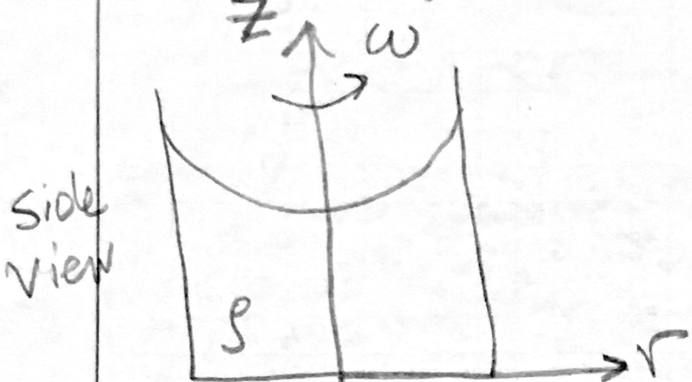
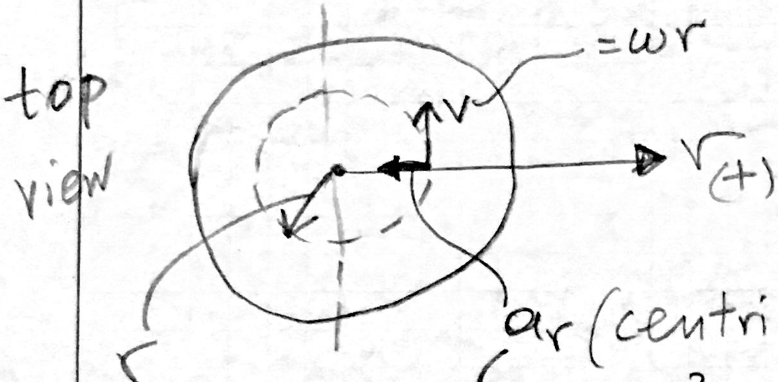
Euler's equ. along r

$$\frac{\partial(p+\rho g z)}{\partial r} = -\rho a_r \quad (1)$$





a_r (centripetal acceleration)

$$= -\frac{v^2}{r} = -\frac{\omega^2 r^2}{r} = -\omega^2 r$$

(1) becomes $\frac{\partial}{\partial r}(p+\rho g z) = \rho \omega^2 r$

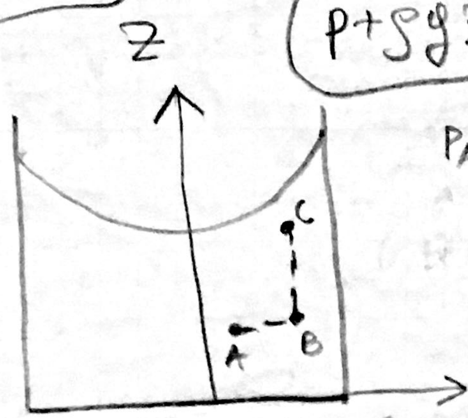
or

$$\frac{\partial}{\partial r}\left(p+\rho g z - \frac{\rho \omega^2 r^2}{2}\right) = 0$$

or

Euler's equ. for liquids in pure rotation

$$\boxed{p+\rho g z - \frac{\rho \omega^2 r^2}{2} = \text{constant}} \quad (2)$$



$$p_A + \cancel{\rho g z_A} - \frac{\rho \omega^2 r_A^2}{2} = p_B + \cancel{\rho g z_B} - \frac{\rho \omega^2 r_B^2}{2}$$

$(z_A = z_B)$

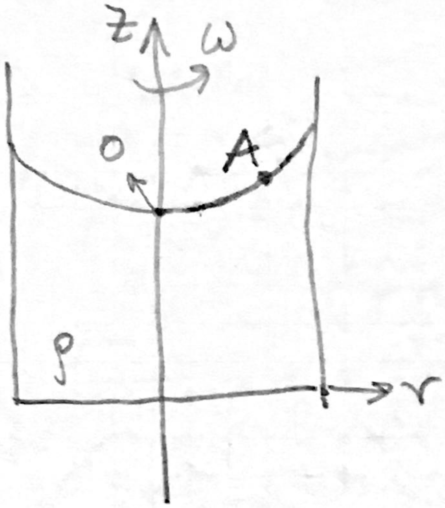
$$p_A - \frac{\rho \omega^2 r_A^2}{2} = p_B - \frac{\rho \omega^2 r_B^2}{2}$$

$r_B > r_A \Rightarrow p_B > p_A$

$$p_B + \rho g z_B - \cancel{\frac{\rho \omega^2 r_B^2}{2}} = p_C + \rho g z_C - \cancel{\frac{\rho \omega^2 r_C^2}{2}} \Rightarrow$$

$(r_B = r_C)$

hydrostatic law still applies on points on the same vertical



$P_A = P_0 = 0$ (gage pressure on the free surface)

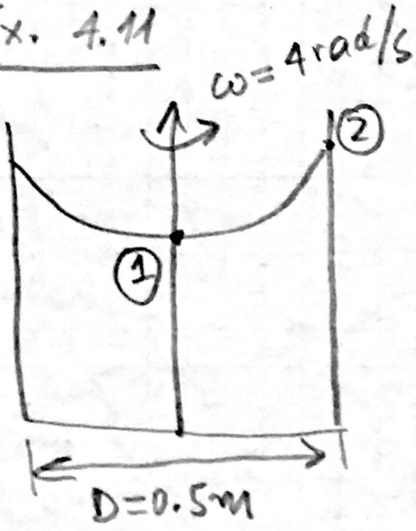
$$\underbrace{\cancel{P_A}}_{=0} + \cancel{\rho} g z_A - \frac{\cancel{\rho}\omega^2 r_A^2}{2} = \underbrace{\cancel{P_0}}_{=0} + \cancel{\rho} g z_0 - \cancel{\frac{\rho\omega^2 r_0^2}{2}} \quad (r_0 = 0)$$

$$\Rightarrow z_A = z_0 + \frac{\omega^2 r_A^2}{2g}$$

$$\text{or} \quad \boxed{(z_A - z_0) = \frac{\omega^2 r_A^2}{2g}} \quad (3)$$

equation for a <u>paraboloid</u>

<u>Ex. 4.11</u>



<u>Find $z_2 - z_1$</u>

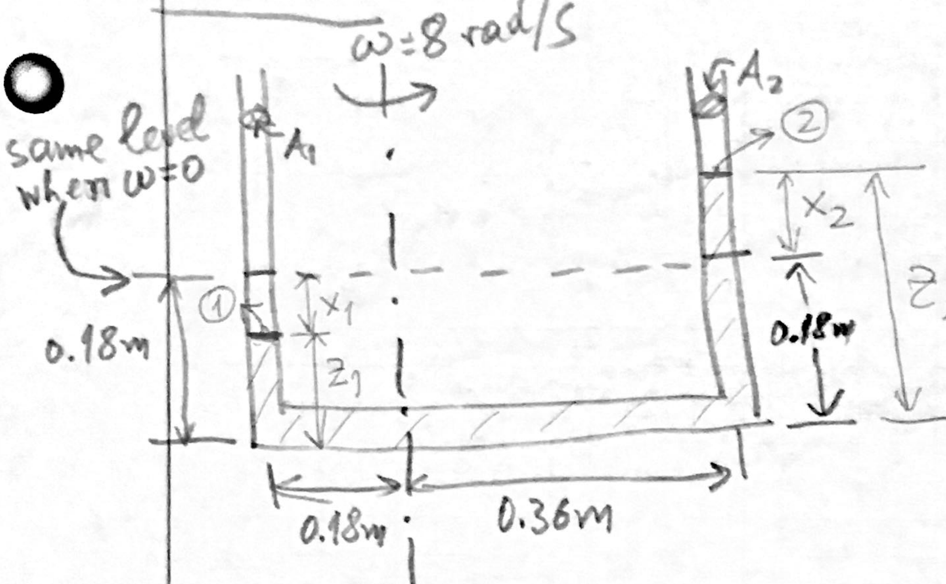
Based on (3)

$$z_2 - z_1 = \frac{\omega^2 r_2}{2g}$$

$$z_2 - z_1 = \frac{(4)^2 \times (0.25)^2}{2 \times 9.81} = 0.051\text{m}$$

$= \underline{5.1\text{cm}}$

Ex. 4.12



$x_1 = x_2$ since $A_1 = A_2$

(since $A_1 x_1 = A_2 x_2$ for liquid to maintain its mass)

A_1, A_2 sectional areas of tubes

$z_1 = 0.18 - x$

$z_2 = 0.18 + x$ $\quad (x_1 = x_2 = x)$

Need to find x!

Euler's equ. between ① & ②:

$$\underbrace{\cancel{p_1}}_{=0} + \cancel{\rho} g z_1 - \frac{\cancel{\rho}\omega^2 r_1^2}{2} = \underbrace{\cancel{p_2}}_{=0} + \cancel{\rho} g z_2 - \frac{\cancel{\rho}\omega^2 r_2^2}{2}$$

$$z_1 - z_2 = \frac{\omega^2}{2g}\left(r_1^2 - r_2^2\right)$$

$$(\cancel{0.18} - x) - (\cancel{0.18} + x) = \frac{8^2}{2 \cdot 9.81}\left((0.18)^2 - (0.36)^2\right) = -0.317\,m$$

$$-2x = -0.317 \Rightarrow x = \frac{0.317}{2} = 0.1585\,m$$

$z_1 = 0.18 - 0.1585 = \mathbf{0.0215}\ m$ ✓

$z_2 = 0.18 + 0.1585 = \mathbf{0.3385}\ m$ ✓

Q: Where do you expect the pressure inside the liquid to be max?

A: $p + \rho g z - \frac{\rho\omega^2 r^2}{2} = \text{const.} \Rightarrow p = \text{const.} - \rho g z + \frac{\rho\omega^2 r^2}{2}$

For $p_{max} \rightsquigarrow z_{min}$ & $r_{max} \Rightarrow$ point $\boxed{}$