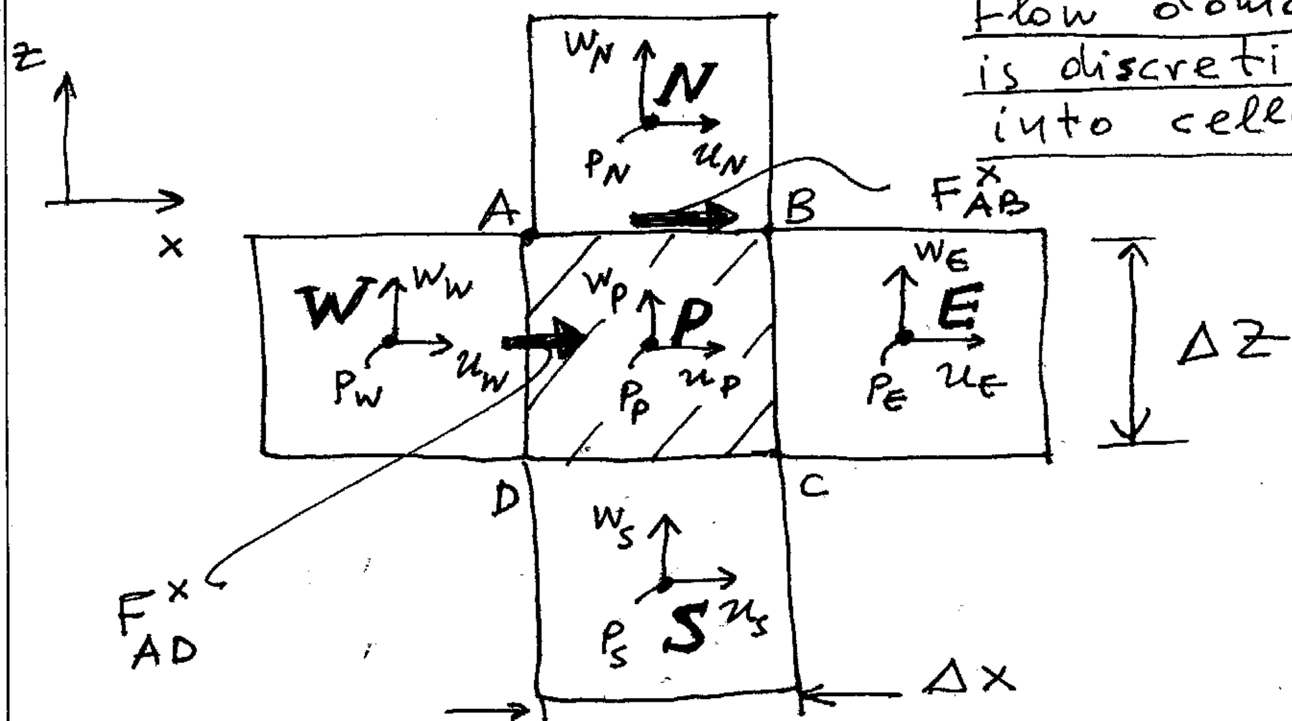
Some elements of CFD (in 2-D)

Finite Volume Method (FVM)

Flow domain is discretized into cells



- Cell P and its neighbors (N, S, E, W)
- At the centroid of each cell we wish to determine the two components of velocity (u, w) and the pressure, p
- For each cell we must apply discretized forms of the continuity and the Navier-Stokes equations, by only involving the unknown (u, w, p) at each cell and its neighbors

For example:

Continuity equ. applied at cell P (ABCD):

In-Flux through face AD: $\dfrac{u_W + u_P}{2} \cdot \Delta z$

" " " DC: $\dfrac{W_P + W_S}{2} \cdot \Delta x$

Out-Flux through face AB: $\dfrac{W_N + W_P}{2} \cdot \Delta x$

" " " BC: $\dfrac{u_E + u_P}{2} \cdot \Delta z$

Continuity (for incompressible fluid):

Total In-Flux = Total Out-Flux

$$\frac{u_W + \cancel{u_P}}{2}\,\Delta z + \frac{\cancel{W_P} + W_S}{2}\,\Delta x = \frac{W_N + \cancel{W_P}}{2}\,\Delta x + \frac{u_E + \cancel{u_P}}{2}\,\Delta z$$

$$\Rightarrow \boxed{\frac{u_W - u_E}{2}\,\Delta z = \frac{W_N - W_S}{2}\,\Delta x} \quad (1)$$

discretized version of continuity equ. applied at cell P

Instead of discretizing the N-S equations we apply Newton's Law at each cell (in both directions). There are FVM details that we will skip here!

Newton's Law along direction x:
(see handout on N-S equations)

$$\sum F_x = (\rho \Delta x \Delta z) a_x \quad (2)$$

a_x: acceleration of fluid along x =

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z}$$

F_x must be determined on all sides (faces) of cell P (ABCD):

For example, pressure force at face AD:

$$F^x_{AD} = \frac{P_W + P_P}{2} \cdot (AD) = \frac{P_W + P_P}{2} \Delta z$$

or shear (friction) force at face AB:

$$F^x_{AB} = \tau_{AB} (AB)$$

$$\tau_{AB} = \mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \rightarrow$$ must be determined at face AB

$$\left.\begin{matrix} \text{1st term} \\ \text{of } \tau_{AB} \end{matrix}\right\} = \mu \frac{\partial u}{\partial z} \approx \mu \frac{u_N - u_P}{(NP)} = \mu \frac{u_N - u_P}{\Delta z}$$ (if Δz uniform along z)

Can you think of how to approximate the second term of τ_{AB} : $\mu \frac{\partial w}{\partial x}$?

<u>HINT:</u> You must involve velocities at cells NE (North-East) and NW (North-West) with respect to cell P!

Doing the same at all faces of cell P (ABCD) we get the discretized version of the LHS of equ. 2.

The RHS (of equ. 2) requires some special treatment, which we will skip here...

→ After we have done the above, we have <u>3 discretized equations and 3 unknowns at each cell</u>

In addition we have <u>boundary conditions</u>