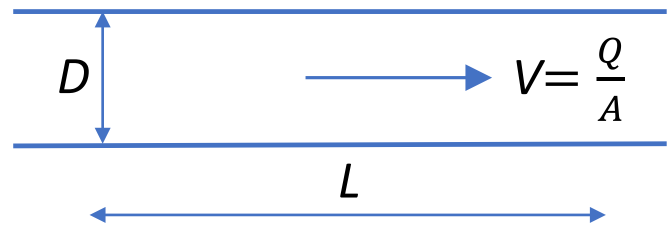
Example 10.2 (Laminar Flow; Re<2,000)

Oil ($SG = 0.85$) with a kinematic viscosity of $6 \times 10^{-4}\ \mathrm{m^2/s}$ flows in a 15 cm diameter pipe at a rate of $0.020\ \mathrm{m^3/s}$. What is the head loss for a 100 m length of pipe?

$$h_L = h_f = f\,\frac{L}{D}\,\frac{V^2}{2g}$$

Reynolds Number: $$Re = \frac{VD}{\nu}$$



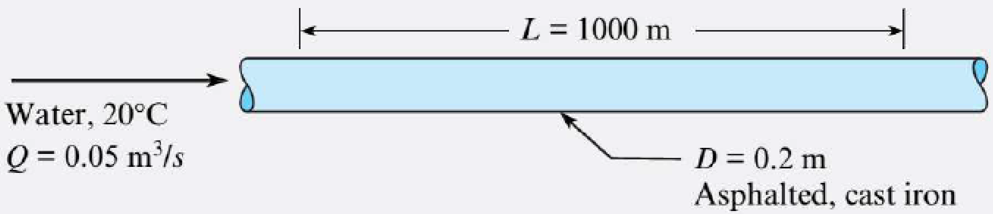
$$f = \frac{64}{Re}$$

Example 10.3 (Turbulent Flow; Re>3,000)

Water ($T = 20°C$) flows at a rate of $0.05\ m^3/s$ in a 20 cm asphalted cast iron pipe. What is the head loss per kilometer of pipe?

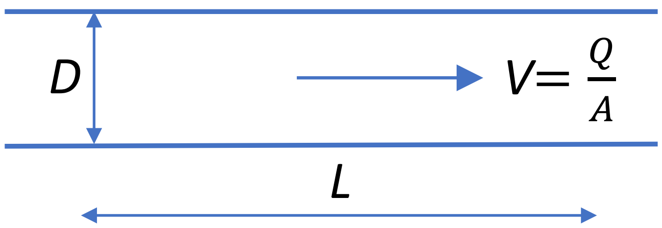
Define the Situation

Water is flowing in a pipe.

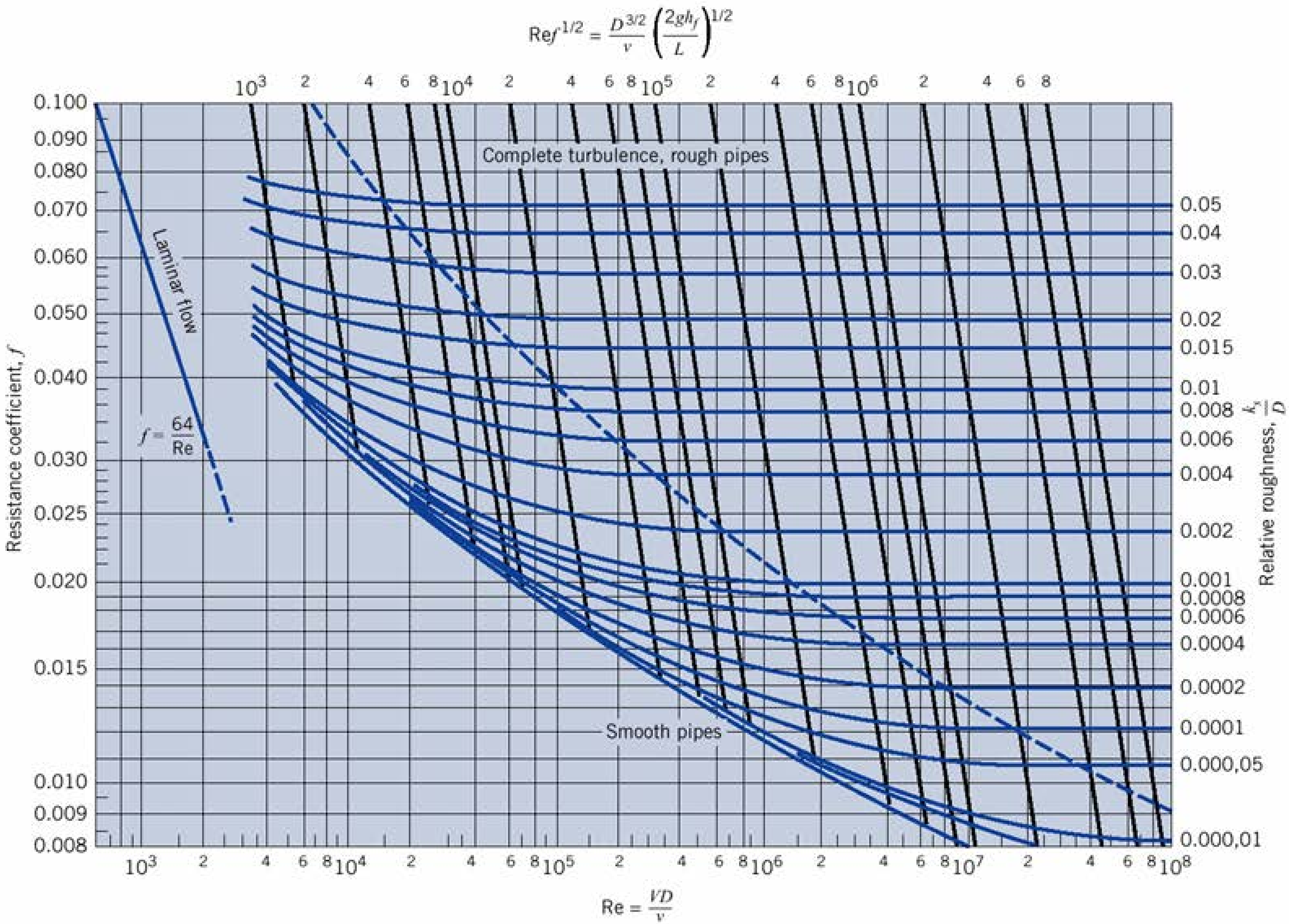


$$h_L = h_f = f\,\frac{L}{D}\,\frac{V^2}{2g}$$

Reynolds Number: $Re = \dfrac{VD}{\nu}$



The Moody Chart (created in 1944!)



Relative roughness k_s/D

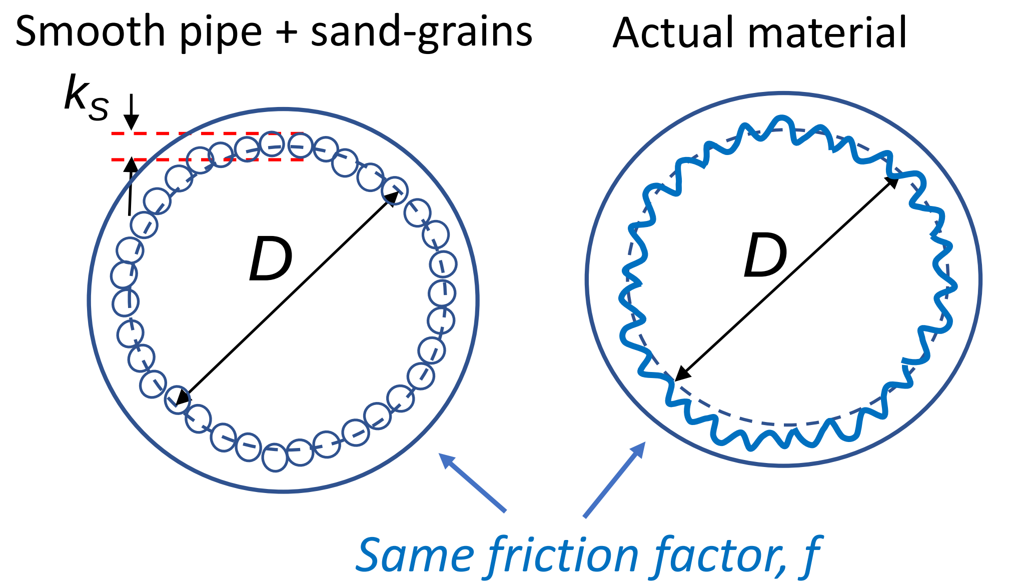


TABLE 10.4 Equivalent Sand-Grain Roughness, (k_s), for Various Pipe Materials

Boundary Material	k_s, Millimeters	k_s, Inches
Glass, plastic	0.00 (smooth)	0.00 (smooth)
Copper or brass tubing	0.0015	6×10^{-5}
Wrought iron, steel	0.046	0.002
Asphalted cast iron	0.12	0.005
Galvanized iron	0.15	0.006
Cast iron	0.26	0.010
Concrete	0.3 to 3.0	0.012–0.12
Riveted steel	0.9–9	0.035–0.35
Rubber pipe (straight)	0.025	0.001

Formulas for smooth and rough pipes

Smooth pipes
(Prandtl formula)

$$\frac{1}{\sqrt{f}} = 2.0\log_{10}(\mathrm{Re}\sqrt{f}\,) - 0.8$$

Rough pipes
(Colebrook-White formula)

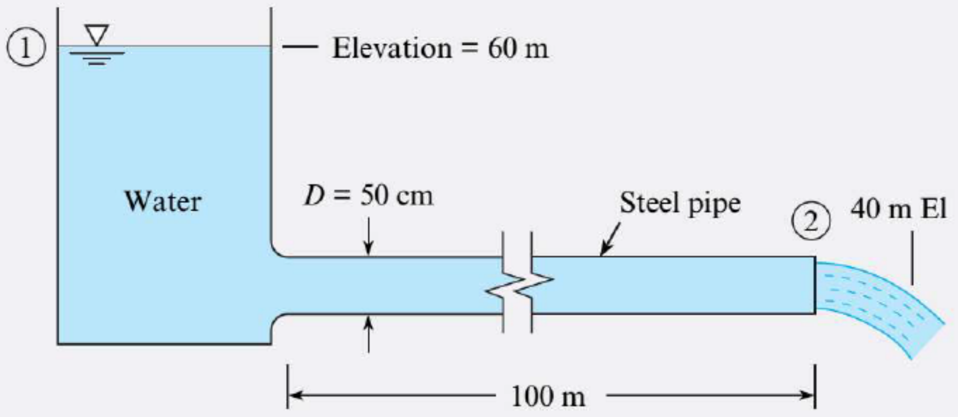
$$\frac{1}{\sqrt{f}} = 2\log_{10}(Re\sqrt{f}) - 0.8 - 2\log_{10}\left[1 + \frac{k_S}{D}\frac{Re\sqrt{f}}{9.287}\right]$$

Rough pipes
(Swamee & Jain formula)

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\mathrm{Re}_D^{0.9}}\right)\right]^2}$$

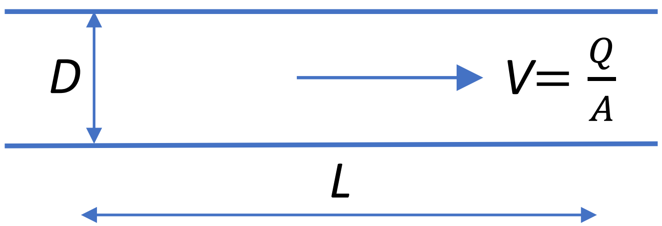
Example 10.5

Water ($T = 20°C$) flows from a tank through a 50 cm diameter steel pipe. Determine the discharge of water.



$$h_L = h_f = f\,\frac{L}{D}\frac{V^2}{2g}$$

Reynolds Number: $Re = \dfrac{VD}{\nu}$



$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} + \cancel{h_p} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + \cancel{h_t} + h_L$$

The Moody Chart (created in 1944!)

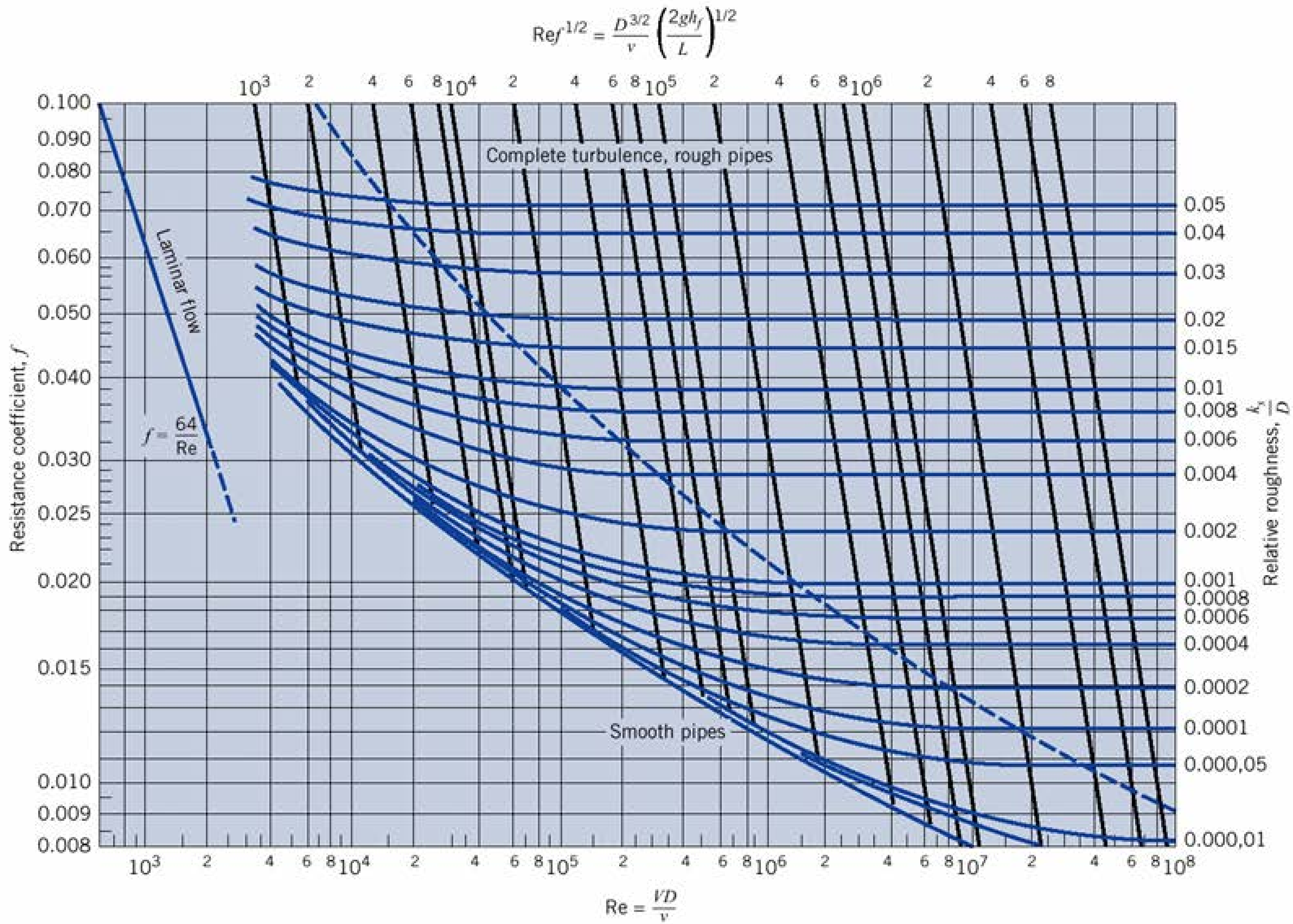
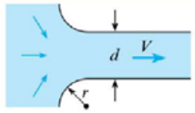
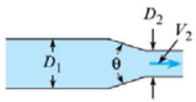
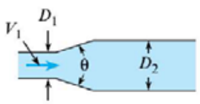
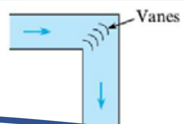
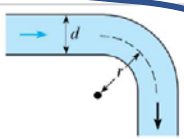
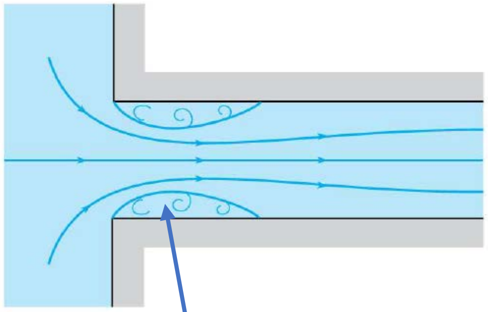


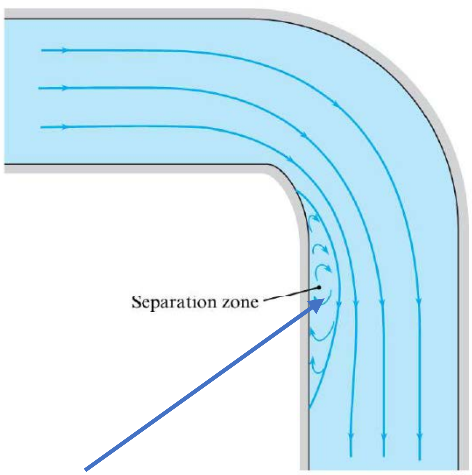
TABLE 10.5 Loss Coefficients for Various Transitions and Fittings

Description	Sketch	Additional Data		K	Source
Pipe entrance $h_L = K_e V^2/2g$		r/d 0.0 0.1 >0.2		K_e 0.50 0.12 0.03	(10)
Contraction $h_L = K_C V_2^2/2g$		D_2/D_1 0.00 0.20 0.40 0.60 0.80 0.90	K_C $\theta = 60°$ 0.08 0.08 0.07 0.06 0.06 0.06	K_C $\theta = 180°$ 0.50 0.49 0.42 0.27 0.20 0.10	(10)
Expansion $h_L = K_E V_1^2/2g$		D_1/D_2 0.00 0.20 0.40 0.60 0.80	K_E $\theta = 20°$ 0.30 0.25 0.15 0.10	K_E $\theta = 180°$ 1.00 0.87 0.70 0.41 0.15	(9)
90° miter bend		Without vanes		$K_b = 1.1$	(15)
		With vanes		$K_b = 0.2$	(15)
90° smooth bend		r/d 1 2 3 4 6 8 10	$K_b = 0.35$ 0.19 0.16 0.21 0.28 0.32		(16) and (9)
Threaded pipe fittings	Globe valve—wide open Angle valve—wide open Gate valve—wide open Gate valve—half open Return bend Tee Straight-through flow Side-outlet flow 90° elbow 45° elbow			$K_v = 10.0$ $K_v = 5.0$ $K_v = 0.2$ $K_v = 5.6$ $K_b = 2.2$ $K_t = 0.4$ $K_t = 1.8$ $K_b = 0.9$ $K_b = 0.4$	(15)

Flow separation increases head losses



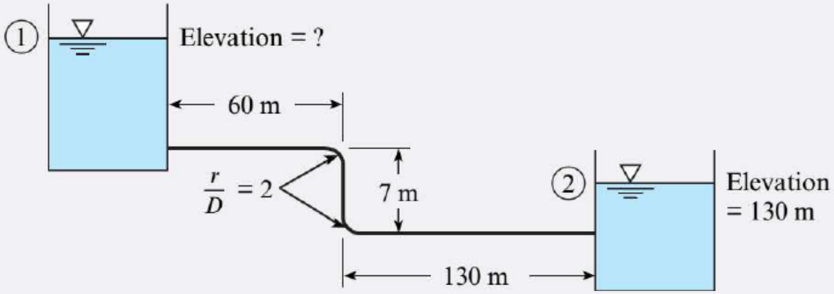
Flow separation in an sharp-edged inlet



Flow separation in an elbow

Example 10.7

If oil ($\nu = 4 \times 10^{-5}$ m^2/s; $SG = 0.9$) flows from the upper to the lower reservoir at a rate of 0.028 m^3/s in the 15 cm smooth pipe, then what is the elevation of the oil surface in the upper reservoir?



1. Energy equation and term-by-term analysis:

$$\frac{p_1}{\gamma} + \alpha_1 \frac{\overline{V}_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{\overline{V}_2^2}{2g} + z_2 + h_t + h_L$$

$$0 + 0 + z_1 + 0 = 0 + 0 + z_2 + 0 + h_L$$

$$z_1 = z_2 + h_L$$

Interpretation: Change in elevation head is balanced by the total head loss.

2. Combined head loss equation:

$$h_L = \sum_{\text{pipes}} f \frac{L}{D} \frac{V^2}{2g} + \sum_{\text{components}} K \frac{V^2}{2g}$$

$$h_L = f \frac{L}{D} \frac{V^2}{2g} + \left(2K_b \frac{V^2}{2g} + K_e \frac{V^2}{2g} + K_E \frac{V^2}{2g} \right)$$

$$= \frac{V^2}{2g} \left(f \frac{L}{D} + 2K_b + K_e + K_E \right)$$

3. Combine Eqs. (1) and (2):

$$z_1 = z_2 + \frac{V^2}{2g} \left(f \frac{L}{D} + 2K_b + K_e + K_E \right)$$

4. Resistance coefficient:

- Flow rate equation:

$$V = \frac{Q}{A} = \frac{(0.028 \text{ m}^3/\text{s})}{(\pi/4)(0.15 \text{ m})^2} = 1.58 \text{ m/s}$$

- Reynolds number:

$$\text{Re}_D = \frac{VD}{\nu} = \frac{1.58 \text{ m/s}(0.15 \text{ m})}{4 \times 10^{-5} \text{ m}^2/\text{s}} = 5.93 \times 10^3$$

Thus, flow is turbulent.

- Swamee-Jain equation (10.39):

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} = \frac{0.25}{\left[\log_{10}\left(0 + \frac{5.74}{5930^{0.9}}\right)\right]^2} = 0.036$$

5. Calculate z_1 using the equation from step (3):

$$z_1 = (130 \text{ m}) + \frac{(1.58 \text{ m/s})^2}{2(9.81)\text{m/s}^2} \left(0.036 \frac{(197 \text{ m})}{(0.15 \text{ m})} + 2(0.19) + 0.5 + 1.0 \right)$$

$$\boxed{z_1 = 136 \text{ m}}$$