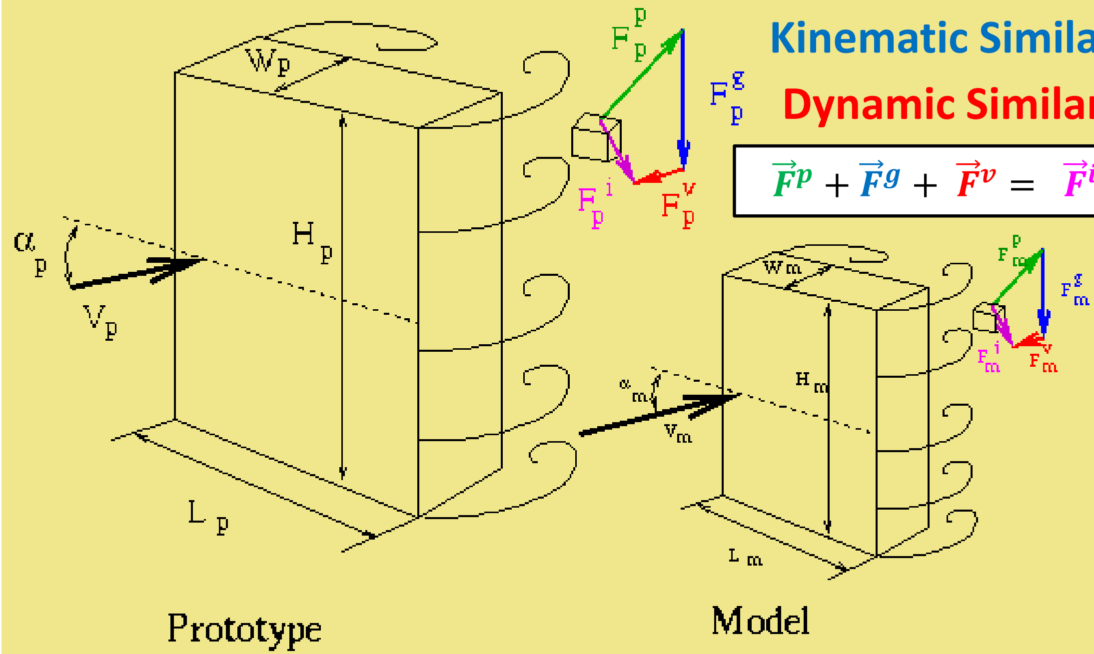
NOTES ON SIMILITUDE



Kinematic Similarity

Dynamic Similarity

$$\boxed{\vec{F}^p + \vec{F}^g + \vec{F}^v = \vec{F}^i = M\vec{a}}$$

Geometric Similarity: Wm/Wp=Hm/Hp=Lm/Lp=scale ratio

$\alpha_m = \alpha_p$

Viscous forces | Pressure forces | Gravity forces | Inertial forces

$$\frac{\vec{F}_p^v}{\vec{F}_m^v} = \frac{\vec{F}_p^p}{\vec{F}_m^p} = \frac{\vec{F}_p^g}{\vec{F}_m^g} = \frac{\vec{F}_p^i}{\vec{F}_m^i}$$

$$F^v \sim \mu \frac{dV}{dy}(Area) \sim \mu \frac{V}{L} L^2 \sim \mu V L$$

$$F^p \sim \Delta p(Area) \sim \Delta p L^2$$

$$F^g \sim \gamma(Volume) \sim \rho g L^3$$

$$F^i \sim Ma \sim \rho L^3 \frac{V}{T} \sim \rho L^3 \frac{V^2}{L} \sim \rho L^2 V^2$$

$$\frac{\mu_p V_p L_p}{\mu_m V_m L_m} = \frac{\rho_p L_p^2 V_p^2}{\rho_m L_m^2 V_m^2}$$

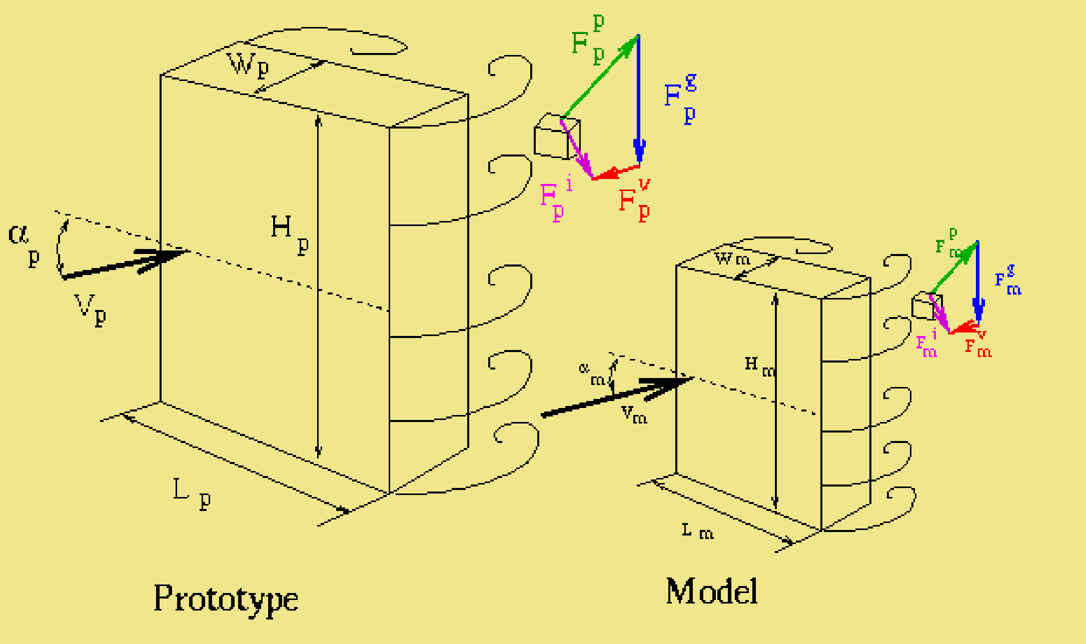
$$Re_p = \frac{\rho_p V_p L_p}{\mu_p} = \frac{\rho_m V_m L_m}{\mu_m} = Re_m$$

$$\frac{\rho_p g_p L_p^3}{\rho_m g_m L_m^3} = \frac{\rho_p L_p^2 V_p^2}{\rho_m L_m^2 V_m^2}$$

$$Fr_p^2 = \frac{V_p^2}{g_p L_p} = \frac{V_m^2}{g_m L_m} = Fr_m^2$$

Reynolds Number: $Re = \frac{V\,L}{\nu} = \frac{\rho L^2 V^2}{\mu V L} = \frac{Inertial\ Forces}{Viscous\ Forces}$

Froude Number: $Fr = \frac{V}{\sqrt{g\,L}} = \sqrt{\frac{\rho L^2 V^2}{\rho g L^3}} = \sqrt{\frac{Inertial\ Forces}{Gravity\ Forces}}$



Geometric Similarity: Wm/Wp=Hm/Hp=Lm/Lp=scale ratio

$\alpha_m = \alpha_p$

Viscous forces	Pressure forces	Gravity forces	Inertial forces

$$\frac{\vec{F}_p^v}{\vec{F}_m^v} = \frac{\vec{F}_p^p}{\vec{F}_m^p} = \frac{\vec{F}_p^g}{\vec{F}_m^g} = \frac{\vec{F}_p^i}{\vec{F}_m^i}$$

$$F^v \sim \mu \frac{dV}{dy}(Area) \sim \mu \frac{V}{L} L^2 \sim \mu V L$$

$$F^p \sim \Delta p (Area) \sim \Delta p L^2$$

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$$F^i \sim Ma \sim \rho L^3 \frac{V}{T} \sim \rho L^3 \frac{V^2}{L} \sim \rho L^2 V^2$$

$$\frac{\Delta p_p L_p^2}{\Delta p_m L_m^2} = \frac{\rho_p L_p^2 V_p^2}{\rho_m L_m^2 V_m^2}$$

Pressure Coefficient: $$C_P = \frac{\Delta p}{\frac{1}{2}\rho V^2} \qquad C_{Pm} = \frac{\Delta p_m}{\frac{1}{2}\rho_m V_m^2} = \frac{\Delta p_p}{\frac{1}{2}\rho_p V_p^2} = C_{Pp}$$

Force Coefficient: $$C_F = \frac{F}{\frac{1}{2}\rho V^2 A} \qquad C_{Fm} = \frac{F_m}{\frac{1}{2}\rho_m V_m^2 A_m} = \frac{F_p}{\frac{1}{2}\rho_p V_p^2 A_p} = C_{Fp}$$

A: Reference Area of the Body

For the Reynolds number to be kept the same for the prototype and the model:

$$Re_p = \frac{\rho_p V_p L_p}{\mu_p} = \frac{\rho_m V_m L_m}{\mu_m} = Re_m \quad \Longrightarrow \quad \boxed{\frac{V_m}{V_p} = \frac{L_p}{L_m}\frac{\nu_m}{\nu_p}}$$

For the Froude number to be kept the same for the prototype and the model:

$$Fr_p^2 = \frac{V_p^2}{g_p L_p} = \frac{V_m^2}{g_m L_m} = Fr_m^2 \quad \Longrightarrow \quad \boxed{\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}}}$$

For both the Reynolds and the Froude number to be kept the same for the prototype and the model:

$$\frac{L_p}{L_m}\frac{\nu_m}{\nu_p} = \sqrt{\frac{L_m}{L_p}} \quad \Longrightarrow \quad \frac{\nu_m}{\nu_p} = \left(\frac{L_m}{L_p}\right)^{3/2}$$

Practically impossible to achieve Reynolds and Froude number equality!

- *If free surface effects are NOT important (fully filled pipe, fully submerged body, fluid is air), then we only equate Re numbers*
- *If free surface effects are important (open channel flow, ship), then we equate Fr numbers and account for effects of different Re numbers*