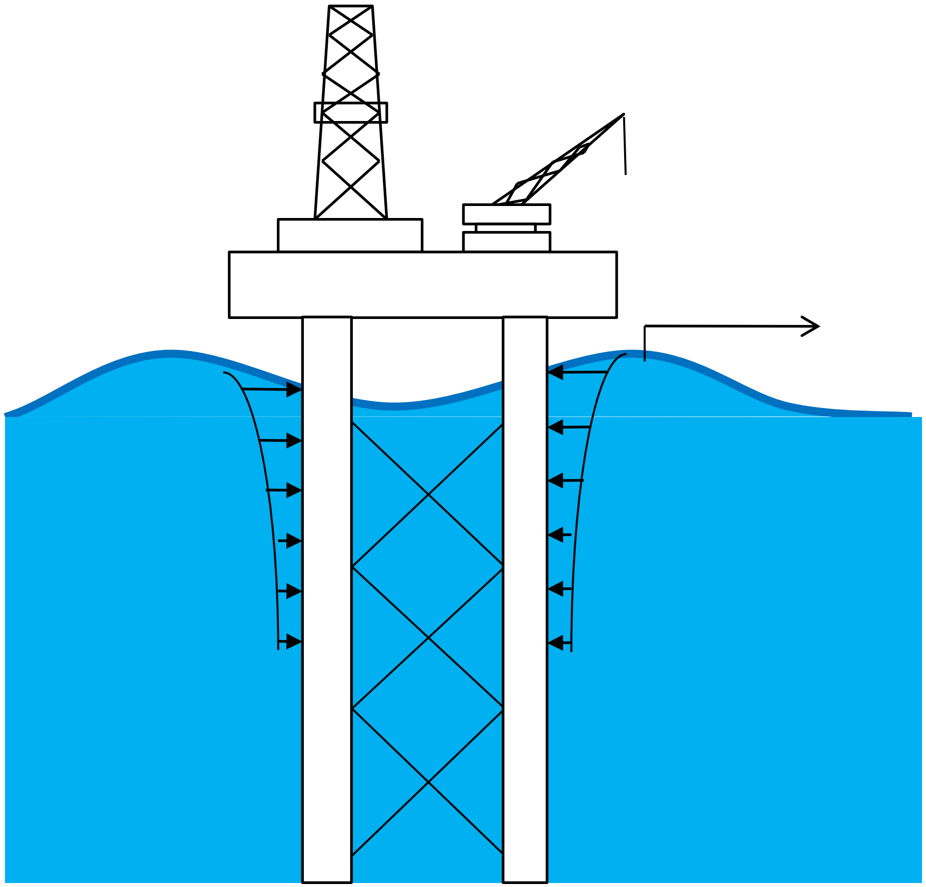
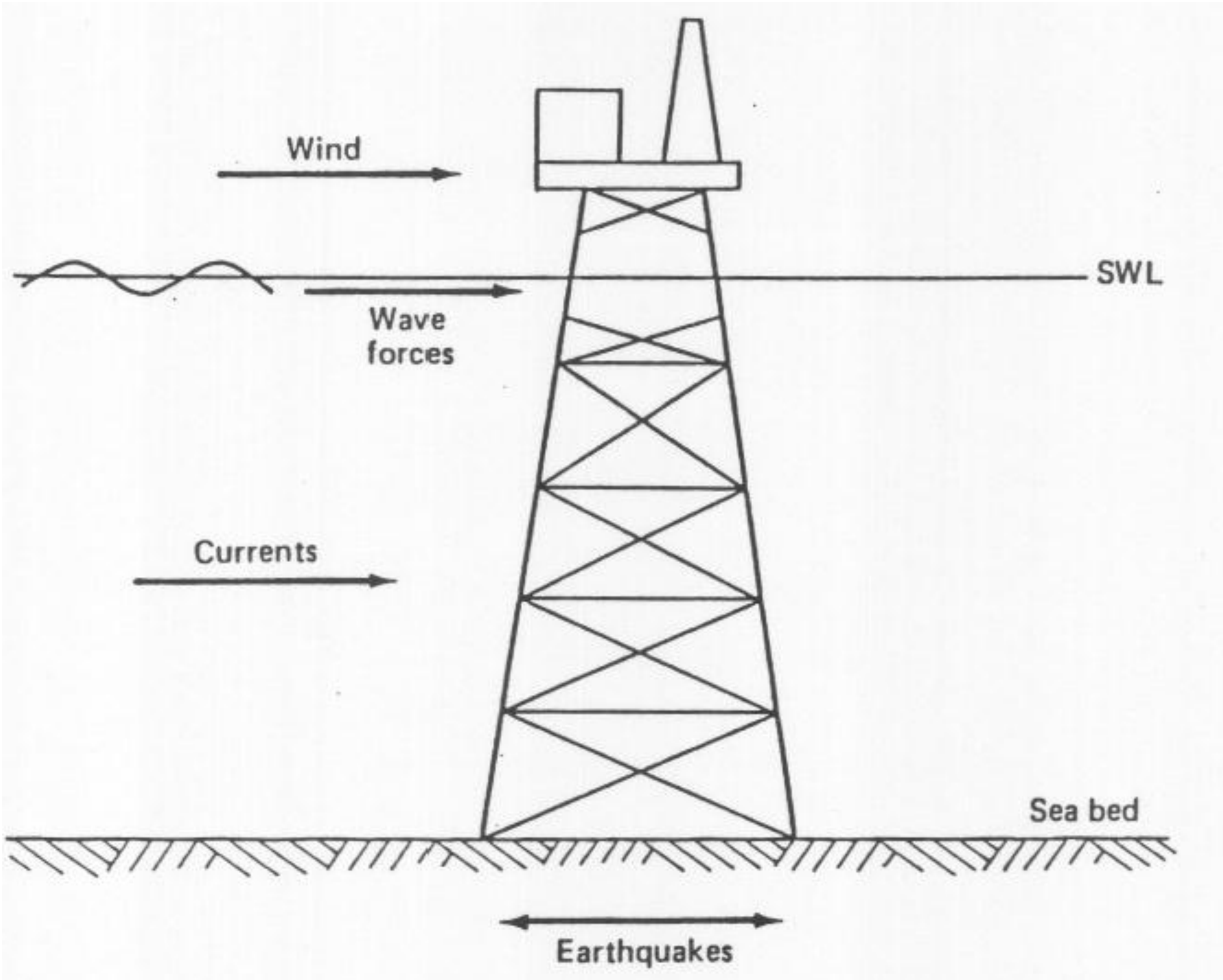
Wave, current, and wind forces



An offshore structure is designed to withstand the 100-year storm (wave/current/wind). A mono-chromatic wave of height H_{max} is assumed.


Wind
SWL
Wave
forces
Currents
Sea bed
Earthquakes

OFFSHORE PLATFORMS are comprised of many cylindrical or prismatic components (structural elements, floatation parts, risers, tendons, mooring lines, etc)

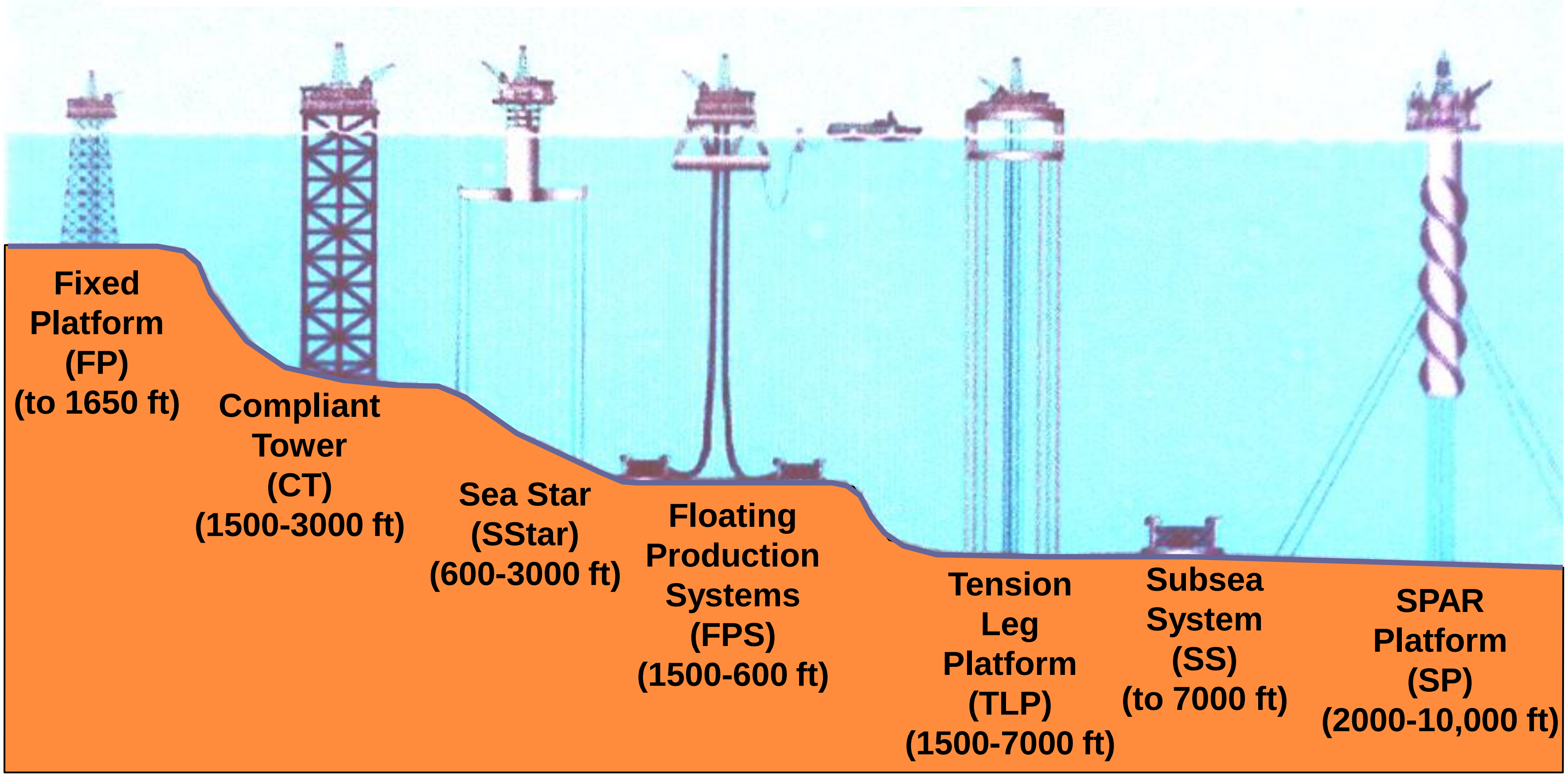
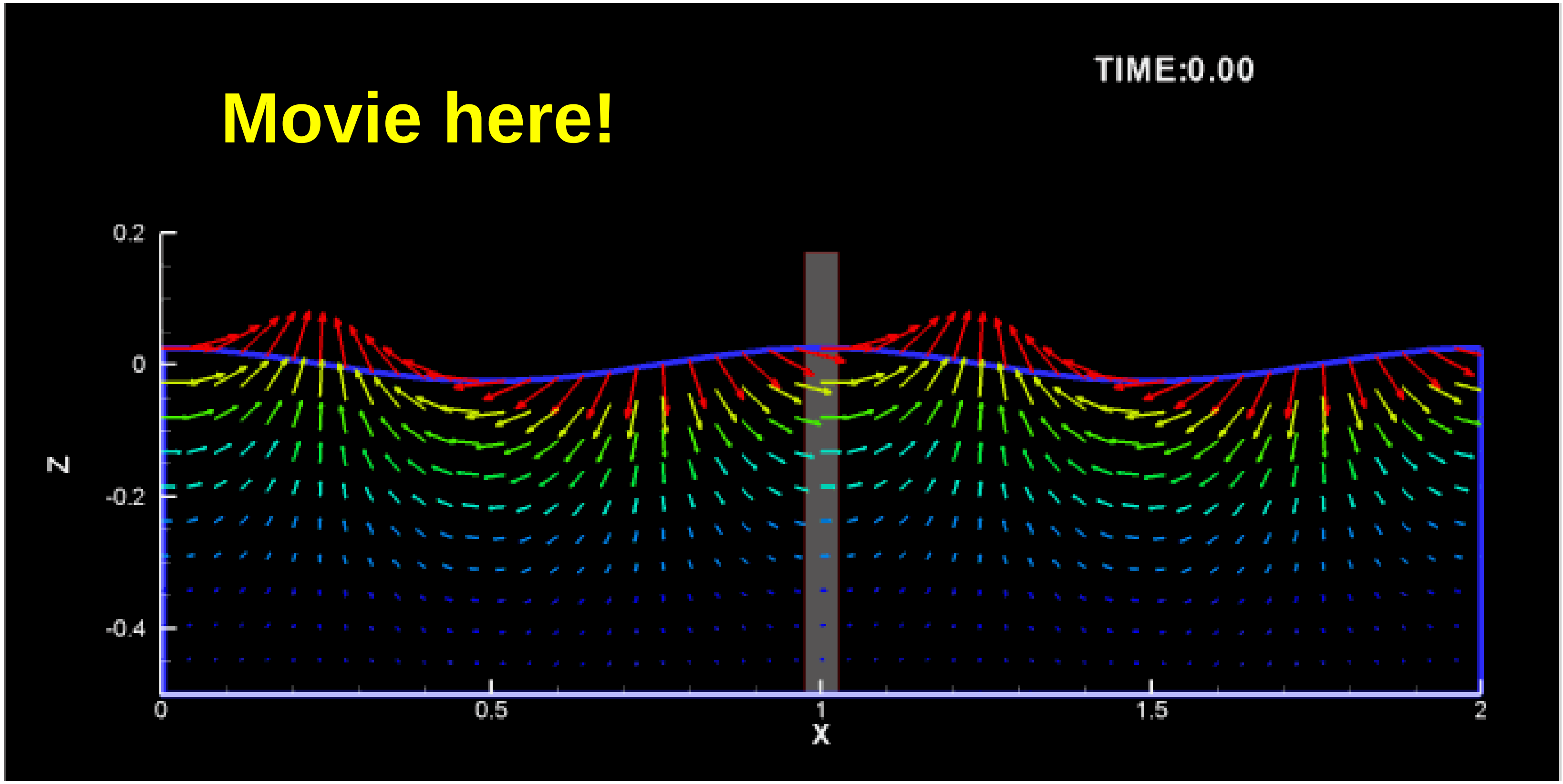
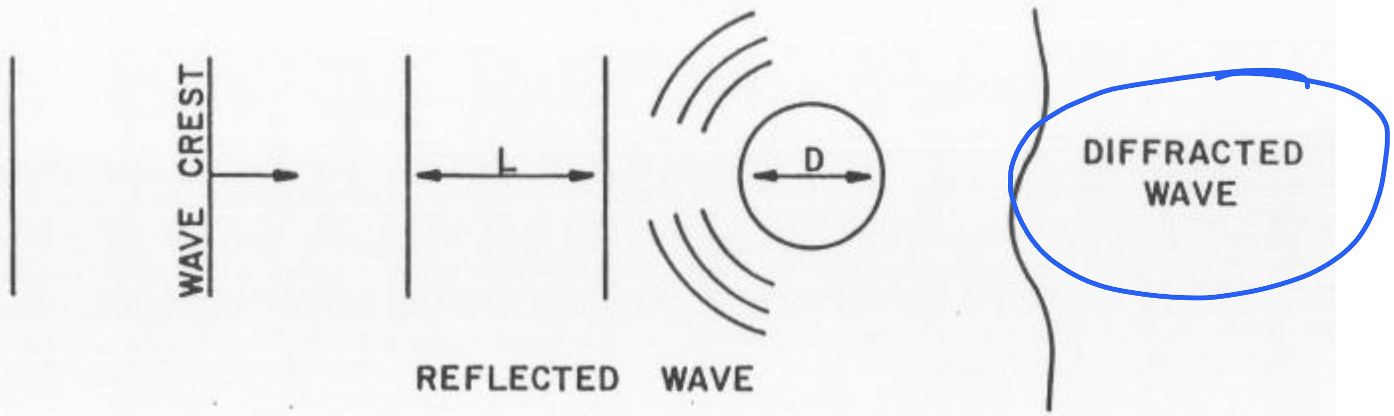


Figure from BOEMRE, U.S. Department of the Interior

What inflow velocity would a pile be subjected to?

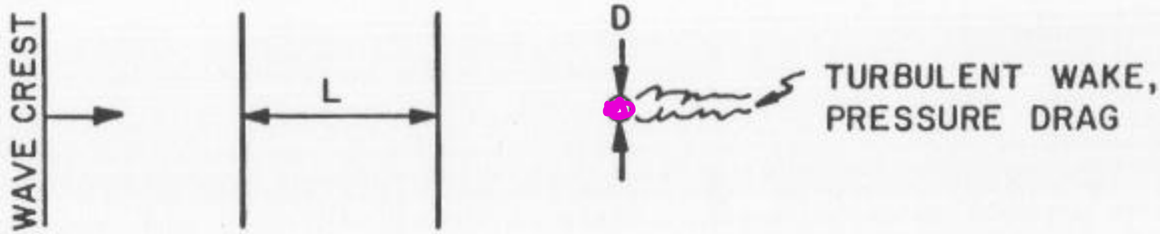


WAVE FORCE LIMITING CASE



WAVES MODIFIED BY OBJECT, $D > \frac{L}{5}$

WAVE FORCE LIMITING CASE



WAVE DOES NOT "FEEL" OBJECT, $D < \frac{L}{6}$

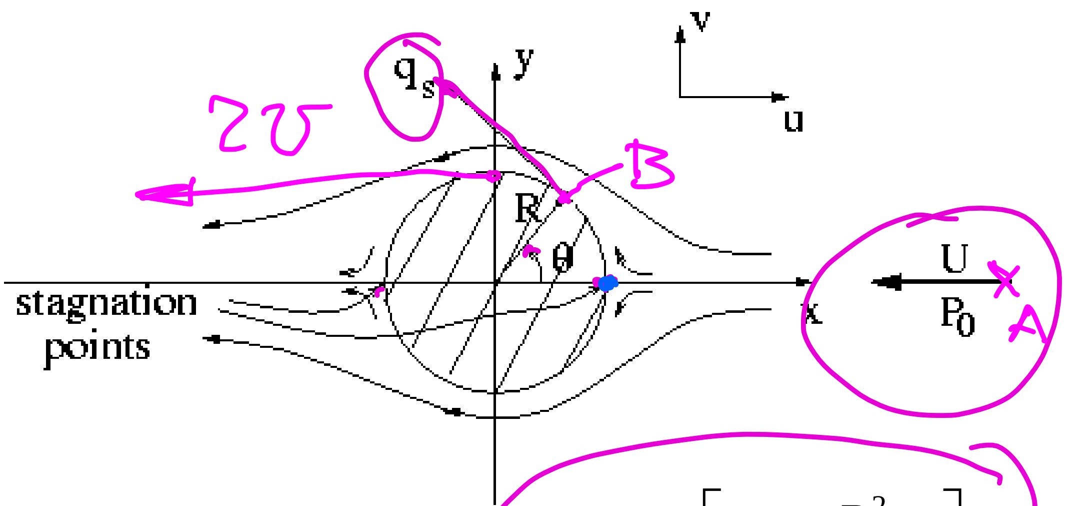
Steps to take:

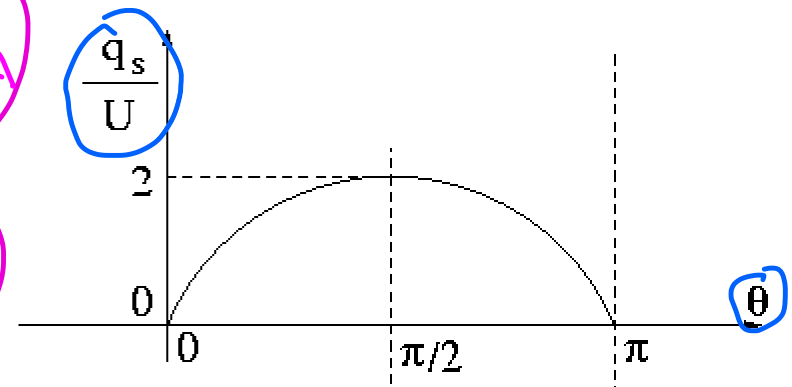
- Study steady flow around 2-D cylinder (circle) subject to **steady inflow**

- Study unsteady flow around 2-D cylinder (circle) subject to **accelerating inflow**

- Apply the study and the formulas developed in the previous steps, on "slices" of the **3-D cylinder, subject to wave and current,** and integrate along its length to determine total forces and moments

Inviscid Flow Around a Circle





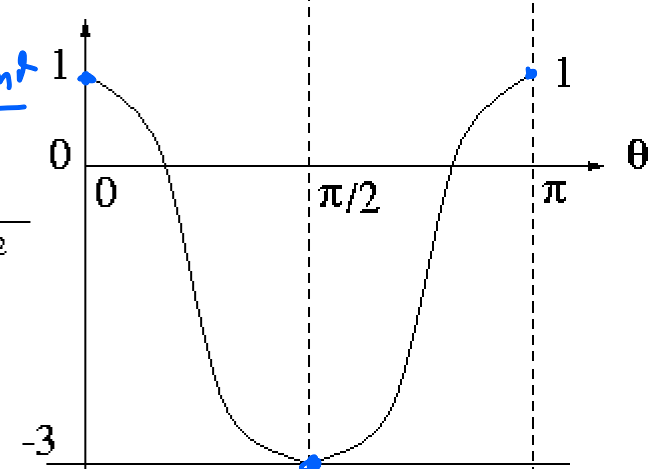
Velocity potential: $\Phi = -Ux\left[1 + \frac{R^2}{x^2 + y^2}\right]$

Surface velocity $q_s = \sqrt{u^2 + v^2} = 2U\sin\vartheta$

Surface pressure coefficient

$$C_P = \frac{P_S - P_0}{\frac{1}{2}\rho U^2} = 1 - \left(\frac{q_S}{U}\right)^2$$

$$C_p = \frac{P_s - P_0}{\frac{1}{2}\rho U^2}$$



D'Alembert "paradox": The force on a body subject to inviscid steady flow is equal to zero!

Bernoulli: $p_A + \frac{\rho}{2} v_A^2 = p_B + \frac{\rho}{2} v_B^2$

$p_0 + \frac{\rho}{2} v^2 = p + \frac{\rho}{2} q_s^2$

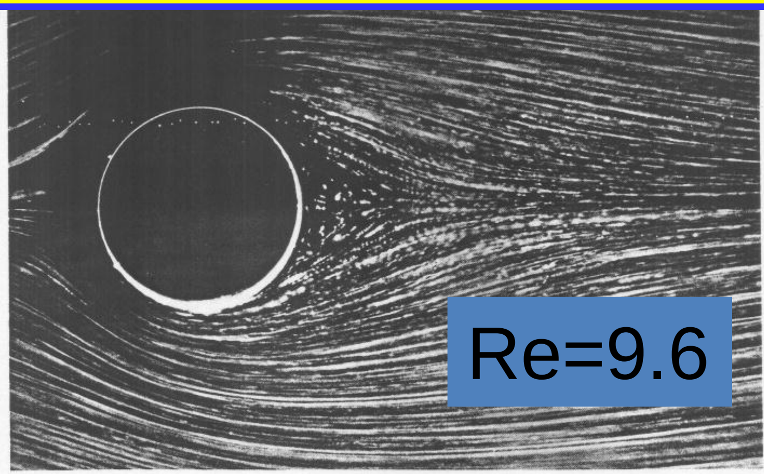
Pressure coefficient }

$$C_p = \frac{p - p_0}{\frac{\rho}{2} v^2} = \frac{\frac{\rho}{2} v^2 - \frac{\rho}{2} q_s^2}{\frac{\rho}{2} v^2}$$

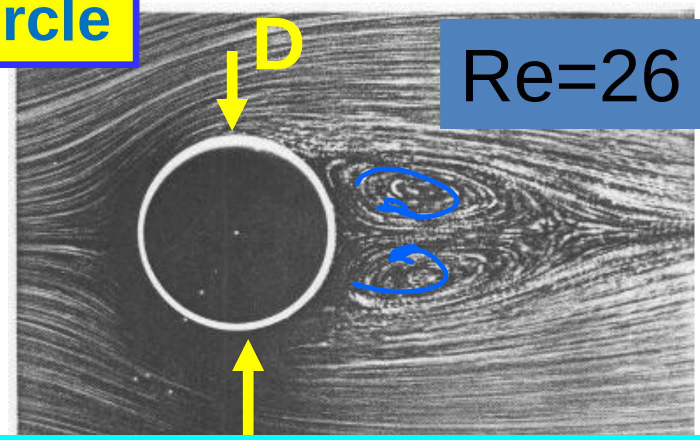
DEF

$$\rightarrow \quad C_p = 1 - \left(\frac{q_s}{v}\right)^2$$

Effect of viscosity on flow around circle

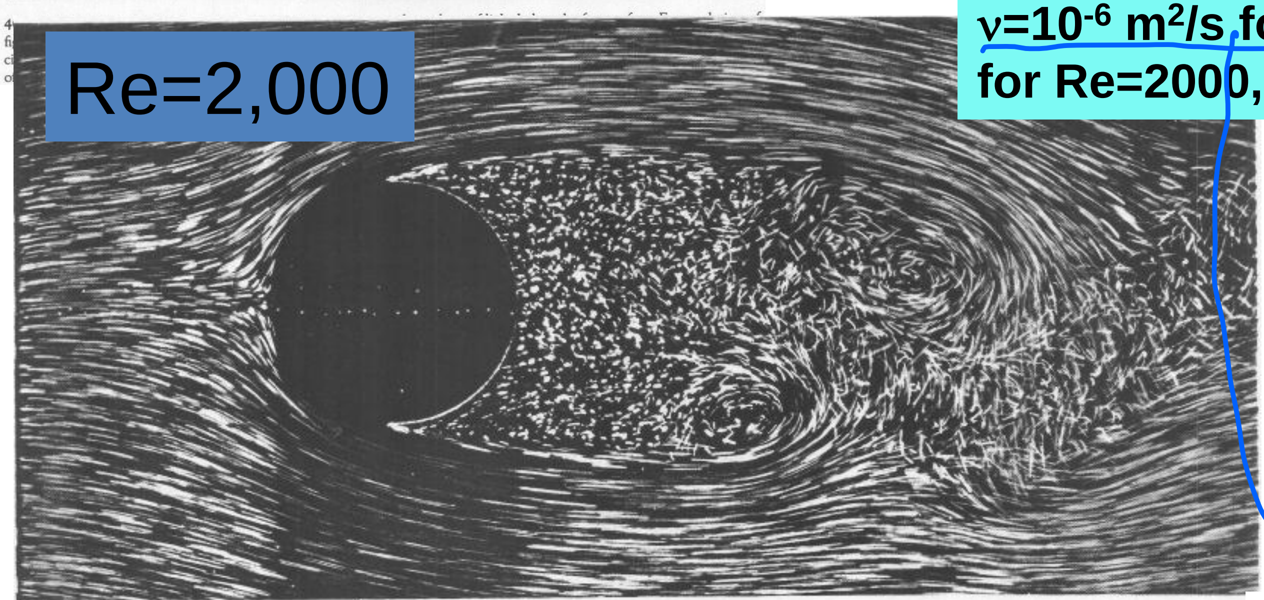


U



$$\nu = \mu / \rho = \text{kinematic viscosity}$$

$\nu = 10^{-6}$ m^2/s for H_2O at 20° C. If D=20cm for Re=2000, U should be 0.01 m/s



47. Circular cylinder at R=2000. At this Reynolds number one may properly speak of a boundary layer. It is laminar over the front, separates, and breaks up into a turbulent wake. The separation points, moving forward as the Reynolds number is increased, have now attained their upstream limit, ahead of maximum thickness. Visualization is by air bubbles in water. *ONERA photograph, Werlé & Gallon 1972*

Reynolds Number

$$Re = \frac{UD}{\nu}$$

Photos from Album of Fluid Motion of M. Vandyke

Physical properties of air and water circle

Table A.3 MECHANICAL PROPERTIES OF AIR AT STANDARD ATMOSPHERIC PRESSURE

Temperature	Density	Specific Weight	Dynamic Viscosity	Kinematic Viscosity
	kg/m³	N/m³	N · s/m²	m²/s
−20°C	1.40	13.70	1.61×10^{-5}	1.16×10^{-5}
−10°C	1.34	13.20	1.67×10^{-5}	1.24×10^{-5}
0°C	1.29	12.70	1.72×10^{-5}	1.33×10^{-5}
10°C	1.25	12.20	1.76×10^{-5}	1.41×10^{-5}
20°C	1.20	11.80	1.81×10^{-5}	1.51×10^{-5}
30°C	1.17	11.40	1.86×10^{-5}	1.60×10^{-5}
40°C	1.13	11.10	1.91×10^{-5}	1.69×10^{-5}
50°C	1.09	10.70	1.95×10^{-5}	1.79×10^{-5}
60°C	1.06	10.40	2.00×10^{-5}	1.89×10^{-5}
70°C	1.03	10.10	2.04×10^{-5}	1.99×10^{-5}
80°C	1.00	9.81	2.09×10^{-5}	2.09×10^{-5}
90°C	0.97	9.54	2.13×10^{-5}	2.19×10^{-5}
100°C	0.95	9.28	2.17×10^{-5}	2.29×10^{-5}
120°C	0.90	8.82	2.26×10^{-5}	2.51×10^{-5}
140°C	0.85	8.38	2.34×10^{-5}	2.74×10^{-5}
160°C	0.81	7.99	2.42×10^{-5}	2.97×10^{-5}
180°C	0.78	7.65	2.50×10^{-5}	3.20×10^{-5}
200°C	0.75	7.32	2.57×10^{-5}	3.44×10^{-5}
	slugs/ft³	lbf/ft³	lbf-s/ft²	ft²/s
0°F	0.00269	0.0866	3.39×10^{-7}	1.26×10^{-4}
20°F	0.00257	0.0828	3.51×10^{-7}	1.37×10^{-4}
40°F	0.00247	0.0794	3.63×10^{-7}	1.47×10^{-4}
60°F	0.00237	0.0764	3.74×10^{-7}	1.58×10^{-4}
80°F	0.00228	0.0735	3.85×10^{-7}	1.69×10^{-4}
100°F	0.00220	0.0709	3.96×10^{-7}	1.80×10^{-4}
120°F	0.00213	0.0685	4.07×10^{-7}	1.91×10^{-4}
150°F	0.00202	0.0651	4.23×10^{-7}	2.09×10^{-4}
200°F	0.00187	0.0601	4.48×10^{-7}	2.40×10^{-4}
300°F	0.00162	0.0522	4.96×10^{-7}	3.05×10^{-4}
400°F	0.00143	0.0462	5.40×10^{-7}	3.77×10^{-4}

SOURCE: Reprinted with permission from R. E. Bolz and G. L. Tuve, *Handbook of Tables for Applied Engineering Science*, CRC Press, Inc., Cleveland, 1973. Copyright © 1973 by The Chemical Rubber Co., CRC Press, Inc.

Table A.5 APPROXIMATE PHYSICAL PROPERTIES OF WATER* AT ATMOSPHERIC PRESSURE

Temperature	Density	Specific Weight	Dynamic Viscosity	Kinematic Viscosity	Vapor Pressure
	kg/m³	N/m³	N · s/m²	m²/s	N/m² abs
0°C	1000	9810	1.79×10^{-3}	1.79×10^{-6}	611
5°C	1000	9810	1.51×10^{-3}	1.51×10^{-6}	872
10°C	1000	9810	1.31×10^{-3}	1.31×10^{-6}	1,230
15°C	999	9800	1.14×10^{-3}	1.14×10^{-6}	1,700
20°C	998	9790	1.00×10^{-3}	1.00×10^{-6}	2,340
25°C	997	9781	8.91×10^{-4}	8.94×10^{-7}	3,170
30°C	996	9771	7.97×10^{-4}	8.00×10^{-7}	4,250
35°C	994	9751	7.20×10^{-4}	7.24×10^{-7}	5,630
40°C	992	9732	6.53×10^{-4}	6.58×10^{-7}	7,380
50°C	988	9693	5.47×10^{-4}	5.53×10^{-7}	12,300
60°C	983	9643	4.66×10^{-4}	4.74×10^{-7}	20,000
70°C	978	9594	4.04×10^{-4}	4.13×10^{-7}	31,200
80°C	972	9535	3.54×10^{-4}	3.64×10^{-7}	47,400
90°C	965	9467	3.15×10^{-4}	3.26×10^{-7}	70,100
100°C	958	9398	2.82×10^{-4}	2.94×10^{-7}	101,300
	slugs/ft³	lbf/ft³	lbf-s/ft²	ft²/s	psia
40°F	1.94	62.43	3.23×10^{-5}	1.66×10^{-5}	0.122
50°F	1.94	62.40	2.73×10^{-5}	1.41×10^{-5}	0.178
60°F	1.94	62.37	2.36×10^{-5}	1.22×10^{-5}	0.256
70°F	1.94	62.30	2.05×10^{-5}	1.06×10^{-5}	0.363
80°F	1.93	62.22	1.80×10^{-5}	0.930×10^{-5}	0.506
100°F	1.93	62.00	1.42×10^{-5}	0.739×10^{-5}	0.949
120°F	1.92	61.72	1.17×10^{-5}	0.609×10^{-5}	1.69
140°F	1.91	61.38	0.981×10^{-5}	0.514×10^{-5}	2.89
160°F	1.90	61.00	0.838×10^{-5}	0.442×10^{-5}	4.74
180°F	1.88	60.58	0.726×10^{-5}	0.385×10^{-5}	7.51
200°F	1.87	60.12	0.637×10^{-5}	0.341×10^{-5}	11.53
212°F	1.86	59.83	0.593×10^{-5}	0.319×10^{-5}	14.70

* Notes: (1) Bulk modulus E_v of water is approximately 2.2 GPa (3.2×10^5 psi); (2) water–air surface tension is approximately 7.3×10^{-2} N/m (5×10^{-3} lbf/ft) from 10°C to 50°C.

SOURCE: Reprinted with permission from R. E. Bolz and G. L. Tuve, *Handbook of Tables for Applied Engineering Science*, CRC Press, Inc., Cleveland, 1973. Copyright © 1973 by The Chemical Rubber Co., CRC Press, Inc.

From Engineering Fluid Mechanics of Crowe et al, 2009

Effect of Re on the pressure distribution on surface of circle

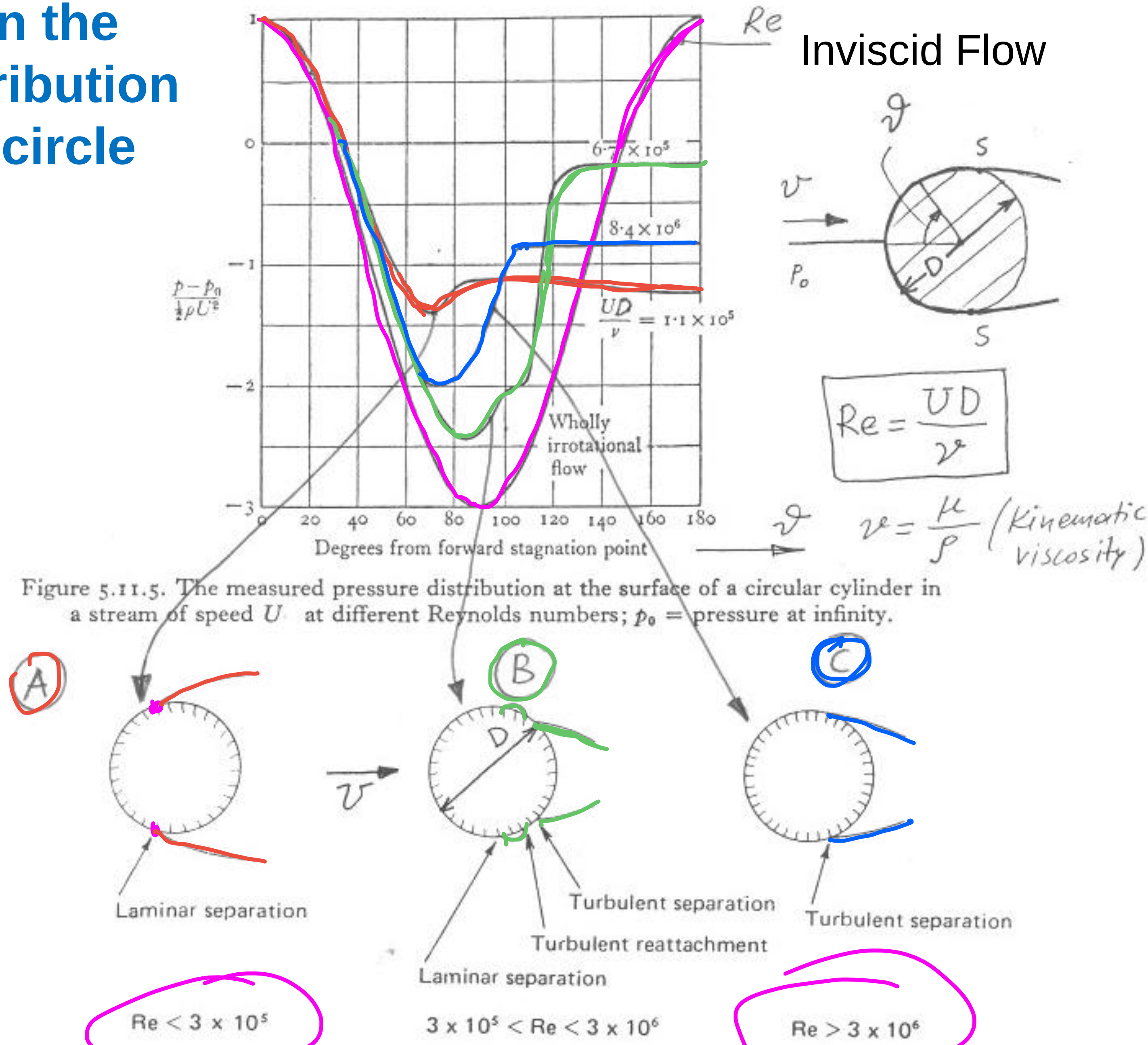
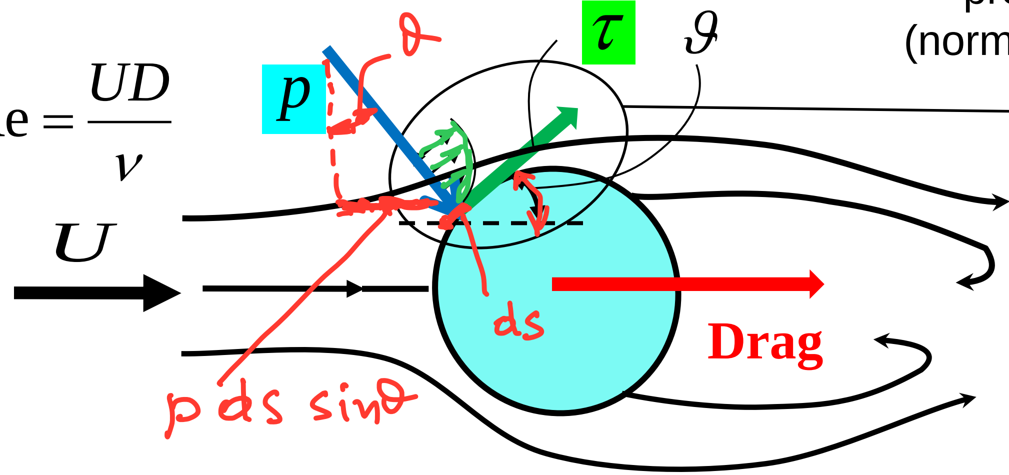
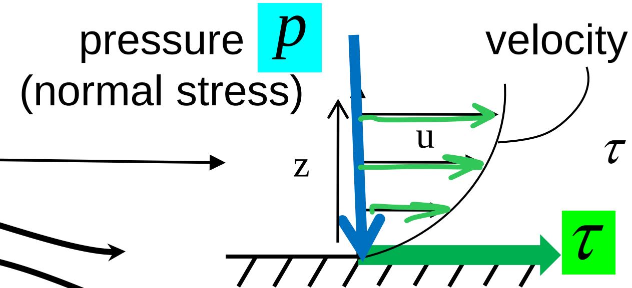


Figure 5.11.5. The measured pressure distribution at the surface of a circular cylinder in a stream of speed U at different Reynolds numbers; p_0 = pressure at infinity.

Drag and Drag coefficient

$$Re = \frac{UD}{\nu}$$





$$\tau = \mu \cdot \frac{du}{dz}$$

μ=dynamic viscosity

τ= friction (shear stress) acting on the body

Total Drag = Friction Drag + Pressure Drag

$$\text{Friction Drag} = \int \tau \, ds \cos\vartheta \neq 0$$

$$\text{Pressure (or form) Drag} = \int p \, ds \sin\vartheta \neq 0$$

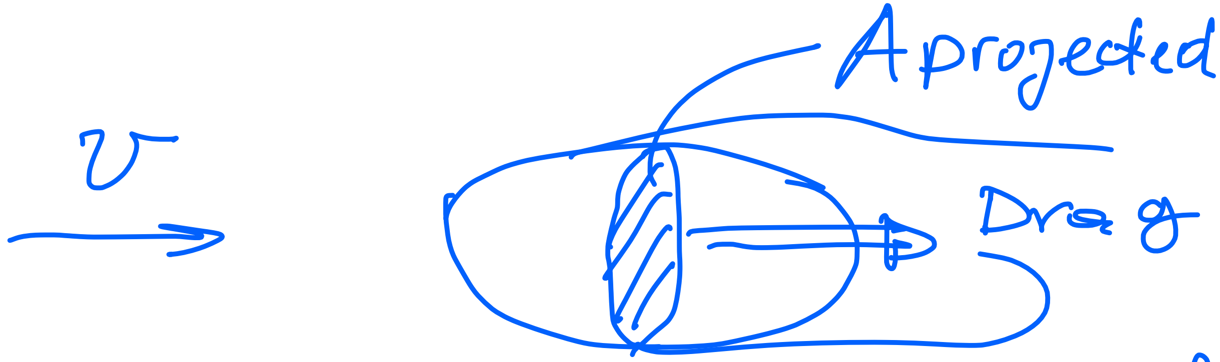
Drag Coefficient (in 2-D):

$$C_D = \frac{\text{Drag force per unit width}}{\frac{1}{2}\rho U^2 D}$$

(in 3-D):

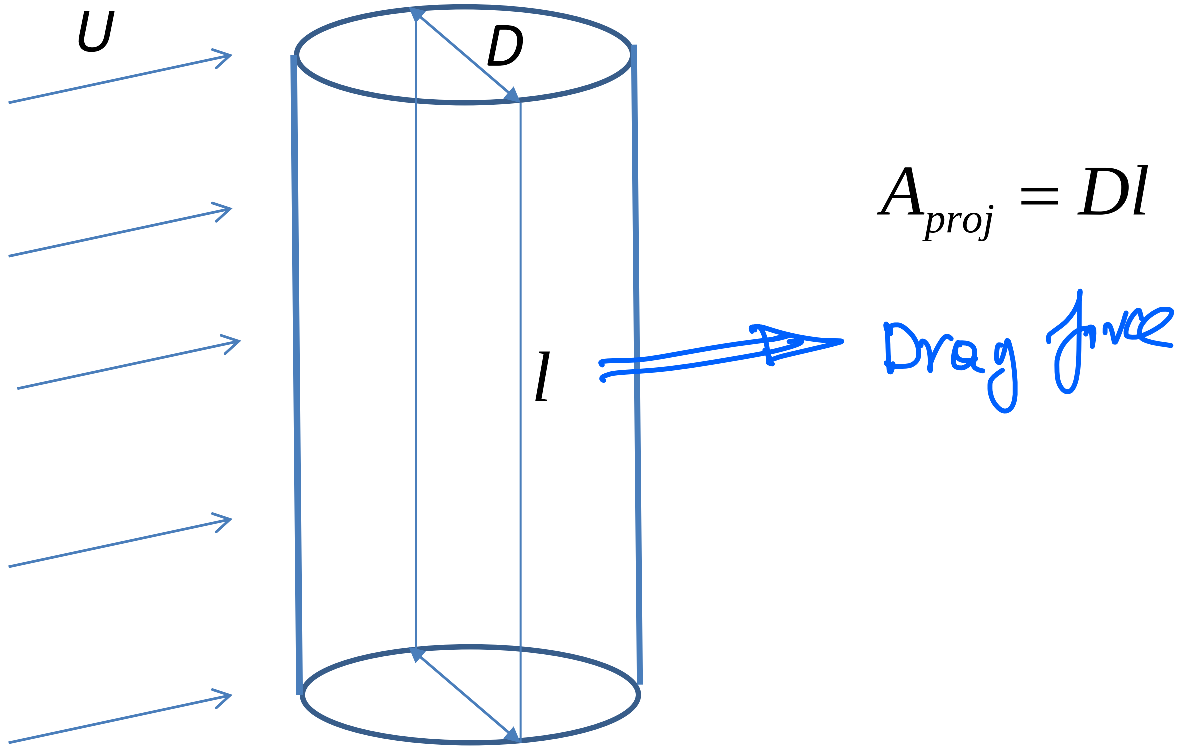
$$C_D = \frac{\text{Drag force}}{\frac{1}{2}\rho U^2 A_{proj}}$$

A_{proj} is the **projected area** of the body on a plane normal to the direction of inflow



Drag coefficient: $C_D = \dfrac{\text{Drag force}}{\frac{\rho}{2} v^2 A_{proj.}}$

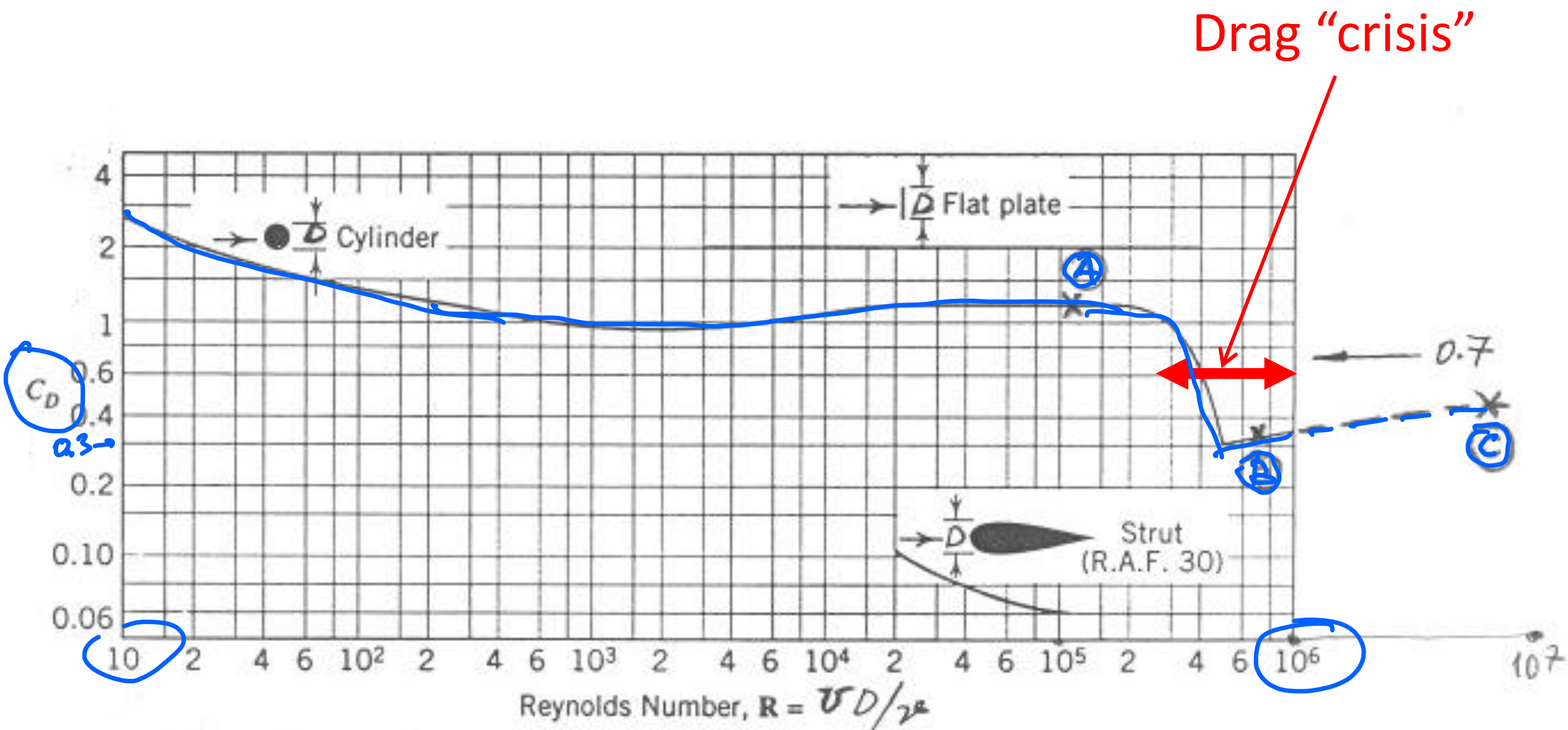
Drag force on a cylinder subject to uniform current *U*



$$A_{proj} = Dl$$

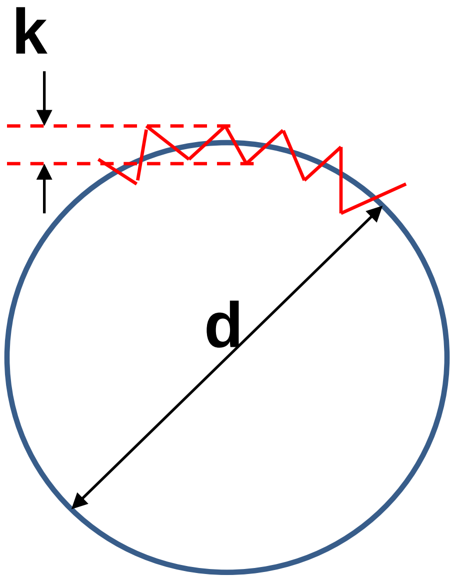
$$C_D = \frac{\text{Drag force}}{\frac{1}{2}\rho U^2 A_{proj}} = \frac{\text{Drag force}}{\frac{1}{2}\rho U^2 Dl} = \frac{\text{Drag force per unit width}}{\frac{1}{2}\rho U^2 D}$$

Effect of Re on Drag coefficient on Cylinder

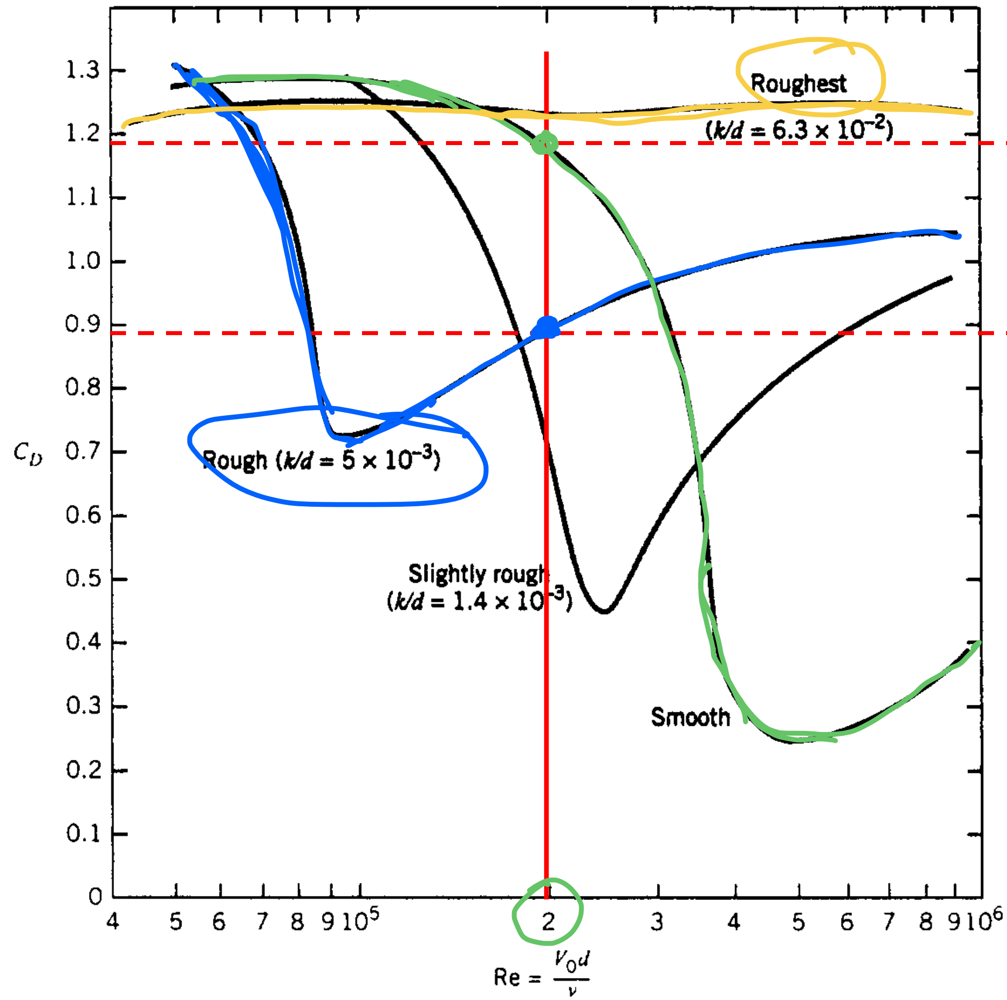


$$C_D = \frac{\text{Drag force per unit width}}{\frac{1}{2}\rho U^2 D}$$

Effect of roughness on Drag coefficient on Cylinder



k/d= relative roughness



$C_{D,smooth}$

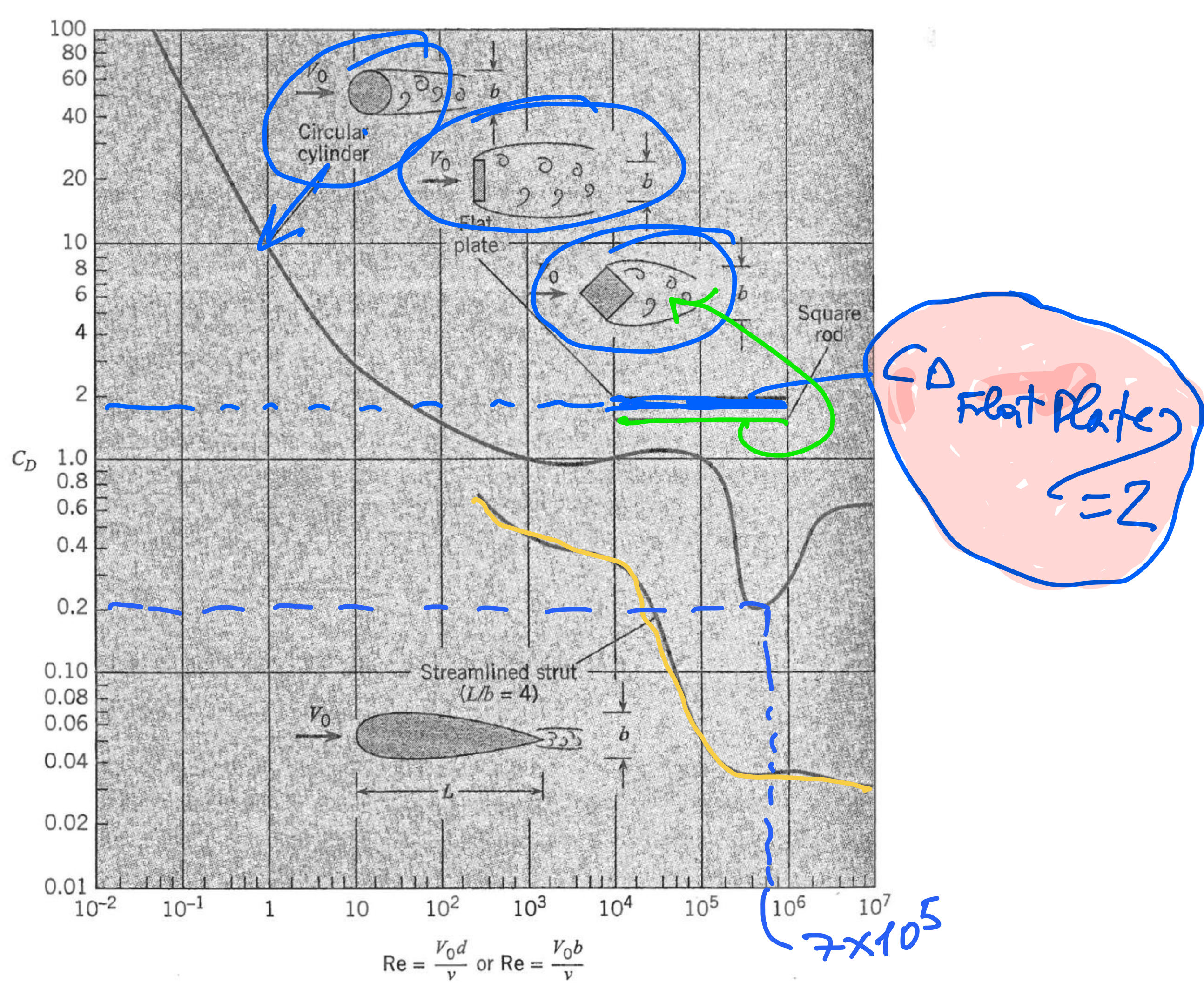
$C_{D,rough}$

From Engineering Fluid Mechanics of Crowe et al, 2009

Drag coefficients for some other 2-D shapes

FIGURE 11.5

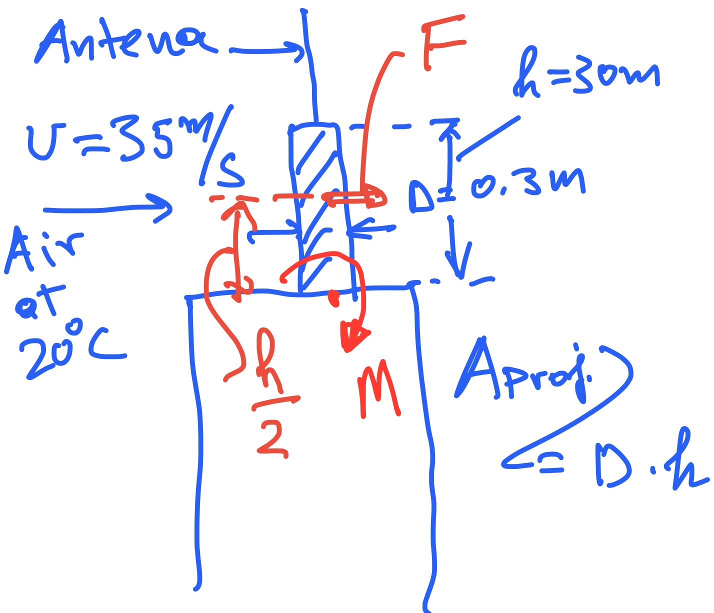
Coefficient of drag versus Reynolds number for two-dimensional bodies. [Data sources: Bullivant (5), Defoe (9), Goett and Bullivant (12), Jacobs (15), Jones (17), and Lindsey (21)]



From Engineering Fluid Mechanics of Crowe et al, 2009

Ex. 11.1 (p.486) from Crowe's Book

Find Force F and Moment M at the base of pile.



$$C_D = \frac{F}{\frac{\rho}{2} V^2 A_{proj.}} \rightsquigarrow$$

$$\rightsquigarrow F = C_D \frac{\rho}{2} V^2 A_{proj}$$

$$Re = \frac{V.D}{\nu} = \frac{35 \times 0.3}{1.51\times 10^{-5}} = 7\times 10^5$$

$$\Longrightarrow C_D = 0.2$$ (see graph above)

For air at 20°C

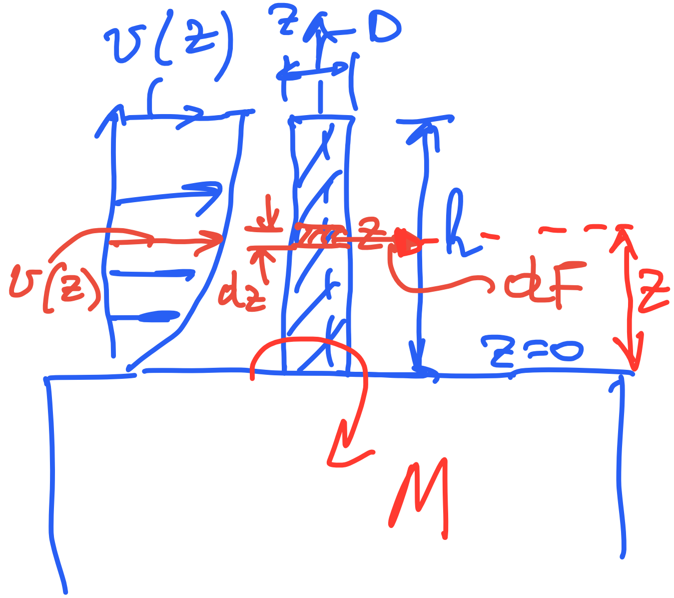
$$\rho = 1.2\ kg/m^3$$

$$\nu = 1.51\times 10^{-5}\ \frac{m^2}{s}$$

$$F = 0.2 \times \frac{1.2}{2} \times (35)^2 (0.3)(30) = 1,323\ N$$

$$M = F \times \frac{h}{2} = 19,845\ N\text{-}m$$

▶ How to evaluate F & M if inflow is not uniform (more realistic case, due to boundary layer profile)? See below ↓



$$dF = C_D \frac{\rho}{2} U^2(z)\, D\, dz$$

We assume C_D is uniform over pile

$$F = \int_0^h dF = C_D \frac{\rho}{2} D \int_0^h U^2(z)\, dz$$

$$dM = z\, dF$$

$$M = \int_0^h dM = \int_0^h z\, dF = C_D \frac{\rho}{2} D \int_0^h z\, U^2(z)\, dz$$