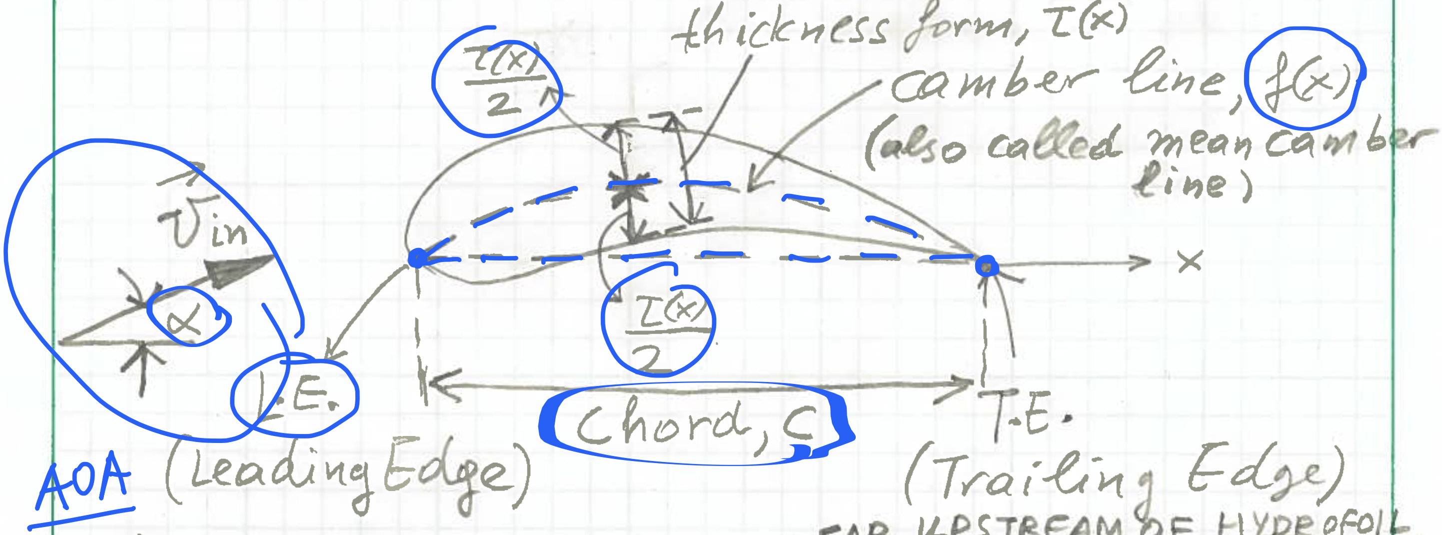
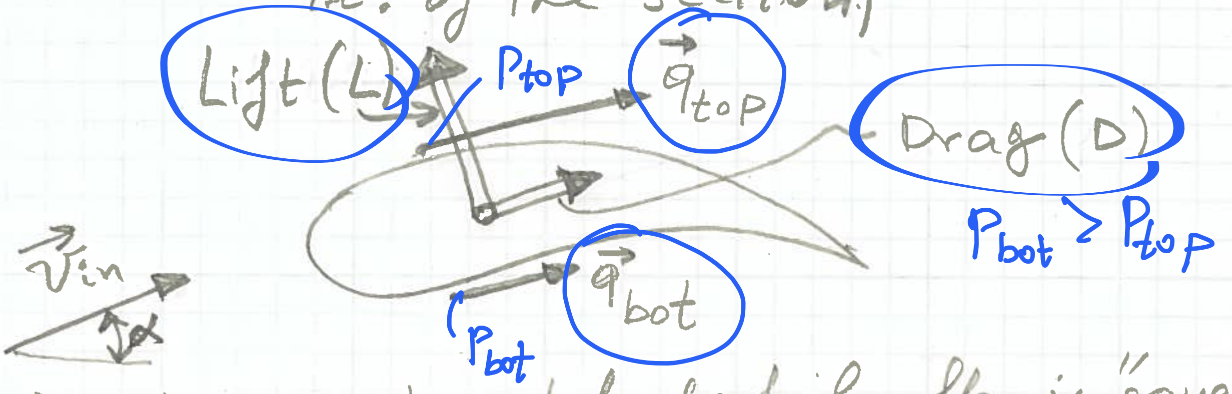
BASICS OF HYDROFOIL & TURBINE DESIGN

How a hydrofoil works:



$\vec{V}_{in}$ = inflow velocity, FAR UPSTREAM OF HYDROFOIL, equal and opposite to airplane speed in the case of the airfoil section of the airplane wing.

α = Angle of Attack (AOA) of the inflow relative to the chord line (defined as the line connecting the L.E. with T.E. of the section)



Due to geometry of hydrofoil flow is "squeezed" at top, but "widened" at the bottom ⇒

$$\Rightarrow |\vec{q}_{top}| > |\vec{q}_{bot}| \Rightarrow P_{top} < P_{bot}$$ (due to Bernoulli equ.)

(pressures)

Pressure differential ($P_{bot} - P_{top}$) results into a Lift force, L, which can be shown that it is perpendicular to the direction of the inflow velocity $\vec{v}_{in}$.

However, due to the effects of viscosity there is also a Drag force (D) defined in the direction of $\vec{v}_{in}$

Usually Lift and Drag are defined via (unitless) coefficients:

Lift coeff. $C_L = \dfrac{L}{\frac{1}{2}\rho v_{in}^2 c}$

Drag coeff. $C_D = \dfrac{D}{\frac{1}{2}\rho v_{in}^2 c}$

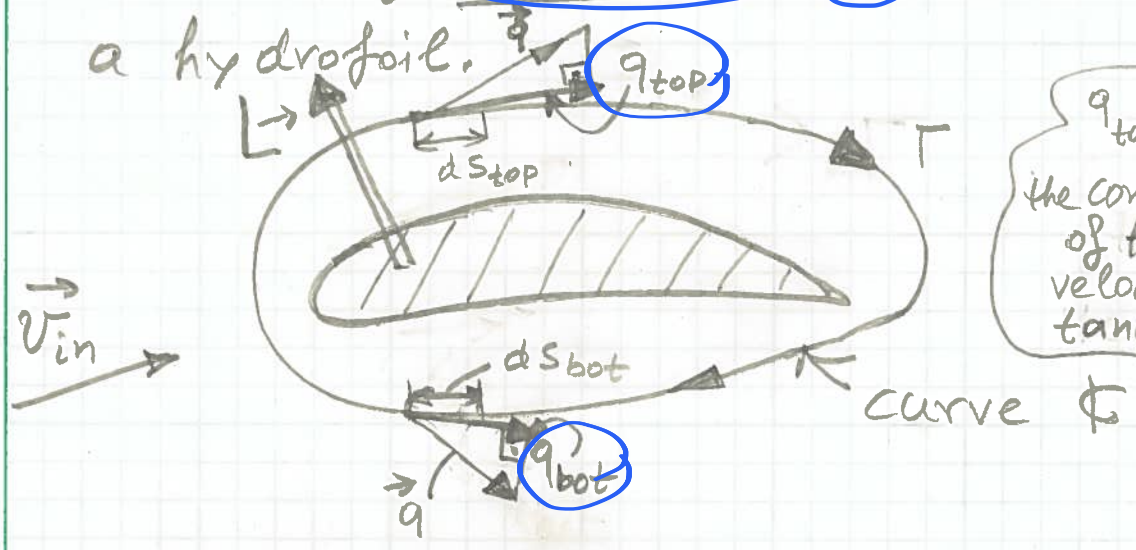
NOTE:
in 2-D
L & D
are forces
per unit
width or span.

c: chord length of the hydrofoil.

In the turbine design we specify the ratio $\boxed{K\,(\text{kappa}) = \dfrac{C_D}{C_L} = \dfrac{D}{L}}$

This ratio depends on the camber, thickness, and AOA, as well as the Reynolds number $\boxed{Re = \dfrac{v_{in} c}{\nu}}$

Definition of circulation, Γ, around a hydrofoil.



q_{top}, q_{bot} are the components of the flow velocity $\vec{q}$ tangent to ₵

For a closed curve ₵ around the hydrofoil

Γ represents a global circular flow around the hydrofoil

$$\Gamma \underset{\uparrow \text{DEF}}{=} \int_{₵} \left(\underline{q_{top}}\, \underline{ds_{top}} - \underline{q_{bot}}\, \underline{ds_{bot}} \right)$$

For inviscid (ideal) flow it can be shown that Γ is independent of curve ₵!

Also it can be shown that:

* Γ is a function of hydrofoil shape and AOA

$$\boxed{L = \rho v_{in} \Gamma}$$ (Joukowski Law)

where ρ is the density of the fluid.

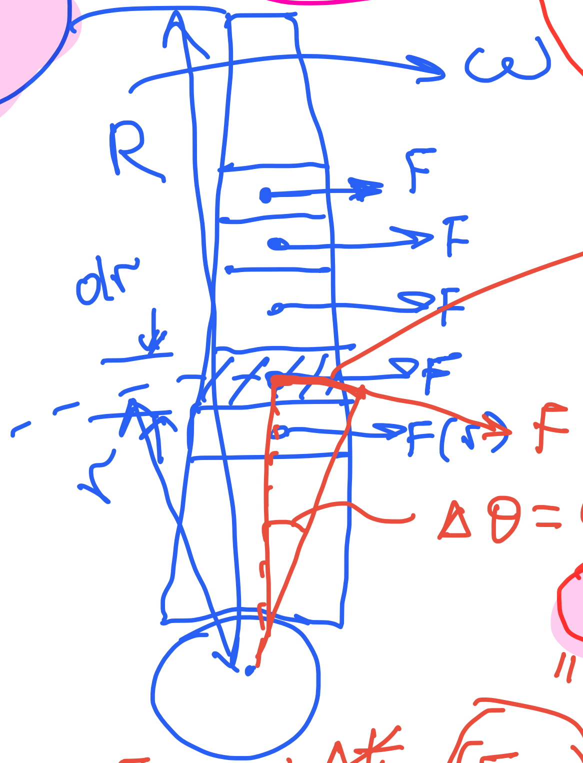
Physically, the above equation means that Γ provides the velocity differential between the top & bottom sides, which in turn produces the pressure differential, which gives rise to Lift, L!

Betz limit

η = efficiency of turbine

$$\eta = \frac{Q\omega}{\frac{1}{2}\rho\pi R^2 v^3} \leq 59.3\%$$

Power = $Q . \omega$ generated by the turbine



$\Delta S = r\,\Delta\theta$

$\Delta\theta = \omega\,\Delta t$

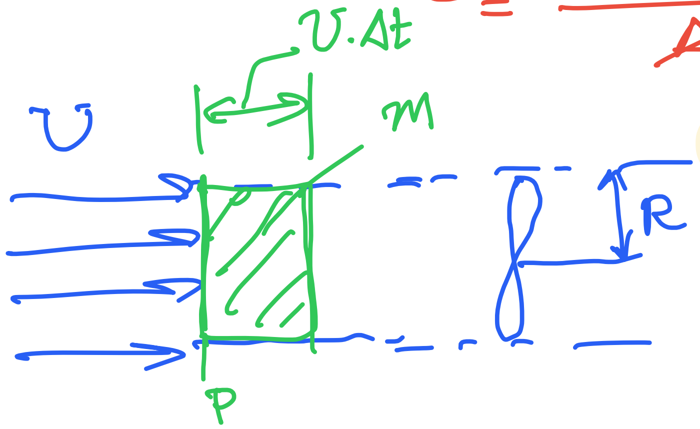
$$\text{Power} = \frac{\text{Work}}{\Delta t} = \frac{F . \Delta s}{\Delta t} = \frac{F . r\,\Delta\theta}{\Delta t} = \frac{F . r . \omega\,\cancel{\Delta t}}{\cancel{\Delta t}} = (F . r)\,\omega = Q . \omega$$

Q = Torque

$\parallel$

Tip Speed Ratio

$$TSR = \frac{\omega R}{v}$$



kinetic energy of wind

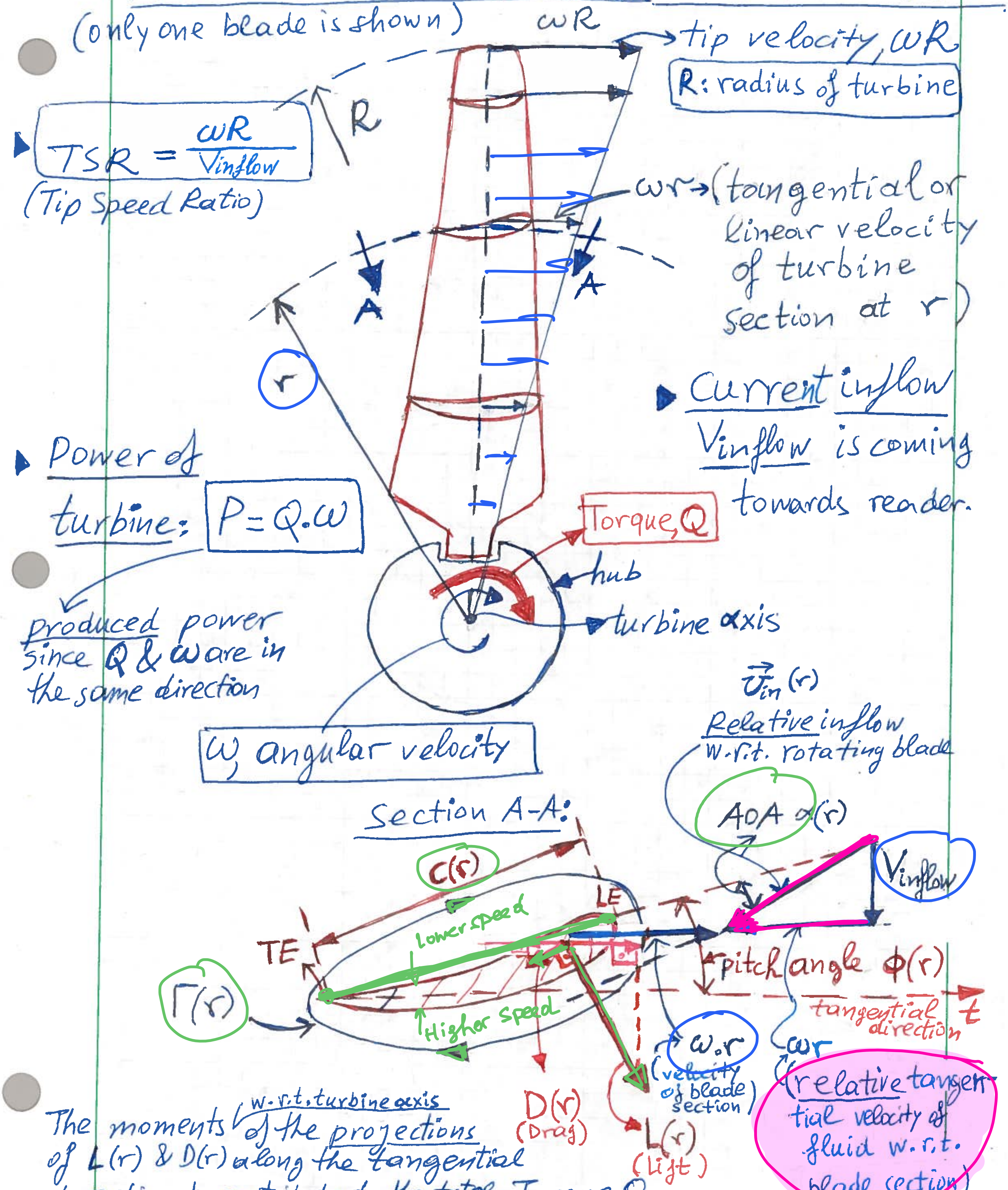
$$\frac{1}{2} m v^2 = \frac{1}{2}\left(\rho\pi R^2 v . \Delta t\right) v^2 \rightarrow KE_{wind}$$

$$m = \rho\pi R^2 v\,\Delta t$$

$$\text{Power}_{wind} = \frac{KE_{wind}}{\Delta t} = \frac{1}{2}\rho\pi R^2 v^3$$

How a turbine works (basic theory with simplificatio

(only one blade is shown)



- $TSR = \frac{\omega R}{V_{inflow}}$ (Tip Speed Ratio)

- Power of turbine: $P = Q \cdot \omega$

produced power since Q & ω are in the same direction

- Current inflow

Vinflow is coming towards reader.

The moments w.r.t. turbine axis of the projections of L(r) & D(r) along the tangential direction, t, contribute to the total Torque, Q, exerted by the fluid on the turbine.

LLOPT*, UT/OE's code for the design of turbine blades: for given V_{inflow}, ω, R, number of blades, and K, determines $\Gamma(r)$, from hub to tip, so that the produced Q (thus Power) is maximized. Then, based on this optimal distribution $\Gamma(r)$ it determines the geometry of each section, and the pitch angle, $\phi(r)$, at various radii, between hub & tip.

LL
Lifting
line

In general, the power of the turbine:

$$\boxed{P_{turb} = \underset{\substack{\uparrow \\ \text{maximize } Q}}{Q}\cdot\omega = \eta\,\frac{1}{2}\rho\, V_{inflow}^{3}\cdot\pi R^{2}}$$

where η is the turbine efficiency.

In the case of open turbines it can be shown that $\eta < 59.3\%$ (Betz limit)

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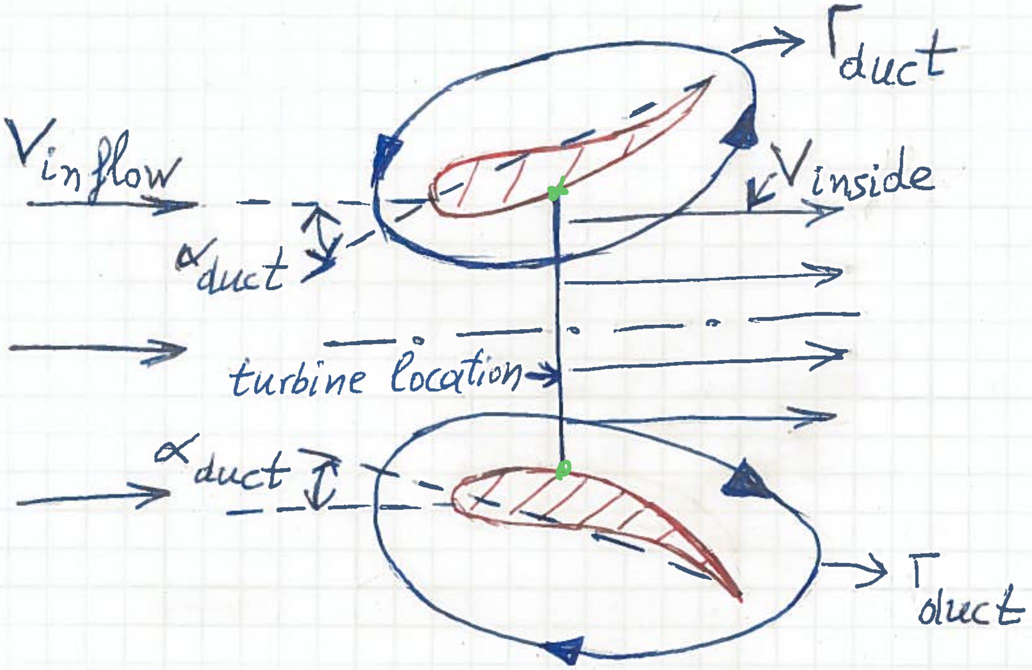
*LLOPT stands for Lifting Line OPTimization. Lifting line theory approximates the blade with a straight line between hub and tip, over which $\Gamma(r)$ is to be found!

Purpose of the duct:



With a duct at an AOA α_{duct}, as shown, we create a circulation Γ_{duct} which accelerates the flow inside the duct. Thus, by placing a turbine inside this duct, it is expected to increase the produced power by $\left(\frac{V_{inside}}{V_{inflow}}\right)^3$.

LLOPT is coupled with Fluent in order to evaluate fully the interaction between the turbine and the duct flow.