 139 of Textbook)

Method for relating pressure measurement to velocity

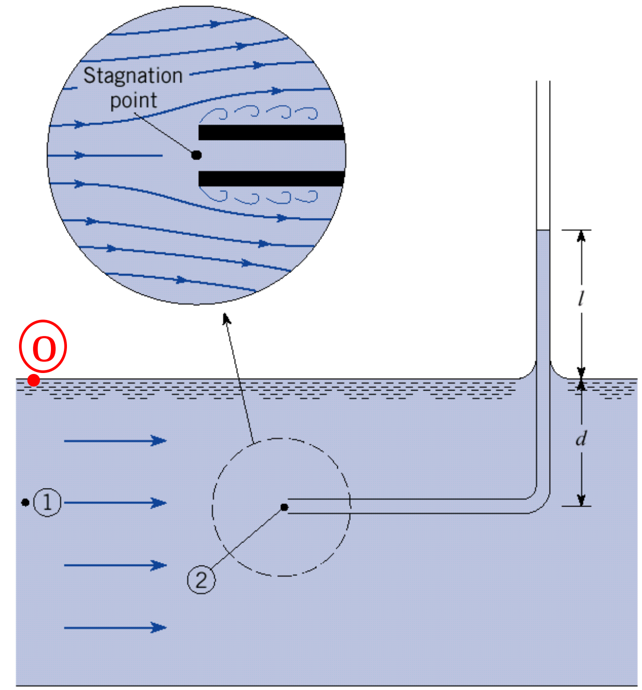
$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma}$$

$$V_1^2 = \frac{2}{\rho}(p_2 - p_1)$$

$$= \frac{2}{\rho}(\gamma(l+d) - \gamma d)$$

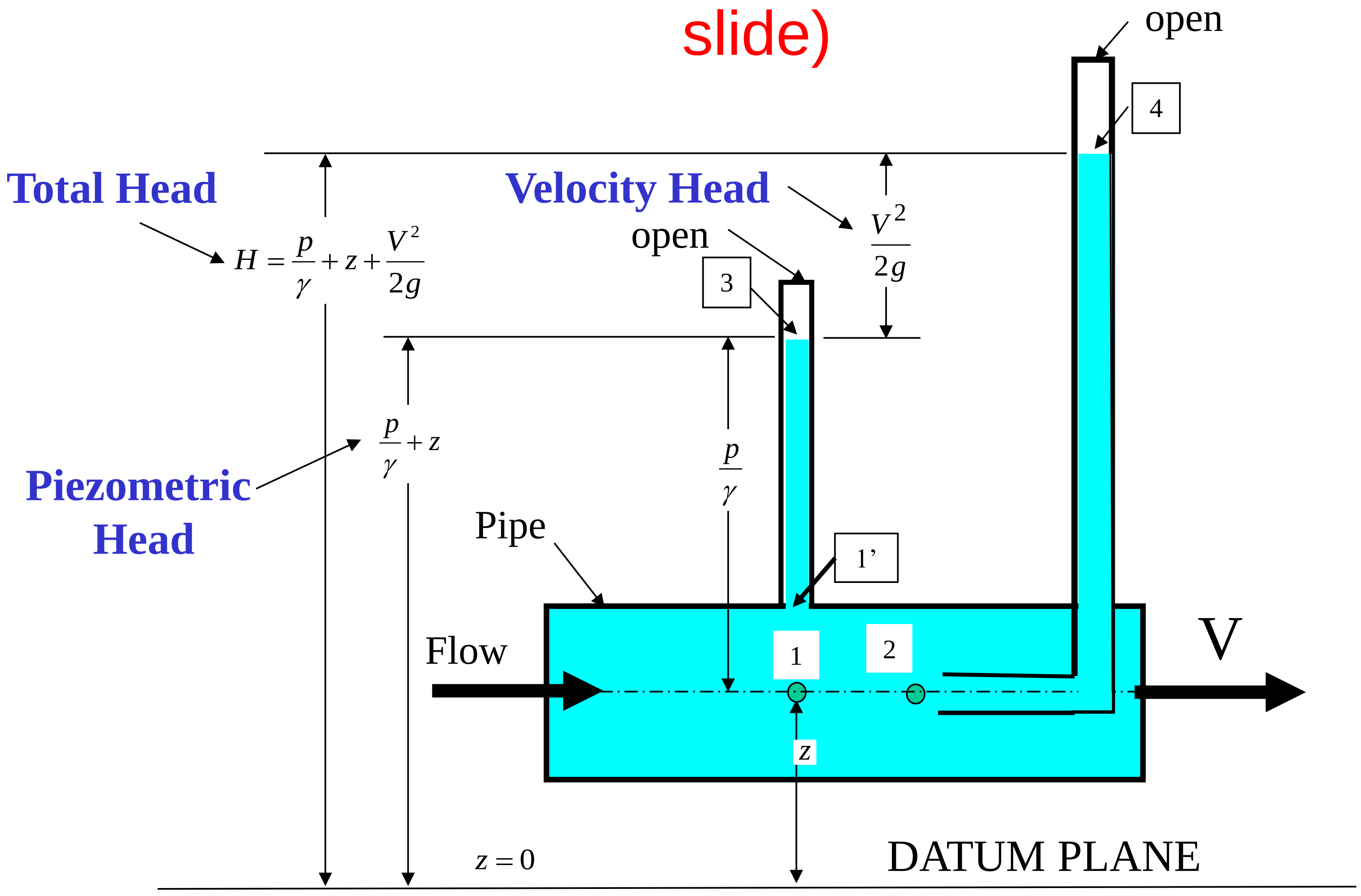
$$V_1 = \sqrt{2gl}$$



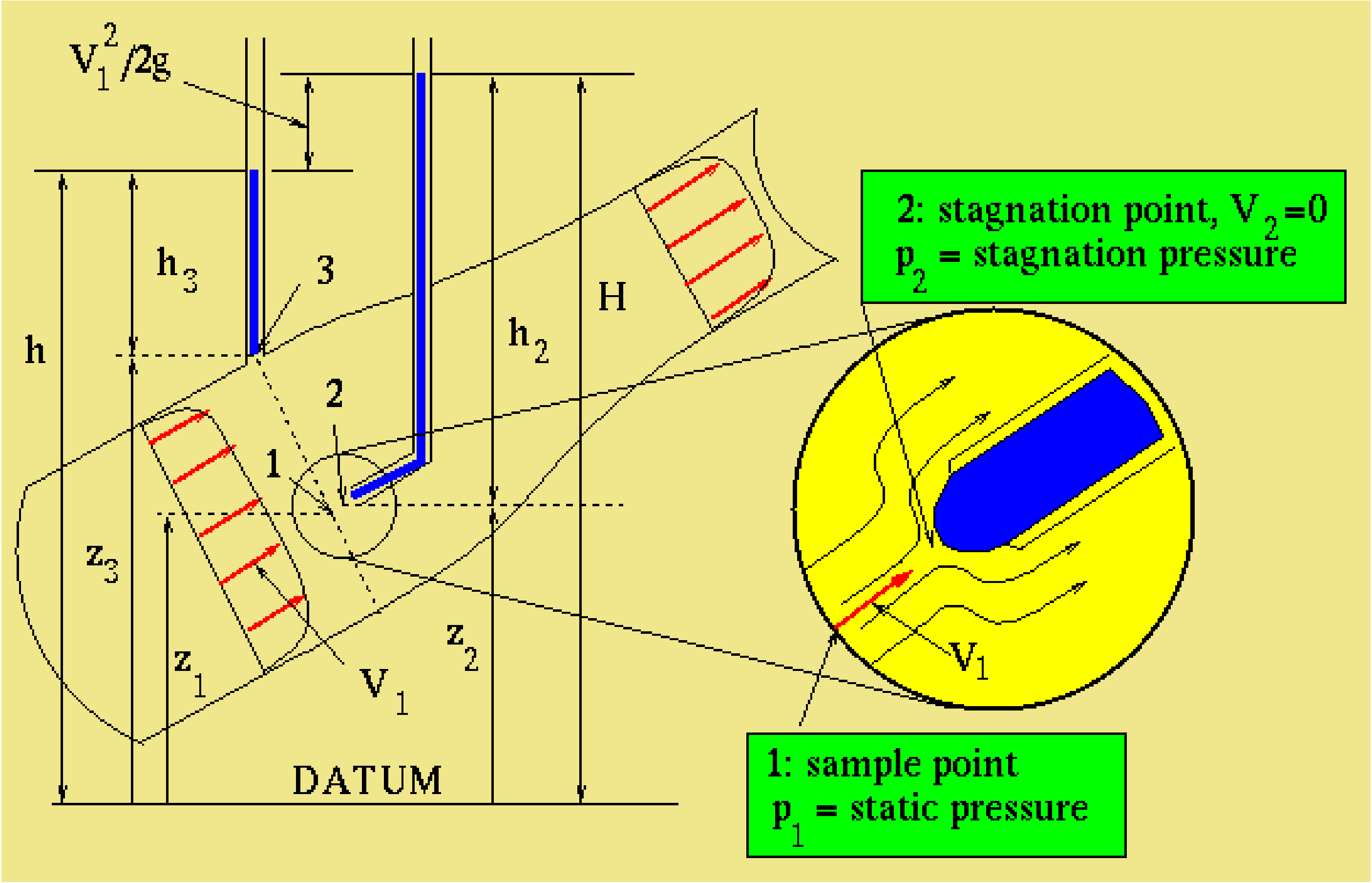
Pressures between points 1 and o (free surface) are related via HYDROSTATIC LAW (WHY?)

Stagnation Tube in a Pipe

(presented for a more general case in the next slide)



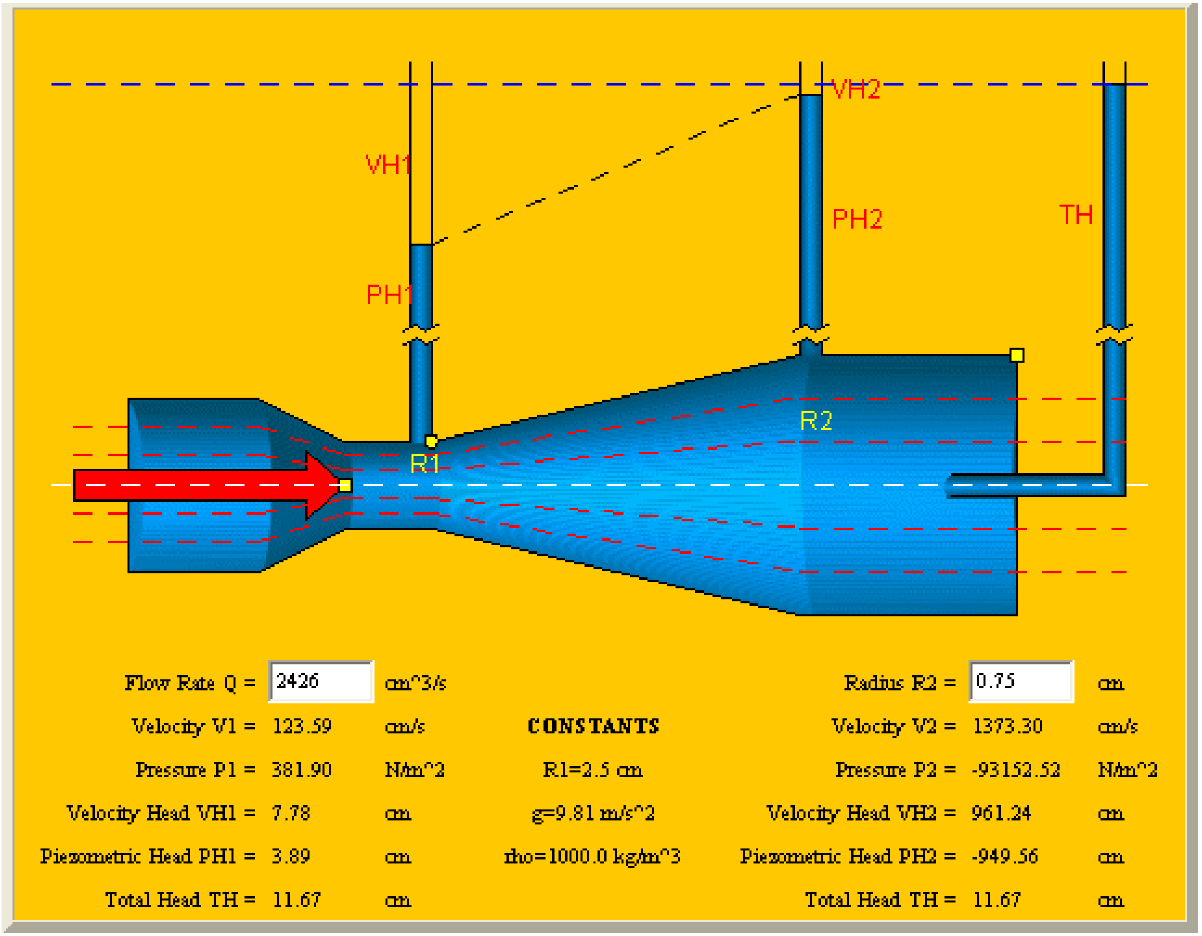
Measurement and physical meaning of total head (H) and piezometric head (h)



For details see:

http://cavity.ce.utexas.edu/kinnas/COURSES/ce319/ebook/head/head.html

The Venturi Applet



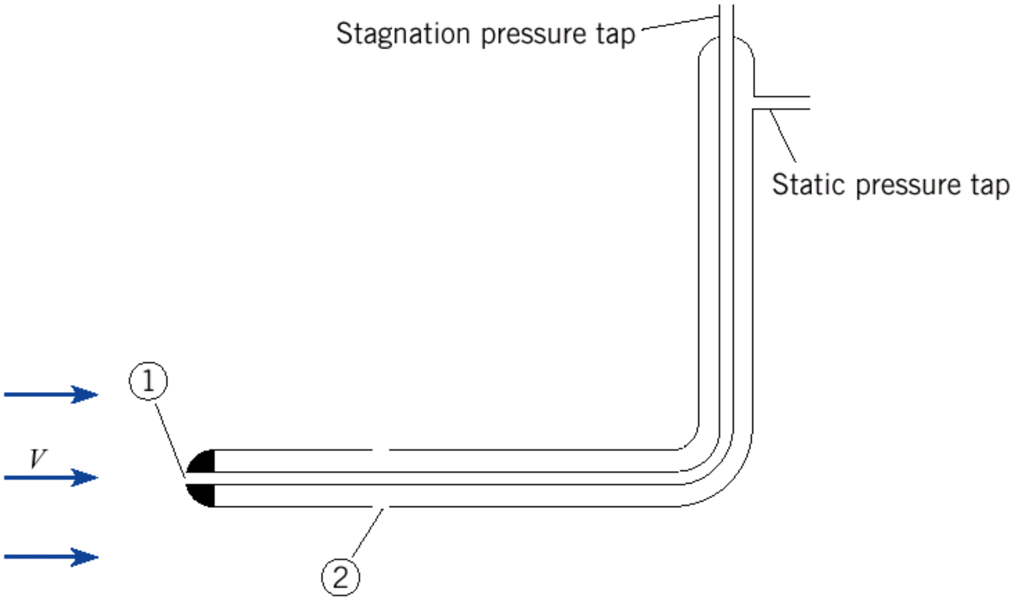
http://www.ce.utexas.edu/prof/kinnas/319LAB/fr_tool.html

Definition of Stagnation point and Stagnation pressure

Q: What pressure would you feel on your palm (or forehead!) if you stick your hand (or head!) out of the window of a car moving at 65 miles/hour, and place it vertical to the direction of motion?

Solution to be presented in class!

Pitot Tube

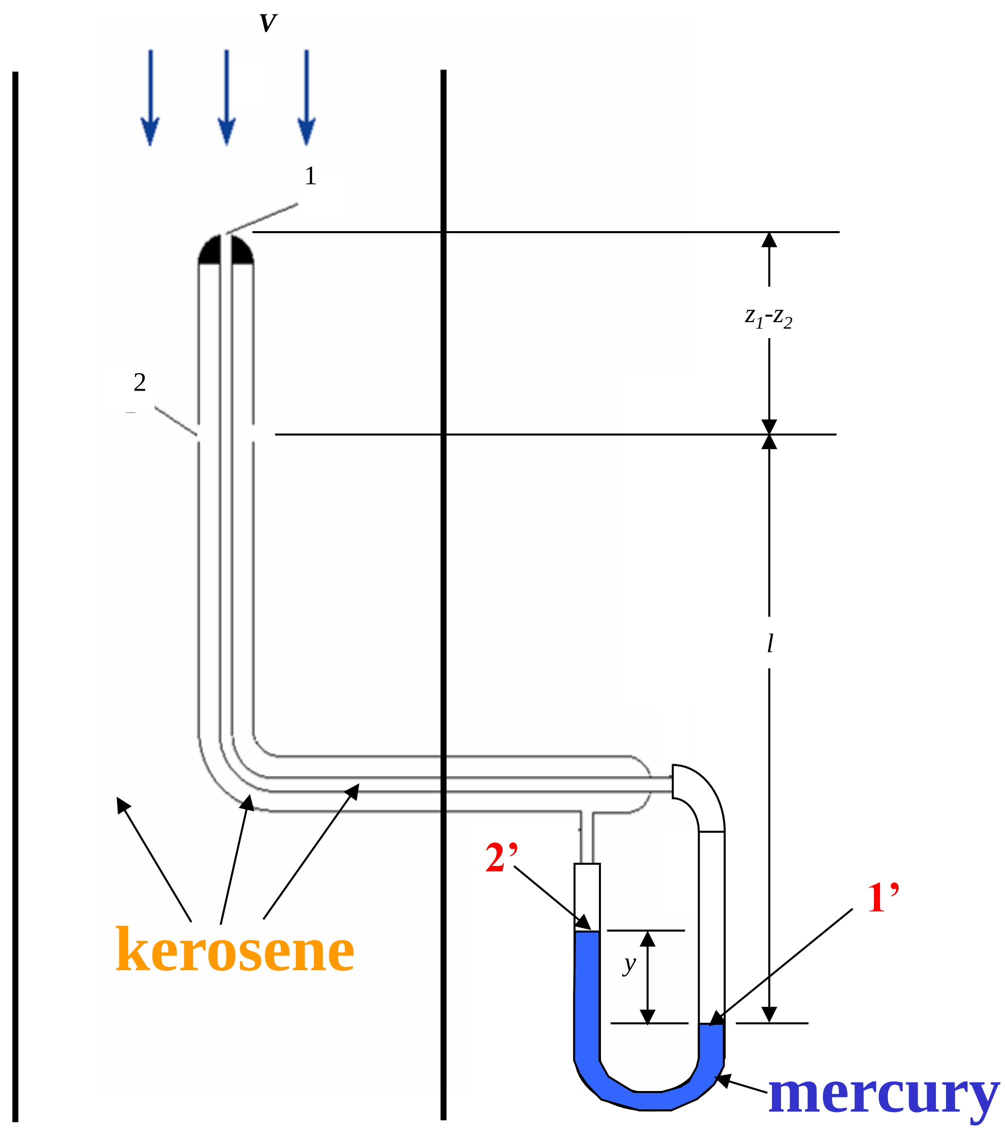


• Very important, easy to use (and cheap!) velocity measurement device.

• Derivation to be done on the board (also given in web-out)

Example 4.7 of Textbook

A **mercury**-**kerosene** manometer is connected to the Pitot tube as shown. If the deflection, y, on the manometer is 7 inches, what is the kerosene velocity in the pipe? Assume that the specific gravity of kerosene is 0.81.

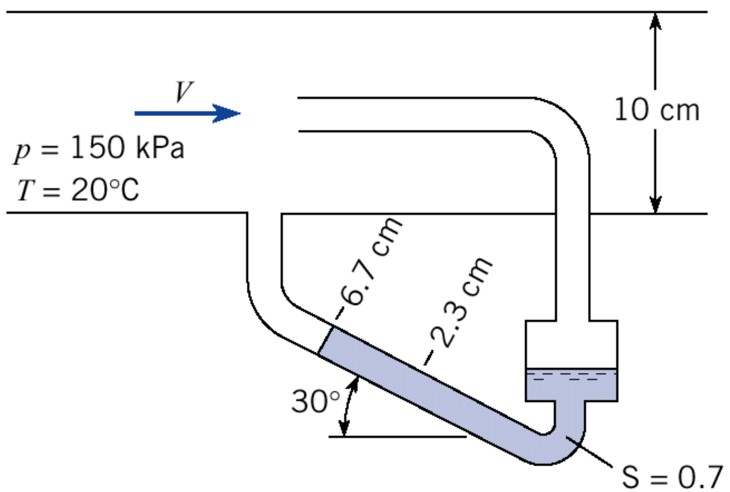


Example (Prob. 4.70 of Textbook)

A tube with a 2 mm diameter is mounted at the center of a duct conveying air. The well of manometer fluid is large enough so that level changes in the well are negligible. With no flow in the duct, the level of the slant manometer is 2.3 cm. With flow in the duct it moves to 6.7 cm on the slant scale.

Find the velocity of air in the duct.

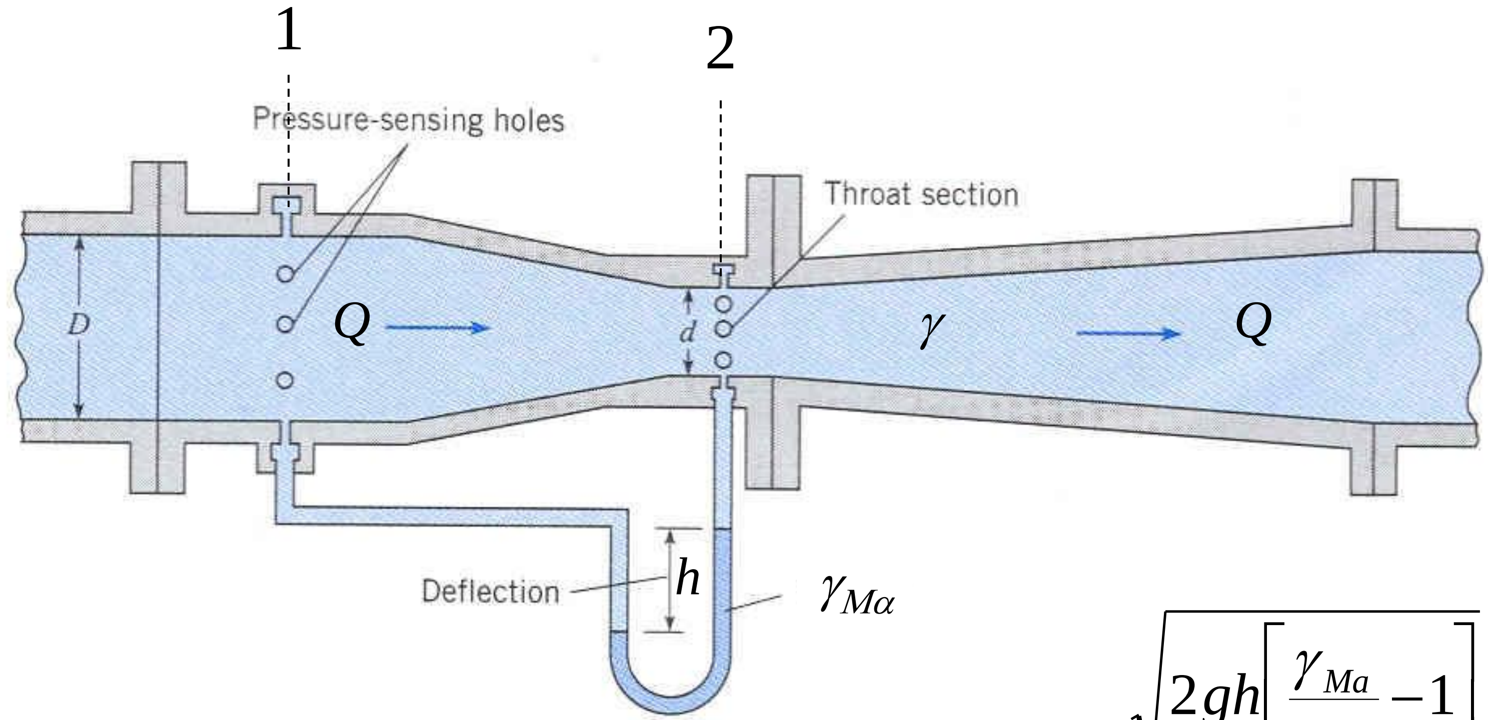
IMPORTANT NOTE: The given pressure of air inside duct is ABSOLUTE. You must use absolute pressure of gas when you apply ideal gas law!



Solution to be presented in class!

The Venturi meter

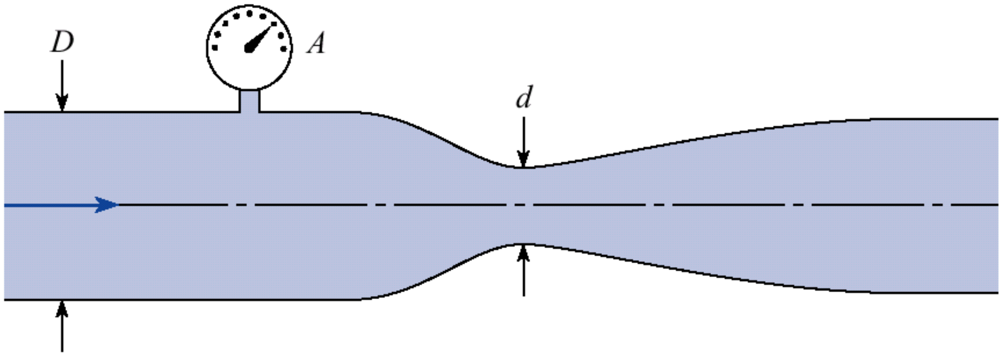
Is a device for measuring flow-rate Q (see p. 494 of the textbook)



Show that: $$Q = \frac{\sqrt{2gh\left[\frac{\gamma_{Ma}}{\gamma} - 1\right]} A_2}{\sqrt{1 - \left(\frac{d}{D}\right)^4}}$$

Example – Venturi & Cavitation

When gage A reads 120 kPa gage, cavitation just starts to occur in the venturi meter. If D = 40 cm and d = 10 cm, what is the water discharge in the system for a condition of incipient cavitation? The atmospheric pressure is 100 kPa. The water temperature is 10 ^{o}C. Neglect gravitational effects.

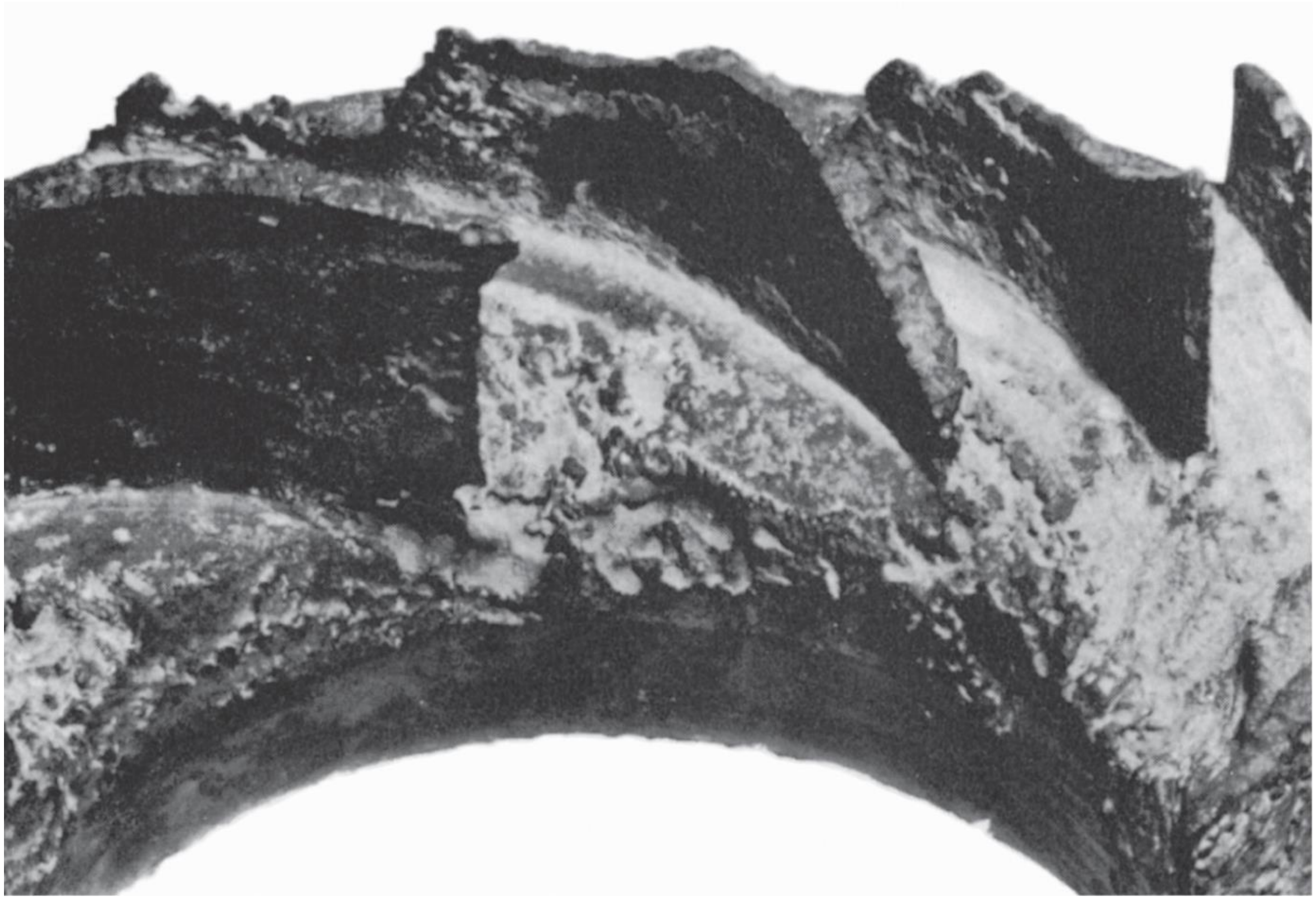


NOTE: Cavitation occurs when the ABSOLUTE pressure reaches vapor pressure, p_v , for the given temperature of operation

Picture of **cavitation** on the upper (suction) side of a hydrofoil placed inside a cavitation tunnel (flow goes from left to right)

For more pictures/theory check UT's Cavitation Home Page at **http://cavity.ce.utexas.edu**

Cavitation damage on impeller of a pump (taken from 9th edition of textbook)



Cavitation damage on dam spillway tunnel (taken from textbook, p. 193)