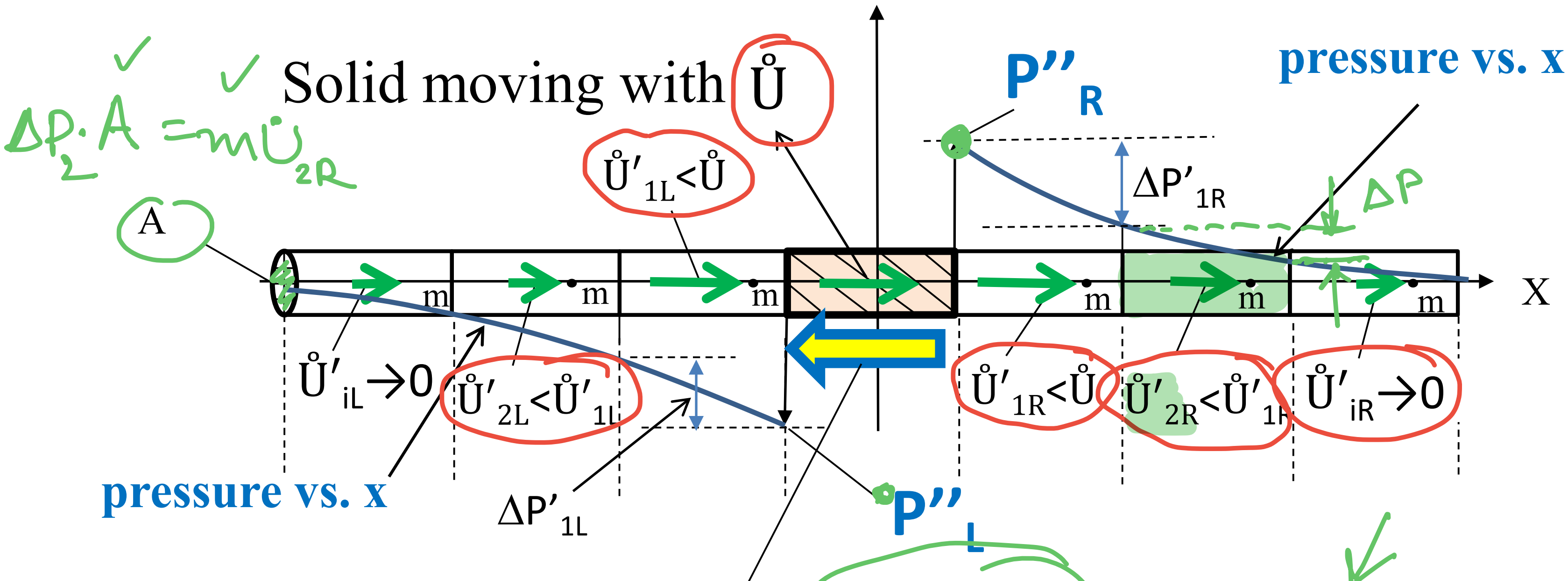
The concept of Added Mass (1/7)

Solid moving with acceleration $\mathring{U}$ inside quiescent fluid



Resistive Force F_{res} : $F_{res} = (P''_R - P''_L)\, A = m_{added}\,\mathring{U}$

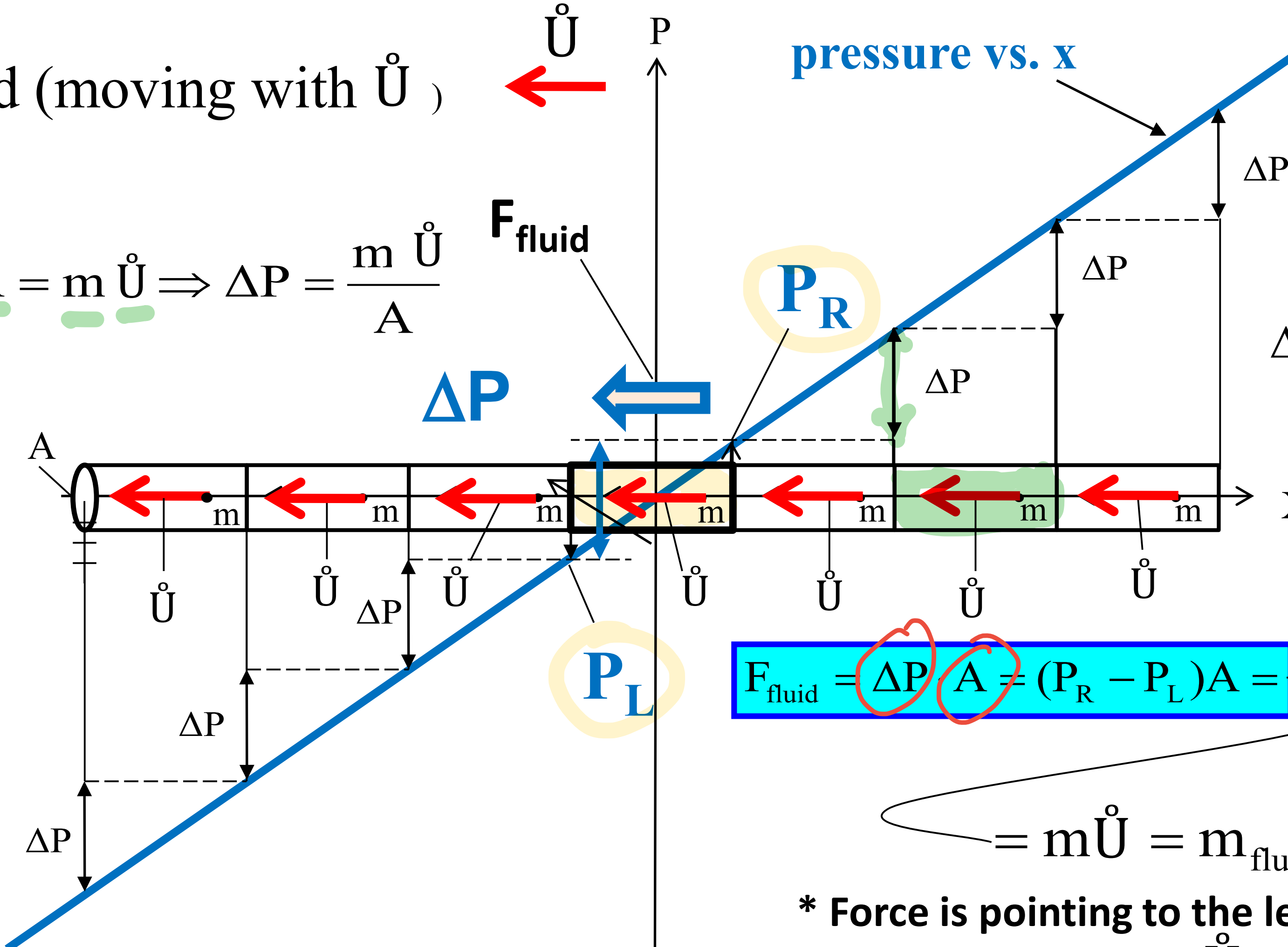
$$F_{res} = (P''_R - P''_L)\, A = \sum \Delta P'_{iR} A + \sum \Delta P'_{iL} A = \sum m\, \dot{U}'_{iR} + \sum m\, \dot{U}'_{iL}$$

The resistive force ($F_{res} = m_{added}\ \mathring{U}$) is needed in order to accelerate parts of the surrounding fluid which move with the body.

The concept of Added Mass (2/7)

Fluid subject to acceleration without a body inside it

Fluid (moving with $\mathring{U}$)

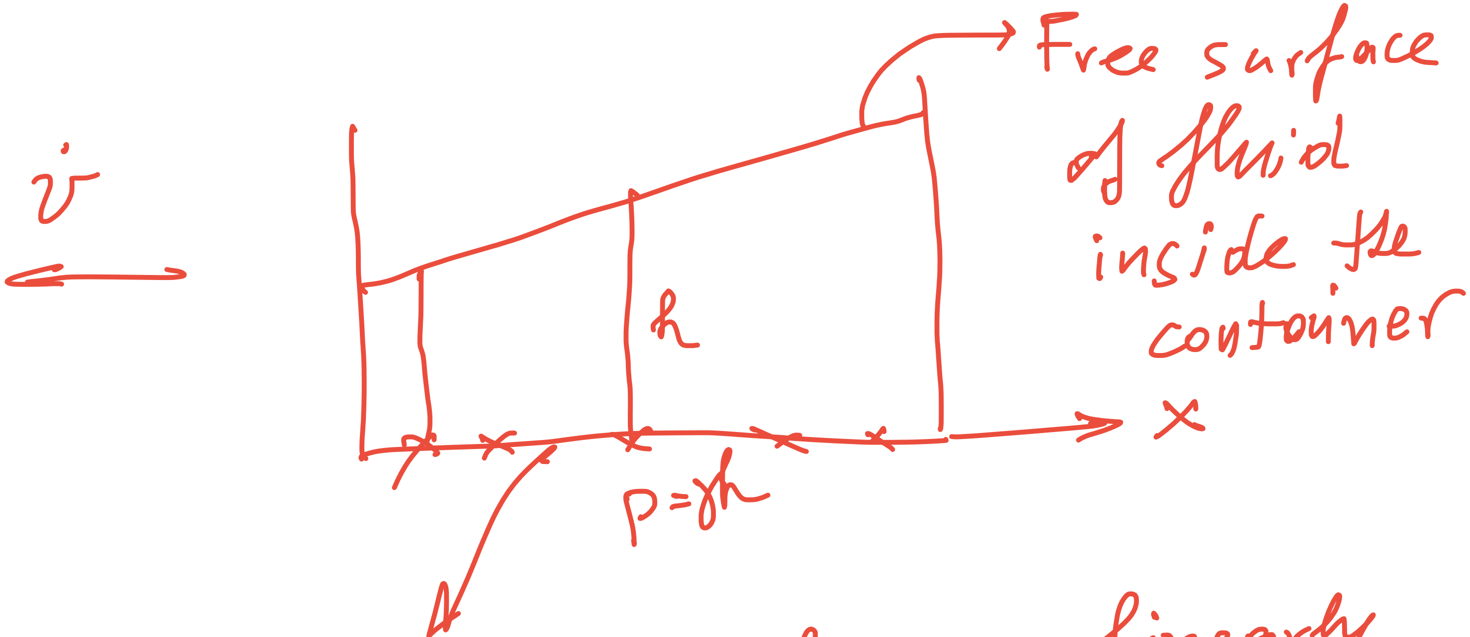


$$\Delta P \cdot A = m\,\mathring{U} \Rightarrow \Delta P = \frac{m\,\mathring{U}}{A}$$

$$\Delta P = P_R - P_L$$

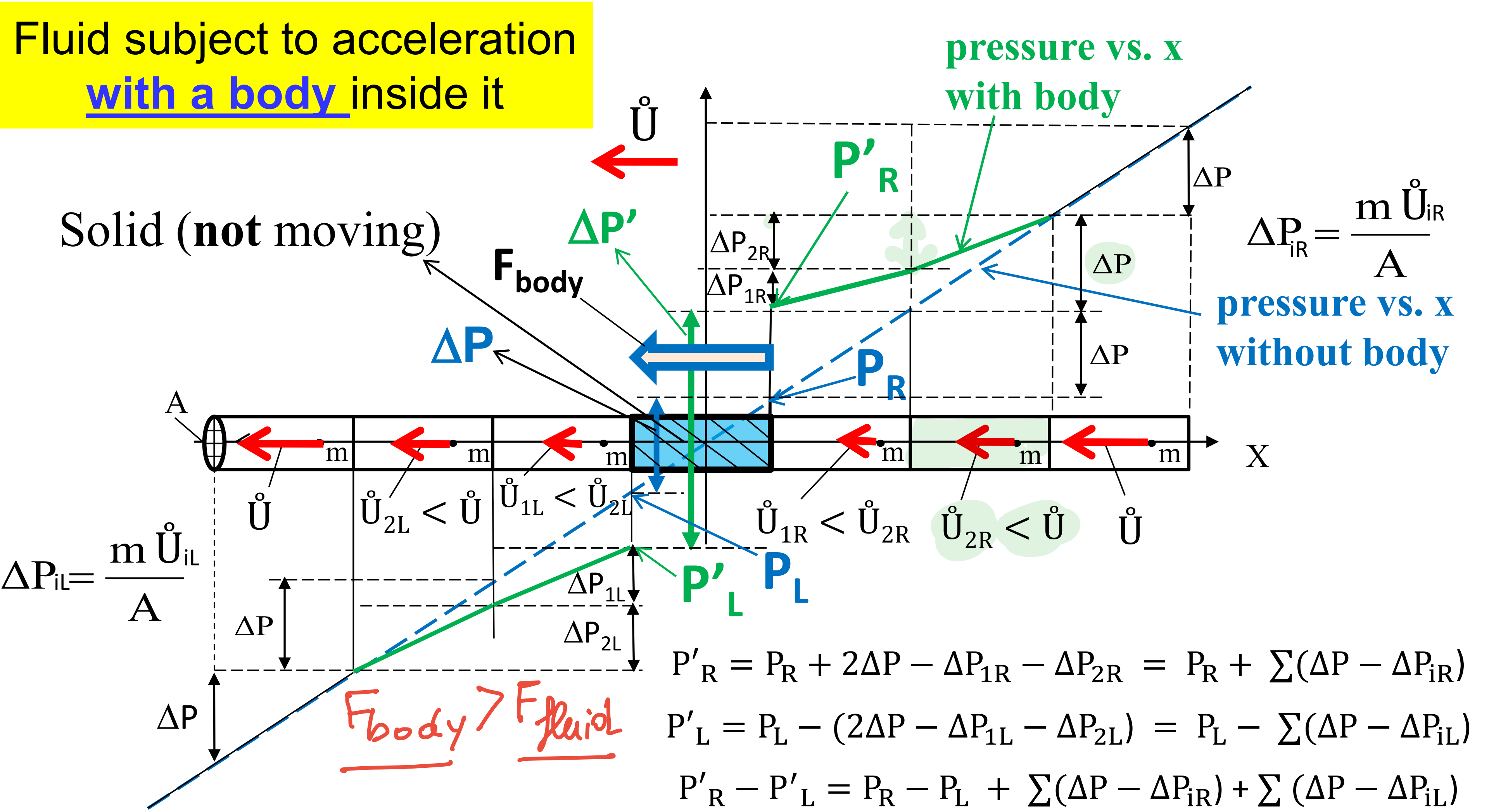
$$F_{fluid} = \Delta P\, A = (P_R - P_L)A = m\mathring{U} = m_{fluid}\,\mathring{U}$$

*** Force is pointing to the left (in the direction of $\mathring{U}$)**



pressure changes linearly along x, since $p = \gamma h$ and h is a linear function of x.

The concept of Added Mass (3/7)



$$P'_R = P_R + 2\Delta P - \Delta P_{1R} - \Delta P_{2R} = P_R + \sum(\Delta P - \Delta P_{iR})$$

$$P'_L = P_L - (2\Delta P - \Delta P_{1L} - \Delta P_{2L}) = P_L - \sum(\Delta P - \Delta P_{iL})$$

$$P'_R - P'_L = P_R - P_L + \sum(\Delta P - \Delta P_{iR}) + \sum(\Delta P - \Delta P_{iL})$$

$$F_{body} = (P'_R - P'_L)\,A = (P_R - P_L)\,A + \sum m\,(\dot{U} - \dot{U}_{iR}) + \sum m\,(\dot{U} - \dot{U}_{iL})$$

$$F_{body} = m_{fluid}\,\dot{U} + \sum m\,(\dot{U} - \dot{U}_{iR}) + \sum m\,(\dot{U} - \dot{U}_{iL})$$

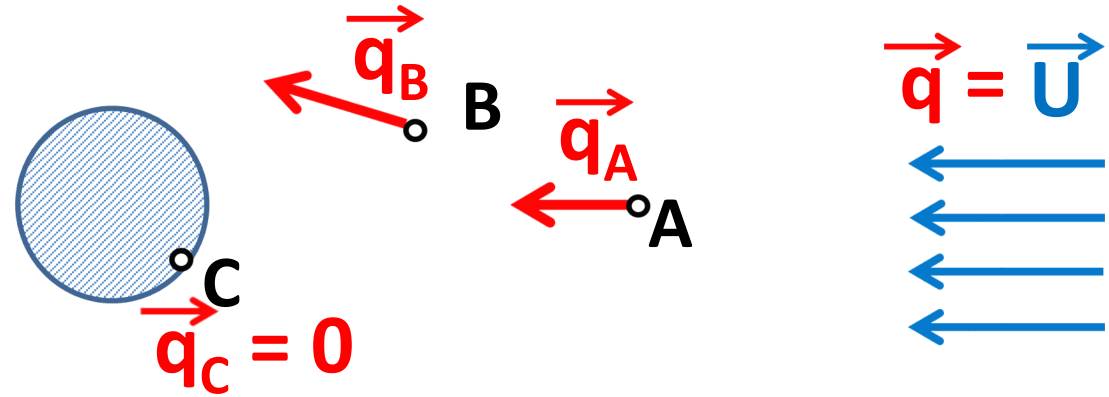
$F_{body} = F_{fluid}$!

The concept of Added Mass (4/7)

Inertia coefficient

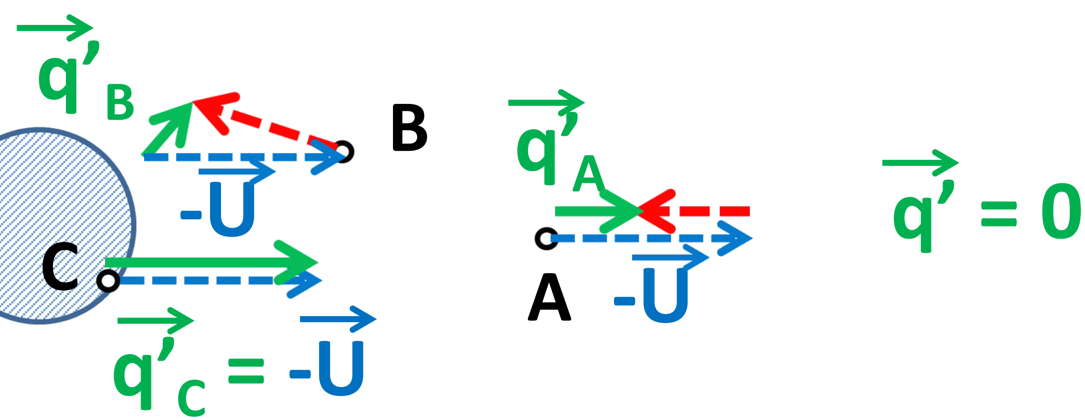
Two Different Frames of Reference (for the same problem!):

Body is stationary and flow is moving to the left with speed U



$$\vec{q}' = \vec{q} - \vec{U}$$

Flow is stationary (far upstream) and body is moving to the right with speed U



The concept of Added Mass (5/7)

$$F_{res} = (P''_R - P''_L)\,A = \sum m\,\dot{U}'_{iR} + \sum m\,\dot{U}'_{iL} = m_{added}\dot{U}$$

$$\dot{U}'_{iR} = -\dot{U}_{iR} - (-\dot{U}) = \dot{U} - \dot{U}_{iR}$$

$$\dot{U}'_{iL} = -\dot{U}_{iL} - (-\dot{U}) = \dot{U} - \dot{U}_{iL}$$

Thus:

$$\sum m\,(\dot{U} - \dot{U}_{iR}) + \sum m\,(\dot{U} - \dot{U}_{iL}) = m_{added}\dot{U}$$

$$F_{body} = m_{fluid}\,\dot{U} + \sum m\,(\dot{U} - \dot{U}_{iR}) + \sum m\,(\dot{U} - \dot{U}_{iL})$$

Thus:

$$\mathbf{F_{body} = m_{fluid}\,\dot{U} + m_{added}\dot{U} = = (m_{fluid} + m_{added})\,\dot{U}}$$

$$\mathbf{F_{body} = F_{inertial} = C_M\,m_{fluid}\,\dot{U}}$$ $= F_{fluid}$

Inertia coefficient C_M

$$C_M = \frac{F_{body}}{F_{fluid}}$$

$$C_M = \frac{m_{fluid} + m_{added}}{m_{fluid}} = 1 + \frac{m_{added}}{m_{fluid}}$$

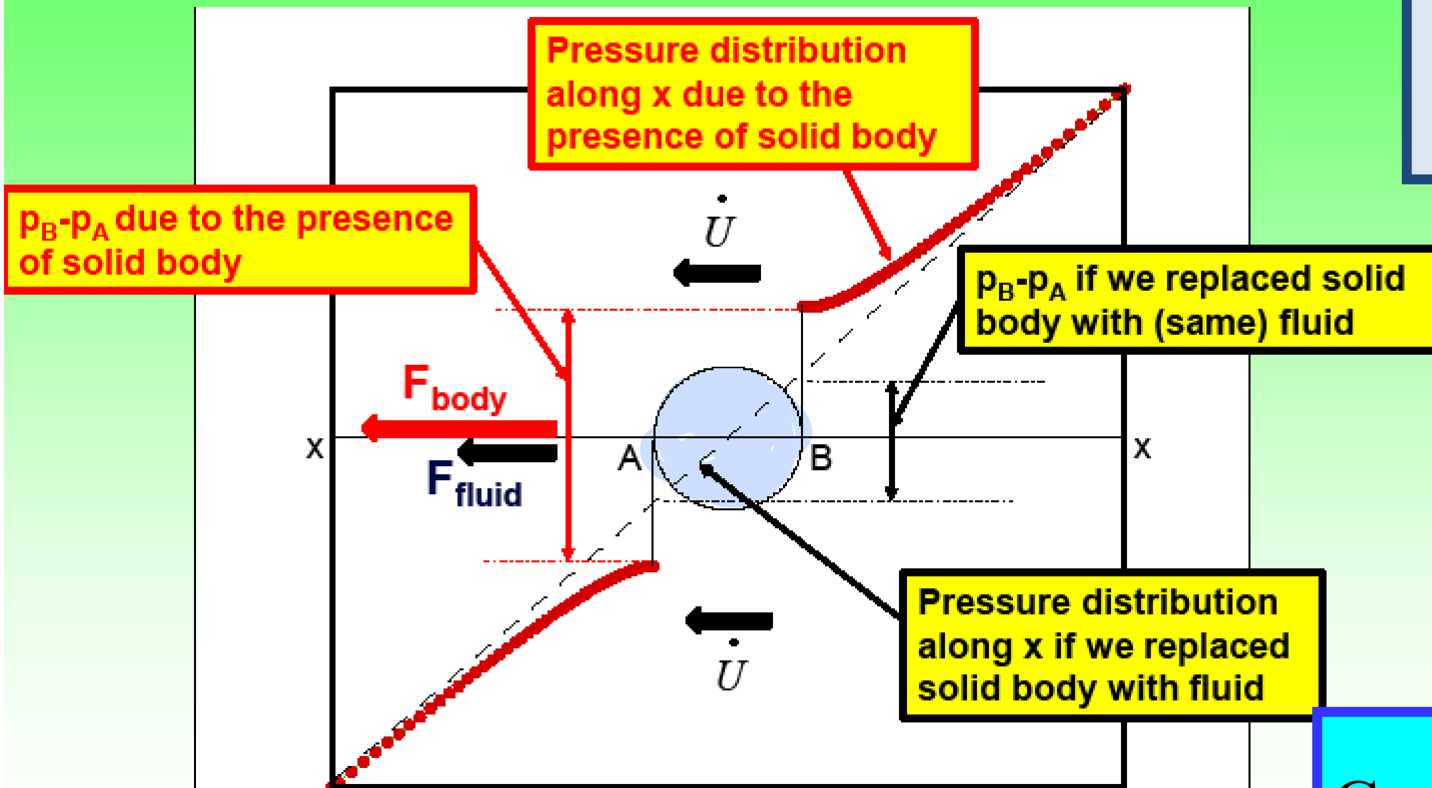
- **m_{fluid}=mass of fluid displaced by body**
- **m_{added}=added mass; function of body shape and inflow direction**

The concept of Added Mass (6/7)

Cylinder subject to accelerated inflow.

Results from Inviscid flow simulation

Definition of inertia coefficient C_M



$$C_M = \frac{F_{body}}{F_{fluid}}$$

$$C_M = \frac{m_{fluid} + m_{added}}{m_{fluid}}$$

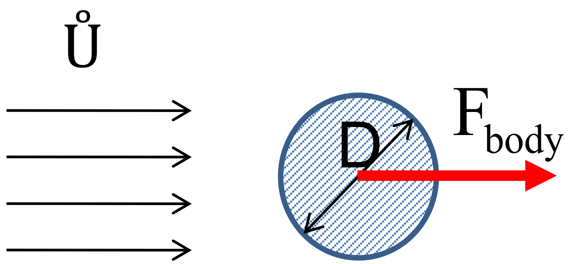
$$F_{fluid} = \rho(\pi D^2 / 4)\dot{U}$$

$$F_{body} = C_M \rho(\pi D^2 / 4)\dot{U}$$

For inviscid flow around 2-D cylinder: C_M=2

The concept of Added Mass (7/7)

Summary: (all quantities are per unit width in 2-D)



$$F_{body} = C_M m_{fluid} \mathring{U}$$ = F_{fluid}

$$m_{fluid} = \rho_{fluid} V_{fluid} = \text{mass of diplaced fluid}$$

$$V_{fluid} = \text{volume of displaced fluid}$$

$$C_M = \text{inertia coefficient} = \frac{m_{fluid} + m_{added}}{m_{fluid}} = 1 + \frac{m_{added}}{m_{fluid}} = 1 + a$$

$$a = \frac{m_{added}}{m_{fluid}} = \text{added mass coefficient (depends on shape + direction of flow)}$$

$a = \frac{m_{added}}{m_{fluid}}$

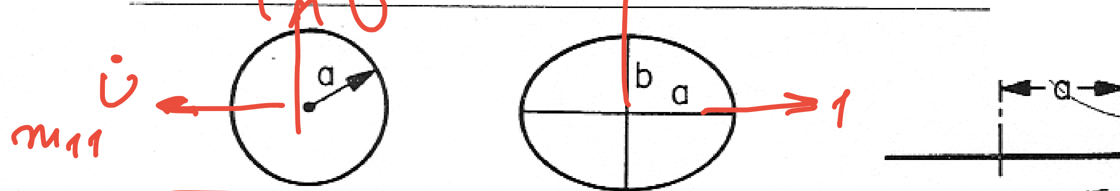
For a cylinder (circle in 2-D) V_{fluid}=area of cross section = $\pi D^2/4$

For a cylinder in **inviscid unbounded flow**: C_M=2 (a=1)

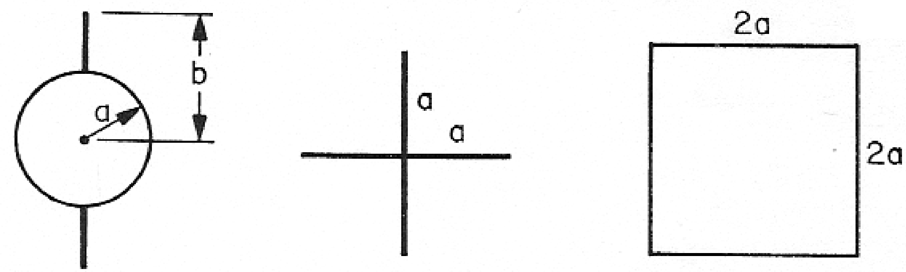
(Inviscid) Added Mass for other shapes:

m_{11} and **m_{22}** are the added masses when the flow is accelerated in the horizontal or the vertical axis, respectively. **m_{66}** is the added moment of inertial when the body rotates around an axis normal to the paper.

Table 4.3
Added-Mass Coefficients for Various Two-Dimensional Bodies.



m_{11}:	$\pi\rho a^2$	$\pi\rho b^2$	0
m_{22}:	$\pi\rho a^2$	$\pi\rho a^2$	$\pi\rho a^2$
m_{66}:	0	$\frac{1}{8}\pi\rho(a^2-b^2)^2$	$\frac{1}{8}\pi\rho a^4$



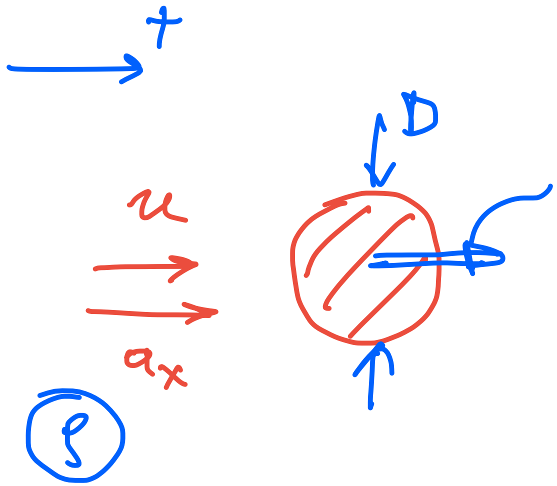
m_{11}:	$\pi\rho[a^2+(b^2-a^2)^2/b^2]$	$\pi\rho a^2$	$4.754\ \rho a^2$
m_{22}:	$\pi\rho a^2$	$\pi\rho a^2$	$4.754\ \rho a^2$
m_{66}:	*	$\frac{2}{\pi}\rho a^4$	$0.725\ \rho a^4$

*For the finned circle the added moment of inertia is given by the formula

$$m_{66} = \rho a^4(\pi^{-1}\csc^4\alpha\,[2\alpha^2 - \alpha\sin 4\alpha + \tfrac{1}{2}\sin^2 2\alpha] - \pi/2)$$

FOR INVISCID FLOW

From Marine Hydrodynamics, Newman, J.N., 1977

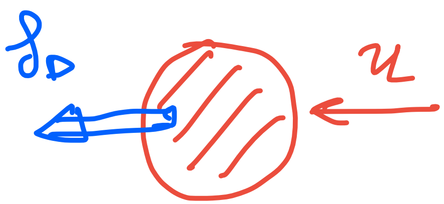


$$f_D = C_D \frac{\rho}{2} u^2 \cdot D \quad \text{(per unit depth)}$$

C_D → drag coeff.

$$f_i = C_M \rho \frac{\pi D^2}{4} a_x \quad \text{(inertial force)}$$

C_M → inertia coeff.



$$f_D = C_D \frac{\rho}{2} u |u| D$$

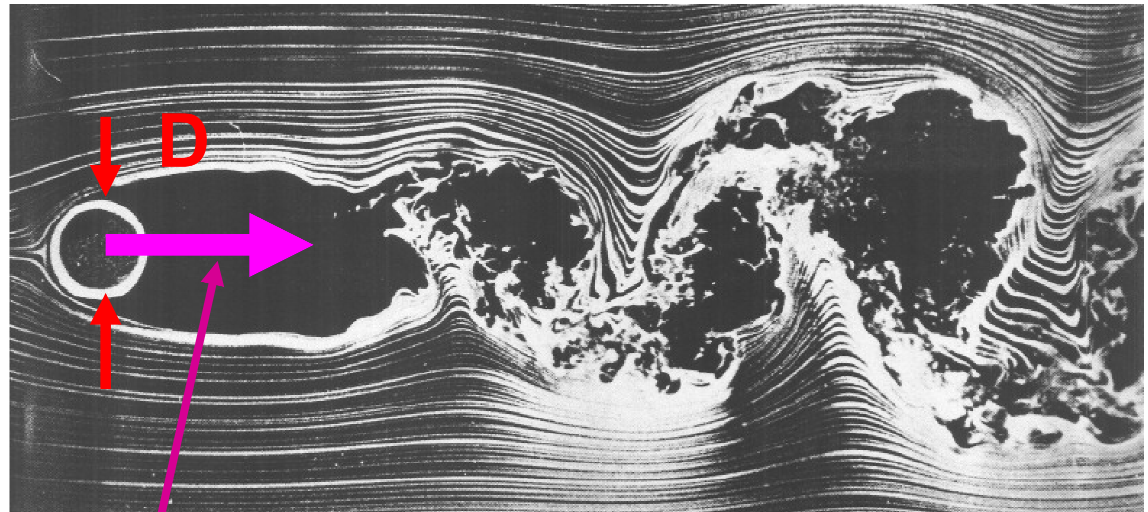
$$f_{total} = f_D + f_i$$

Morison’s equation for total force in the direction of wave propagation

[Morison, J. R.; O'Brien, M. P.; Johnson, J. W.; Schaaf, S. A. (1950), "The force exerted by surface waves on piles", Petroleum Transactions (American Institute of Mining Engineers) 189: 149–154]

Velocity, u

Acceleration, a

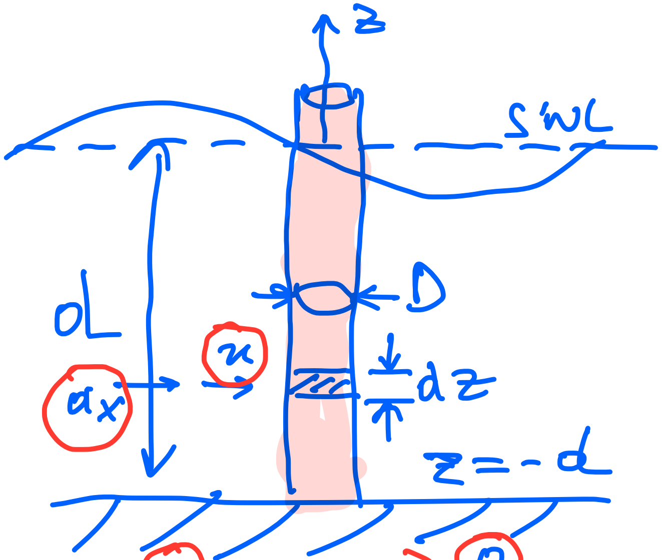


Total force = Viscous force + Inertial force

Total force (per unit width) → $$C_D \frac{1}{2} \rho D u\,|\,u\,| + C_M \rho \frac{\pi D^2}{4} a$$

C_D: **Drag coefficient**

C_M: **Inertia coefficient**



$$dF = f_D\, dz + f_i\, dz$$

$$\rightarrow f_D = C_D \frac{\rho}{2} u|u| D \checkmark$$

$$\rightarrow f_i = C_M \rho \frac{\pi D^2}{4} a_x \checkmark$$

$$F_t = \int_{-d}^{\eta} dF = \underbrace{\int_{-d}^{0} f_D\, dz}_{F_D} + \underbrace{\int_{-d}^{0} f_i\, dz}_{F_i}$$

a_x from Fig 2.6

$\theta = kx - \omega t$

$$F_i(t) = \int_{-d}^{0} C_M \frac{\pi D^2}{4} \rho\, a_x\, dz = C_M \rho \frac{\pi D^2}{4} \int_{-d}^{0} \frac{g\pi H}{L} \frac{\cosh[k(z+d)]}{\cosh[kd]} dz \sin\theta$$

$$= C_M \rho \frac{\pi D^2}{4} H g \left[\int_{-d}^{0} \frac{\pi}{L} \frac{\cosh[k(z+d)]}{\cosh[kd]} dz \right] \sin\theta$$

$$K_{im} = \frac{1}{2}\tanh\left(\frac{2\pi d}{L}\right)$$

$$F_i(t) = F_{im} \sin\theta$$

$$F_{im} = C_M \rho g \frac{\pi D^2}{4} H K_{im}$$

max value of inertial force

Similarly:

$$F_D(t) = \int_{-d}^{0} f_D \, dz = F_{Dm} \left|\cos\theta\right| \cos\theta$$

$$F_{Dm} = C_D \frac{1}{2} \rho g D H^2 K_{Dm}$$

max value of drag (viscous) force

$$K_{Dm} = \frac{1}{8}\left[1 + \frac{4\pi d/L}{\sinh\left[4\pi d/L\right]}\right] = \frac{n}{4}$$

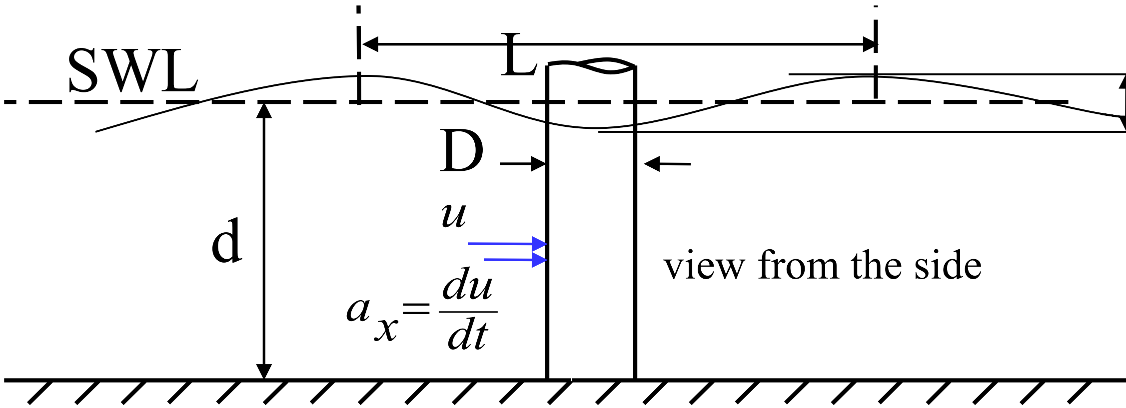
where $n = \frac{C_g}{C}$

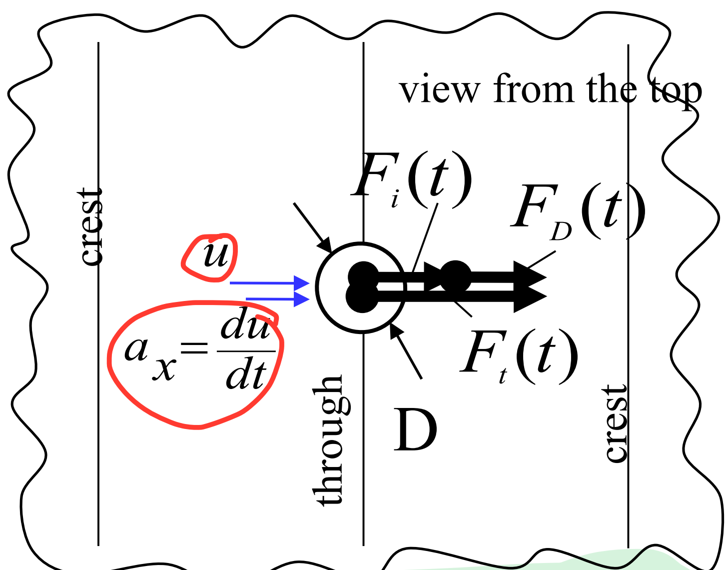
RELATIVE DEPTH	SHALLOW WATER $\frac{d}{L} < \frac{1}{25}$	TRANSITIONAL WATER $\frac{1}{25} < \frac{d}{L} < \frac{1}{2}$	DEEP WATER $\frac{d}{L} > \frac{1}{2}$
1. Wave profile	Same As →	$\eta = \frac{H}{2} \cos\left[\frac{2\pi x}{L} - \frac{2\pi t}{T}\right] = \frac{H}{2} \cos\theta$	← Same As
2. Wave celerity	$C = \frac{L}{T} = \sqrt{gd}$	$C = \frac{L}{T} = \frac{gT}{2\pi} \tanh\left(\frac{2\pi d}{L}\right)$	$C = C_0 = \frac{L}{T} = \frac{gT}{2\pi}$
3. Wavelength	$L = T\sqrt{gd} = CT$	$L = \frac{gT^2}{2\pi} \tanh\left(\frac{2\pi d}{L}\right)$	$L = L_0 = \frac{gT^2}{2\pi} = C_0 T$
4. Group velocity	$C_g = C = \sqrt{gd}$	$C_g = nC = \frac{1}{2}\left[1 + \frac{4\pi d/L}{\sinh(4\pi d/L)}\right] \cdot C$	$C_g = \frac{1}{2} C = \frac{gT}{4\pi}$
5. Water Particle Velocity (a) Horizontal	$u = \frac{H}{2}\sqrt{\frac{g}{d}} \cos\theta$	$u = \frac{H}{2} \frac{gT}{L} \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \cos\theta$	$u = \frac{\pi H}{T} e^{\frac{2\pi z}{L}} \cos\theta$
(b) Vertical	$w = \frac{H\pi}{T}\left(1 + \frac{z}{d}\right) \sin\theta$	$w = \frac{H}{2} \frac{gT}{L} \frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \sin\theta$	$w = \frac{\pi H}{T} e^{\frac{2\pi z}{L}} \sin\theta$
6. Water Particle Accelerations (a) Horizontal	$a_x = \frac{H\pi}{T}\sqrt{\frac{g}{d}} \sin\theta$	$a_x = \frac{g\pi H}{L} \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \sin\theta$	$a_x = 2H\left(\frac{\pi}{T}\right)^2 e^{\frac{2\pi z}{L}} \sin\theta$
(b) Vertical	$a_z = -2H\left(\frac{\pi}{T}\right)^2\left(1 + \frac{z}{d}\right) \cos\theta$	$a_z = -\frac{g\pi H}{L} \frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \cos\theta$	$a_z = -2H\left(\frac{\pi}{T}\right)^2 e^{\frac{2\pi z}{L}} \cos\theta$
7. Water Particle Displacements (a) Horizontal	$\xi = -\frac{HT}{4\pi}\sqrt{\frac{g}{d}} \sin\theta$	$\xi = -\frac{H}{2} \frac{\cosh[2\pi(z+d)/L]}{\sinh(2\pi d/L)} \sin\theta$	$\xi = -\frac{H}{2} e^{\frac{2\pi z}{L}} \sin\theta$
(b) Vertical	$\zeta = \frac{H}{2}\left(1 + \frac{z}{d}\right) \cos\theta$	$\zeta = \frac{H}{2} \frac{\sinh[2\pi(z+d)/L]}{\sinh(2\pi d/L)} \cos\theta$	$\zeta = \frac{H}{2} e^{\frac{2\pi z}{L}} \cos\theta$
8. Subsurface Pressure	$p = \rho g(\eta - z)$	$p = \rho g \eta \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} - \rho g z$	$p = \rho g \eta e^{\frac{2\pi z}{L}} - \rho g z$

Figure 2-6. Summary of linear (Airy) wave theory--wave characteristics.

Application of Morison's equation to determine forces on vertical piles

Morison's equation is integrated over the length of the pile, after the values for **u** and **a_x** have been determined by either using linear or **non-linear wave theories**





$$\vartheta = -\omega\ t$$

$$F_{total}(t) = F_i(t) + F_D(t)$$

$$F_i(t) = F_{im} \cdot \sin(\vartheta)$$

$$F_{im} = C_M \cdot \rho g \cdot \frac{\pi D^2}{4} H \cdot K_{im}$$

$$F_D(t) = F_{Dm} \cdot |\cos\vartheta| \cos\vartheta$$

$$F_{Dm} = C_D \frac{1}{2} \rho g\ D H^2 \cdot K_{Dm}$$

$$K_{im} = \frac{1}{2} \tanh\left(\frac{2\pi d}{L}\right)$$

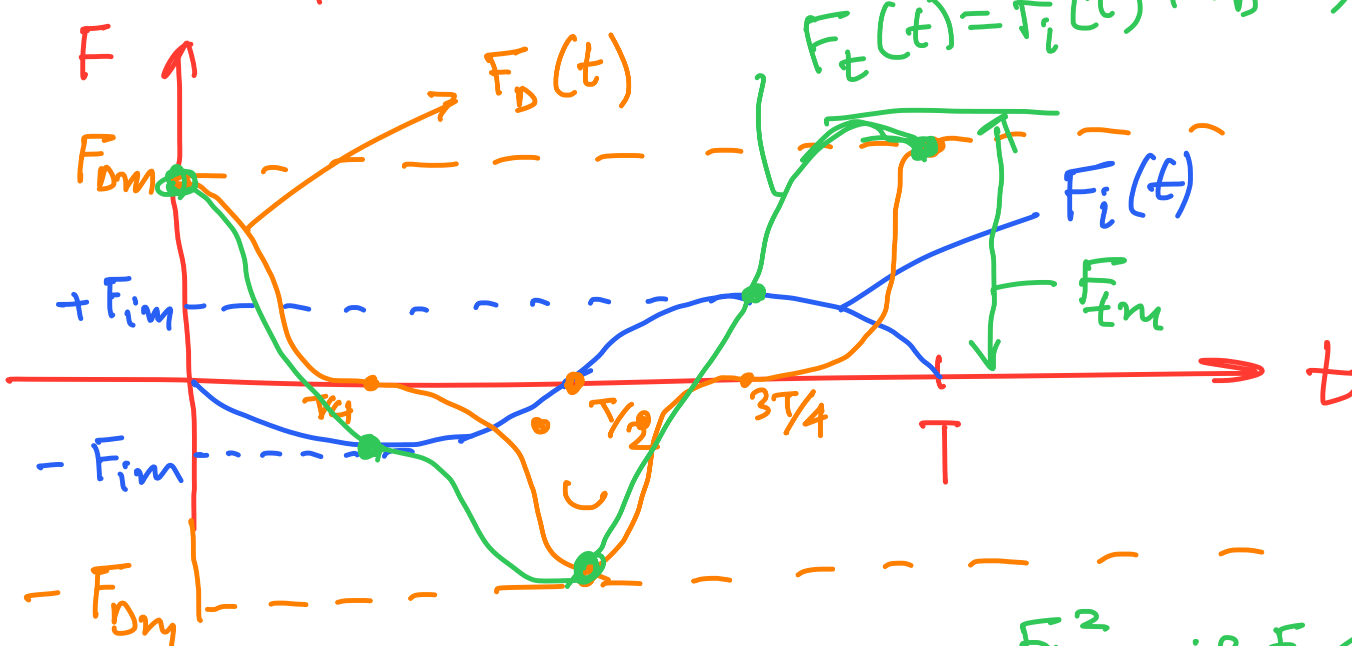
$$\frac{\pi}{4} = K_{DM} = \frac{1}{8}\left(1 + \frac{4\pi d / L}{\sinh[4\pi d / L]}\right)$$

For deep H_2O: $K_{im} = \frac{1}{2}$ & $K_{Dm} = \frac{1}{8}$

Draw forces vs. t for $x=0 \rightsquigarrow \theta = -\omega t$
(crest at location of pile at $t=0$)

→ $F_i(t) = F_{im} \sin(-\omega t) = -F_{im} \sin(\omega t)$

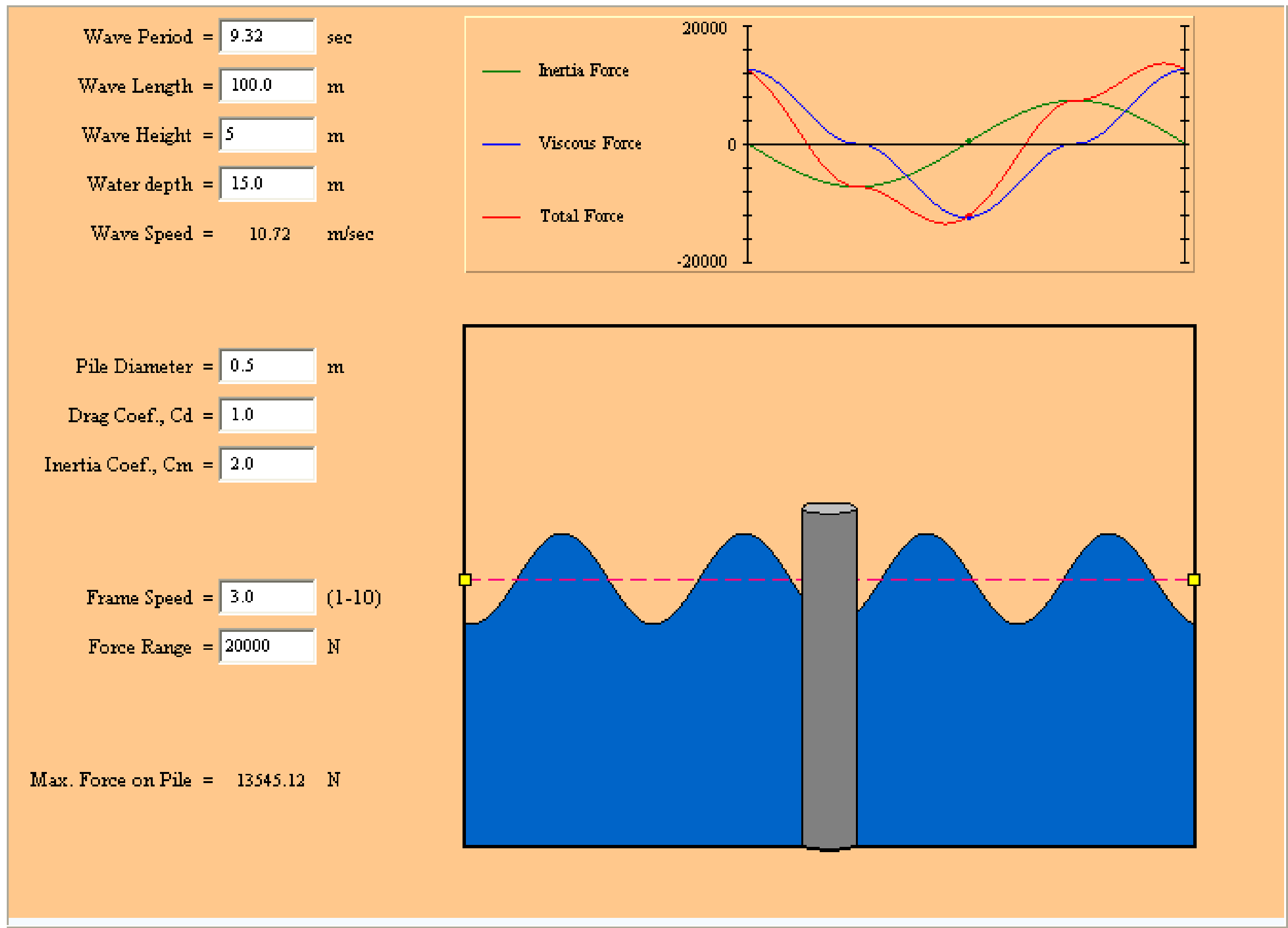
→ $F_D(t) = F_{Dm} |\cos(\omega t)| \cos(\omega t)$



- F_{tm} = max total force $= F_{Dm} + \dfrac{F_{im}^2}{4F_{Dm}}$ if $F_{im} < 2F_{Dm}$
- F_{tm} = " $= F_{im}$ if $F_{im} > 2F_{Dm}$

The wave forces applet sums-up the forces per pile slice over the pile length

http://cavity.ce.utexas.edu/kinnas/wow/public_html/waveroom/Applet/WaveForces/WaveForces.html

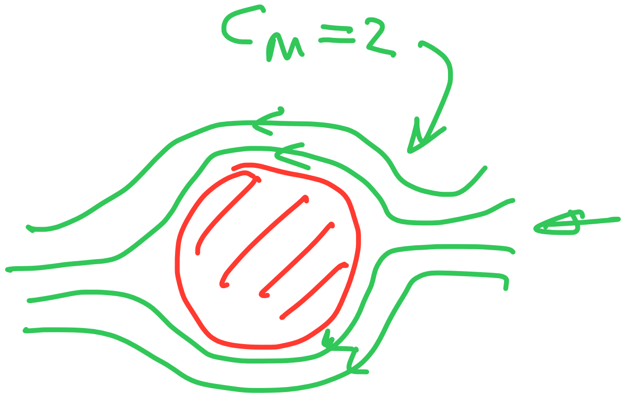


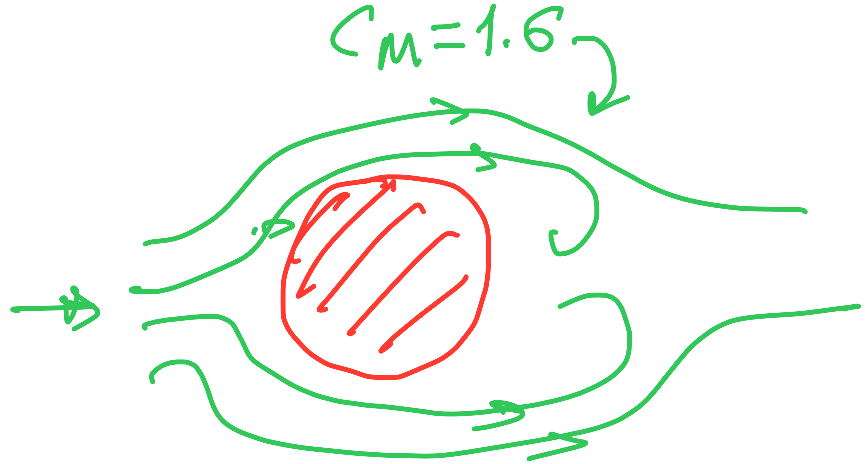
Typical values of the drag and inertia coefficients

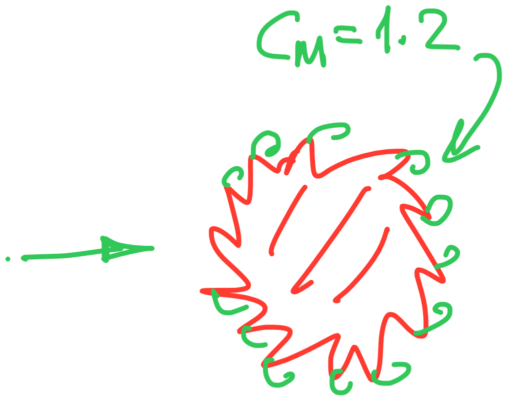
From API's (American Petroleum Institute) Recommended Practice 2A-WSD (Dec. 2000)

- $C_D = 0.65$ and $C_M = 1.6$ for smooth piles
- $C_D = 1.05$ and $C_M = 1.2$ for rough piles (due to marine growth)

Note: The diameter of the pile, D, also increases with marine growth







$$C_M = 1 + \frac{m_{added}}{m_{fluid}}$$

$$m_{added} < m_{fluid}$$

less fluid is accelerated by the rough cylinder

Total Force on Pile
(in the direction of wave propagation)

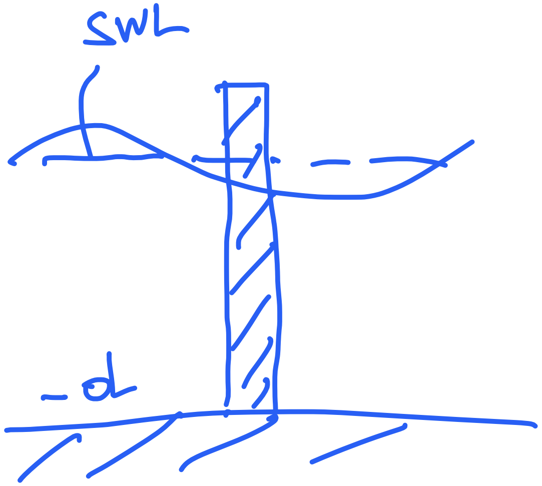
Total force = Viscous force + Inertial force

Morison's equation → $\sim C_D \rho D H^2$ + $\sim C_M \rho D^2 H$

Drag coefficient (C_D) **Inertia coefficient** (C_M)

$$\frac{\text{Viscous force}}{\text{Inertial force}} \sim \frac{C_D}{C_M} \frac{H}{D}$$

As H↑ or D↓ or H/D↑ the viscous forces become more important

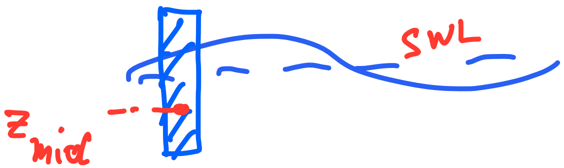


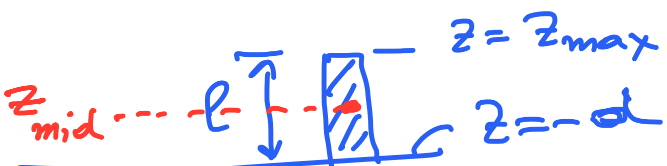
Formulas we got for F_{im}, F_{Dm} are **ONLY** for piles that extend ALL THE WAY from the sea-floor ($z=-d$) to $z=0$.

$$F_i = \int_{-d}^{z_{max}} f_i \, dz$$

$$F_D = \int_{-d}^{z_{max}} f_D \, dz$$

These integrals can be carried out. However we present an easier approach below.





$z_{mid} = z$ at midpoint of pile

It is easier to use the "short pile" assumption.

$$F_i = \int_{-d}^{z_{max}} f_i \, dz = f_{i,mid} \cdot \ell$$

$$F_D = \int_{-d}^{z_{max}} f_D \, dz = f_{D,mid} \cdot \ell$$

We assume f_i, f_D uniform with value that at the middle of the pile

$$f_{i,mid} = C_M \rho \frac{\pi D^2}{4} a_{x,mid} \rightarrow a_x \text{ at } z_{mid}$$

$$f_{D,mid} = C_D \frac{1}{2} \rho u |u| D \rightarrow u \text{ at } z_{mid}$$

$$F_{im} = C_M \rho \frac{\pi D^2}{4} a_{x,max}$$

$$F_{Dm} = C_D \frac{1}{2} \rho D u_{max}^2$$

at z_{mid}

$$F_i(t) = F_{im} \sin(\theta)$$

$$F_D(t) = F_{Dm} |\cos\theta| \cos\theta$$

$$\theta = kx - \omega t$$

- The formulas for the maximum value of the total force, F_{tm}, provided a few slides back, still apply

Formulas for $F_i(t)$ and $F_D(t)$ for "short" piles

$$F_i(t) = f_i \cdot \ell \quad \& \quad F_D(t) = f_D \cdot \ell \quad (1)$$

ℓ = length of column

(2a) $f_i = C_M \rho \frac{\pi D^2}{4} a$; a: acceleration

(2b) $f_D = C_D \frac{1}{2} \rho D u|u|$; u = velocity

As also mentioned in class, we will use the approximation that f_i and f_D are uniform over the pile with values those at midpoint of the pile.

Based on this assumption equ. (1) applies. Then we evaluate f_i & f_D at the midpoint of the pile by using formulas from Fig 2-6 of SPM: $a_x = a_{max} \sin\theta$ and $u = u_{max}\cos\theta$ (the expressions of a_{max}, u_{max} are given on Fig. 2-6)

Then: $\theta = -\omega t \Rightarrow a_x = -a_{max}\sin(\omega t)$ & $u = u_{max}\cos(\omega t)$

Replacing a_x and u in (2a) & (2b) and f_i & f_D in (1) we can get expressions for F_{im} and F_{Dm} (max values of $F_i(t)$ & $F_D(t)$) and then apply formulas to determine F_{Tm} (max total force).

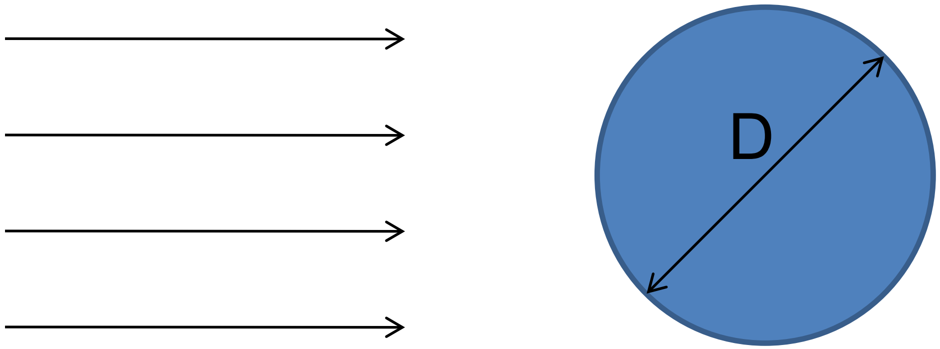
RELATIVE DEPTH	SHALLOW WATER $\frac{d}{L} < \frac{1}{25}$	TRANSITIONAL WATER $\frac{1}{25} < \frac{d}{L} < \frac{1}{2}$	DEEP WATER $\frac{d}{L} > \frac{1}{2}$
1. Wave profile	Same As →	$\eta = \frac{H}{2} \cos\left[\frac{2\pi x}{L} - \frac{2\pi t}{T}\right] = \frac{H}{2}\cos\theta$	← Same As
2. Wave celerity	$C = \frac{L}{T} = \sqrt{gd}$	$C = \frac{L}{T} = \frac{gT}{2\pi}\tanh\left(\frac{2\pi d}{L}\right)$	$C = C_o = \frac{L}{T} = \frac{gT}{2\pi}$
3. Wavelength	$L = T\sqrt{gd} = CT$	$L = \frac{gT^2}{2\pi}\tanh\left(\frac{2\pi d}{L}\right)$	$L = L_o = \frac{gT^2}{2\pi} = C_o T$
4. Group velocity	$C_g = C = \sqrt{gd}$	$C_g = nC = \frac{1}{2}\left[1 + \frac{4\pi d/L}{\sinh(4\pi d/L)}\right] \cdot C$	$C_g = \frac{1}{2}C = \frac{gT}{4\pi}$
5. Water Particle Velocity (a) Horizontal	$u = \frac{H}{2}\sqrt{\frac{g}{d}}\cos\theta$	$u = \frac{H}{2}\frac{gT}{L}\frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\cos\theta$ $= u_{max}$	$u = \frac{\pi H}{T} e^{\frac{2\pi z}{L}}\cos\theta$
(b) Vertical	$w = \frac{H\pi}{T}\left(1 + \frac{z}{d}\right)\sin\theta$	$w = \frac{H}{2}\frac{gT}{L}\frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\sin\theta$	$w = \frac{\pi H}{T} e^{\frac{2\pi z}{L}}\sin\theta$
6. Water Particle Accelerations (a) Horizontal	$a_x = \frac{H\pi}{T}\sqrt{\frac{g}{d}}\sin\theta$	$a_x = \frac{g\pi H}{L}\frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\sin\theta$ $= a_{x,max}$	$a_x = 2H\left(\frac{\pi}{T}\right)^2 e^{\frac{2\pi z}{L}}\sin\theta$
(b) Vertical	$a_z = -2H\left(\frac{\pi}{T}\right)^2\left(1 + \frac{z}{d}\right)\cos\theta$	$a_z = -\frac{g\pi H}{L}\frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)}\cos\theta$	$a_z = -2H\left(\frac{\pi}{T}\right)^2 e^{\frac{2\pi z}{L}}\cos\theta$
7. Water Particle Displacements (a) Horizontal	$\xi = -\frac{HT}{4\pi}\sqrt{\frac{g}{d}}\sin\theta$	$\xi = -\frac{H}{2}\frac{\cosh[2\pi(z+d)/L]}{\sinh(2\pi d/L)}\sin\theta$	$\xi = -\frac{H}{2} e^{\frac{2\pi z}{L}}\sin\theta$
(b) Vertical	$\zeta = \frac{H}{2}\left(1 + \frac{z}{d}\right)\cos\theta$	$\zeta = \frac{H}{2}\frac{\sinh[2\pi(z+d)/L]}{\sinh(2\pi d/L)}\cos\theta$	$\zeta = \frac{H}{2} e^{\frac{2\pi z}{L}}\cos\theta$
8. Subsurface Pressure	$p = \rho g(\eta - z)$	$p = \rho g \eta \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} - \rho g z$	$p = \rho g \eta e^{\frac{2\pi z}{L}} - \rho g z$

Figure 2-6. Summary of linear (Airy) wave theory--wave characteristics.

An assessment of Morison's equation using CFD (Computational Fluid Dynamics)

Two Dimensional Cylinder in Oscillatory Flow

$$U = U_m \cdot \cos(\omega t)$$



<u>Two important numbers:</u>

- **Re (Reynolds No)=$U_m D/\nu$**

- **KC (Keulegan-Carpenter No)=$U_m T/D$ ($T=2\pi/\omega$)**
  ~(distance the particles travel in T)/D

Morison's Equation

The inline force (force in the direction of the flow) is the sum of the drag force and the inertia force (per unit width)

$$F = \frac{1}{2}\rho C_D D|U|U + \frac{1}{4}\rho\pi D^2 C_M \frac{dU}{dt}$$

$$U = U_m \cdot \cos(\omega t)$$

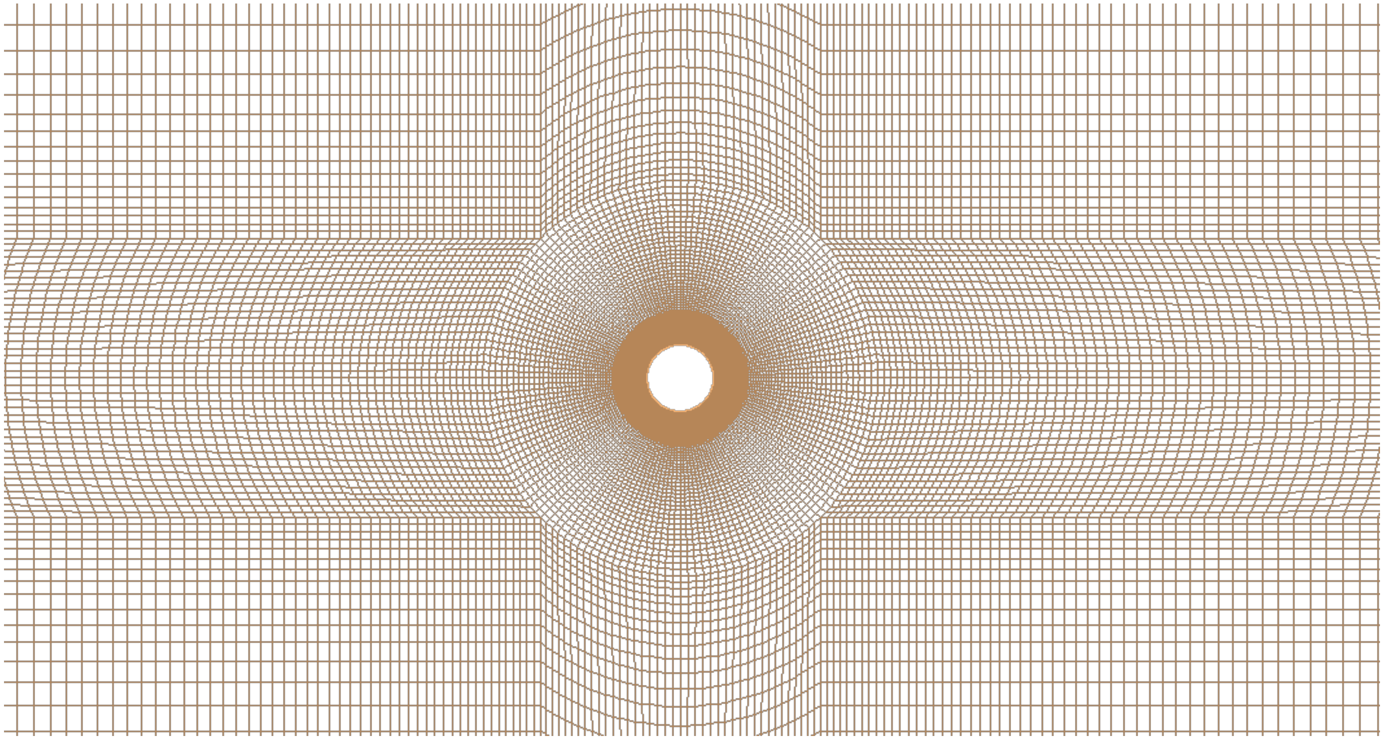
C_M is the inertia coefficient

C_D is the drag coefficient

- we also define: $$C_x = \frac{F}{\frac{\rho}{2} U_m{}^2 D}$$

Grid in Fluent

Structured mesh is used in the calculation domain



Mesh Info:

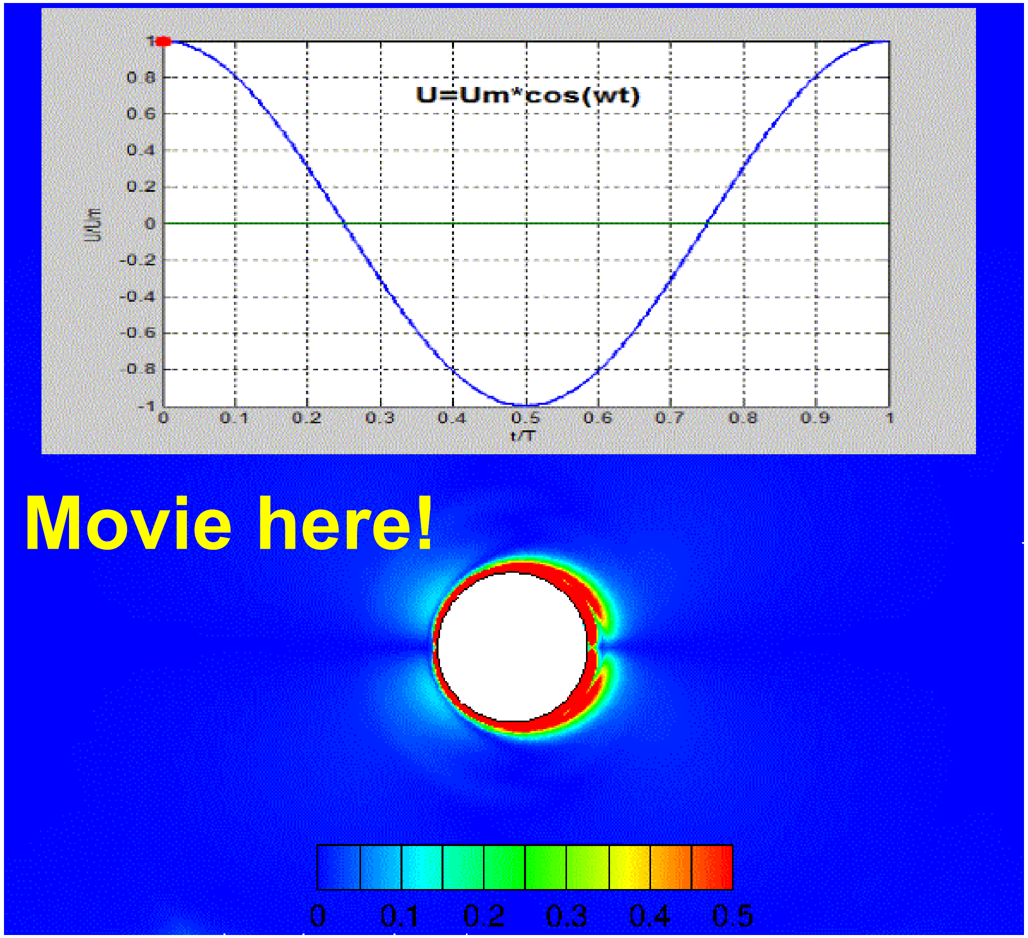
Cells: 76680

Faces: 154190

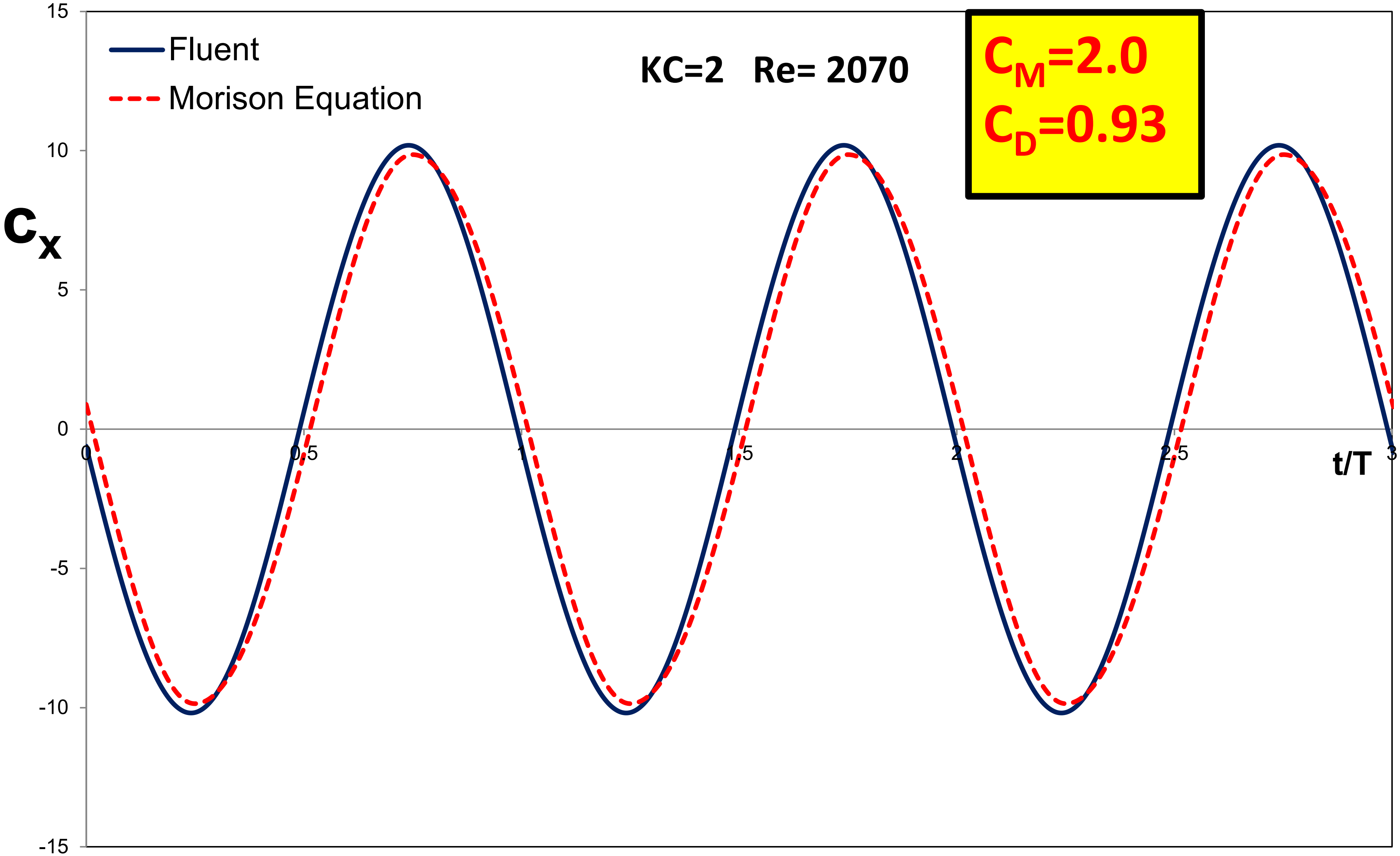
Nodes: 77510

Predicted flow (vorticity) by Fluent: KC=2, Re=1070

(click on the movie to play)

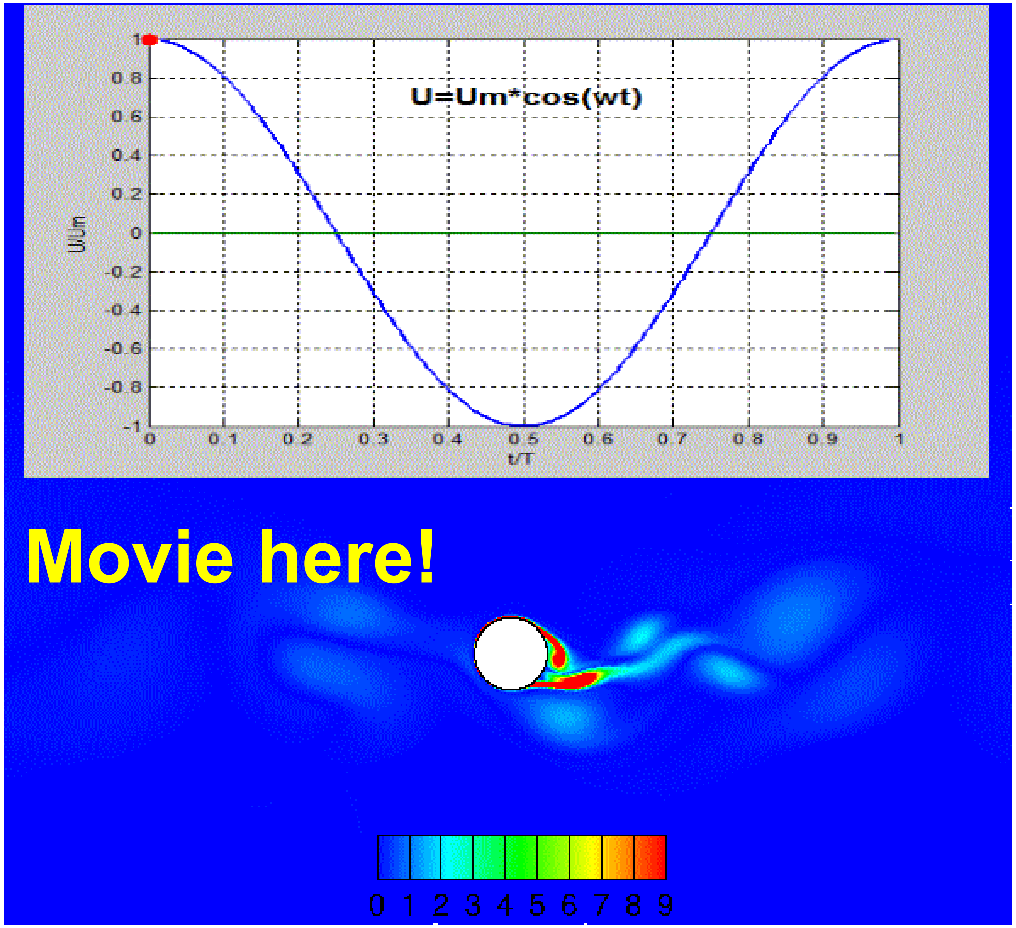


Case I: KC=2 Re=1070

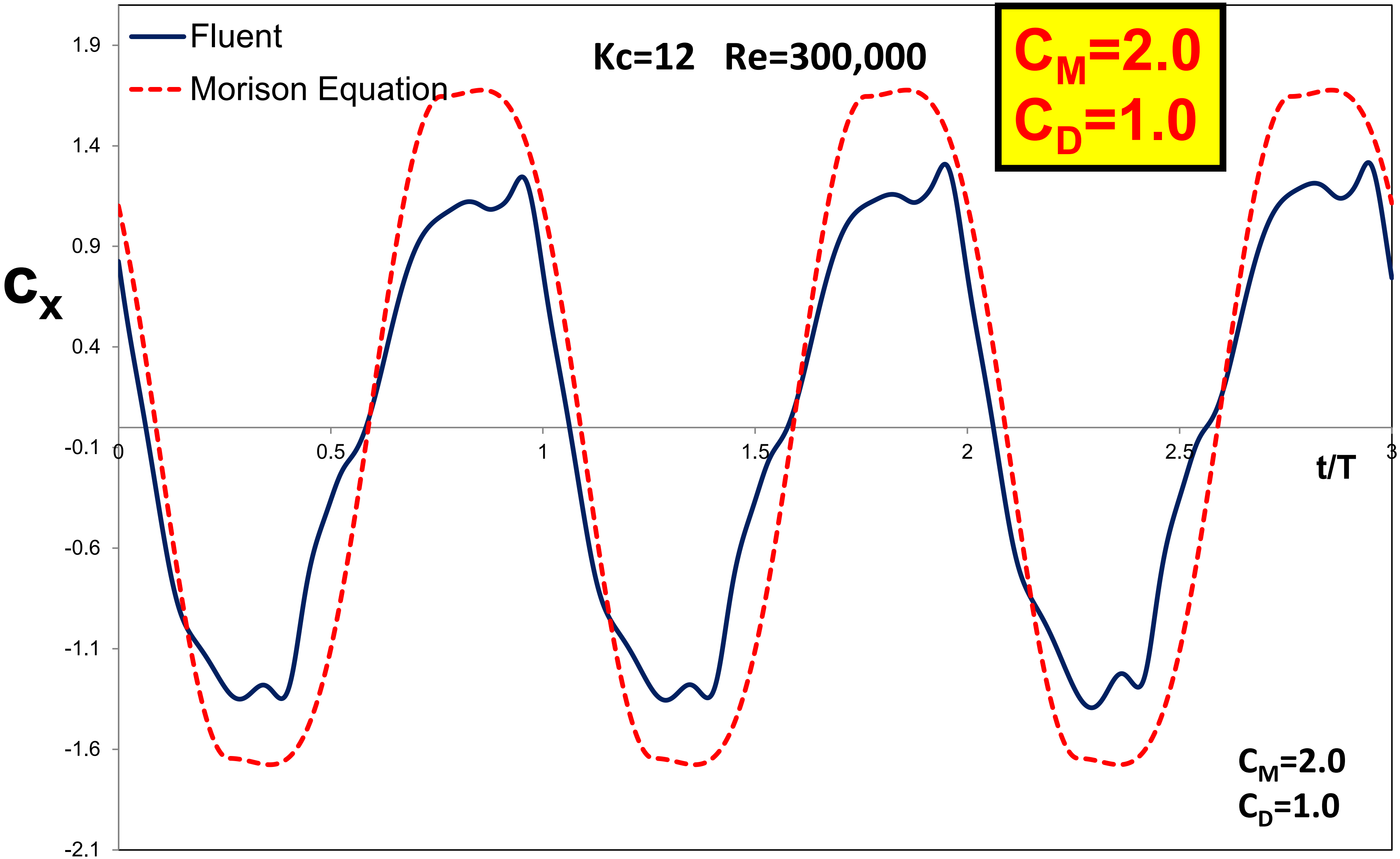


Predicted flow (vorticity) by Fluent: KC=12 Re=300,000

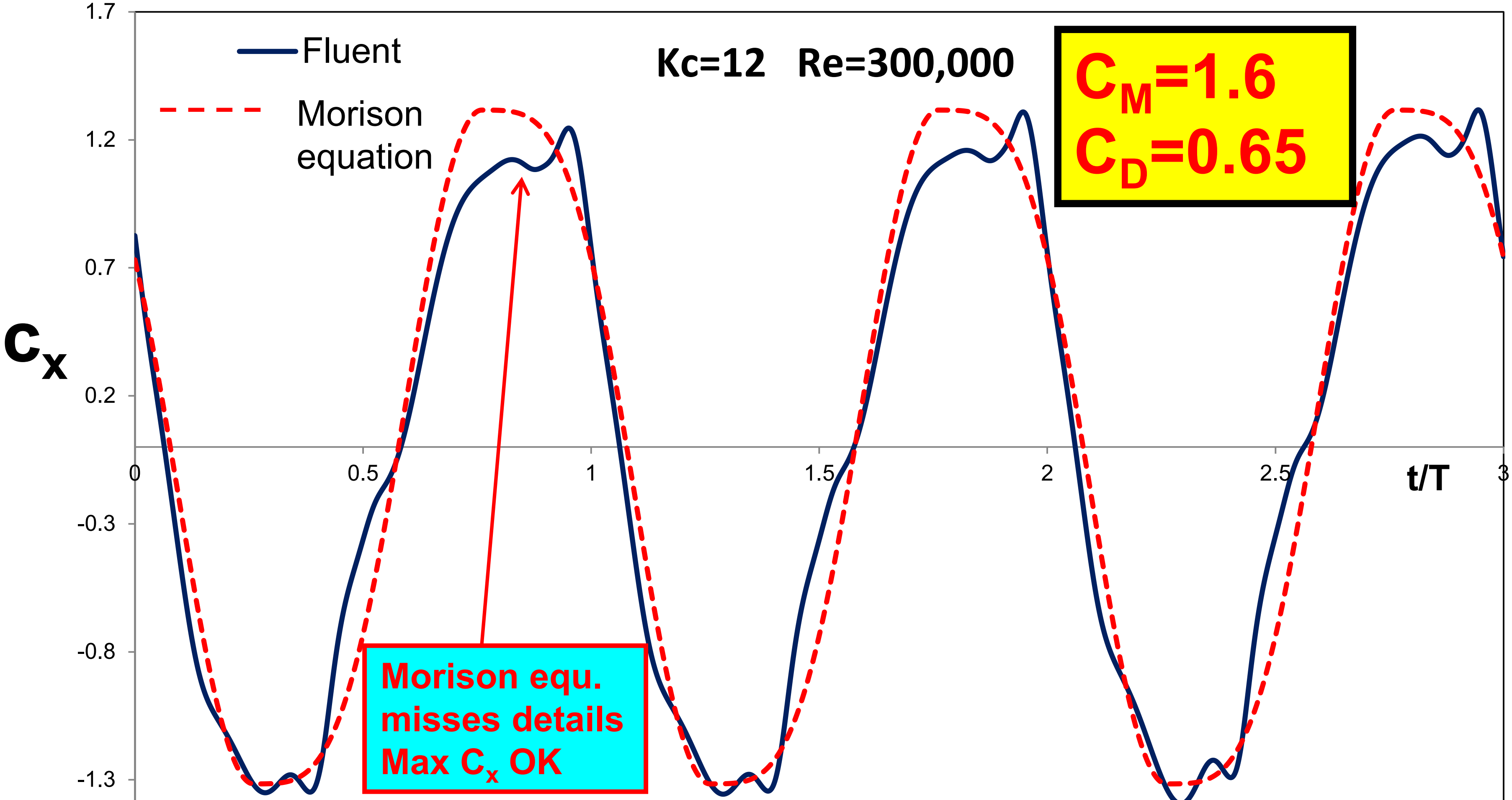
(click on the movie to play)



Case II: KC=12 Re=300,000

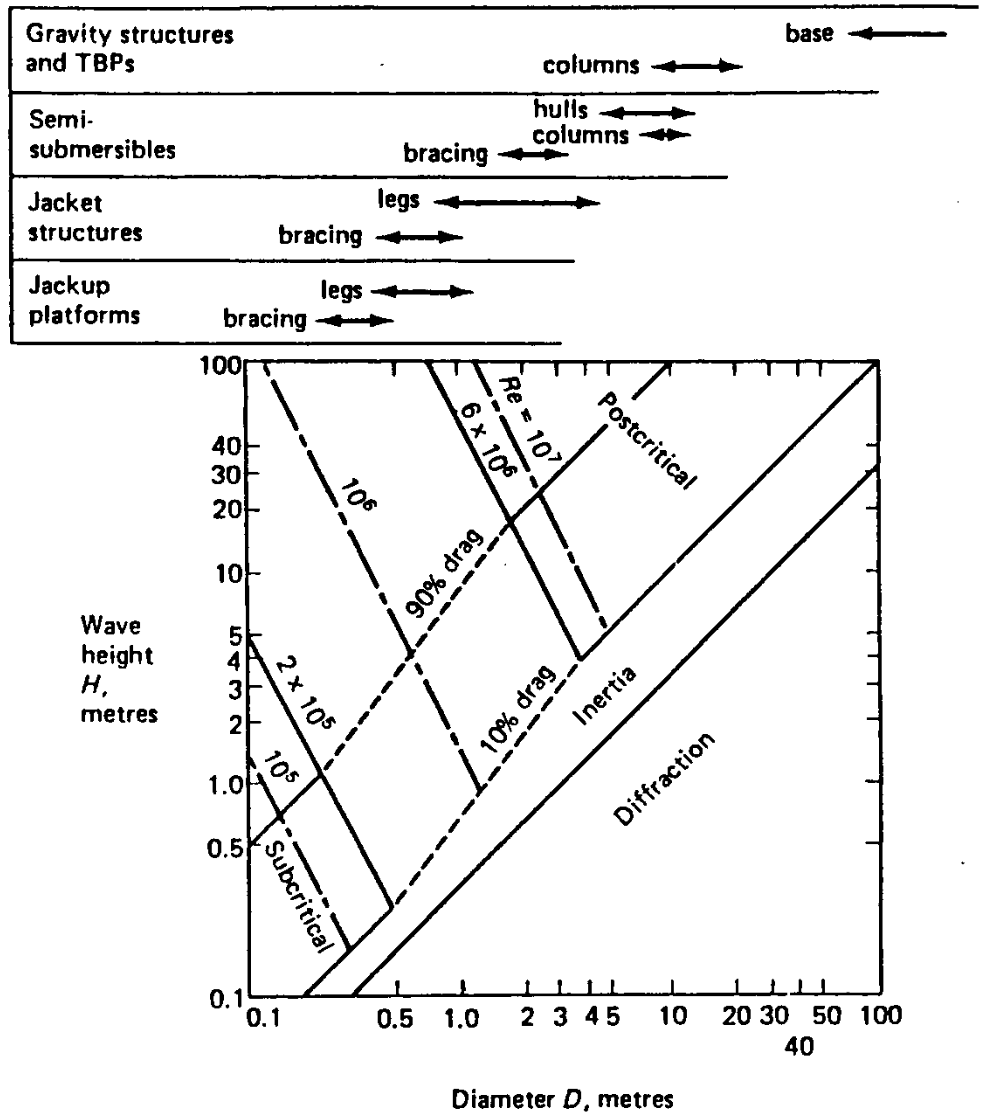


Case II: KC=12 Re=300,000



Maybe due to luck…more needs to be done!!! Some newer simulations will be shown in the lecture on Computational Hydrodynamics

Effect of wave height H and diameter of element D on importance of viscous forces



Loading regimes at still water level (from Hogben[4])