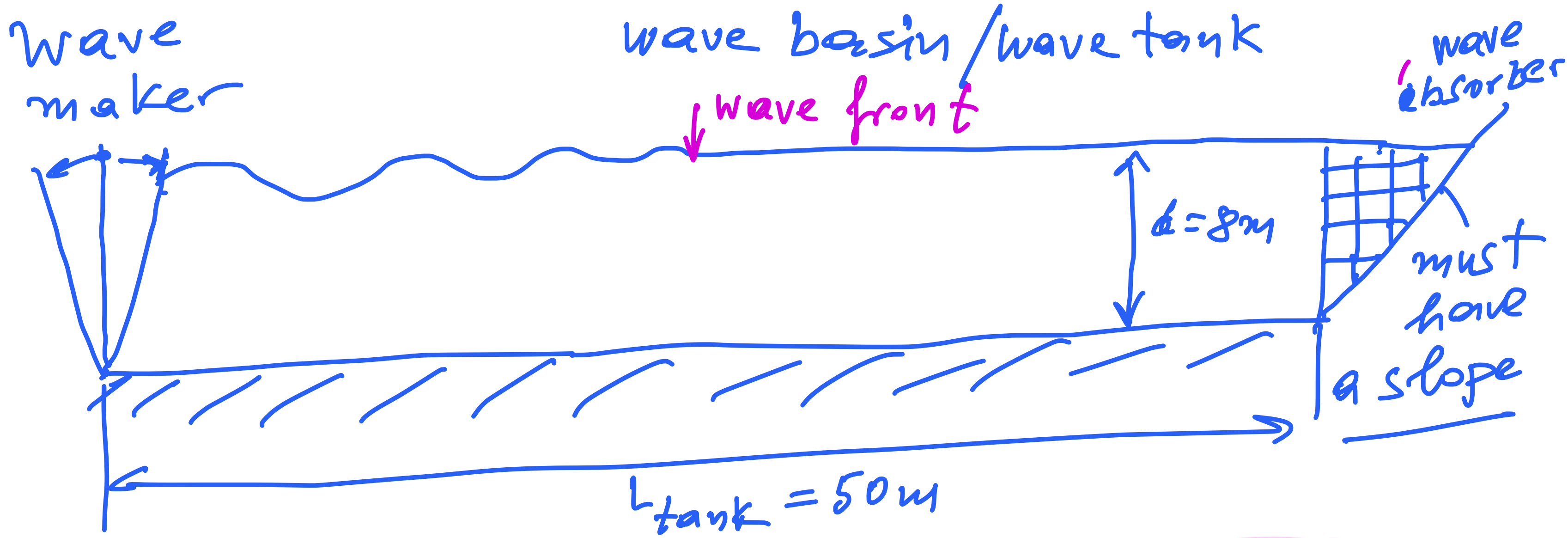
SPEED OF WAVE FRONT



T = 3 sec

How long it will take for the wave front to reach the wave absorber?

$$C = \frac{L}{T} = \frac{14.05}{3} = 4.68\ \frac{m}{s}$$

$$L_0 = \frac{gT^2}{2\pi} = \frac{9.81 \times 3^2}{2\pi} = 14.05\ m \qquad \frac{d}{L_0} = \frac{8}{14.05} > \frac{1}{2}$$

deep water
$L = L_0$

$$\boxed{t_{cross}} = \frac{L_{tank}}{C} = \frac{50}{4.68} = \boxed{10.7\ sec}$$

In reality $t_{cross} = 21.4\ sec$
(TWICE as much as our prediction!!!!)

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$H_1 > H_2 > H_3$

WHY?

2 WAVES WITH SLIGHLTY DIFFERENT LENGTH/PERIOD

DEEP H_2O

① $T_1 = 10\ sec \quad H_1 = 2ft$

$$L_1 = \frac{gT_1^2}{2\pi} = \frac{32.2 \times 10^2}{2\pi} = 512.5\ ft$$

$$\eta_1 = \frac{H_1}{2}\cos(k_1 x - \omega_1 t)$$

② $L_2 = (1.05)L_1 = 538.1\ ft \quad H_2 = 2ft$

$$L_2 = \frac{gT_2^2}{2\pi} \rightsquigarrow T_2 = \sqrt{\frac{2\pi L_2}{g}} = \underline{10.25\ sec}$$

$$\eta_2 = \frac{H_2}{2}\cos(k_2 x - \omega_2 t)$$

$$k_1 = \frac{2\pi}{L_1} = 0.01226\ ft^{-1}; \quad \omega_1 = \frac{2\pi}{T_1} = 0.628\ sec^{-1}$$

$$k_2 = \frac{2\pi}{L_2} = 0.01168\ ft^{-1}; \quad \omega_2 = \frac{2\pi}{T_2} = 0.613\ sec^{-1}$$

compound $\eta = \eta_1 + \eta_2 = \dfrac{H}{2}\left[\cos(k_1 x - \omega_1 t) + \cos(k_2 x - \omega_2 t)\right]$

$(H_1 = H_2 = H = 2ft)$

From trig: $\cos(\alpha) + \cos(\beta) = 2\cos\left(\dfrac{\alpha+\beta}{2}\right)\cos\left(\dfrac{\alpha-\beta}{2}\right)$

Apply the above with $\alpha = k_1 x - \omega_1 t$, $\beta = k_2 x - \omega_2 t$

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GROUP OF 2 WAVES WITH SLIGHLTY DIFFERENT L or T (DEEP WATER)

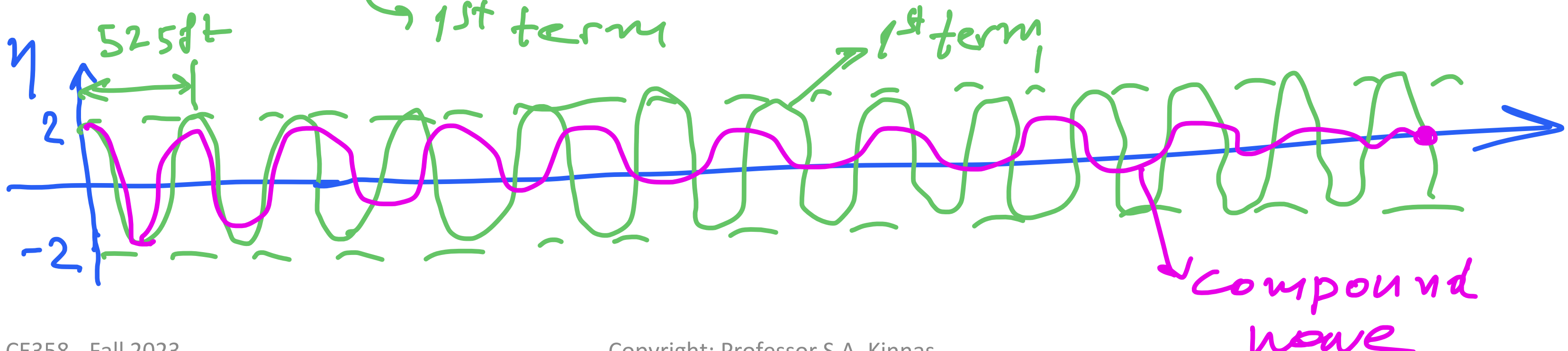
$$\eta = \frac{H}{\cancel{2}}\,\cancel{2}\cos\left(\frac{k_1x-\omega_1t+k_2x-\omega_2t}{2}\right)\cos\left(\frac{k_1x-\omega_1t-k_2x+\omega_2t}{2}\right) =$$

$$= H\cos\left[\underbrace{\frac{k_1+k_2}{2}}_{k_p}x-\underbrace{\frac{\omega_1+\omega_2}{2}}_{\omega_p}t\right]\cos\left[\underbrace{\frac{k_1-k_2}{2}}_{k_m}x-\underbrace{\frac{\omega_1-\omega_2}{2}}_{\omega_m}t\right]$$

$$k_p = \frac{k_1+k_2}{2} = 0.01197\ ft^{-1} \quad ; \quad \omega_p = \frac{\omega_1+\omega_2}{2} = 0.6205\ sec^{-1}$$

$$k_m = \frac{k_1-k_2}{2} = 0.00029\ ft^{-1} \quad ; \quad \omega_m = \frac{\omega_1-\omega_2}{2} = 0.0075\ sec^{-1}$$

$$\eta = \underbrace{H\cos\left[k_p x-\omega_p t\right]}_{1^{st}\ \text{term}}\ \underbrace{\cos\left[k_m x-\omega_m t\right]}_{2^{nd}\ \text{term}}$$



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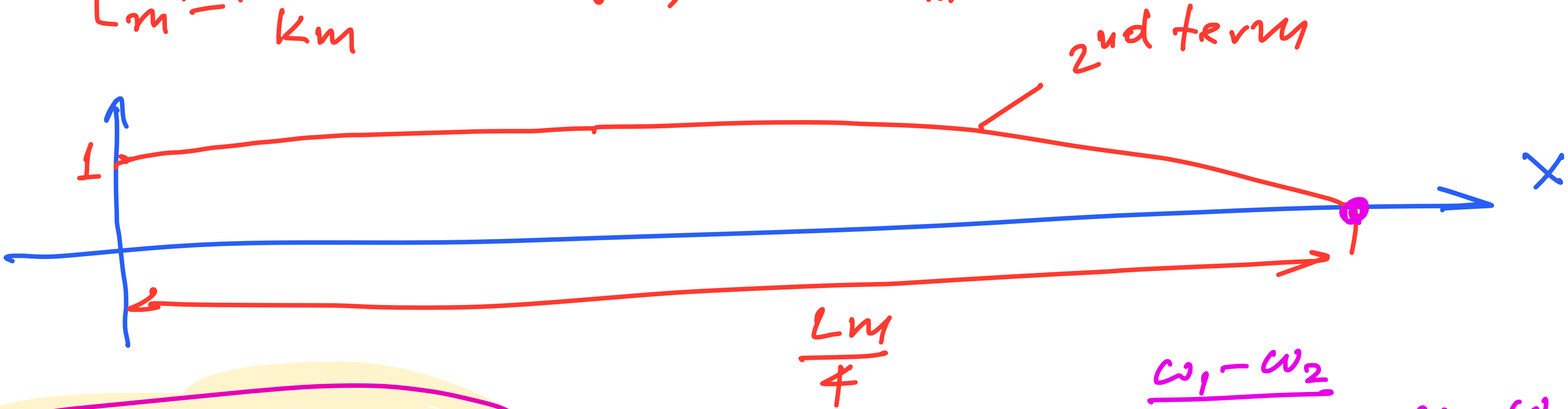
$L_p = \frac{2\pi}{k_p} = 525\,ft$; $T_p = \frac{2\pi}{\omega_p} = 10.123\,sec$

$L_m = \frac{2\pi}{k_m} = 21,545\,ft$; $T_m = \frac{2\pi}{\omega_m} = 820\,sec$



Group velocity

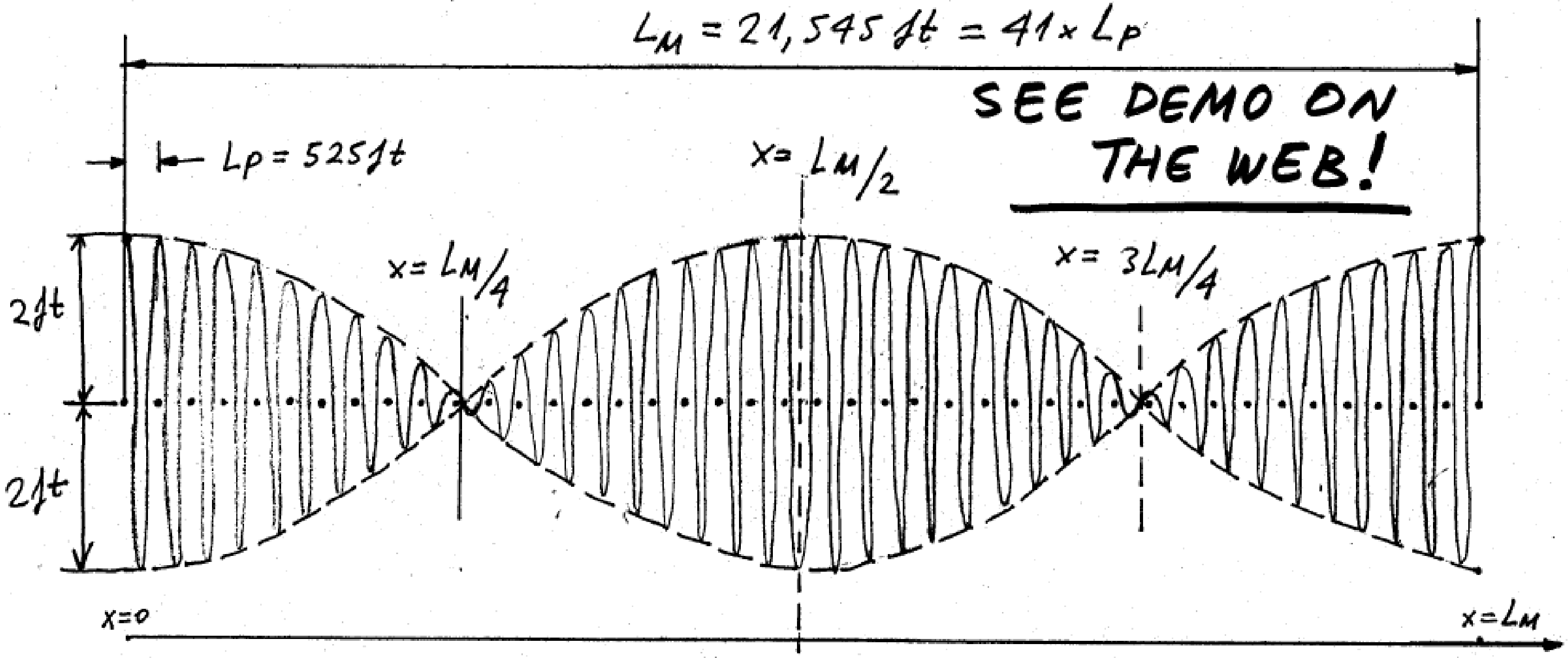
$$C_m = \frac{\omega_m}{k_m} = \frac{\frac{\omega_1-\omega_2}{2}}{\frac{k_1-k_2}{2}} = \frac{\omega_1-\omega_2}{k_1-k_2}$$

$$= \lim_{\Delta k \to 0} \frac{\Delta \omega}{\Delta k} = \frac{d\omega}{dk} = C_g$$

Wave speed: $\boxed{C = \frac{\omega}{k}}$

2 WAVES WITH SLIGHLTY DIFFERENT LENGTH/PERIOD

NOTE: ~~THERE~~ ARE 41 waves of wave length L_P inside wave length L_M



Envelope: $\eta_e = H \cos\left[\frac{2\pi x}{L_M} - \frac{2\pi t}{T_M}\right]$ (propagates in the same direction as original)

Wave speed of envelope is: $C_e = L_M/T_M = 21{,}545/820 = 26.27$ ft/sec

group velocity of wave (1) (for deep water) $C_g = C_1/2 = 0.5\, L_1/T_1 = 25.62$ ft/sec.

$C_e \approx C_g$ Actually definition of $C_g = d\omega/dk = \lim_{dk \to 0} (C_e)$.

Since $\Delta k = k_1 - k_2 = 0.01226 - 0.01168 = 5.8 \times 10^{-4}$ not quite "0"

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DEFINITION OF GROUP VELOCITY

Deep H_2O

$$\frac{\omega^2}{k} = g \rightsquigarrow \omega = \sqrt{gk};$$

$$C_g = \frac{d\omega}{dk} = \frac{d}{dk}\sqrt{gk} = \sqrt{g}\,\frac{1}{2}\,k^{-\frac{1}{2}} = \sqrt{\frac{g}{k}}\cdot\frac{1}{2} = \boxed{\frac{C}{2}}$$

$$C = \frac{\omega}{k} = \frac{\sqrt{gk}}{k} = \sqrt{\frac{g}{k}}$$

$$C_g = C_{front} = \frac{C}{2} \quad (\text{in deep } H_2O)$$

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FORMULAS FOR GROUP VELOCITY

$C_g = n \cdot C$

RELATIVE DEPTH	SHALLOW WATER $\frac{d}{L} < \frac{1}{25}$	TRANSITIONAL WATER $\frac{1}{25} < \frac{d}{L} < \frac{1}{2}$	DEEP WATER $\frac{d}{L} > \frac{1}{2}$
1. Wave profile	Same As →	$\eta = \frac{H}{2} \cos \left[\frac{2\pi x}{L} - \frac{2\pi t}{T} \right] = \frac{H}{2} \cos \theta$	← Same As
2. Wave celerity	$C = \frac{L}{T} = \sqrt{gd}$	$C = \frac{L}{T} = \frac{gT}{2\pi} \tanh \left(\frac{2\pi d}{L} \right)$	$C = C_0 = \frac{L}{T} = \frac{gT}{2\pi}$
3. Wavelength	$L = T\sqrt{gd} = CT$	$L = \frac{gT^2}{2\pi} \tanh \left(\frac{2\pi d}{L} \right)$	$L = L_0 = \frac{gT^2}{2\pi} = C_0 T$
4. Group velocity	$C_g = C = \sqrt{gd}$	$C_g = nC = \frac{1}{2} \left[1 + \frac{4\pi d/L}{\sinh(4\pi d/L)} \right] \cdot C$	$C_g = \frac{1}{2} C = \frac{gT}{4\pi}$
5. Water Particle Velocity (a) Horizontal	$u = \frac{H}{2} \sqrt{\frac{g}{d}} \cos \theta$	$u = \frac{H}{2} \frac{gT}{L} \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \cos \theta$	$u = \frac{\pi H}{T} e^{\frac{2\pi z}{L}} \cos \theta$
(b) Vertical	$w = \frac{H\pi}{T} \left(1 + \frac{z}{d} \right) \sin \theta$	$w = \frac{H}{2} \frac{gT}{L} \frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \sin \theta$	$w = \frac{\pi H}{T} e^{\frac{2\pi z}{L}} \sin \theta$
6. Water Particle Accelerations (a) Horizontal	$a_x = \frac{H\pi}{T} \sqrt{\frac{g}{d}} \sin \theta$	$a_x = \frac{g\pi H}{L} \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \sin \theta$	$a_x = 2H \left(\frac{\pi}{T} \right)^2 e^{\frac{2\pi z}{L}} \sin \theta$
(b) Vertical	$a_z = -2H \left(\frac{\pi}{T} \right)^2 \left(1 + \frac{z}{d} \right) \cos \theta$	$a_z = -\frac{g\pi H}{L} \frac{\sinh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} \cos \theta$	$a_z = -2H \left(\frac{\pi}{T} \right)^2 e^{\frac{2\pi z}{L}} \cos \theta$
7. Water Particle Displacements (a) Horizontal	$\xi = -\frac{HT}{4\pi} \sqrt{\frac{g}{d}} \sin \theta$	$\xi = -\frac{H}{2} \frac{\cosh[2\pi(z+d)/L]}{\sinh(2\pi d/L)} \sin \theta$	$\xi = -\frac{H}{2} e^{\frac{2\pi z}{L}} \sin \theta$
(b) Vertical	$\zeta = \frac{H}{2} \left(1 + \frac{z}{d} \right) \cos \theta$	$\zeta = \frac{H}{2} \frac{\sinh[2\pi(z+d)/L]}{\sinh(2\pi d/L)} \cos \theta$	$\zeta = \frac{H}{2} e^{\frac{2\pi z}{L}} \cos \theta$
8. Subsurface Pressure	$p = \rho g (\eta - z)$	$p = \rho g \eta \frac{\cosh[2\pi(z+d)/L]}{\cosh(2\pi d/L)} - \rho g z$	$p = \rho g \eta e^{\frac{2\pi z}{L}} - \rho g z$

Figure 2-6. Summary of linear (Airy) wave theory--wave characteristics.

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HOW TO DETERMINE GROUP VELOCITY FROM TABLE C-1

$= \frac{C_g}{C}$

Table C-1. Continued.

d/L_o	d/L	$2\pi d/L$	TANH $2\pi d/L$	SINH $2\pi d/L$	COSH $2\pi d/L$	H/H_o'	K	$4\pi d/L$	SINH $4\pi d/L$	COSH $4\pi d/L$	n	C_G/C_o	M
.1500	.1833	1.152	.8183	1.424	1.740	.9133	.5748	2.303	4.954	5.054	.7325	.5994	7.369
.1510	.1841	1.157	.8200	1.433	1.747	.9133	.5723	2.314	5.007	5.106	.7311	.5994	7.339
.1520	.1850	1.162	.8217	1.442	1.755	.9132	.5699	2.324	5.061	5.159	.7296	.5995	7.309
.1530	.1858	1.167	.8234	1.451	1.762	.9132	.5675	2.335	5.115	5.212	.7282	.5996	7.279
.1540	.1866	1.173	.8250	1.460	1.770	.9132	.5651	2.345	5.169	5.265	.7268	.5996	7.250
.1550	.1875	1.178	.8267	1.469	1.777	.9131	.5627	2.356	5.225	5.320	.7254	.5997	7.221
.1560	.1883	1.183	.8284	1.479	1.785	.9130	.5602	2.366	5.283	5.376	.7240	.5998	7.191
.1570	.1891	1.188	.8301	1.488	1.793	.9129	.5577	2.377	5.339	5.432	.7226	.5999	7.162
.1580	.1900	1.194	.8317	1.498	1.801	.9130	.5552	2.387	5.398	5.490	.7212	.5998	7.134
.1590	.1908	1.199	.8333	1.507	1.809	.9130	.5528	2.398	5.454	5.544	.7198	.5998	7.107
.1600	.1917	1.204	.8349	1.517	1.817	.9130	.5504	2.408	5.513	5.603	.7184	.5998	7.079
.1610	.1925	1.209	.8365	1.527	1.825	.9130	.5480	2.419	5.571	5.660	.7171	.5998	7.052
.1620	.1933	1.215	.8381	1.536	1.833	.9130	.5456	2.429	5.630	5.718	.7157	.5998	7.026
.1630	.1941	1.220	.8396	1.546	1.841	.9130	.5432	2.440	5.690	5.777	.7144	.5998	7.000
.1640	.1950	1.225	.8411	1.555	1.849	.9130	.5409	2.450	5.751	5.837	.7130	.5998	6.975
.1650	.1958	1.230	.8427	1.565	1.857	.9131	.5385	2.461	5.813	5.898	.7117	.5997	6.949
.1660	.1966	1.235	.8442	1.574	1.865	.9132	.5362	2.471	5.874	5.959	.7103	.5996	6.924
.1670	.1975	1.240	.8457	1.584	1.873	.9132	.5339	2.482	5.938	6.021	.7090	.5996	6.900
.1680	.1983	1.246	.8472	1.594	1.882	.9133	.5315	2.492	6.003	6.085	.7076	.5995	6.876
.1690	.1992	1.251	.8486	1.604	1.890	.9133	.5291	2.503	6.066	6.148	.7063	.5994	6.853

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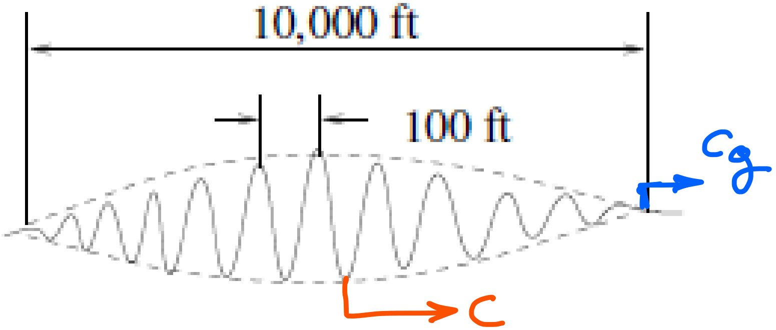
EXAMPLE ON GROUP VELOCITY

A group of waves is 10,000 feet in length in deep water. The waves within the group are 100 feet in length.

$$C_g = \frac{C}{2}$$

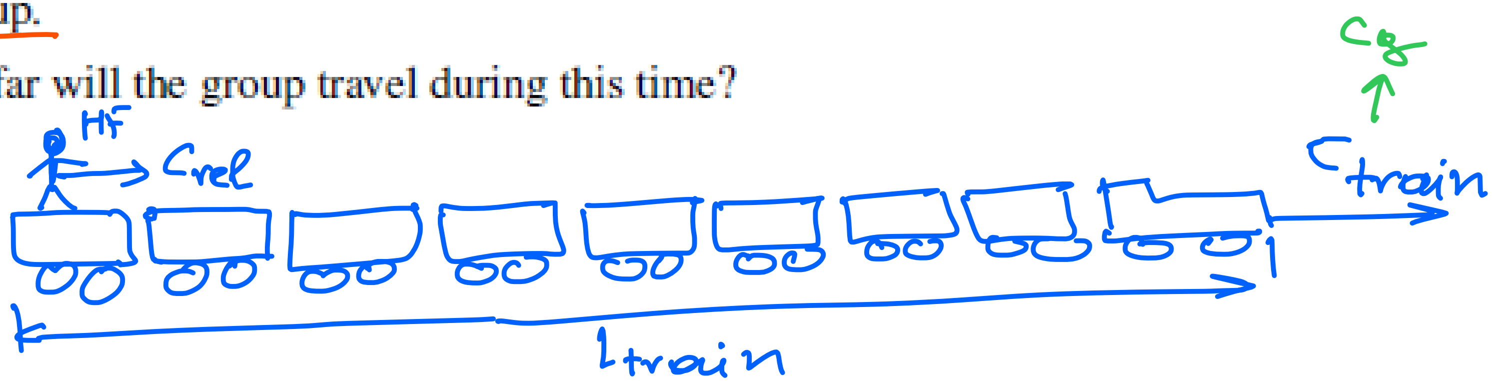


$$C = C_{train} + C_{rel}$$

$$C_{rel} = C - C_{train}$$

a) Calculate the time required for a component wave to travel from the rear to the front of the group.

b) How far will the group travel during this time?



$$t_{cross} = \frac{L_{train}}{C_{rel}}$$

Relative to outside observer HF travels with

$$C = C = C_{train} + C_{rel}$$

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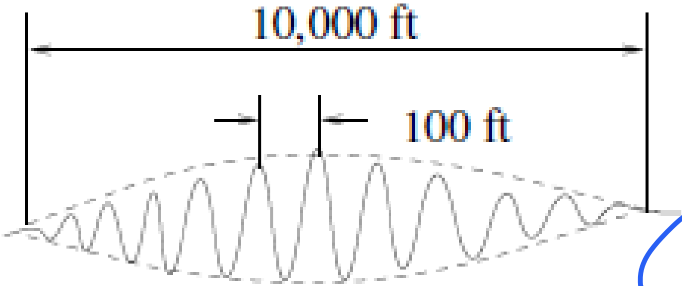
EXAMPLE ON GROUP VELOCITY

A group of waves is 10,000 feet in length in deep water. The waves within the group are 100 feet in length.



$$\boxed{t_{cross}} = \frac{10{,}000\ ft}{C_{rel}} \qquad \left(c_g = \frac{c}{2}\right)$$

$$C_{rel} = C - c_g = C - \frac{C}{2} = \frac{c}{2}$$

$$\frac{10{,}000}{11.32} = \boxed{883.4\ sec}$$

a) Calculate the time required for a component wave to travel from the rear to the front of the group.

b) How far will the group travel during this time?

$$L_{travel} = C_g \cdot t_{cross} = $$
$$= 10{,}000\ ft$$

$$C = \frac{L}{T} = \frac{100\ ft}{4.42} = 22.64\ ft/s$$

$$L = \frac{gT^2}{2\pi} \quad (\text{deep } H_2O) \Longrightarrow T = \sqrt{\frac{2\pi L}{g}} = 4.42\ sec$$

$$c_g = \frac{c}{2} = 11.32\ ft/s$$

$$C_{rel} = C - c_g = 11.32\ ft/s$$

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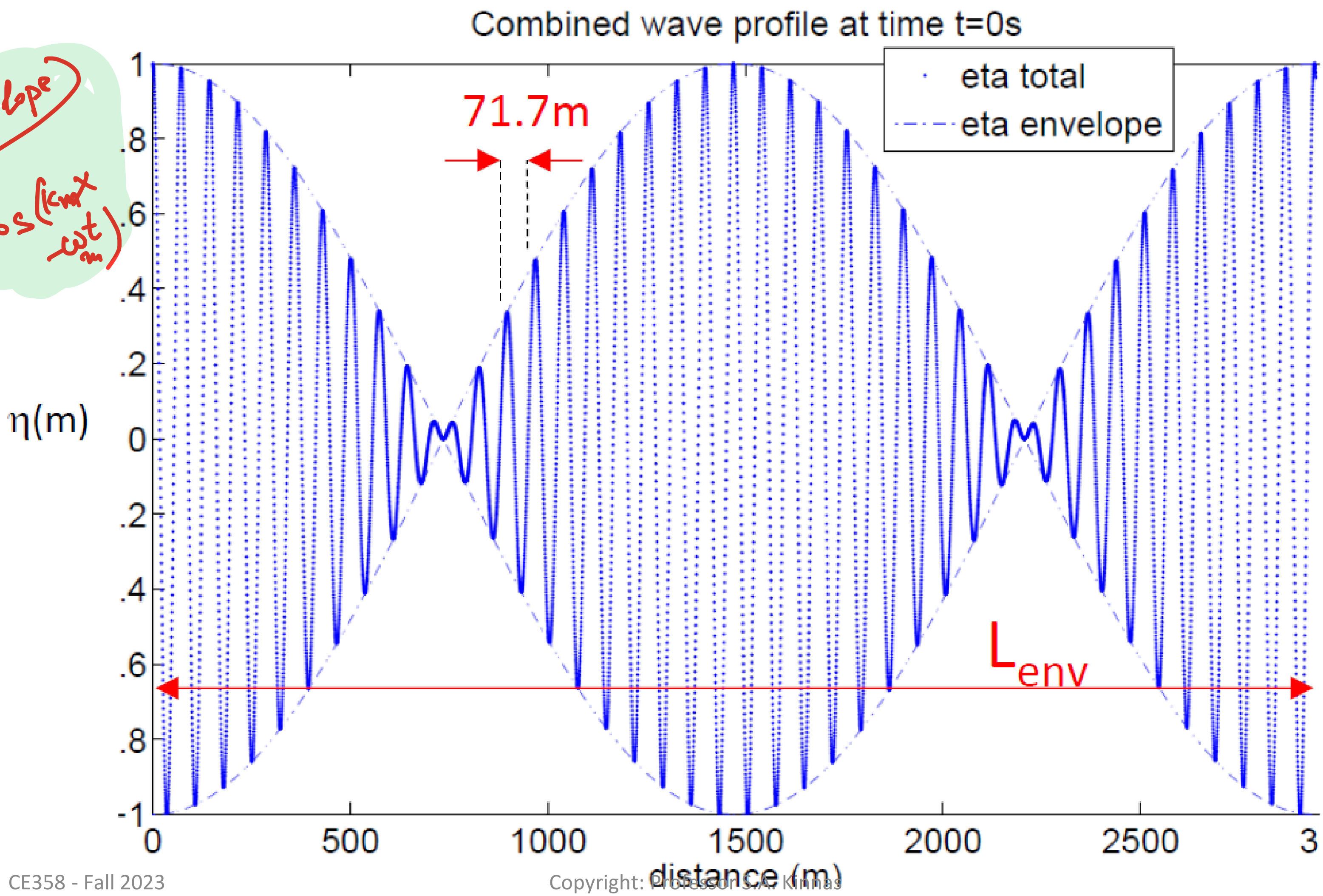
GROUP OF 2 WAVES IN TRANSITIONAL WATER

Total wave profile and wave envelop for a group of two waves with a height of 1m each, and with wave lengths L_1=70 m (T_1=8.973 s) and L_2=73.5 m (T_2=9.37 s), at a depth of 7 m [$L_{env}=2L_1L_2/(L_2-L_1)$=2,940m]

$\eta_{envelope}$ $(= H\cos(k_m x - \omega_m t))$



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