

Name \_\_\_\_\_ Date \_\_\_\_\_ Lab time \_\_\_\_\_

**DIMENSIONAL ANALYSIS APPLIED TO DRAG ON SPHERES**

- 1) The benefit of using dimensional analysis to reduce the number of variables in a problem will be illustrated using drag on spheres. Because of the limitations of the laboratory equipment, only low Reynolds numbers can be used. Nevertheless, as the lab handout shows, five dimensional variables can be combined into only two dimensionless variables.

In the experiments, spheres of different diameters and different densities (two variables) will be dropped in fluids with different densities and different viscosities (two more variables). As a result, there will be different fall velocities (the fifth variable). All of the variations can be represented with only two dimensionless variables.

The viscosity of the two fluids will first be calculated using measurements from falling ball viscometers. The time to fall in the viscometers will be measured, and the viscosity can be calculated using the fluid density, density of the sphere, and viscometer constant (which accounts for the additional drag on the sphere due to the small diameter of the tube). *Note that the viscometer time to fall must be in minutes for the viscosity calculation.* Once the viscosity is known, the fall velocities will be measured in the larger columns of fluid. These velocities together with properties of the fluids and the spheres will be used to calculate the drag coefficients and Reynolds numbers (the two dimensionless variables, which are defined in the lab handout). The notes and table below can be used for calculating the drag coefficients ( $C_D$ ) and the Reynolds numbers ( $Re$ ). These values are then to be plotted on a graph of standard values for comparison and the percent difference between the measured  $C_D$  and the calculated  $C_D$  for the same  $Re$  is to be calculated for each measurement.

Information from TA:

$D_s$  = diameter of spheres

$\rho_s$  = density of spheres

$\rho_f$  = density of fluid

$\rho_v$  = density of spheres (for viscometer)

$L$  = distance spheres fall = 1 ft

$K$  = viscometer constant = 0.035

Measure:

$t_f$  = time that sphere falls through the distance  $L$

Test	$\rho_f$	$\rho_v$	$t_{\text{viscometer}}$	$\mu$	$t_f$	$D_s$	$V_t$	Re	$\rho_s$	$W_{\text{subm}}$	$A_{\text{proj}}$	$C_D$		
	kg/m <sup>3</sup>	kg/m <sup>3</sup>	min	N·s/m <sup>2</sup>	s	m	m/s		kg/m <sup>3</sup>	N	m <sup>2</sup>	meas	calc	% diff
1	1235	X	X	1.261	6.9	0.00394			7820					
2	1235	X	X	1.261	31.1	0.00394			2830					
3	1192	X	X	0.0055	0.6	0.00394			7820					
4	1192	X	X	0.0055	2.5	0.00394			2830					

For runs 1 and 2, the fluid is glycerine, for runs 3 & 4 it is a mixture of glycerine and water (about 50% of each fluid).

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Calculate:

$V_t$  = terminal velocity of sphere =  $L/t_f$

Re = Reynolds number =  $\rho_f D_s V_t / \mu$

$W_{\text{subm}}$  = submerged weight of sphere = weight - buoyant force  
 $= \gamma_s \nabla_s - \gamma \nabla_s = g(\rho_s - \rho_f) \pi D_s^3 / 6$  where  $\nabla$  = volume

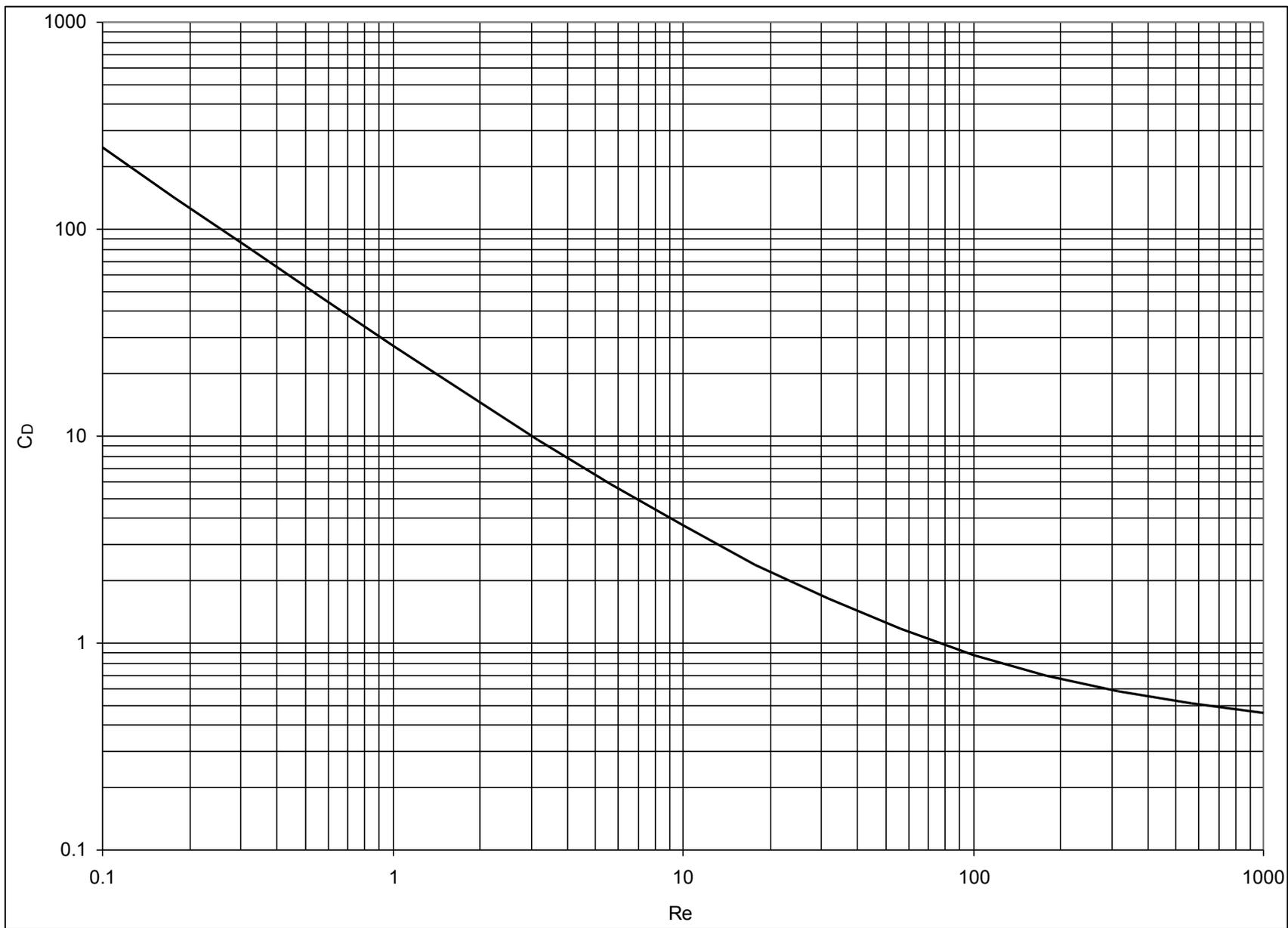
$A_{\text{proj}}$  = projected area of sphere =  $\pi D_s^2 / 4$

$C_{D_{\text{meas}}}$  = measured drag coefficient =  $\frac{F_D}{\rho_f A_{\text{proj}} V_t^2 / 2} = \frac{2W_{\text{subm}}}{\rho_f A_{\text{proj}} V_t^2}$

$C_{D_{\text{calc}}}$  = calculated drag coefficient =  $\frac{24}{\text{Re}} + \frac{3.0}{\sqrt{\text{Re}}} + 0.34$

% diff = percent difference in  $C_D$  values =  $\frac{C_{D_{\text{meas}}} - C_{D_{\text{calc}}}}{C_{D_{\text{calc}}}} 100\%$

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## APPLICATION OF THE BUCKINGHAM PI THEOREM

- 2) The pressure gradient ( $\partial p/\partial s$ , [F/L<sup>3</sup>]) for laminar flow of a viscous fluid through horizontal, small diameter tubes is a function of the cross sectional average velocity ( $V$ , [L/T]), the tube diameter ( $D$ , [L]), and the dynamic viscosity ( $\mu$ , [FT/L<sup>2</sup>]). This relationship can be written as

$$\frac{\partial p}{\partial s} = f(V, D, \mu)$$

where  $s$  is the coordinate in the flow direction. Determine the number of dimensionless groups required. Develop the dimensionless groups. How close can you get to the actual equation, namely

$$\frac{\partial p}{\partial s} = -\frac{32\mu V}{D^2}$$

As we will see in Chapter 10, this expression is limited to  $Re < 2000$ , where  $Re = \rho DV/\mu$  (and to horizontal tubes).

*Procedure:*

- a) Determine the number of dimensional variables.  $\partial p/\partial s$  is one variable.  $n =$  \_\_\_\_\_
- b) Determine the number of dimensions.  $m =$  \_\_\_\_\_
- c) Determine the number of dimensionless variables.  $n - m =$  \_\_\_\_\_
- d) Obtain the dimensionless variables using either the exponent method or the step-by-step method in the textbook.
- e) What can be deduced about the functional relationship from the number of dimensionless variables?