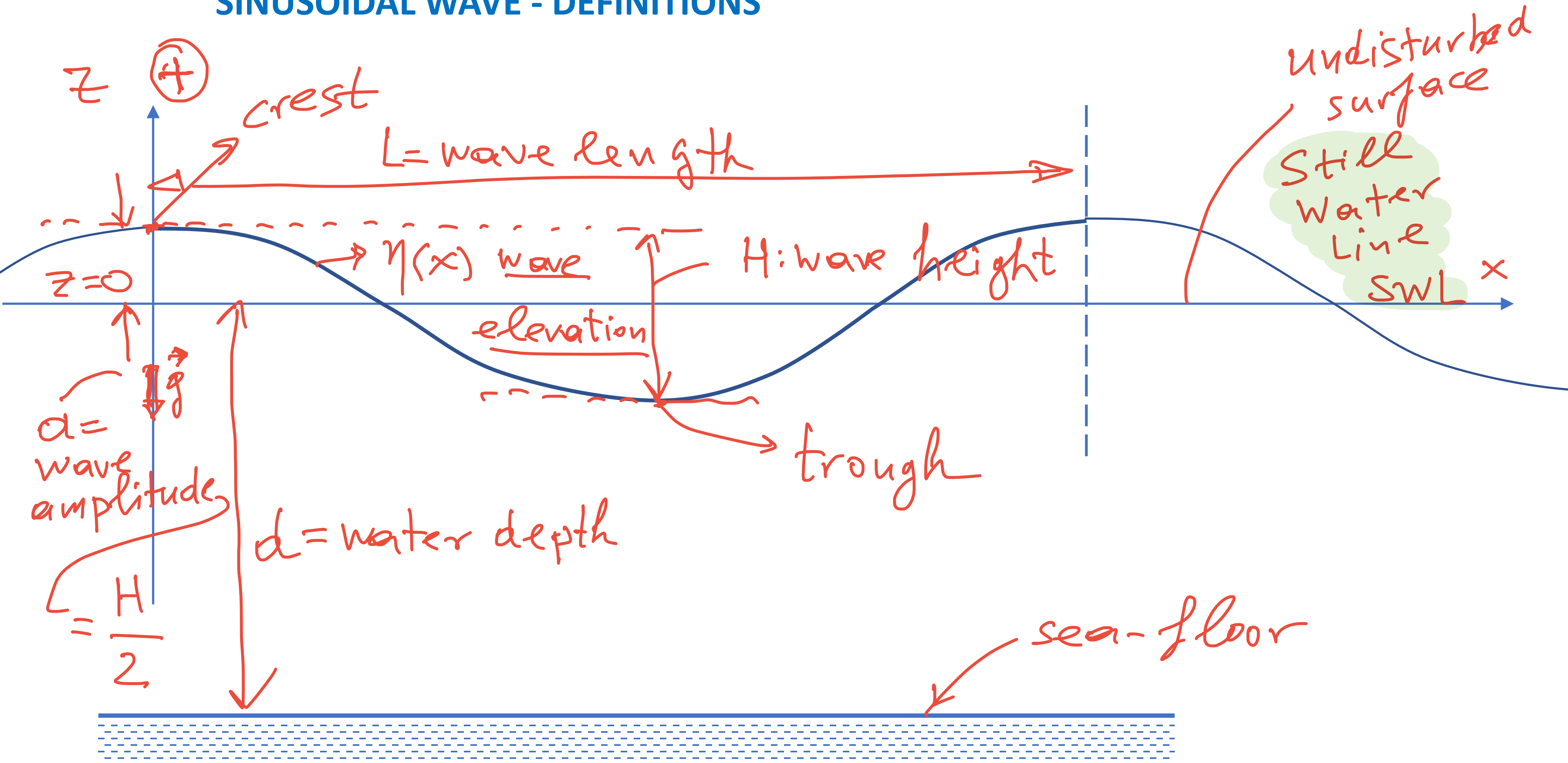
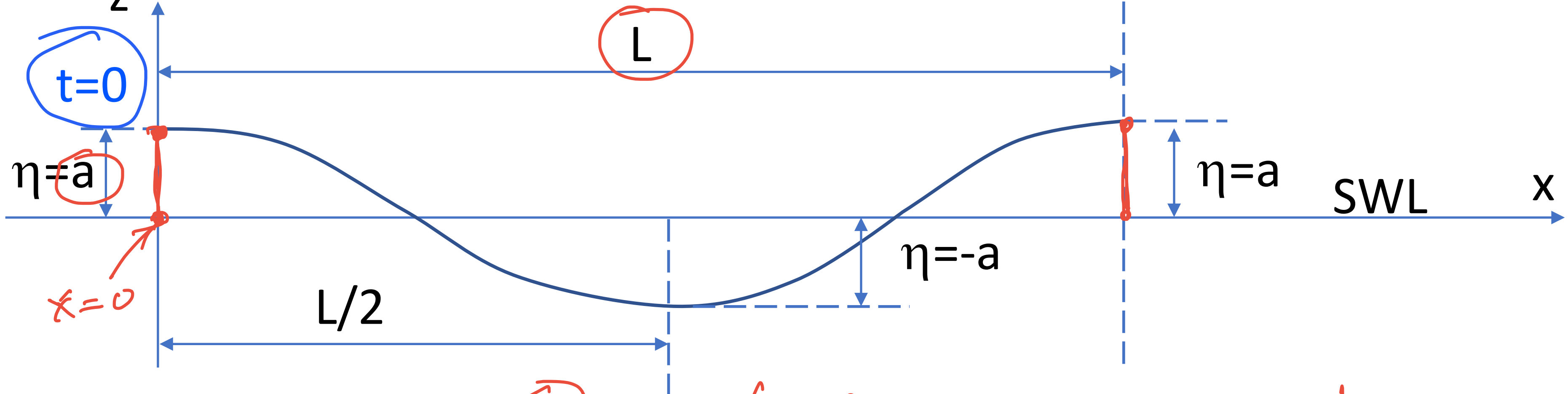
SINUSOIDAL WAVE - DEFINITIONS



SINUSOIDAL WAVE - FORMULAS



$$\eta(x) = A\cos(kx) \quad ; \quad A, k \ \underline{\text{TBD}}!$$

$$x=0 \quad \underline{\eta(0)} = A\cos(k0) = \boxed{A = a}$$

$$x=L \quad \eta(L) = \cancel{a}\cos(kL) = \cancel{a} \Rightarrow \cos(kL) = 1$$

$$kL = 2\pi \Longrightarrow \boxed{k = \frac{2\pi}{L}} \qquad \boxed{\eta(x) = \underline{a}\cos\left(\frac{2\pi x}{L}\right)}$$

wave number (SI: m^{-1}, US: ft^{-1})

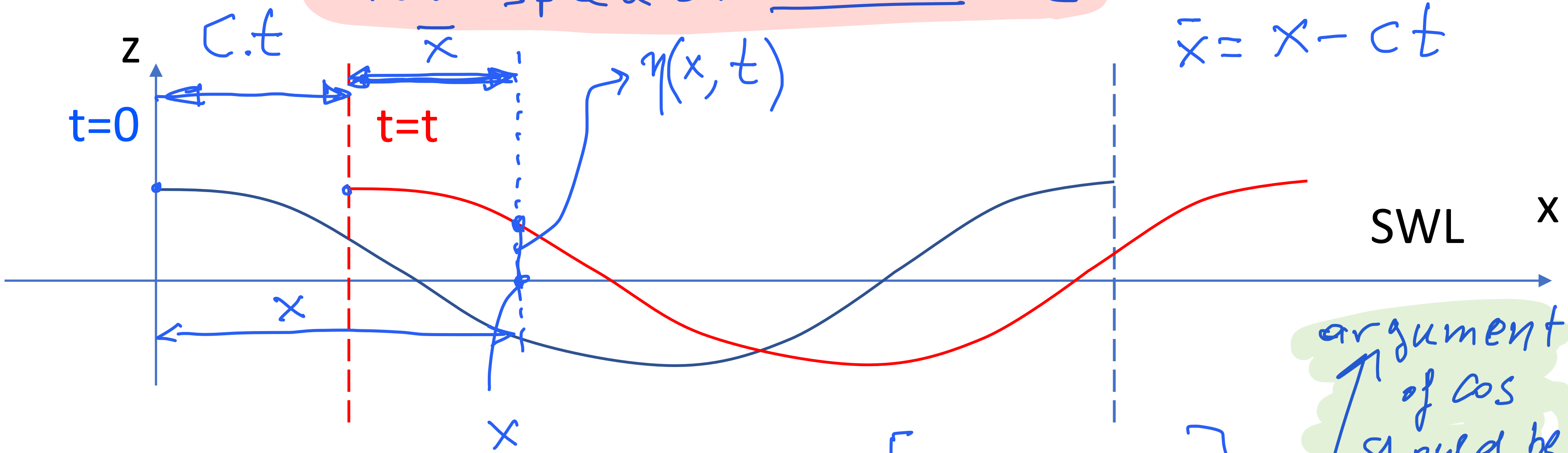
Has to be in RADIANS

SINUSOIDAL WAVE – GOING TO THE "RIGHT"

(goes to +x)

Wave speed or celerity : C

$\bar{x} = x - ct$



$$\boxed{\eta} = a\cos\left(\frac{2\pi\bar{x}}{L}\right) = a\cos\left[\frac{2\pi}{L}\cdot(x-ct)\right] =$$

$$= a\cos\left[\frac{2\pi x}{L} - \frac{2\pi}{L}ct\right] = \boxed{a\cos\left[\frac{2\pi x}{L} - \frac{2\pi t}{T}\right]}$$

argument of cos should be in RADIANS!

T = wave period is defined as the time a crest will move by L

$$L = CT \Rightarrow \boxed{T = \frac{L}{C}} \rightarrow \boxed{a\cos(kx - \omega t)}$$

$\omega = 2\pi f$

wave (angular) frequency : $\omega = \frac{2\pi}{T}$ (rad/sec)

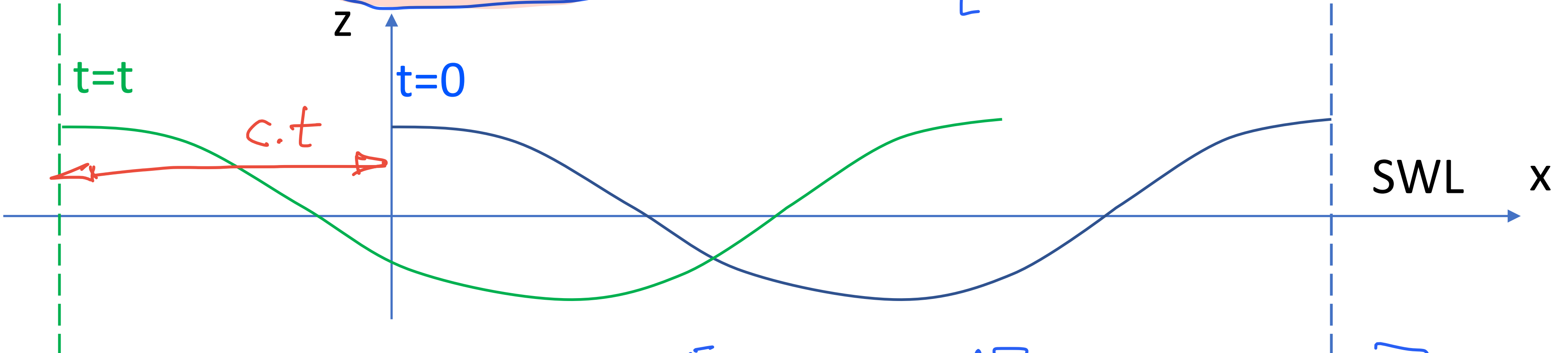
temporal frequency : $f = \frac{1}{T}$ (cycles/sec or Hertz)

SINUSOIDAL WAVE – GOING TO THE "LEFT"

$$\boxed{C = \frac{L}{T} = \frac{\omega}{k}}$$

$$\frac{\omega}{k} = \frac{\cancel{2\pi}/T}{\cancel{2\pi}/L} = \frac{L}{T} \,!$$

goes to $-x$



$$\eta(x,t) = a\cos\left[\frac{2\pi x}{L} + \frac{2\pi t}{T}\right] = a\cos\left[kx + \omega t\right]$$

Expression for wave elevation $\eta(x,t)$

$$\boxed{\eta(x,t) = a\cos\left[kx \pm \omega t\right]}$$

Should be in RADIANS!

$+$ wave goes to $-x$ (to the left)

$-$ wave goes to $+x$ (to the right)

H L T **EXAMPLE PROBLEMS** C

Find the height, the length, the period, the celerity (=wave speed) of this wave, and in which direction it goes, where η and x are in meters, and t in seconds

$$\eta(x,t) = \cos(x - t)$$

$a = 1m$

- $H = 2a = 2 \times 1 = 2m$

$k = 1 = \frac{2\pi}{L} \sim L = 2\pi \, (m)$

$\omega = 1 = \frac{2\pi}{T} \sim T = 2\pi \, (sec)$

$C = \frac{L}{T} = \frac{2\pi}{2\pi} = 1 \, m/s$ $\left(C = \frac{\omega}{k} = \frac{1}{1} = 1 \, m/s\right)$

H, L, a, T, C, ω, k are all positive numbers!

direction: goes to +x (or to the "right")

Do the same if: $\eta(x,t) = \cos(x + t)$

H, L, T, C the same

goes to the −x (to the "left")

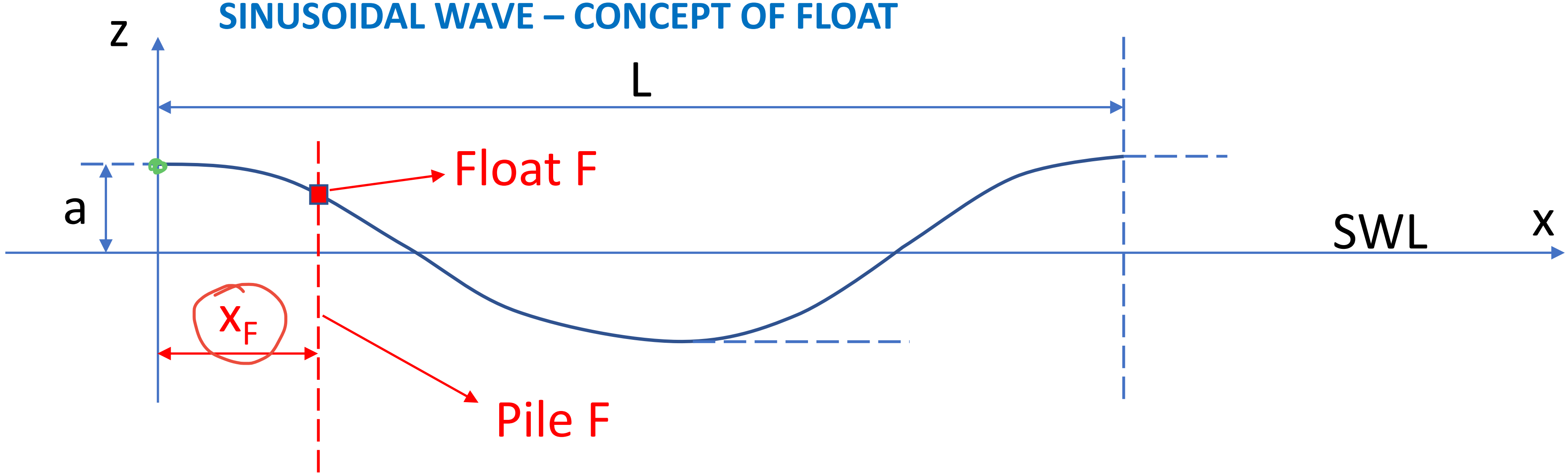
What about if: $\eta(x,t) = \cos(t - x)$ $= \cos(x - t)$

$k = 1$ $\omega = 1$

H, L, T, C the same

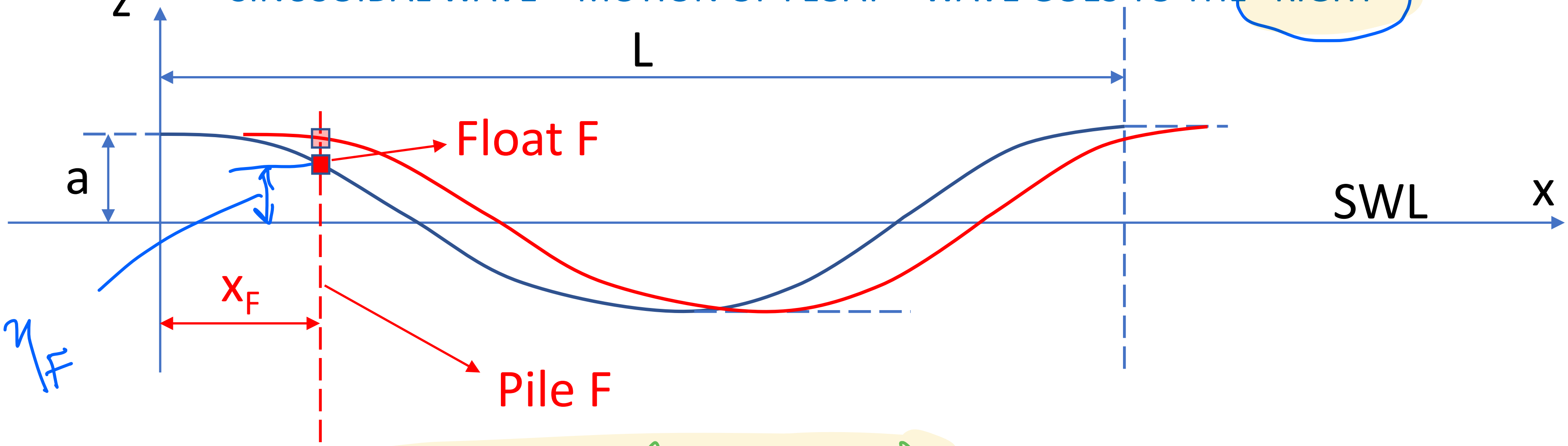
direction: goes to +x (to the "right")

SINUSOIDAL WAVE – CONCEPT OF FLOAT



Float F can travel freely (without friction) along Pile F.

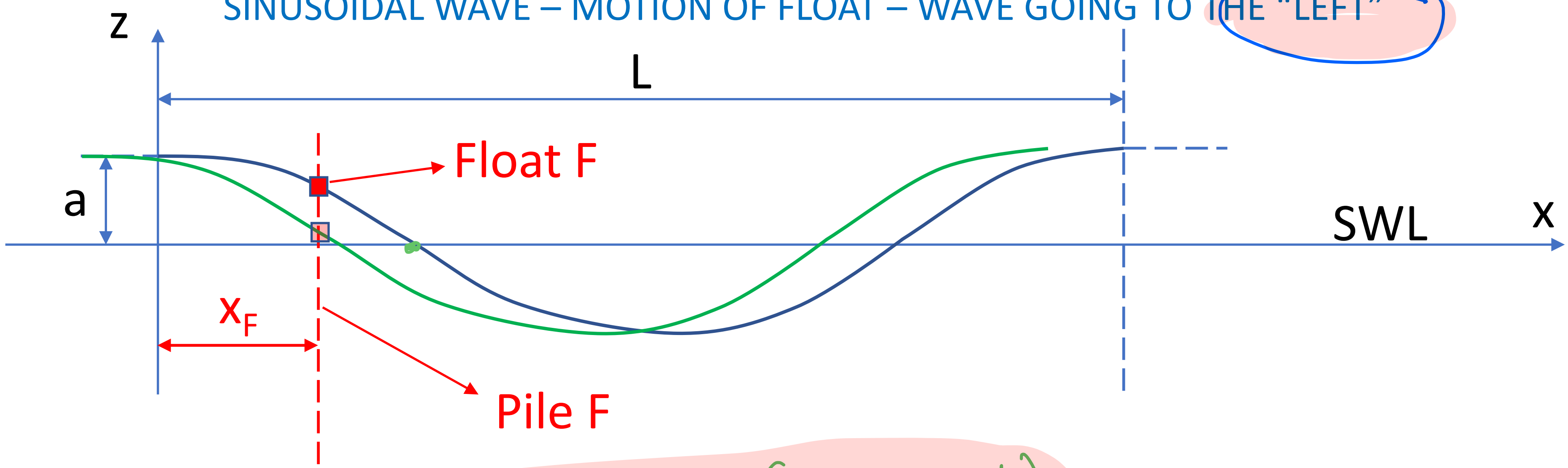
SINUSOIDAL WAVE – MOTION OF FLOAT – WAVE GOES TO THE "RIGHT"



$$\eta_F = a\cos(kx_F - \omega t) \quad *$$

equation for float displacement (or deflection) as a function of time (x_F is fixed!)

SINUSOIDAL WAVE – MOTION OF FLOAT – WAVE GOING TO THE "LEFT"



$$\eta_F = a\cos(kx_F + \omega t)$$

equation for float displacement as a function of time (x_F is fixed!)

EXAMPLE PROBLEM ON FLOATS

A wave is traveling from pile A to B with a speed $C = 55 ft/sec$ (floats A and B move freely along the piles). The distance between piles A and B is $S = 170$ ft.

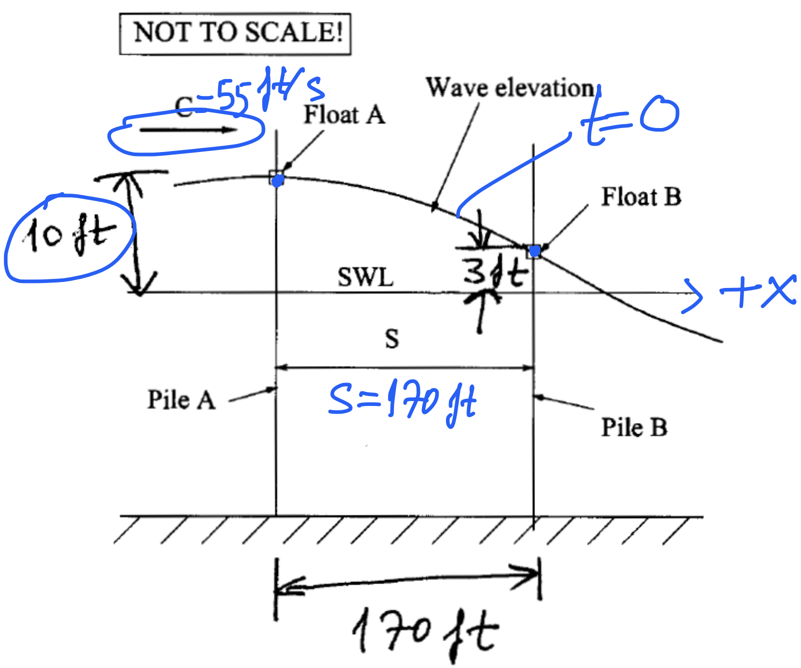
At a particular time we know that float A is at its maximum level (with respect to the Still Water Line, SWL, level), equal to 10ft. At the same instance float B is 3 ft **over** the SWL level.

~~**2a)**~~ Find the height, H, of the wave (5 points)

~~**2b)**~~ Find the maximum wave length, L (NOTE: There is a multiplicity of solutions for L from which only the maximum is requested) (25 points)

~~**2c)**~~ The period of the wave, T (5 points)

2d) How long after A and B will have the same elevation? What is the value of this elevation? (25 points)



a) $a = 10ft \Rightarrow \boxed{H = 2a = 20ft}$

b) $\eta(x,t) = a\cos(kx - \omega t) =$

$= a\cos(kx)$

apply η at $x = S$ $(=170ft)$

$\eta = \underset{10}{a}\cos(kS) = 3$

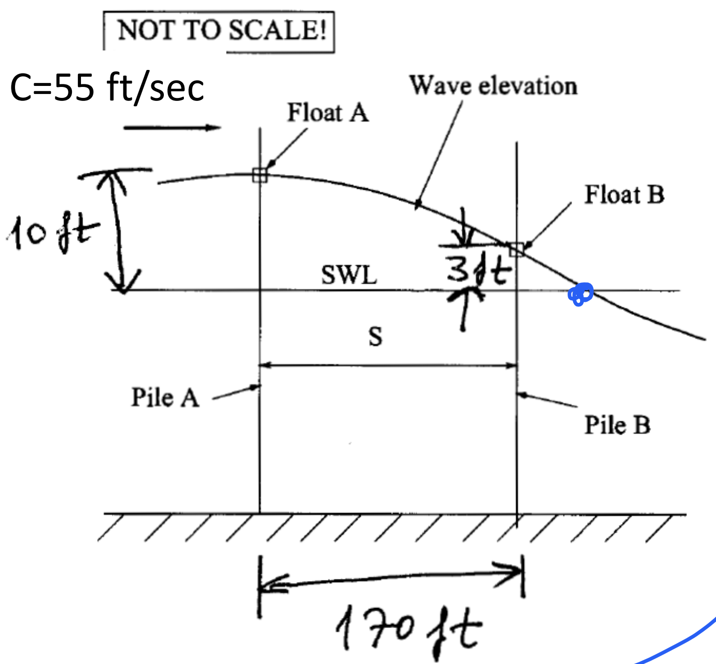
$\Rightarrow 10\cos(ks) = 3$

$\boxed{\cos(ks) = 0.3}$

EXAMPLE PROBLEM ON FLOATS

a) H=? b) L=? c) T=? d) t=?

C=55 ft/sec



$\cos(ks) = 0.3$

find $\cos^{-1}(0.3) = 1.266$ rad

$\cos(ks) = \cos(1.266)$

Trig: $\cos(\alpha) = \cos(\beta)$

$\alpha = \pm\beta + 2\pi n$

$n = 0, \pm 1, \pm 2, \ldots$

$ks = \pm 1.266 + 2\pi n$

$+$: $ks = 1.266 + 2\pi n \Rightarrow k = \dfrac{1.266 + 2\pi n}{S}$

$k = \dfrac{2\pi}{L}$ I want min k (since max L)

$k > 0$

$n = 0$ for min

max L

$k = \dfrac{1.266}{S} = \dfrac{2\pi}{L} \Rightarrow \boxed{L = 843.7\,ft}$

$S = 170$

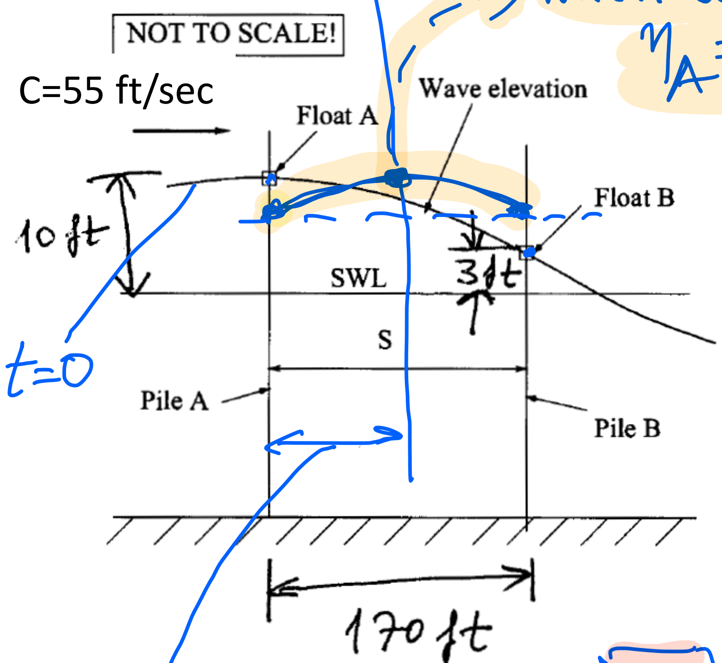
EXAMPLE PROBLEM ON FLOATS

crest

When at middle $\eta_A = \eta_B$!

a) H=?   b) L=?   c) T=?   d) t=?

C=55 ft/sec



$$\ominus \quad ks = -1.266 + 2\pi n$$

k has to be min

$k > 0$

$n = 1$

$$k = \frac{-1.266 + 2\pi}{S} \Rightarrow \boxed{L = 212.9 ft}$$

$$L = \frac{2\pi}{L}$$

not maximum L

$$\frac{S}{2} = 85 ft$$

$$\boxed{t =} \frac{\text{distance}}{C} = \frac{85 ft}{55 ft/s} = \boxed{1.545 sec}$$

Intuitive solution

$\eta_A = \eta_B$

From trig.

$$\cancel{a} \cos(k \cancel{x_A} - \omega t) = \cancel{a} \cos(k x_B - \omega t)$$

$x_A = 0$, $x_B = S$

$$\cos(-\omega t) = \cos(kS - \omega t)$$

$$\cos(\omega t) = \cos(ks - \omega t)$$

$$\omega t = \pm (ks - \omega t) + 2\pi n$$

$+$: $\omega t = ks - \omega t + 2\pi n \rightsquigarrow 2\omega t = ks + 2\pi n$

$t > 0$

t min

$n = 0$

$$2\omega t = ks$$

$$\boxed{t = \frac{ks}{2\omega}} = 1.545\,\underline{sec}$$

$$k = \frac{2\pi}{L}$$

$$ks = 2\pi \frac{170}{843.7} < 2\pi$$

$-$: $\cancel{\omega t = -ks + \omega t + 2\pi n}$

does not provide equ. for t!

$$k = \frac{2\pi}{L} = \frac{2\pi}{843.7} = 0.00745\,(ft^{-1})$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{15.34} = 0.4096\,(rad/sec)$$

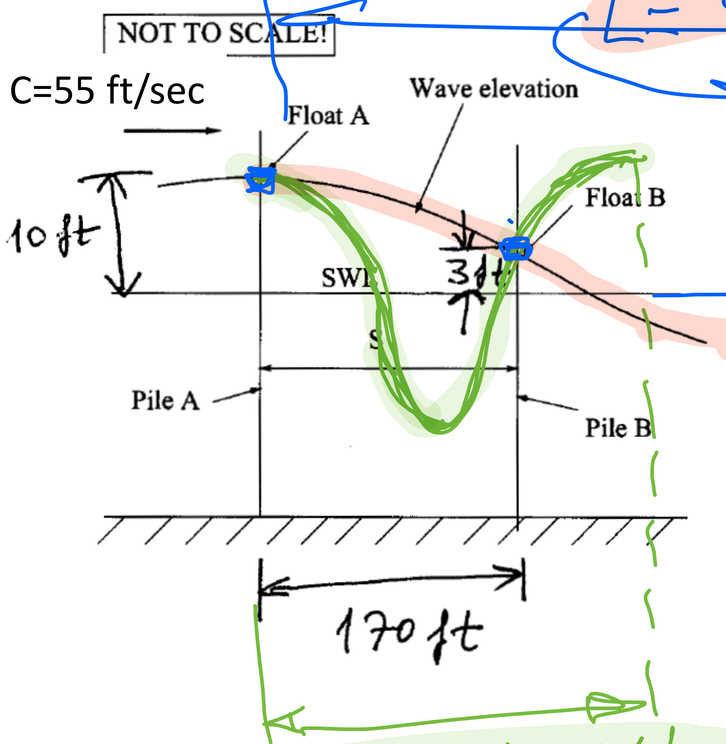
$$s = 170\,ft$$

EXAMPLE PROBLEM ON FLOATS

$L = 843.7\,ft$

a) H=? b) L=? c) T=? d) t=?

Largest L
(which was requested)



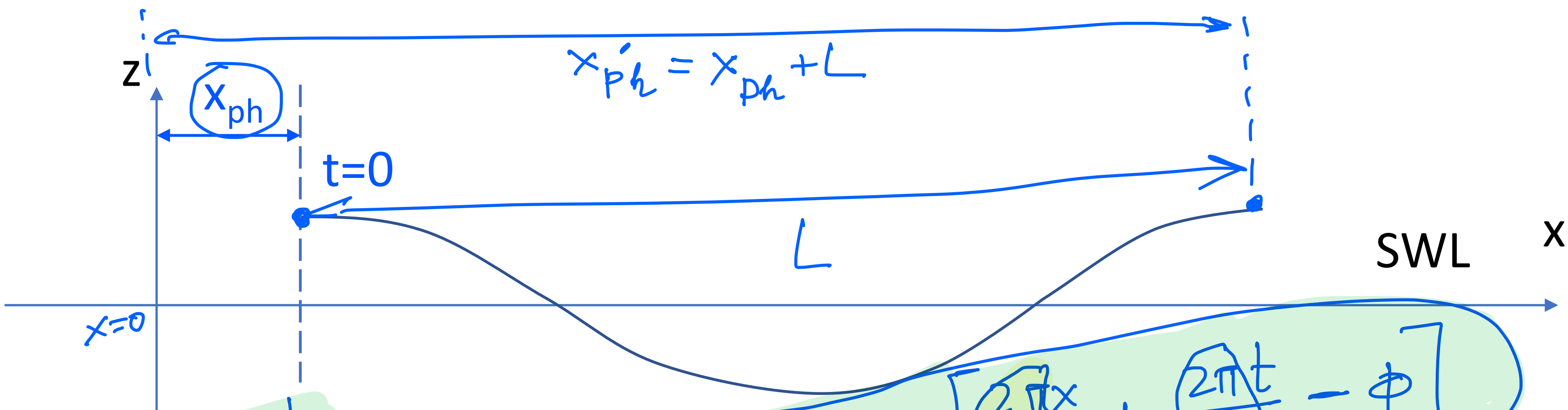
$L = 212.9\,ft$
(2nd Largest, NOT requested)

2c) $T = ?$

$$C = \frac{L}{T}$$

$$\rightsquigarrow \boxed{T =} \frac{L}{C} = \frac{843.7\,(ft)}{55\,(ft/s)} = \boxed{15.34\,sec}$$

SINUSOIDAL WAVE – PHASE OF A WAVE



standard (or canonical) form of a wave profile

$$\eta(x,t) = a\cos\left[\frac{2\pi x}{L} \pm \frac{2\pi t}{T} - \phi\right]$$

ϕ : phase of the wave (in radians)

$\phi > 0$; $\phi < 0$

$$\phi = \frac{x_{ph}}{L}\,2\pi$$

$$\phi' = \frac{x'_{ph}}{L}\,2\pi = \frac{x_{ph}+L}{L}\,2\pi = \left(\frac{x_{ph}}{L}+1\right)2\pi =$$

$$= 2\pi + \frac{x_{ph}}{L}\,2\pi = 2\pi + \phi$$

We can add or subtract 2π (or multitudes of it) to the phase, ϕ, and get the <u>same profile</u>!

In the following examples we find the <u>phase</u> by bringing the provided formulas in their <u>standard form</u>, by using trig. identities.

EXAMPLES ON PHASE OF A WAVE

Put the following wave profiles into their "canonical" form and determine their phase and their direction of propagation. Plot the wave profiles at $t = 0$ and verify that the phases you determined *make sense*. Remember a, H, L, k, T, ω, and C, are, by definition, **positive** numbers.

(a) $\eta = \sin(x - 2t)$

(b) $\eta = -\cos(3x + t)$

$$\eta = \sin(x-2t) = \cos\left(\frac{\pi}{2} - (x-2t)\right) =$$

$$= \cos\left(\frac{\pi}{2} - x + 2t\right) =$$

$$= \cos\left(-\left(\frac{\pi}{2} - x + 2t\right)\right) =$$

$$= \cos\left(-\frac{\pi}{2} + x - 2t\right) =$$

$$= \boxed{\cos\left(x - 2t - \frac{\pi}{2}\right)}$$

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$$

$$\cos(\theta) = \cos(-\theta)$$

Wave goes to the "right" or $+x$

phase: $\boxed{\phi = \frac{\pi}{2}}$

EXAMPLES ON PHASE OF A WAVE

Put the following wave profiles into their "canonical" form and determine their phase and their direction of propagation. Plot the wave profiles at $t = 0$ and verify that the phases you determined *make sense*. Remember a, H, L, k, T, ω, and C, are, by definition, **positive** numbers.

(a) $\eta = \sin(x - 2t)$

(b) $\eta = -\cos(3x + t)$

$$= \cos[\pi - (3x + t)] = \cos[\pi - 3x - t] =$$

$$-\cos(\theta) = \cos(\pi - \theta)$$

$$= \cos[-(\pi - 3x - t)] =$$

$$= \cos[-\pi + 3x + t] =$$

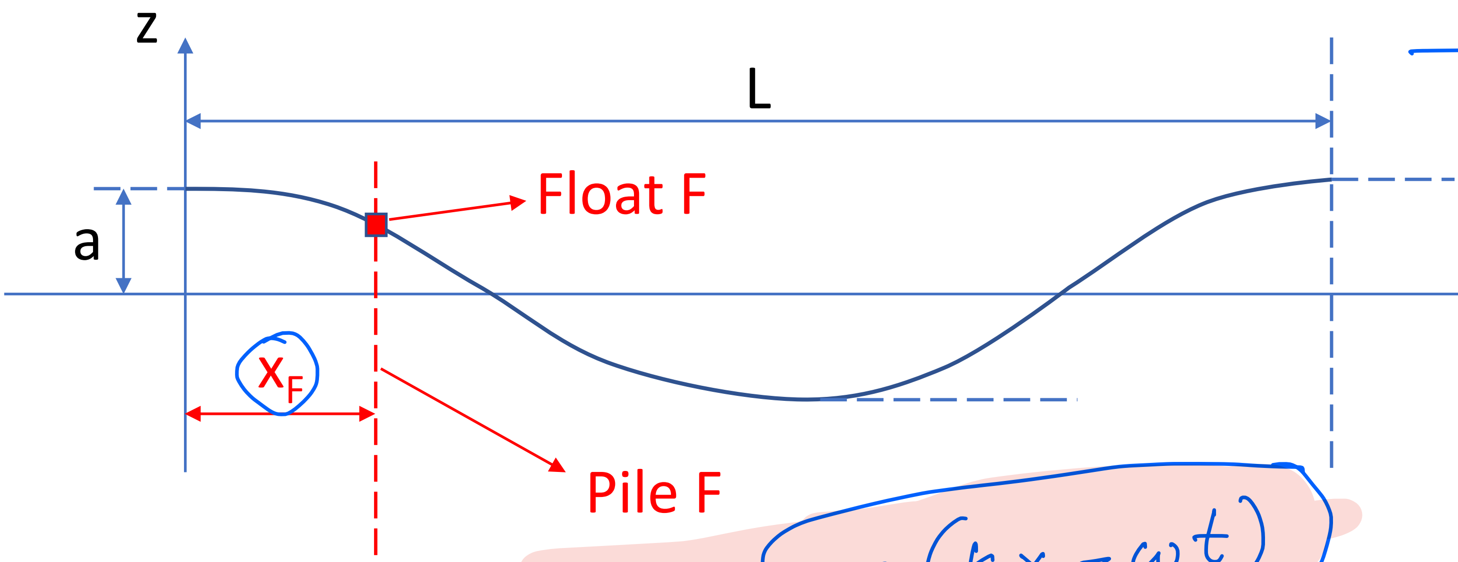
$$\cos(\theta) = \cos(-\theta)$$

$$= \boxed{\cos[3x + t - \pi]}$$

wave goes to the "left" or to $-x$

Phase: $\boxed{\phi = \pi}$

DISPLACEMENT, VELOCITY, AND ACCELERATION OF A FLOAT



direction of wave

$$\frac{d\cos x}{dx} = -\sin x$$

$$\frac{d\sin x}{dx} = \cos x$$

$$* \quad \eta_F = a\cos(kx_F - \omega t)$$

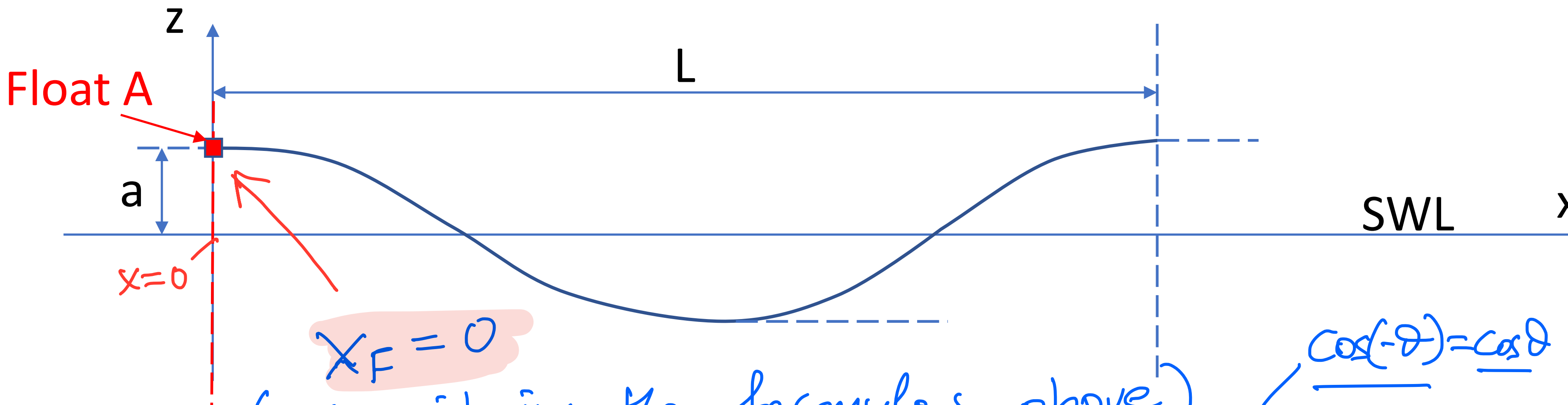
velocity of float

$$V_F = \frac{\partial \eta_F}{\partial t} = a\{-\sin(kx_F - \omega t)\}(-\omega) = a\omega\sin(kx_F - \omega t)$$

acceleration of float

$$a_F = \frac{\partial V_F}{\partial t} = a\omega\cos(kx_F - \omega t)(-\omega) = -\omega^2 \underbrace{a\cos(kx_F - \omega t)}_{\eta_F} = -\omega^2 \eta_F$$

EXAMPLE ON DISPLACEMENT, VELOCITY, AND ACCELERATION OF A FLOAT



$x_F = 0$

(plug it in the formulas above)

$\cos(-\theta) = \cos\theta$

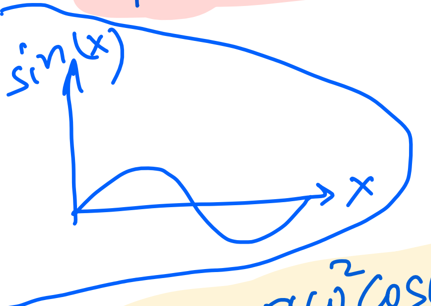
$$\underline{\eta_F} = a\cos(0-\omega t) = a\cos(-\omega t) = \underline{a\cos(\omega t)}$$

$$\underline{V_F} = a\omega\sin(0-\omega t) = a\omega\sin(-\omega t) = \underline{-a\omega\sin(\omega t)}$$

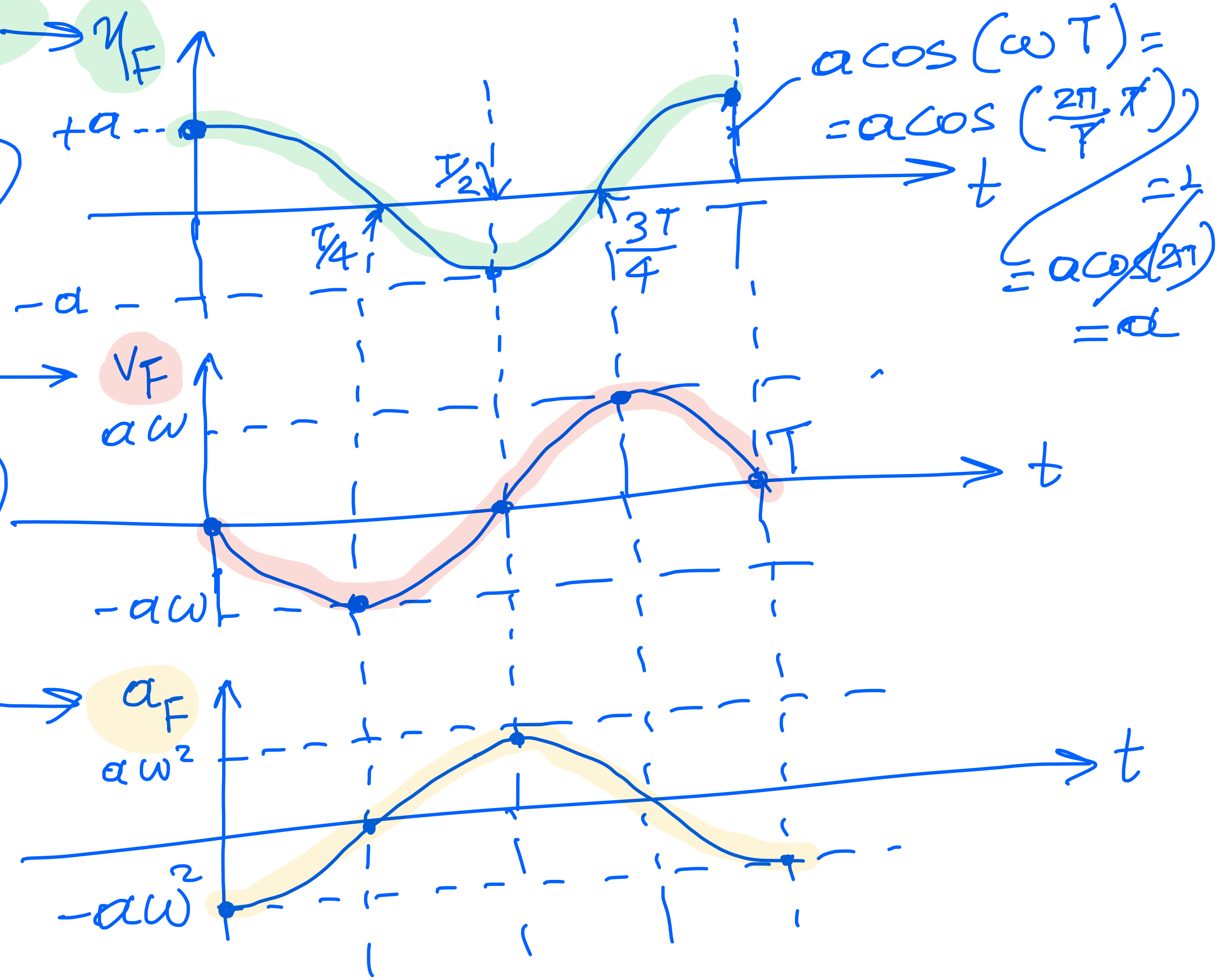
$$\underline{a_F} = -\omega^2\eta_F = \underline{-a\omega^2\cos(\omega t)}$$

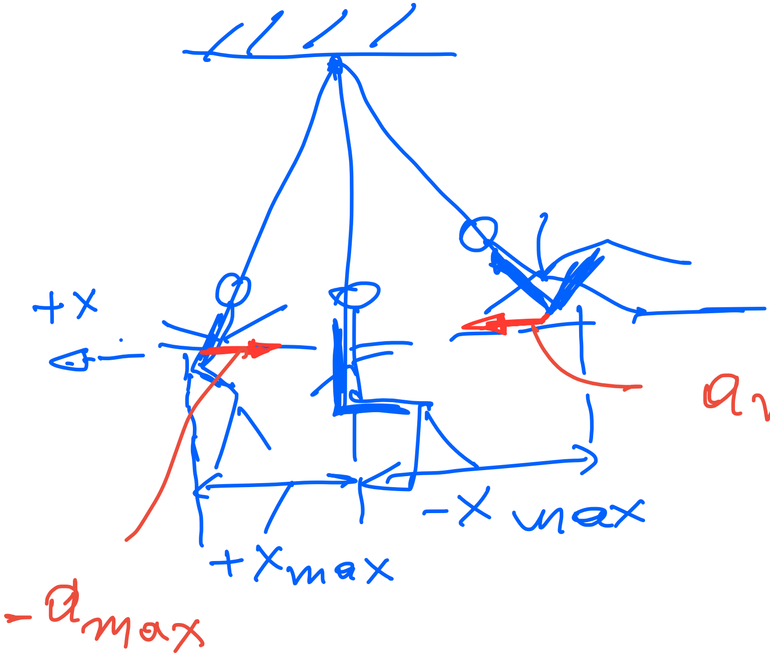
$\eta_F = a\cos(\omega t)$

$V_F = -a\omega \sin(\omega t)$



$a_F = -a\omega^2 \cos(\omega t)$





Motion of a person on swing is analogous to motion of float!