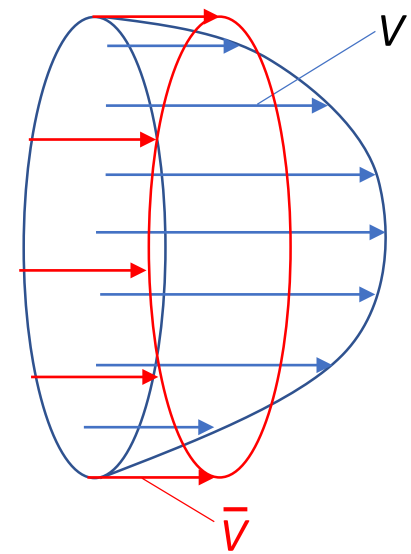
# *ENERGY EQUATION for Non-uniform Flows*




**Mean Velocity:** $$\bar{V} = \frac{Q}{A} = \frac{1}{A}\int_A V\, dA$$

**Kinetic Energy Correction Factor:** $$\alpha = \frac{1}{A}\int_A \left(\frac{V}{\bar{V}}\right)^3 dA$$

**Modified Energy Equation (by using Reynolds Transport Theorem for energy)**

$$\frac{p_1}{\gamma} + z_1 + \frac{\alpha_1 V_1^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \frac{\alpha_2 V_2^2}{2g} + h_t + h_L$$

***$V_1$ and $V_2$ are the <u>mean velocities</u> at stations 1 and 2***

$$h_p = \frac{\dot{W}_p}{\gamma Q} = \frac{\dot{W}_p}{\dot{m} g} \qquad h_t = \frac{\dot{W}_t}{\gamma Q} = \frac{\dot{W}_t}{\dot{m} g}$$

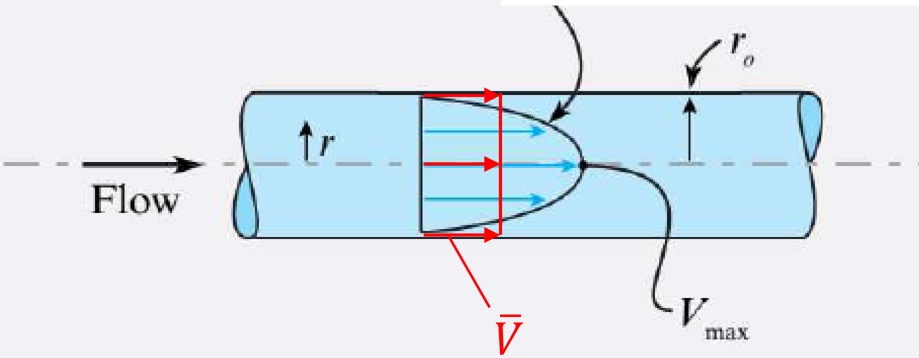
# $\alpha$ for Laminar or Turbulent Pipe Flows

For laminar flows:

Mean Velocity: $\bar{V} = \frac{1}{A}\int_A V\,dA$

**Example 7.1**

$$V = V_{max}\left[1 - \left(\frac{r}{r_o}\right)^2\right]$$

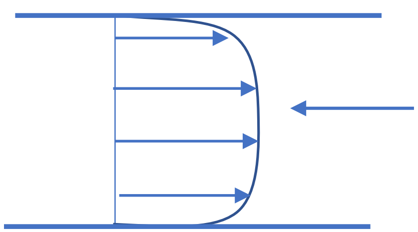



$$\bar{V} = \frac{1}{\pi r_o^2}\int_o^{r_o} V\,2\pi r\,dr = \frac{V_{max}}{2}$$

Kinetic Energy Correction Factor:

$$\alpha = \frac{1}{A}\int_A \left(\frac{V}{\bar{V}}\right)^3 dA$$

$$\alpha = \frac{1}{\pi r_o^2}\int_0^{r_o} \left(\frac{V}{\bar{V}}\right)^3 2\pi r\,dr = 2$$



For uniform flows: $\alpha = 1$

For turbulent flows: $\alpha = 1.05 - 1.07$

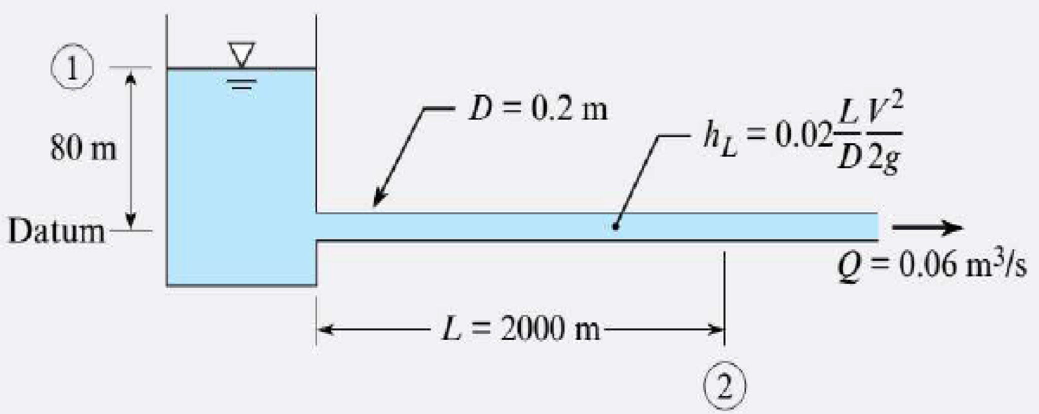
(we often use $\alpha = 1$ for simplicity)

## Example 7.2

A horizontal pipe carries cooling water at 10°C for a thermal power plant. The head loss in the pipe is

$$h_L = \frac{0.02(L/D)V^2}{2g}$$

where $L$ is the length of the pipe from the reservoir to the point in question, $V$ is the mean velocity in the pipe, and $D$ is the diameter of the pipe. If the pipe diameter is 20 cm and the rate of flow is 0.06 m³/s, what is the pressure in the pipe at $L = 2000$ m? Assume $\alpha_2 = 1$.




## *ENERGY EQUATION*

$$\frac{p_1}{\gamma} + z_1 + \frac{\alpha_1 V_1^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \frac{\alpha_2 V_2^2}{2g} + h_t + h_L$$

$$h_p = \frac{\dot{W}_p}{\gamma Q} = \frac{\dot{W}_p}{\dot{m} g} \qquad h_t = \frac{\dot{W}_t}{\gamma Q} = \frac{\dot{W}_t}{\dot{m} g}$$

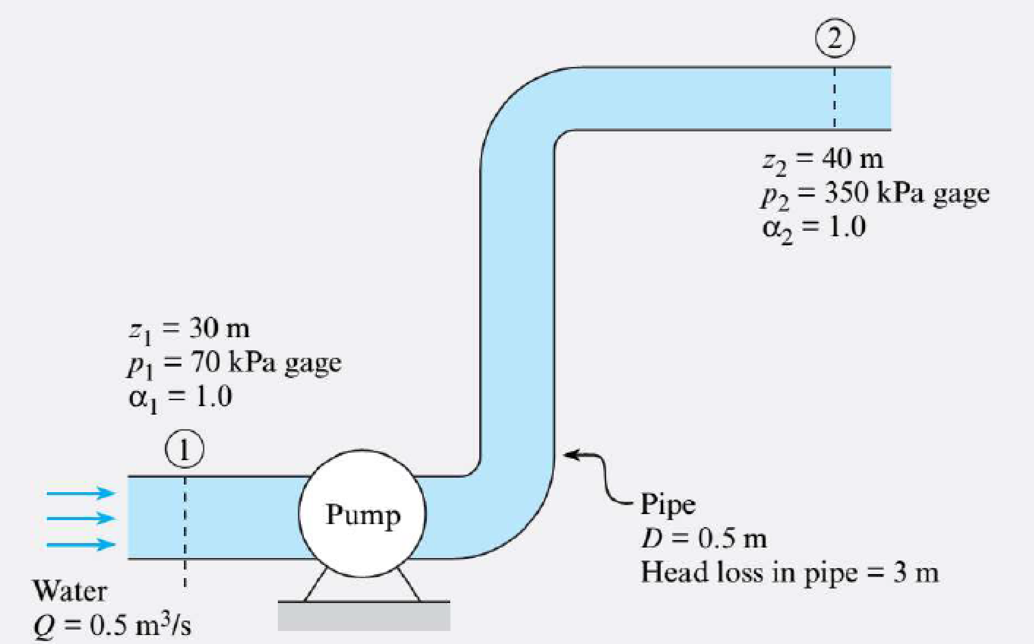
$$h_L = h_f = f\,\frac{L}{D}\,\frac{V^2}{2g} \quad \text{(for pipes)}$$

***Darcy-Weisbach equation***

*f: resistance coefficient or friction factor*

## Example 7.3

A pipe 50 cm in diameter carries water (10°C) at a rate of 0.5 $m^3/s$. A pump in the pipe is used to move the water from an elevation of 30 m to 40 m. The pressure at section 1 is 70 kPa gage, and the pressure at section 2 is 350 kPa gage. What power in kilowatts and in horsepower must be supplied to the flow by the pump? Assume $h_L = 3$ m of water and $\alpha_1 = \alpha_2 = 1$.




## *ENERGY EQUATION*

$$\frac{p_1}{\gamma} + z_1 + \frac{\alpha_1 V_1^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \frac{\alpha_2 V_2^2}{2g} + h_t + h_L$$

$$h_p = \frac{\dot{W}_p}{\gamma Q} = \frac{\dot{W}_p}{\dot{m} g} \qquad h_t = \frac{\dot{W}_t}{\gamma Q} = \frac{\dot{W}_t}{\dot{m} g}$$

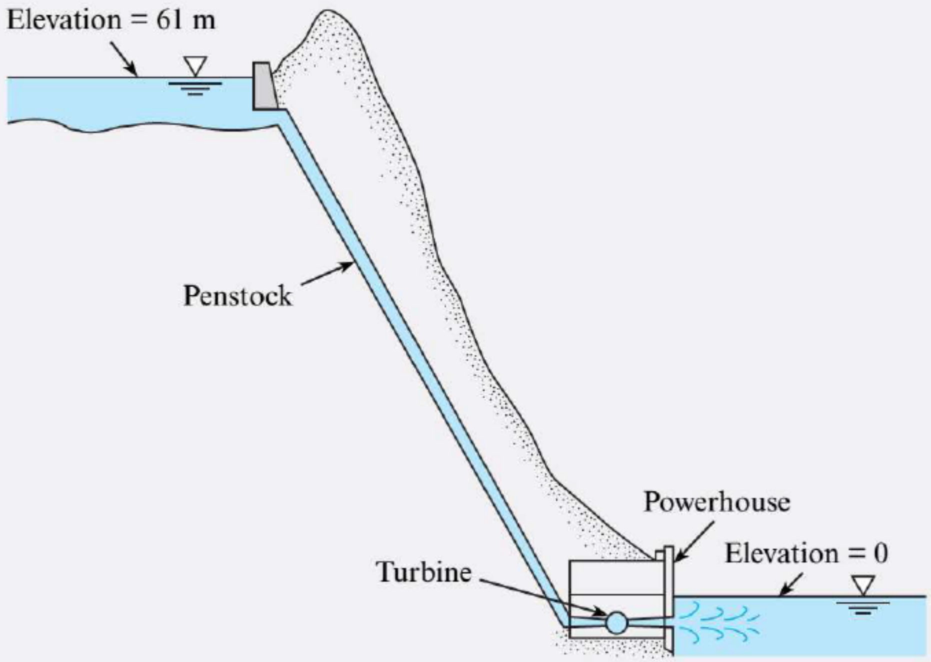
1 hp= 550 lbf ft/sec

1 W = 1 N m/sec

1 hp = 0.746 kW

## Example 7.4

At the maximum rate of power generation, a small hydroelectric power plant takes a discharge of 14.1 m$^3$/s through an elevation drop of 61 m. The head loss through the intakes, penstock, and outlet works is 1.5 m. The combined efficiency of the turbine and electrical generator is 87%. What is the rate of power generation?




## *ENERGY EQUATION*

$$\frac{p_1}{\gamma} + z_1 + \frac{\alpha_1 V_1^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \frac{\alpha_2 V_2^2}{2g} + h_t + h_L$$

$$h_p = \frac{\dot{W}_p}{\gamma Q} = \frac{\dot{W}_p}{\dot{m} g} \qquad h_t = \frac{\dot{W}_t}{\gamma Q} = \frac{\dot{W}_t}{\dot{m} g}$$

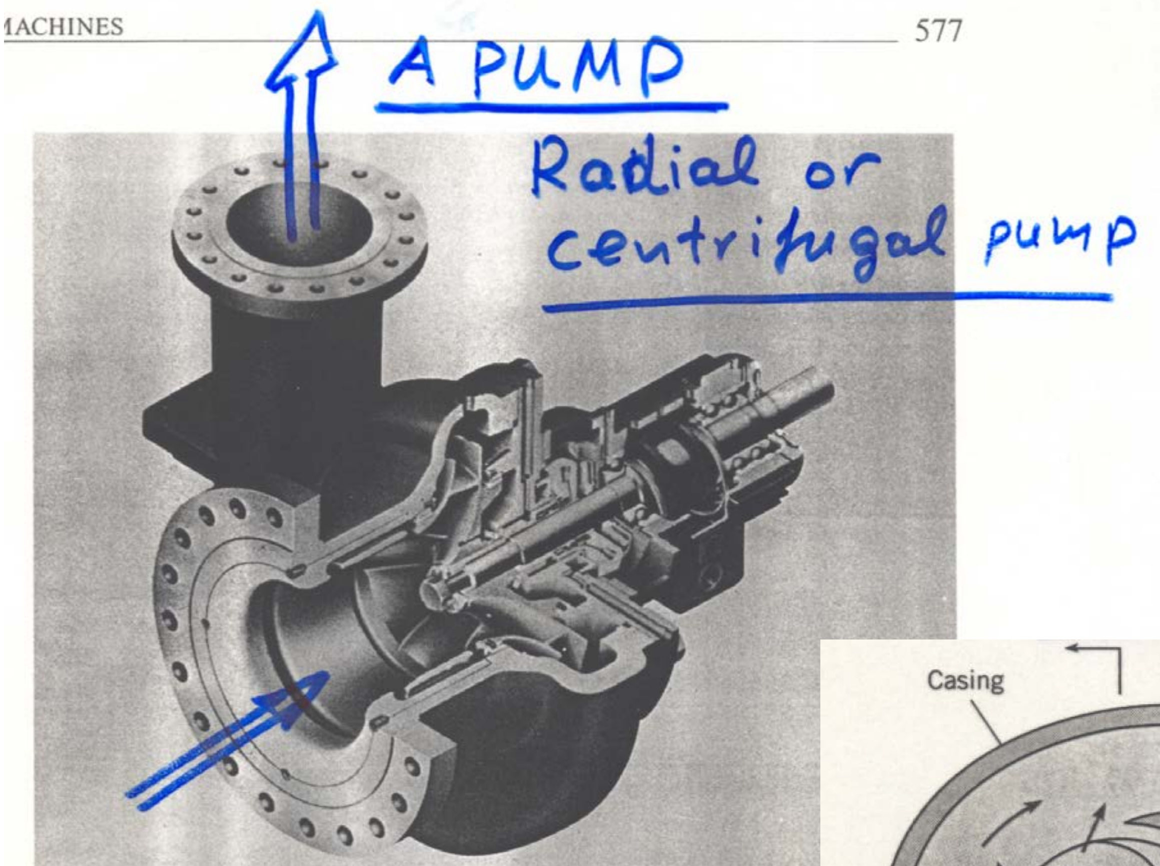
# Efficiency, $\eta$, of a machine

$$\eta_{machine} = \frac{Useful\ power\ produced\ by\ the\ machine}{Given\ Power\ to\ the\ machine}$$

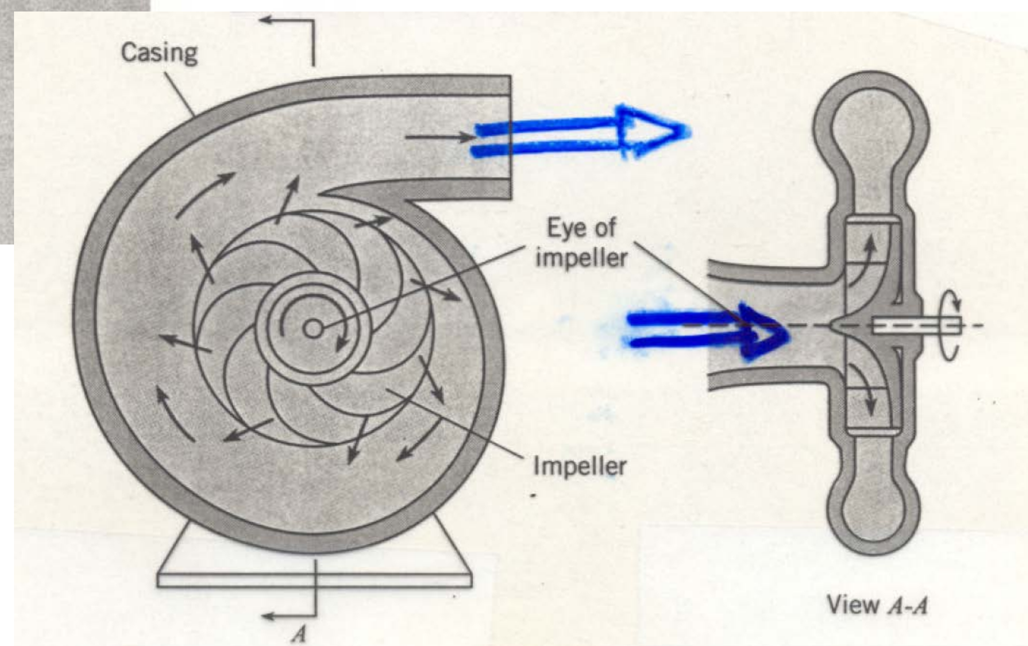
$$\eta_{pump} = \frac{Power\ provided\ to\ the\ water\ by\ the\ pump}{Electrical\ Power\ given\ to\ motor\ of\ pump}$$

$$\eta_{turbine} = \frac{Electrical\ Power\ produced\ by\ the\ generator\ of\ the\ turbine}{Power\ taken\ from\ the\ water\ by\ the\ turbine}$$

# Photos/Images of some types of pumps and turbines

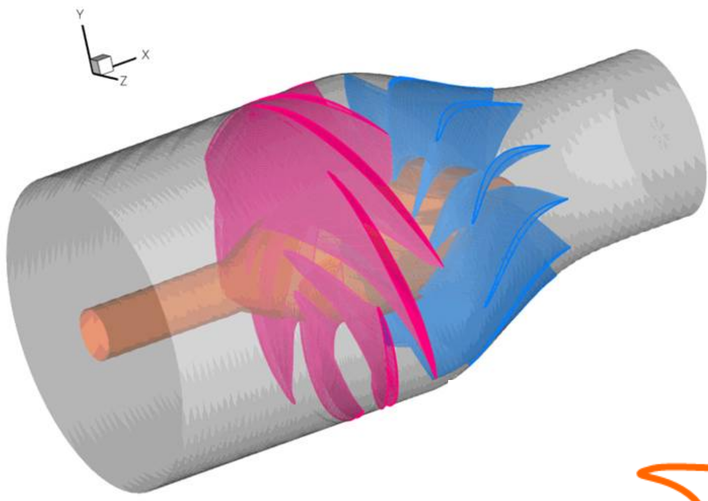


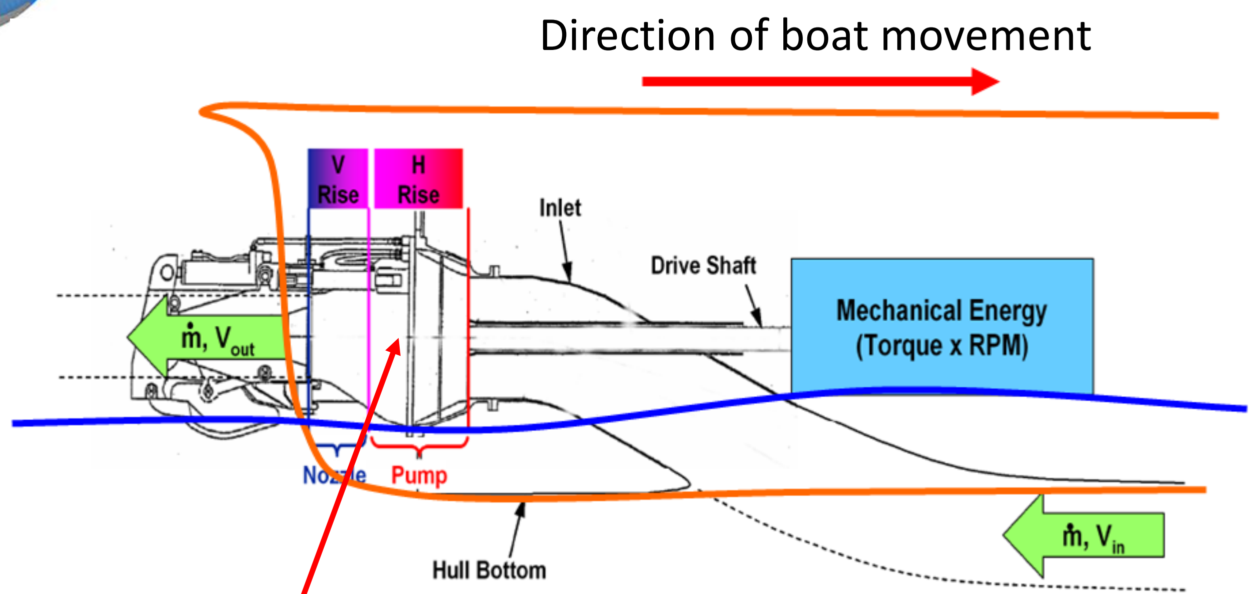
ACHINES
577
A PUMP
Radial or centrifugal pump

Casing
Eye of impeller
Impeller
View A-A
A

**An axial water-jet pump used for the propulsion of a high-speed boat:**
**A 7-bladed rotor , at the front (in red), is followed by an 11-bladed stator**




Water-jet used for the propulsion of a high-speed boat

# A TURBINE

14.17
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Brazil.
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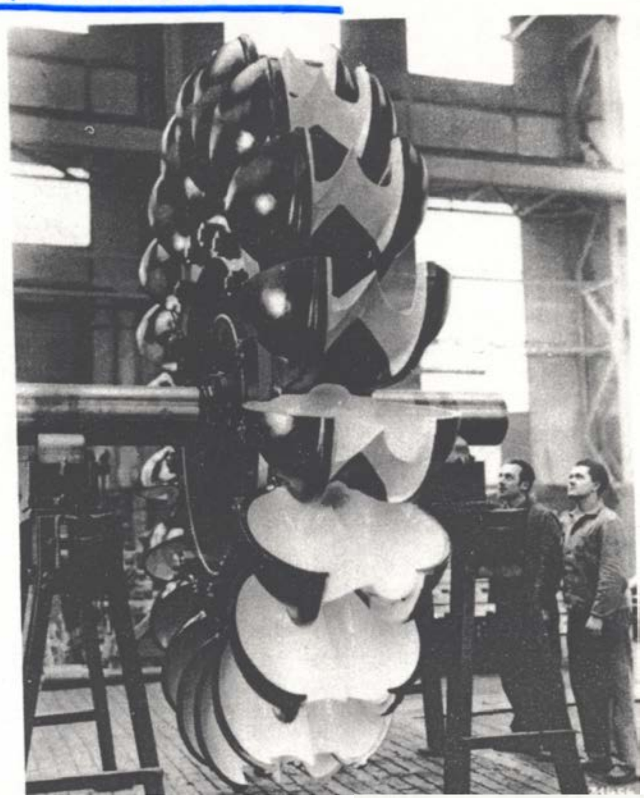
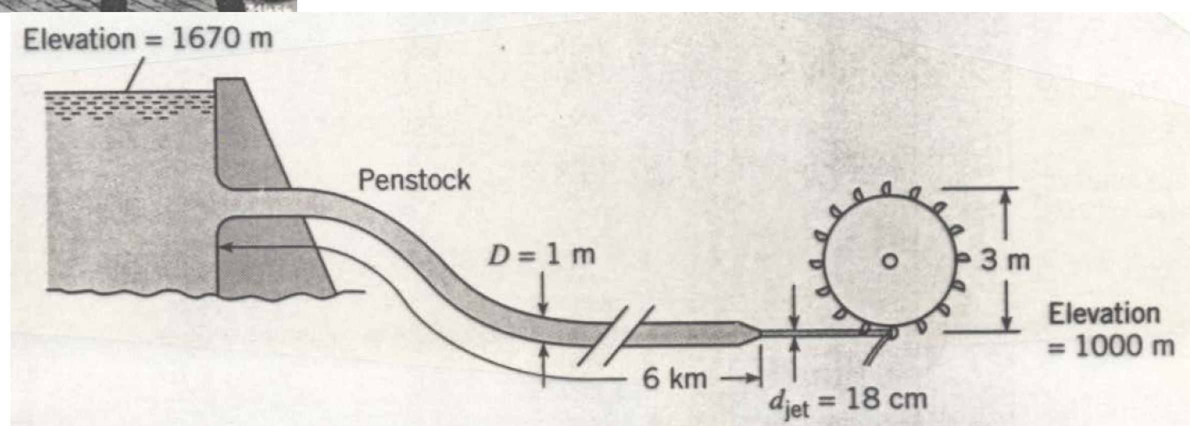
Impulse
or
Pelton
type

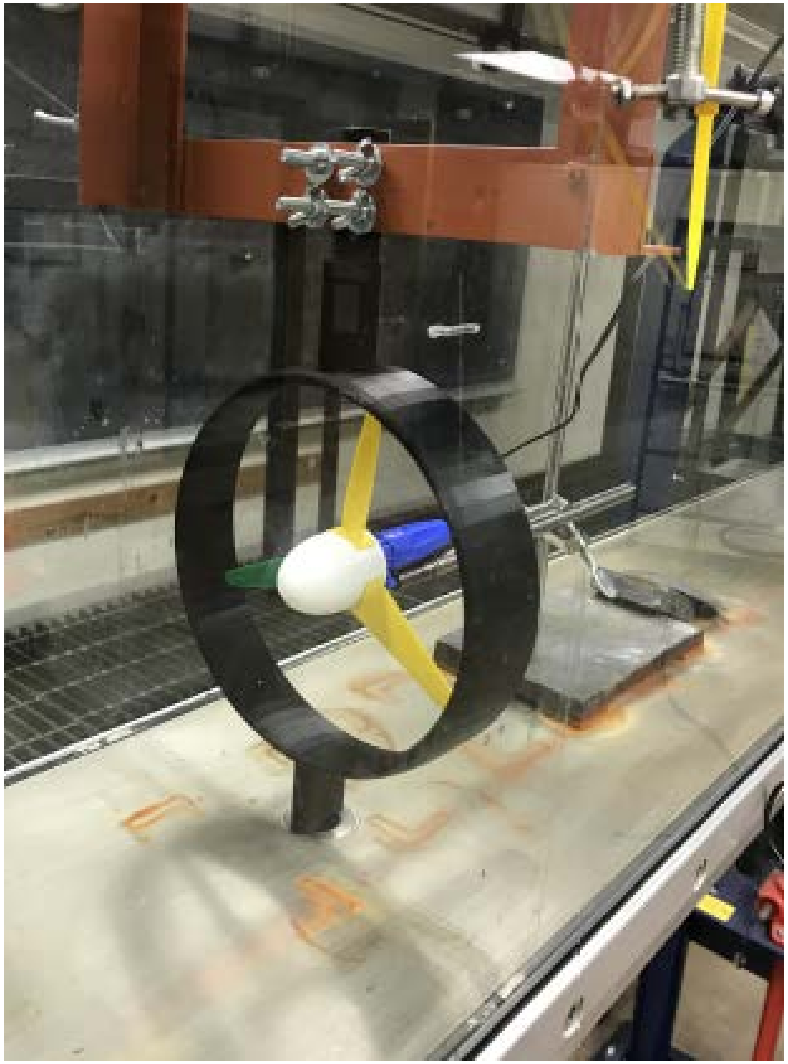




# Wind turbine farm

# Ocean current (or tidal) turbine

# A ducted marine turbine tested inside CAEE's Flume



Check: https://twitter.com/spyroskinnas
For more info, photos, movies of the experiment performed by students in CE358 (Ocean Engineering) class

# Steam turbine (rotor part)

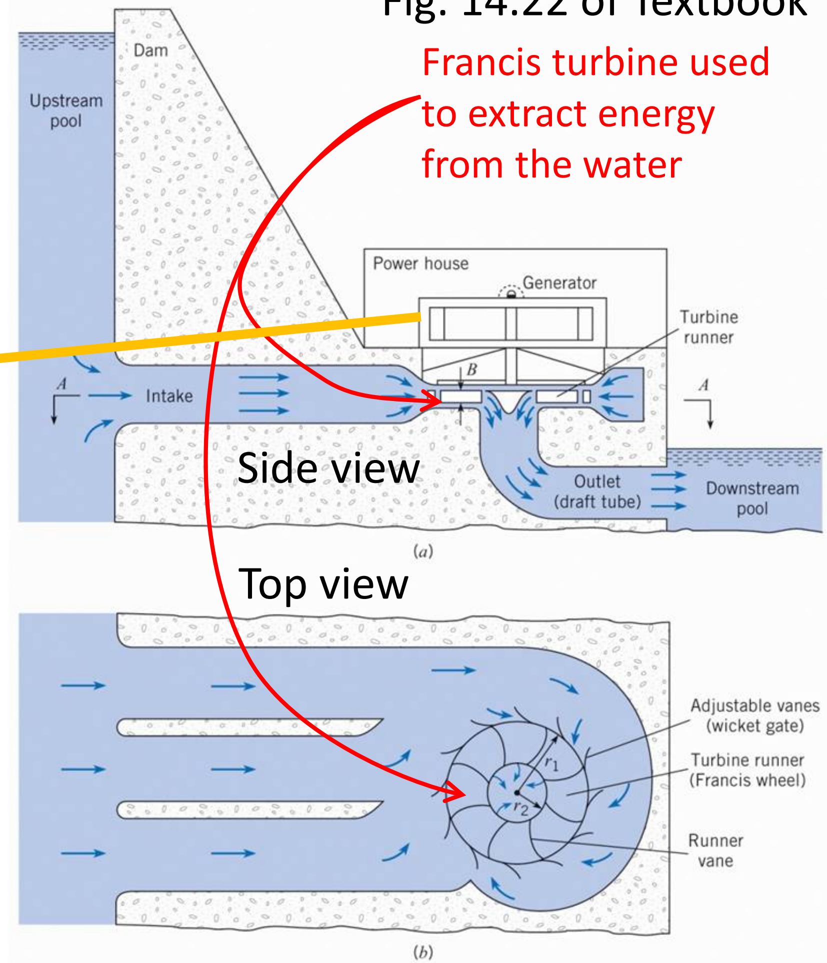
Hoover Dam Power Plant (17 turbines/generators capable of producing 2GWs=2,000 MWs of electrical power)

# A hydro-electric power plant

https://en.wikipedia.org/wiki/File:Hoover_Dam_generators.jpg

Fig. 14.22 of Textbook

Francis turbine used to extract energy from the water



# Example 7.5

## *ENERGY EQUATION*

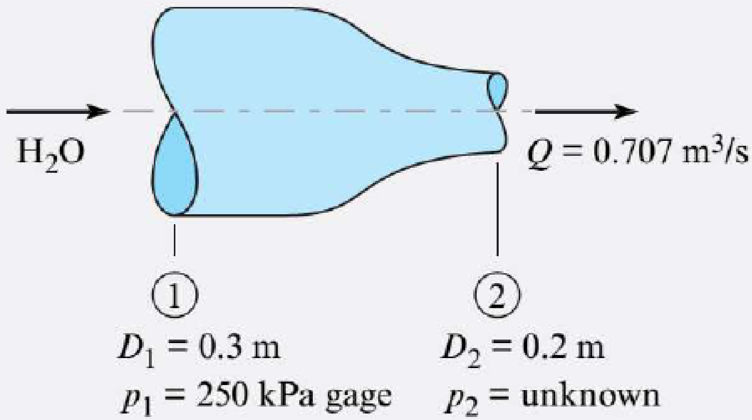
$$\frac{p_1}{\gamma} + z_1 + \frac{\alpha_1 V_1^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \frac{\alpha_2 V_2^2}{2g} + h_t + h_L$$

A pipe 30 cm in diameter carries water (10°C, 250 kPa) at a rate of 0.707 $m^3/s$. The pipe contracts to a diameter of 20 cm. The head loss through the contraction is given by

$$h_L = 0.1\frac{V_2^2}{2g}$$

where $V_2$ is the velocity in the 20 cm pipe. What horizontal force is required to hold the transition in place? Assume the kinetic energy correction factor is 1.0 at both the inlet and exit.

- $\alpha_1 = \alpha_2 = 1.0$
- $h_L = 0.1\ (V_2^2/(2g))$



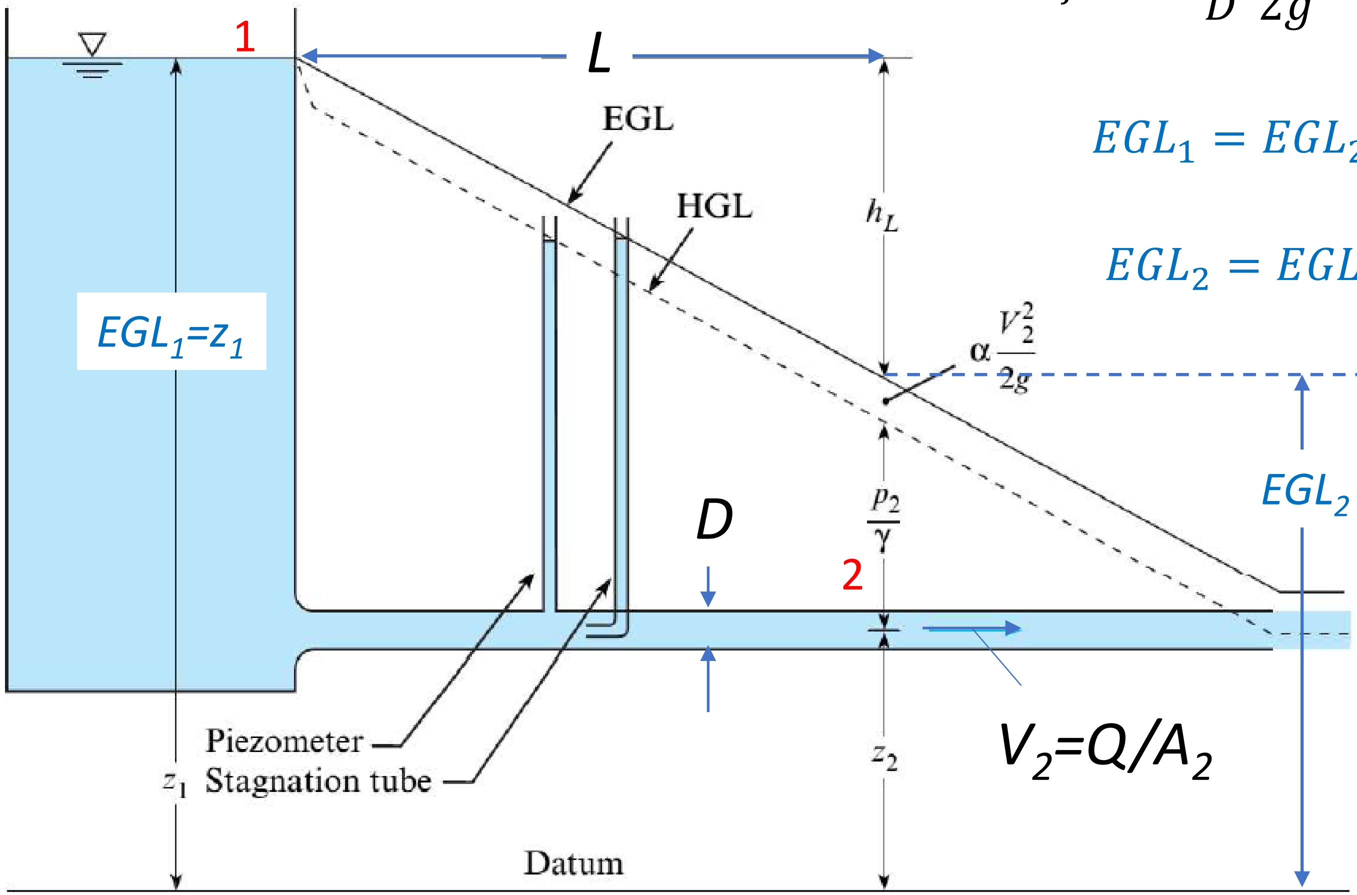
## EGL (Energy Grade Line) and HGL (Hydraulic Grade Line)

$$EGL_1 + h_p = EGL_2 + h_t + h_L \iff \frac{p_1}{\gamma} + z_1 + \frac{\alpha_1 V_1^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \frac{\alpha_2 V_2^2}{2g} + h_t + h_L$$

$$EGL = \frac{p}{\gamma} + z + \frac{\alpha V^2}{2g} \qquad HGL = \frac{p}{\gamma} + z \qquad h_p = \frac{\dot{W}_p}{\gamma Q} = \frac{\dot{W}_p}{\dot{m} g} \qquad h_t = \frac{\dot{W}_t}{\gamma Q} = \frac{\dot{W}_t}{\dot{m} g}$$

$$HGL = EGL - \frac{\alpha V^2}{2g}$$

$$h_L = h_f = f \frac{L}{D} \frac{V^2}{2g} \quad \text{(for pipes)}$$




$$EGL_1 = EGL_2 + h_L$$

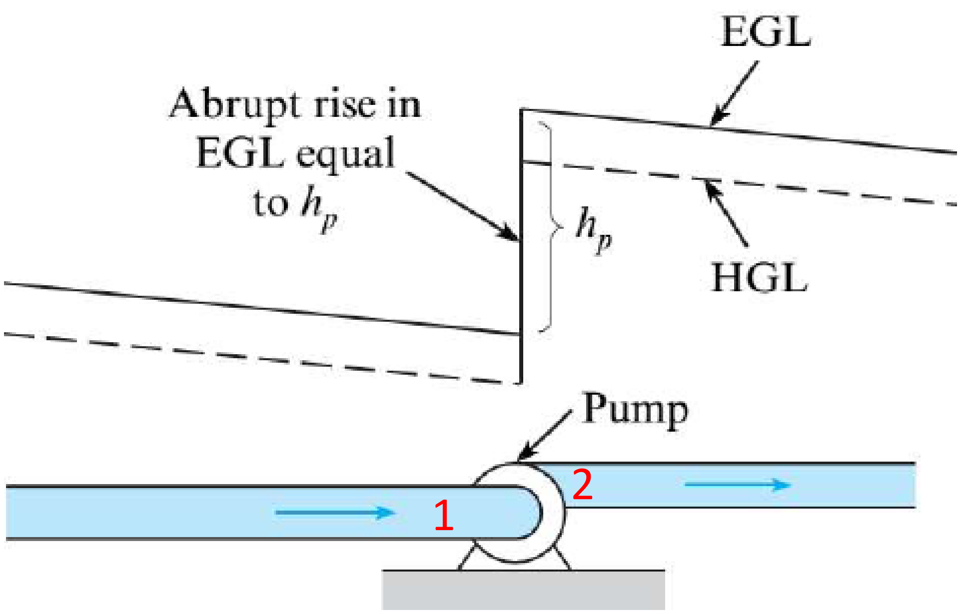
$$EGL_2 = EGL_1 - h_L$$

## EGL and HGL for pump

$$EGL_1 + h_p = EGL_2 + h_t + h_L \quad \Leftrightarrow \quad \frac{p_1}{\gamma} + z_1 + \frac{\alpha_1 V_1^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \frac{\alpha_2 V_2^2}{2g} + h_t + h_L$$

$$EGL = \frac{p}{\gamma} + z + \frac{\alpha V^2}{2g} \qquad HGL = \frac{p}{\gamma} + z \qquad h_p = \frac{\dot{W}_p}{\gamma Q} = \frac{\dot{W}_p}{\dot{m} g} \qquad h_t = \frac{\dot{W}_t}{\gamma Q} = \frac{\dot{W}_t}{\dot{m} g}$$

$$h_L = h_f = f\,\frac{L}{D}\,\frac{V^2}{2g} \quad \text{(for pipes)}$$



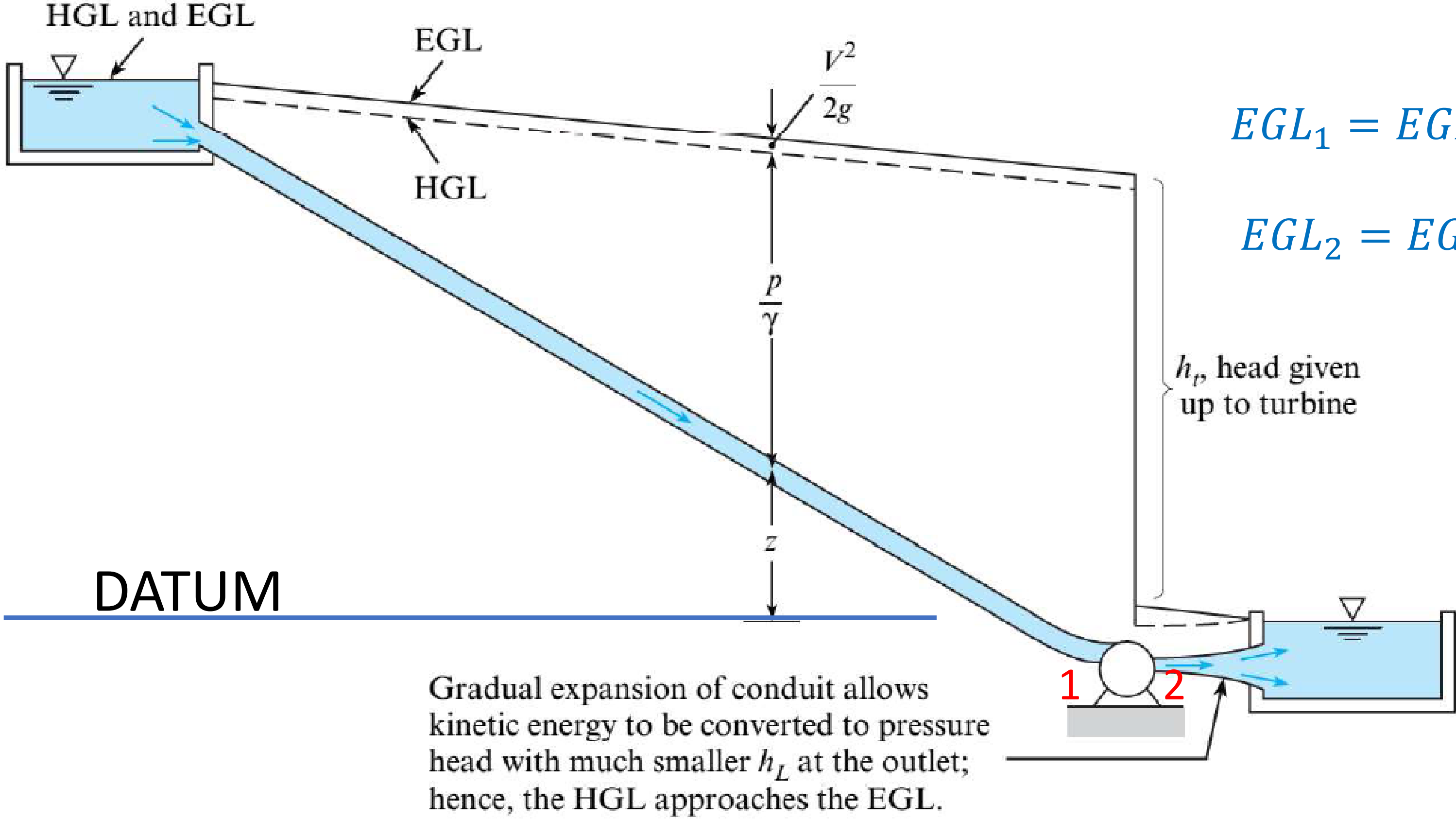

$$EGL_1 + h_p = EGL_2$$

## EGL and HGL for turbine

$$EGL_1 + h_p = EGL_2 + h_t + h_L \quad \Leftrightarrow \quad \frac{p_1}{\gamma} + z_1 + \frac{\alpha_1 V_1^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \frac{\alpha_2 V_2^2}{2g} + h_t + h_L$$

$$EGL = \frac{p}{\gamma} + z + \frac{\alpha V^2}{2g} \qquad HGL = \frac{p}{\gamma} + z \qquad h_p = \frac{\dot{W}_p}{\gamma Q} = \frac{\dot{W}_p}{\dot{m} g} \qquad h_t = \frac{\dot{W}_t}{\gamma Q} = \frac{\dot{W}_t}{\dot{m} g}$$

$$h_L = h_f = f\,\frac{L}{D}\,\frac{V^2}{2g} \quad \text{(for pipes)}$$




$$EGL_1 = EGL_2 + h_t$$

$$EGL_2 = EGL_1 - h_t$$

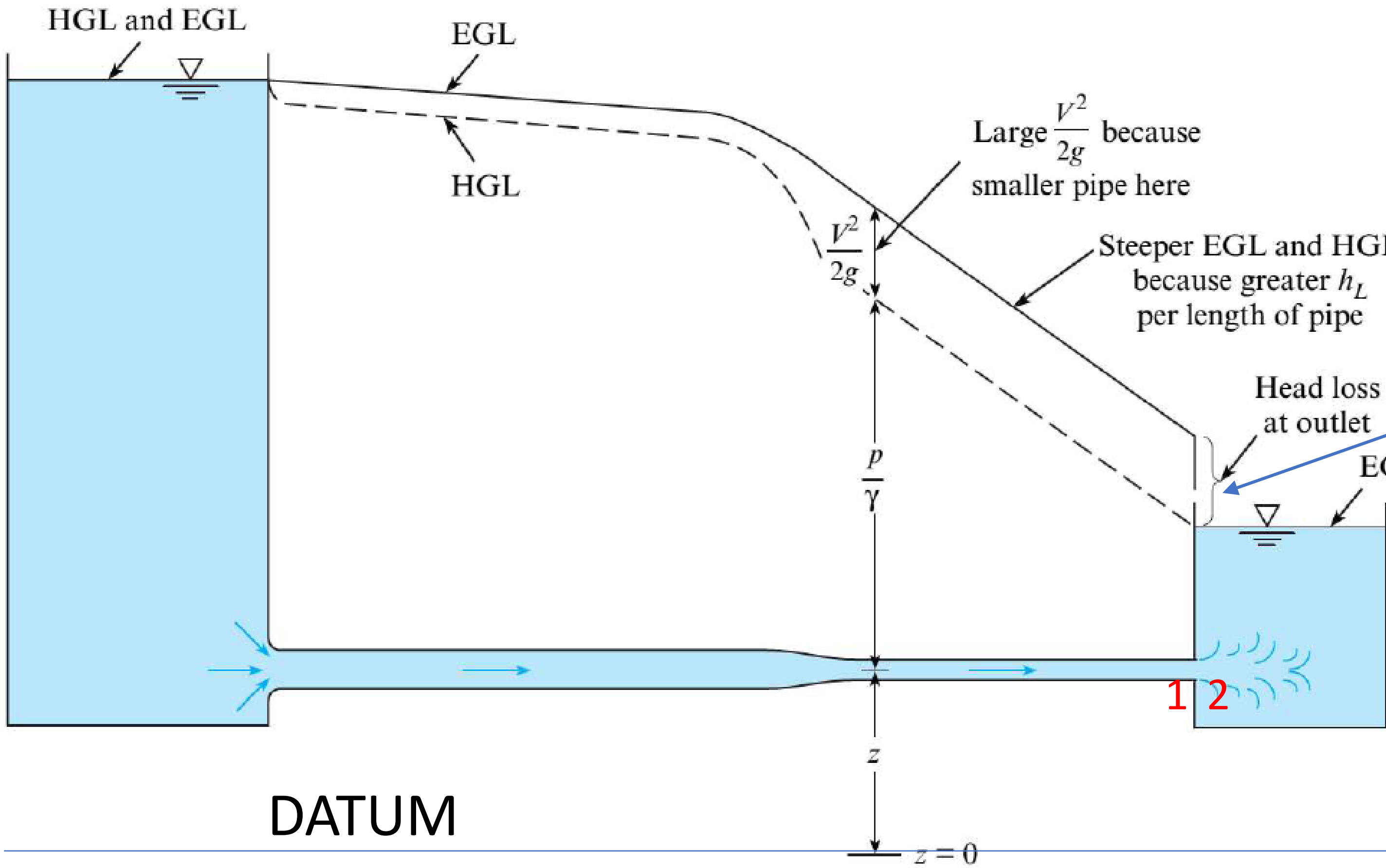
## EGL and HGL between reservoirs

$$EGL_1 + h_p = EGL_2 + h_t + h_L \quad \Leftrightarrow \quad \frac{p_1}{\gamma} + z_1 + \frac{\alpha_1 V_1^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \frac{\alpha_2 V_2^2}{2g} + h_t + h_L$$

$$EGL = \frac{p}{\gamma} + z + \frac{\alpha V^2}{2g} \qquad HGL = \frac{p}{\gamma} + z \qquad h_p = \frac{\dot{W}_p}{\gamma Q} = \frac{\dot{W}_p}{\dot{m} g} \qquad h_t = \frac{\dot{W}_t}{\gamma Q} = \frac{\dot{W}_t}{\dot{m} g}$$

$$h_L = h_f = f \frac{L}{D} \frac{V^2}{2g} \quad \text{(for pipes)}$$

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + h_e$$




$$\boxed{\frac{V_1^2}{2g} = h_e}$$

$z_1 = z_2$
$V_2 = 0$
$p_1 = p_2$

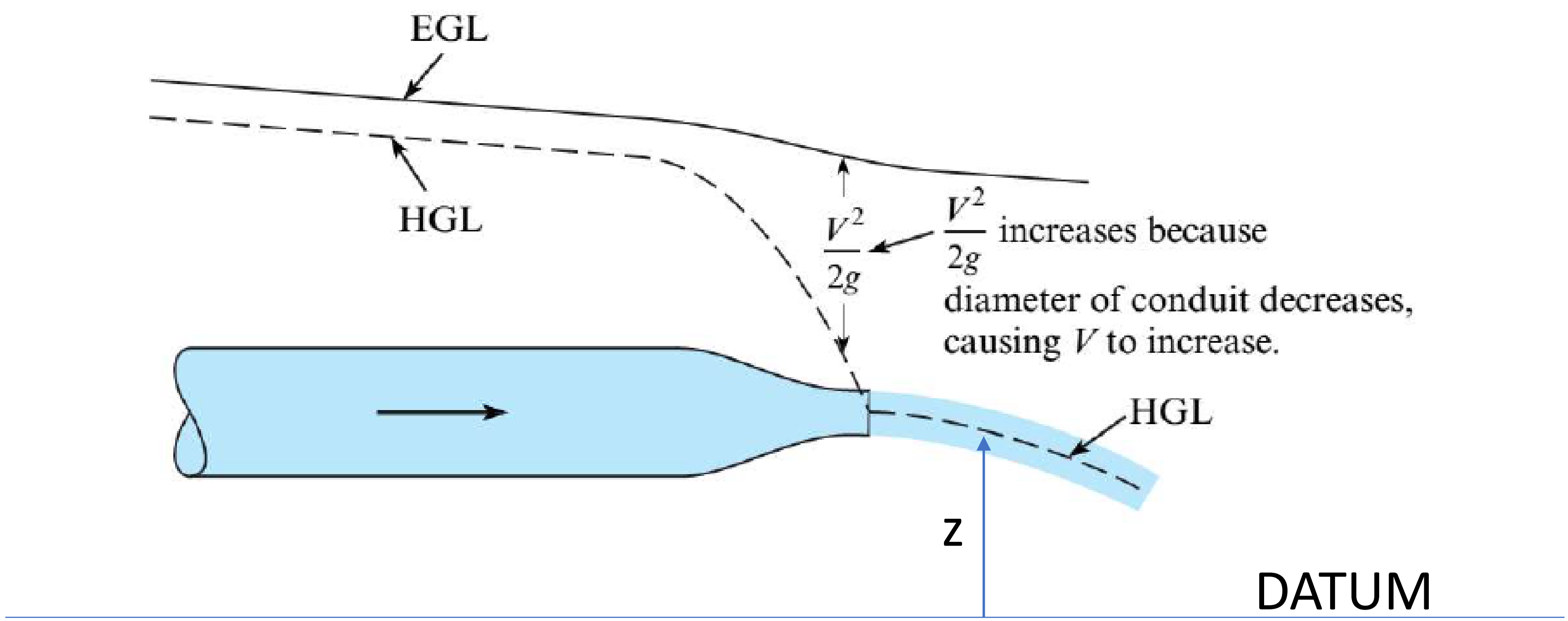
## EGL and HGL for nozzle



$$EGL_1 + h_p = EGL_2 + h_t + h_L \quad \Leftrightarrow \quad \frac{p_1}{\gamma} + z_1 + \frac{\alpha_1 V_1^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \frac{\alpha_2 V_2^2}{2g} + h_t + h_L$$

$$EGL = \frac{p}{\gamma} + z + \frac{\alpha V^2}{2g} \qquad HGL = \frac{p}{\gamma} + z \qquad h_p = \frac{\dot{W}_p}{\gamma Q} = \frac{\dot{W}_p}{\dot{m} g} \qquad h_t = \frac{\dot{W}_t}{\gamma Q} = \frac{\dot{W}_t}{\dot{m} g}$$

$$h_L = h_f = f\,\frac{L}{D}\,\frac{V^2}{2g} \quad \text{(for pipes)}$$



## Example 7.6



A pump draws water (50°F) from a reservoir, where the water surface elevation is 520 ft, and forces the water through a pipe 5000 ft long and 1 ft in diameter. This pipe then discharges the water into a reservoir with water surface elevation of 620 ft. The flow rate is 7.85 cfs, and the head loss in the pipe is given by

$$h_L = 0.01\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right)$$

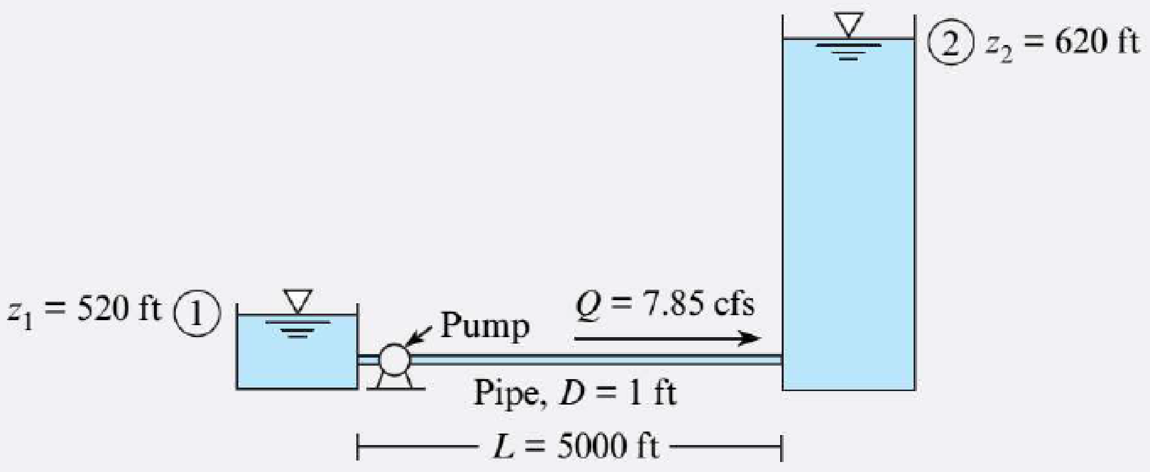
Determine the head supplied by the pump, $h_p$, and the power supplied to the flow, and draw the HGL and EGL for the system. Assume that the pipe is horizontal and is 510 ft in elevation.

$$EGL_1 + h_p = EGL_2 + h_t + h_L$$

$$EGL = \frac{p}{\gamma} + z + \frac{\alpha V^2}{2g} \qquad HGL = \frac{p}{\gamma} + z$$

$$\frac{p_1}{\gamma} + z_1 + \frac{\alpha_1 V_1^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \frac{\alpha_2 V_2^2}{2g} + h_t + h_L$$

$$h_L = h_f = f\,\frac{L}{D}\,\frac{V^2}{2g}$$



- Improved solution of Ex. ~~7.7~~ 7.6 in the textbook (as presented in class)

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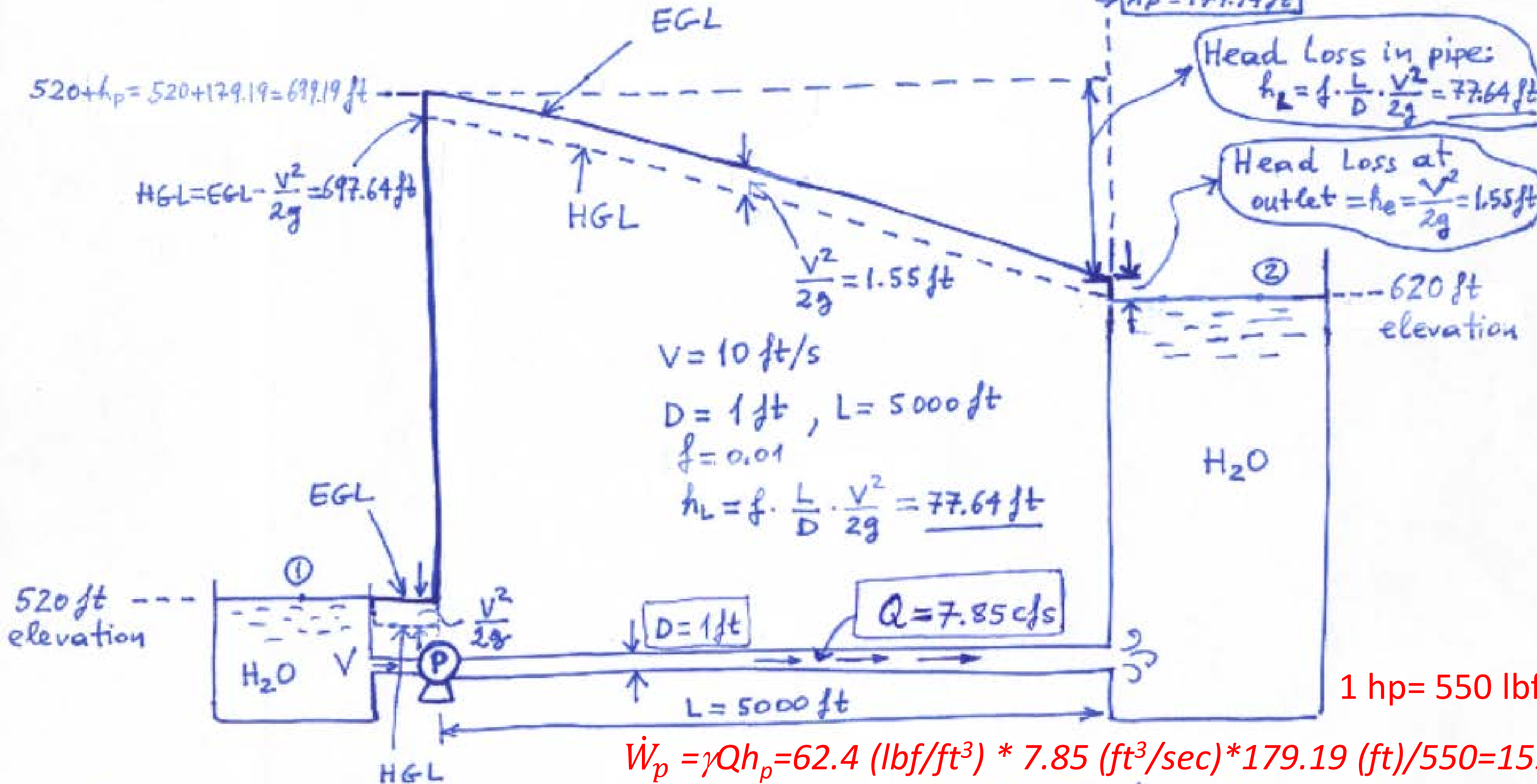
Energy equation between ① and ②: $EGL_2 - EGL_1 = h_p - h_L - h_e \Rightarrow 620 - 520 = h_p - 77.64 - 1.55$

$\Rightarrow \boxed{h_p = 179.19\,ft}$




1 hp= 550 lbf ft/sec

$\dot{W}_p = \gamma Q h_p = 62.4\,(lbf/ft^3) * 7.85\,(ft^3/sec) * 179.19\,(ft)/550 = 159.59\,hp$

Energy equation is shown graphically through the EGL/HGL diagram:

$EGL_2 = EGL_1 + h_p - h_L - h_e$ or $620 = 520 + 179.19 - 77.64 - 1.55$ (check this!)

Note: We assume the pump is very close to the tank on the left, so that the head losses on the pipe connecting it with the pump are ignored.