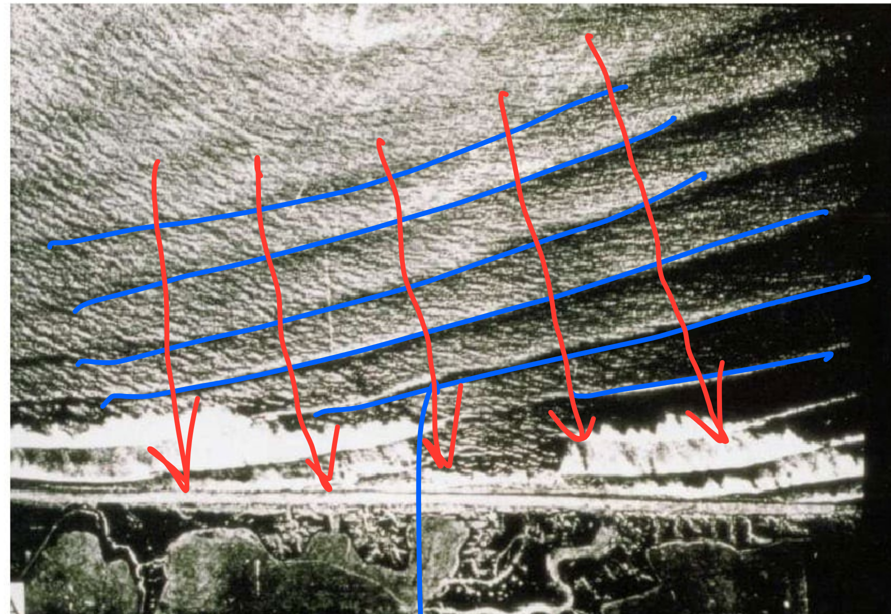
WAVE REFRACTION

Wave Refraction

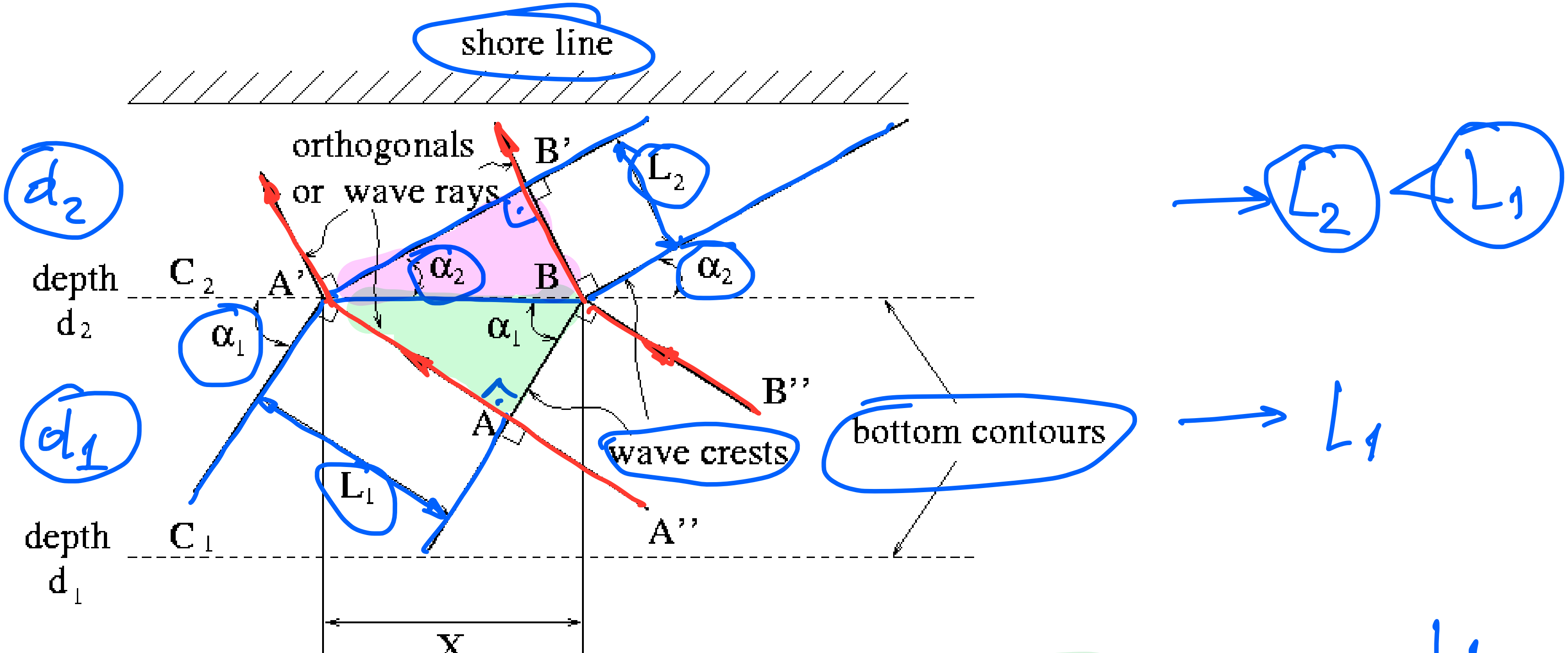


crests

wave orthogonals / wave rays

wave rays ⊥ crests

WAVE REFRACTION



From orthogonal triangle $AA'B \Rightarrow A'B = \dfrac{L_1}{\sin\alpha_1}$

" " " $A'B'B \Rightarrow A'B = \dfrac{L_2}{\sin\alpha_2}$

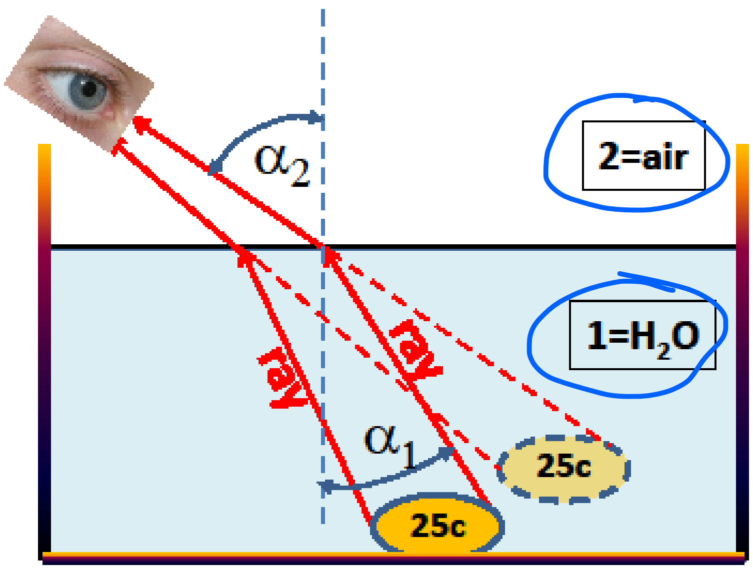
$$\Rightarrow \frac{L_1}{\sin\alpha_1} = \frac{L_2}{\sin\alpha_2} \Longrightarrow \boxed{\frac{\sin\alpha_1}{\sin\alpha_2} = \frac{L_1}{L_2} = \frac{C_1}{C_2}}$$

Snell's Law

WAVE REFRACTION

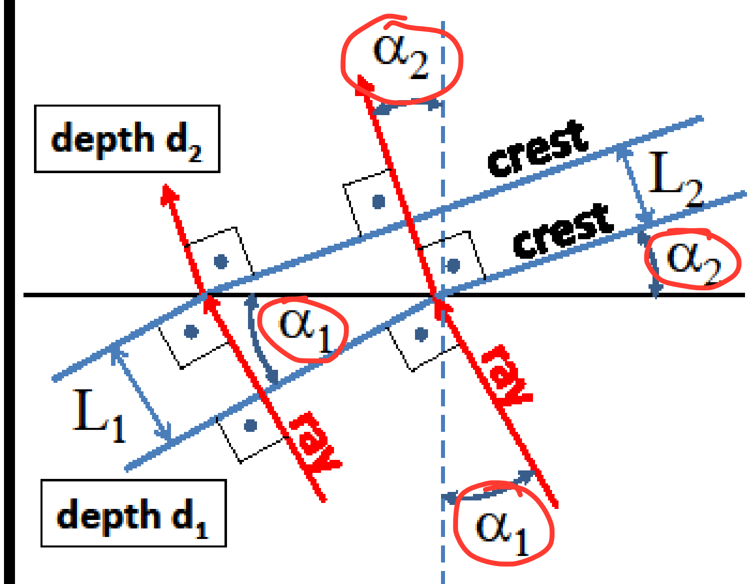
Refraction

Optics



$$C_{H_2O} \approx 0.75 C_{air} \Rightarrow C_2 > C_1 \Rightarrow \alpha_2 > \alpha_1$$

Wave Mechanics



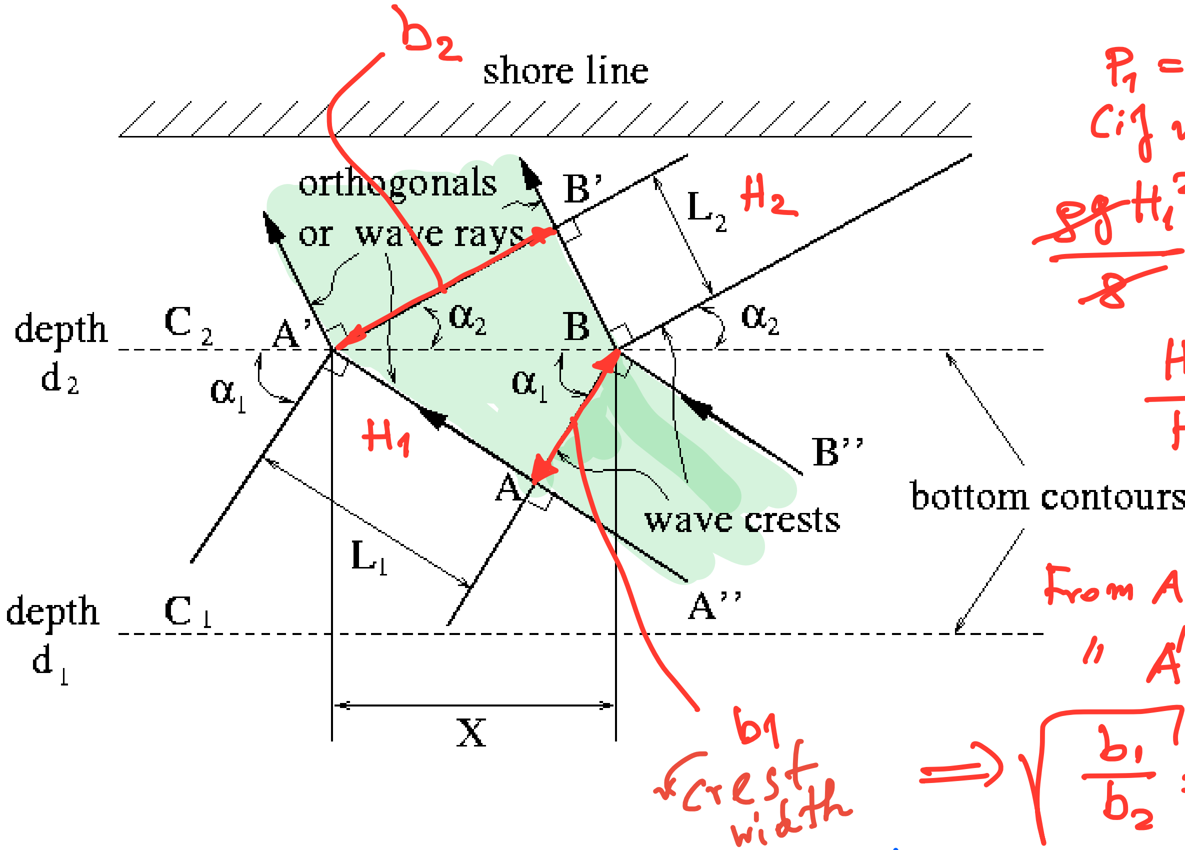
$$d_2 < d_1 \Rightarrow C_2 < C_1 \Rightarrow \alpha_2 < \alpha_1$$

$$\text{Snell's Law}: \frac{\sin(\alpha_1)}{\sin(\alpha_2)} = \frac{C_1}{C_2} = \frac{L_1}{L_2}$$

▶ In optics refraction is due to different speed of light in water vs. air!

▶ In free surface waves refraction is due to different C in different depths!

WAVE REFRACTION



$P_1 = P_2$ (if no losses)

$$\frac{\rho g H_1^2}{8} C_{g_1} b_1 = \frac{\rho g H_2^2}{8} C_{g_2} b_2$$

$$\frac{H_2}{H_1} = \sqrt{\frac{C_{g_1}}{C_{g_2}}} \sqrt{\frac{b_1}{b_2}}$$

From $A'AB$: $b_1 = (A'B)\cos\alpha_1$

" $A'B'B$: $b_2 = (A'B)\cos\alpha_2$

$$\Longrightarrow \sqrt{\frac{b_1}{b_2}} = \sqrt{\frac{\cos\alpha_1}{\cos\alpha_2}}$$

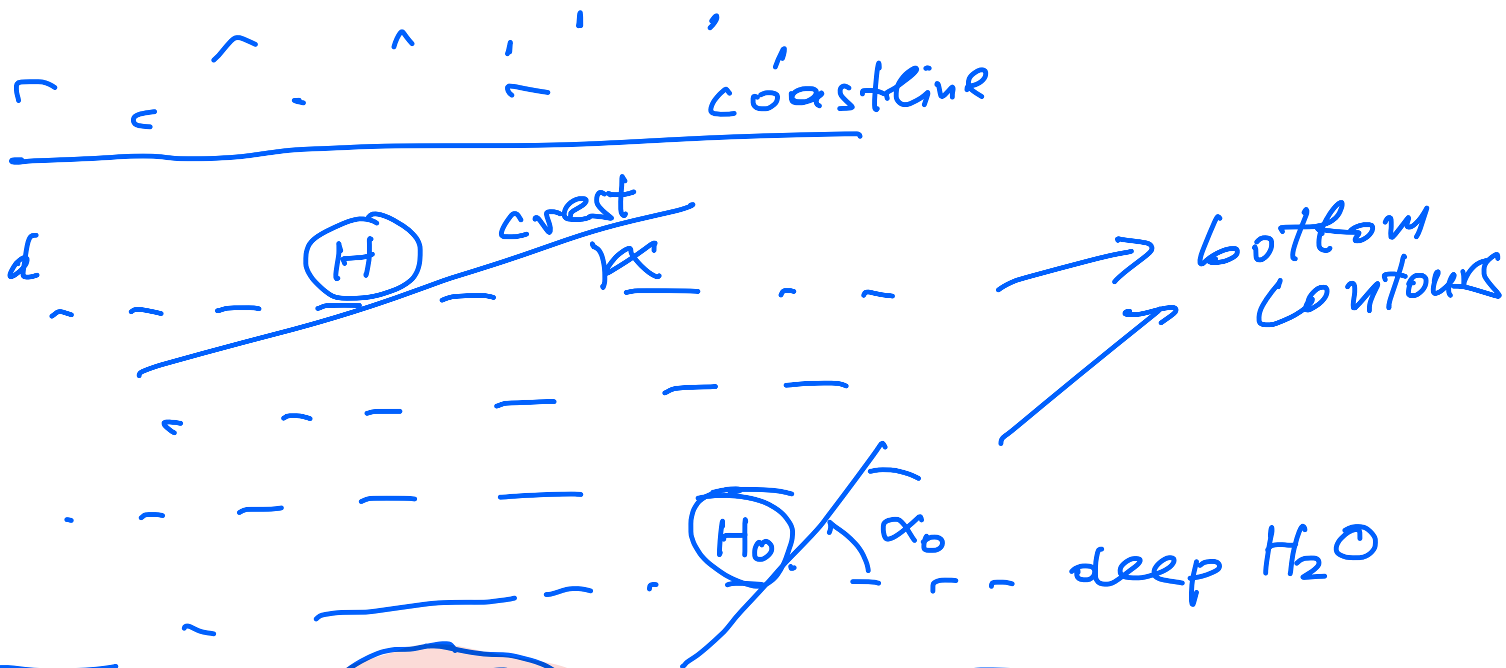
If $d_1 \to d_0$ and $d_2 \to d$

$$\frac{H}{H_0} = \sqrt{\frac{C_{g_0}}{C_g}} \sqrt{\frac{b_0}{b}}$$

$$\sqrt{\frac{C_0}{2\,n\,C}} \sqrt{\frac{\cos\alpha_0}{\cos\alpha}}$$

shoaling coeff. K_s

can be found in C-1 $\frac{H}{H_0'}$



$$\frac{H}{H_0} = K_s \sqrt{\frac{\cos\alpha_0}{\cos\alpha}} = K_s K_R$$

K_R = refraction coeff.

H_0' = unrefracted deep water wave height

$K_s = \frac{H}{H_0'}$ and $K_R = \frac{H_0'}{H_0}$

$$K_s K_R = \frac{H}{H_0'} \frac{H_0'}{H_0} = \frac{H}{H_0}$$

WAVE REFRACTION - EXAMPLES

1. A 4.83 sec plane mono-chromatic wave approaches the beach with its crests in deep water at an angle of 40^o with respect to the straight shoreline, and a wave height of 30 cm. Determine the angle of the crests and the wave height at a depth of $3m$. Consider that the bottom contours are parallel to the shoreline and that the effects of reflection are negligible.

$= T$, H_o, α_o

$\alpha = ?$ $H = ?$ at $d = 3m$

Snell's Law

a) $\frac{\sin\alpha}{\sin\alpha_0} = \frac{L}{L_0}$

$L_0 = \frac{gT^2}{2\pi} = 36.42m$

$\frac{d}{L_0} = \frac{3}{36.42} = 0.0824 \xrightarrow{C\text{-}1} \frac{d}{L} = 0.1255 \Rightarrow L = 23.9m$

$\Rightarrow \frac{\sin\alpha}{\sin\alpha_0} = \frac{23.9}{36.42} \Rightarrow \sin\alpha = 0.422 \Rightarrow \alpha = 25^o$

b) $\frac{H}{H_0} = K_s\sqrt{\frac{\cos\alpha_0}{\cos\alpha}} = 0.951\sqrt{\frac{\cos 40^o}{\cos 25^o}} = 0.951 \times 0.92 = 0.875$

$K_s \left(= \frac{H}{H_0'}\right) = 0.951$

$H = H_0 \times 0.875 = 0.2625m$

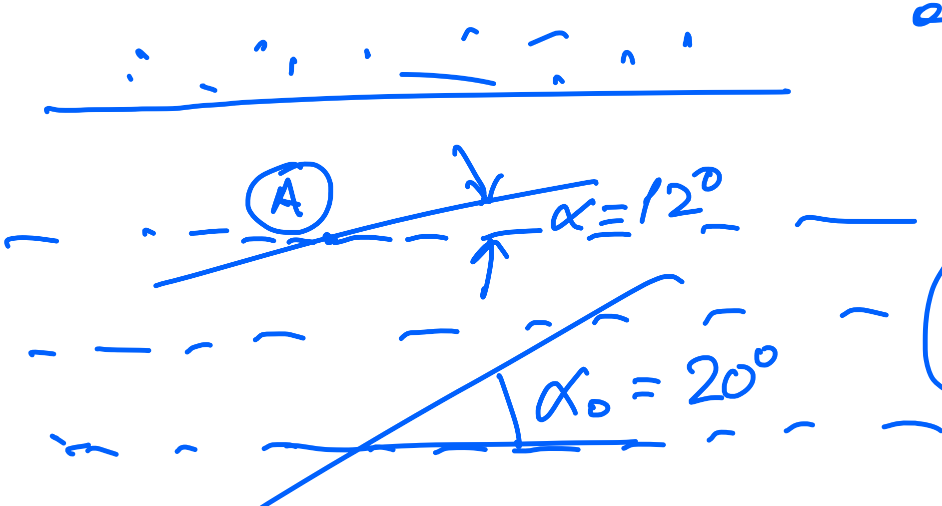
WAVE REFRACTION - EXAMPLES

2. A 4.5 sec sinusoidal wave is approaching the beach with its crests in deep water at an angle of 20^o with respect to the straight shoreline. At a certain distance from the beach the wave crests have been refarcted and form an angle of $\alpha = 12^o$ with respect to the shoreline. Assuming that the bottom contours are parallel to the shoreline and that the effects of reflection are negligible, find the following:

   (a) The wave length at the location where $\alpha = 12^o$

   (b) The depth of the water at the location where $\alpha = 12^o$

   (c) If the wave height at the same depth as that in (b) is 40 cm, what would the height of the wave be in deep water?



a) Snell's Law

$$\frac{\sin\alpha}{\sin\alpha_o} = \frac{L}{L_o}$$

$$L_o = \frac{gT^2}{2\pi} = 31.6\,m$$

$$L_A = 31.6 \times \frac{\sin(12^o)}{\sin(20^o)} = 19.2\,m$$

$$L = L_o \tanh\left(\frac{2\pi d}{L}\right) \quad (1)$$

b) $d_A = ?$

$$\frac{d_A}{L_A} \qquad \frac{d_A}{L_o}$$

WAVE REFRACTION - EXAMPLES

From (1) $\tanh\left(\frac{2\pi d}{L}\right) = \frac{L}{L_0} = \frac{19.2}{31.6} = 0.608 \Rightarrow \frac{2\pi d}{L} = \tanh^{-1}(0.608)$

↑ inverse function of tanh

$\Rightarrow \frac{2\pi d}{L} = 0.7057 \Rightarrow$ $\boxed{d = 2.16\,m}$

c) $H = 0.4\,m$ $H_0 = ?$

$$\frac{H}{H_0} = K_s K_R$$

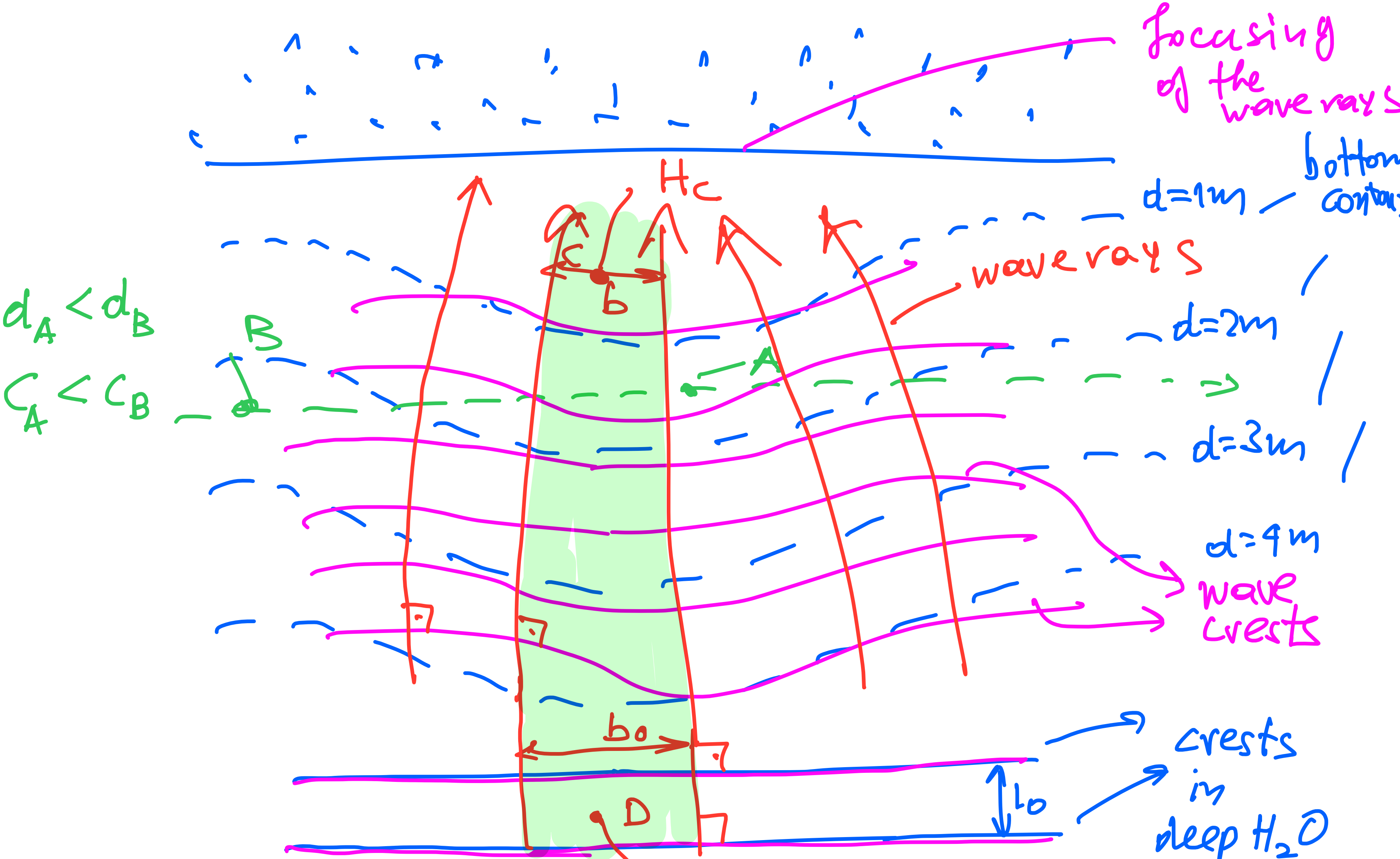
K_s from C-1 and $\frac{d}{L} = \frac{2.16}{19.2} = 0.113 > 0.04$ NOT Shallow H_2O

→ $K_s = 0.9752$

$$K_R = \sqrt{\frac{\cos\alpha_0}{\cos\alpha}} = \sqrt{\frac{\cos(20°)}{\cos(12°)}} = 0.98$$

$\Rightarrow \frac{H}{H_0} = 0.9732 \times 0.98 = 0.956 \Rightarrow$ $\boxed{H_0 = 0.418\,m}$

WAVE REFRACTION - EXAMPLES

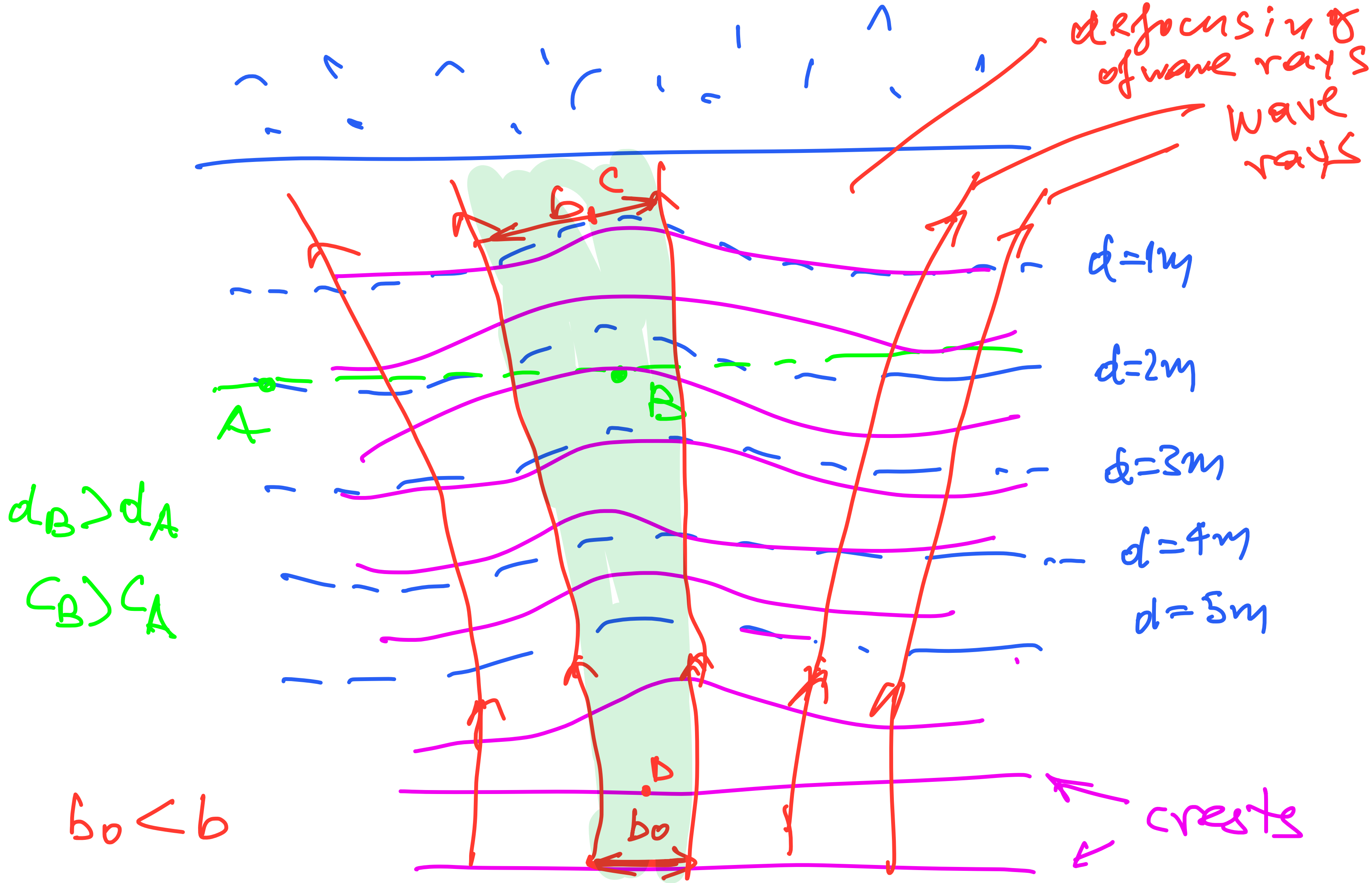


$b < b_0$

$$\frac{\rho g H_0^2}{8} C_{g_0} b_0 = \frac{\rho g H_c^2}{8} C_g b \Rightarrow$$

WAVE REFRACTION - EXAMPLES

$$\Rightarrow \frac{H_c}{H_0} = \underbrace{\sqrt{\frac{c_{g0}}{c}}}_{K_S}\ \underbrace{\sqrt{\frac{b_0}{b}}}_{K_R > 1}$$



$$\frac{H_c}{H_o} = K_S K_R \rightarrow \sqrt{\frac{b_o}{b}} < 1$$

WAVE REFRACTION - EXAMPLES

INTRODUCTION 15

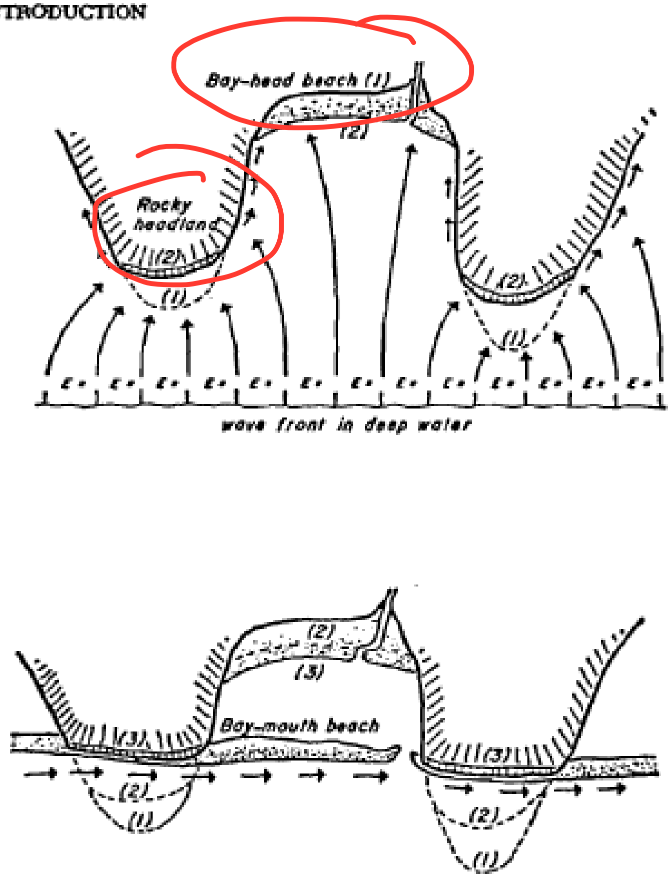


FIG. 6. Waves straighten a rocky coast. *Top:* Zones of equal wave energy in deep water are concentrated by wave refraction so that headlands are attacked. *Bottom:* Eventually headlands are cut back and furnish enough sand to build a straight continuous beach.

5. For the image below of a headland with wave refraction around it qualititatively explain what the underlying depth contours must be and why the wave crests are bending as they do. Draw the ray lines. Where is wave energy more and less concentrated.

WAVE REFRACTION - EXAMPLES

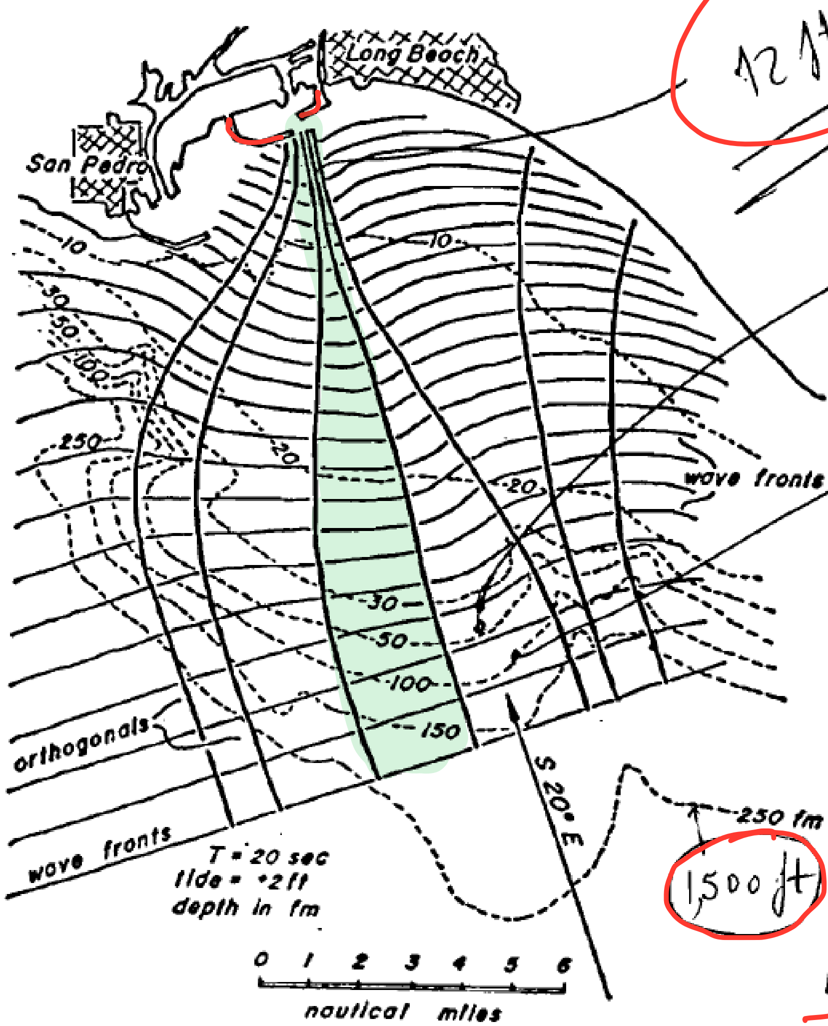


FIG. 30. Refraction diagram for destructive waves at Long Beach, California, showing how underwater topography several hundred feet deep and a dozen miles offshore focused wave energy on the breakwater.

12 ft high

due to significant focusing of wave rays.

50 x 6 = 300 ft

600 ft

1 fm = 6 feet

fathom

1,500 ft

$L_0 = 2,000$ ft → very long wave length in deep H_2O which makes waves feel sea-floor in deeper H_2O