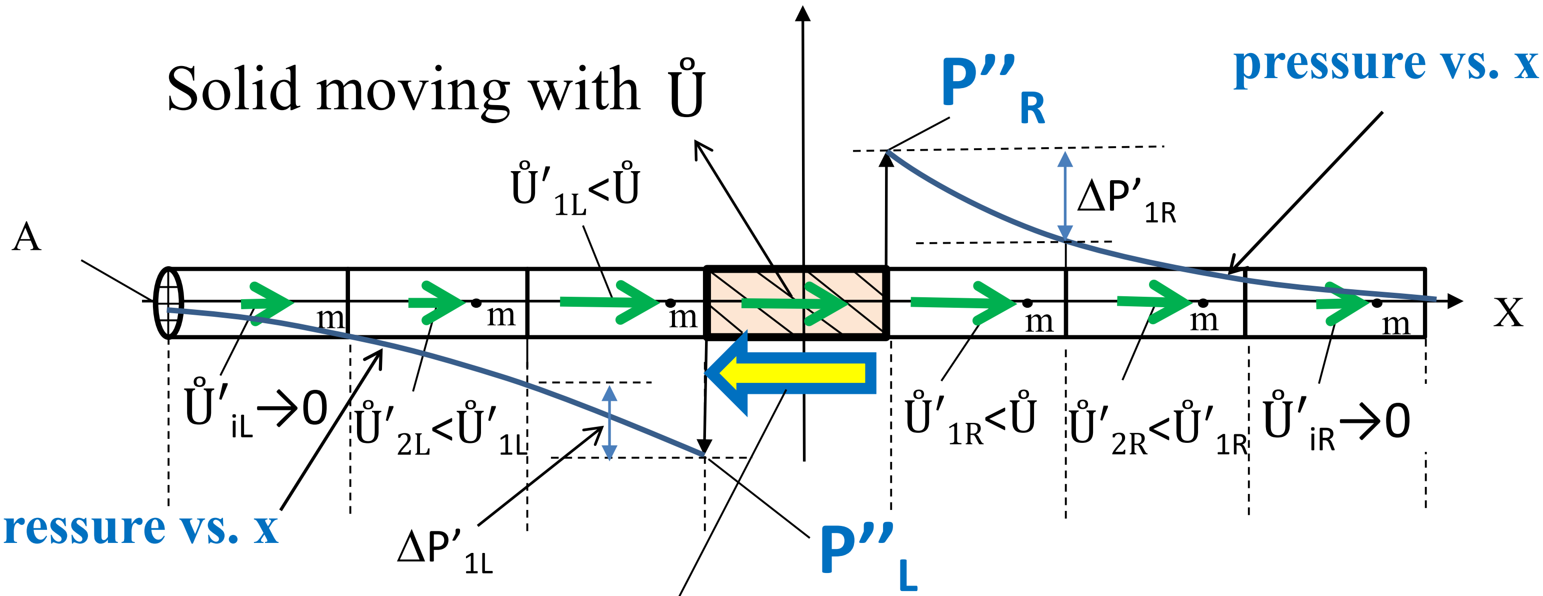
The concept of Added Mass (1/7)

Solid moving with acceleration Ů inside quiescent fluid



Resistive Force F_{res} : $F_{res} = (P''_R - P''_L)\,A = m_{added}\,\dot{U}$

$$F_{res} = (P''_R - P''_L)\,A = \sum \Delta P'_{iR}\,A + \sum \Delta P'_{iL}\,A = \sum m\,\dot{U}'_{iR} + \sum m\,\dot{U}'_{iL}$$

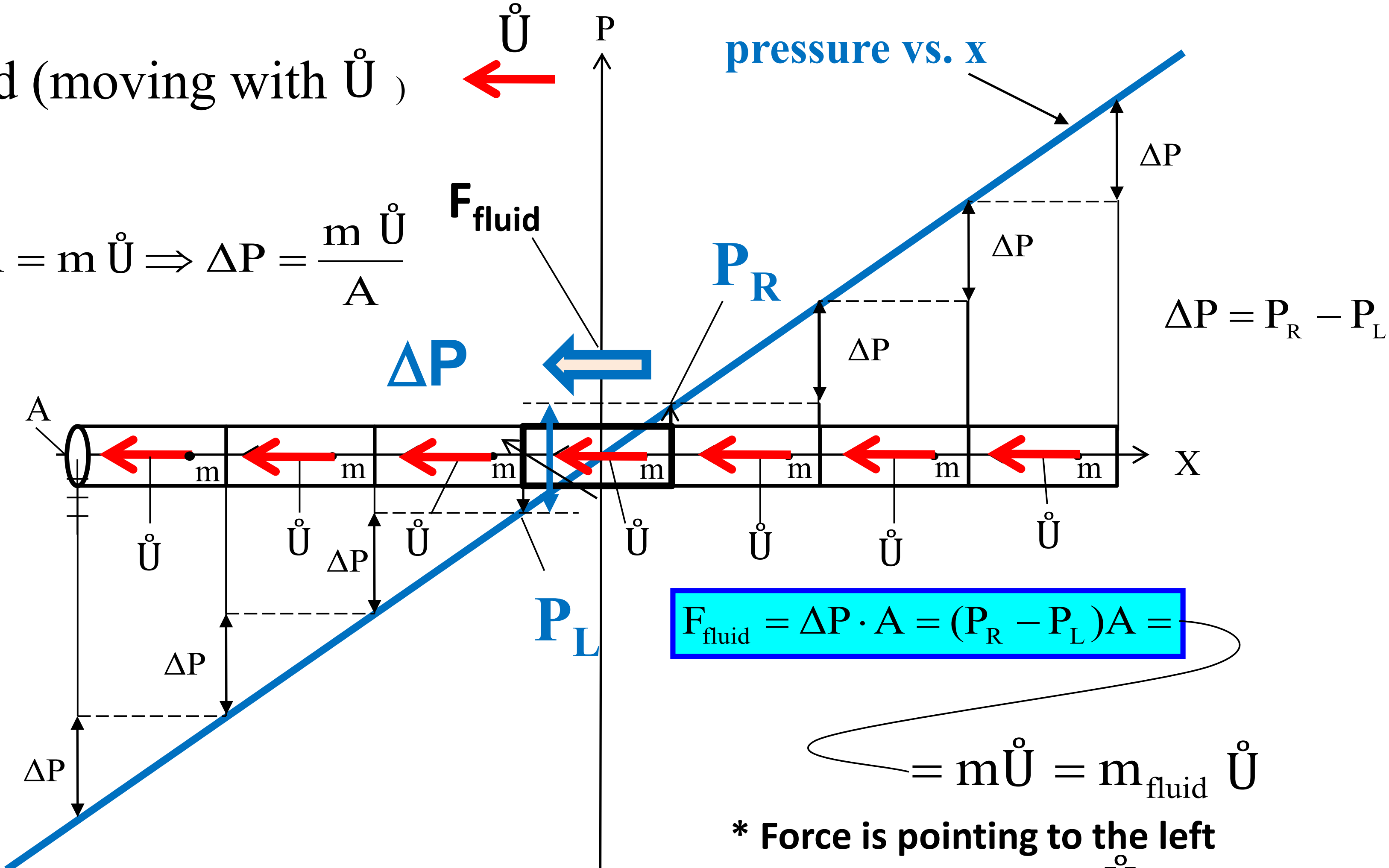
The resistive force ($F_{res} = m_{added}\,\dot{U}$) is needed in order to accelerate parts of the surrounding fluid which move with the body.

The concept of Added Mass (2/7)

Fluid subject to acceleration *without a body* inside it

Fluid (moving with $\mathring{U}$)

$$\Delta P \cdot A = m\,\mathring{U} \Rightarrow \Delta P = \frac{m\,\mathring{U}}{A}$$

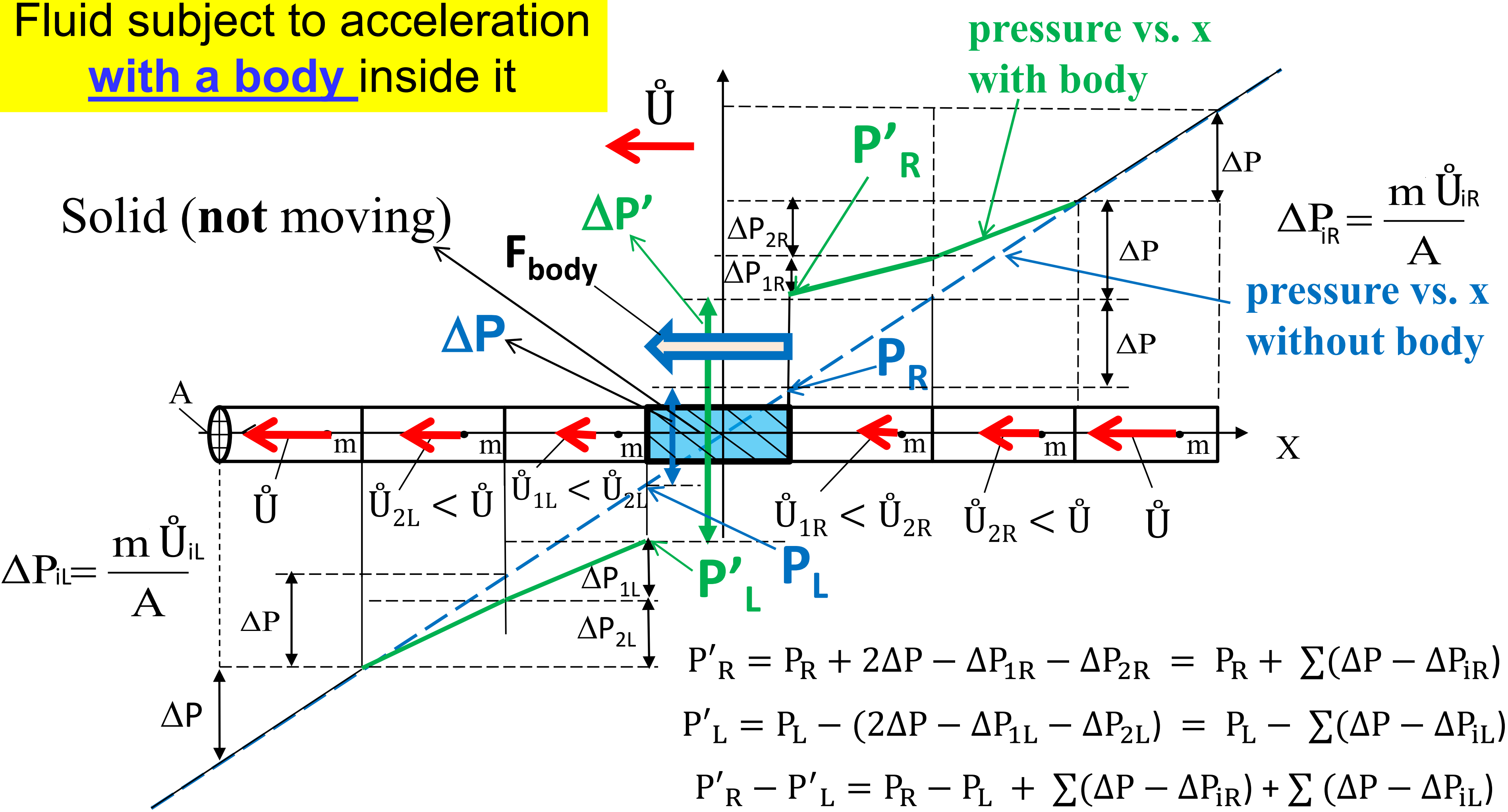


$$F_{fluid} = \Delta P \cdot A = (P_R - P_L)A = m\mathring{U} = m_{fluid}\,\mathring{U}$$

*** Force is pointing to the left (in the direction of $\mathring{U}$)**

The concept of Added Mass (3/7)

Fluid subject to acceleration **with a body** inside it



$$\Delta P_{iR} = \frac{m\,\mathring{U}_{iR}}{A}$$

$$\Delta P_{iL} = \frac{m\,\mathring{U}_{iL}}{A}$$

$$P'_R = P_R + 2\Delta P - \Delta P_{1R} - \Delta P_{2R} = P_R + \sum(\Delta P - \Delta P_{iR})$$

$$P'_L = P_L - (2\Delta P - \Delta P_{1L} - \Delta P_{2L}) = P_L - \sum(\Delta P - \Delta P_{iL})$$

$$P'_R - P'_L = P_R - P_L + \sum(\Delta P - \Delta P_{iR}) + \sum(\Delta P - \Delta P_{iL})$$

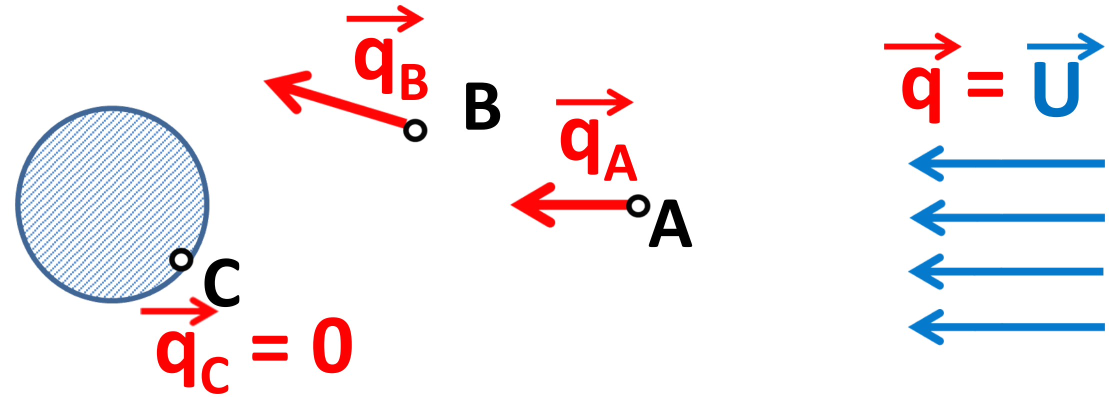
$$F_{body} = (P'_R - P'_L)\,A = (P_R - P_L)\,A + \sum m\,(\dot{U} - \dot{U}_{iR}) + \sum m\,(\dot{U} - \dot{U}_{iL})$$

$$\mathbf{F_{body} = m_{fluid}\,\dot{U} + \sum m\,(\dot{U} - \dot{U}_{iR}) + \sum m\,(\dot{U} - \dot{U}_{iL})}$$

The concept of Added Mass (4/7)

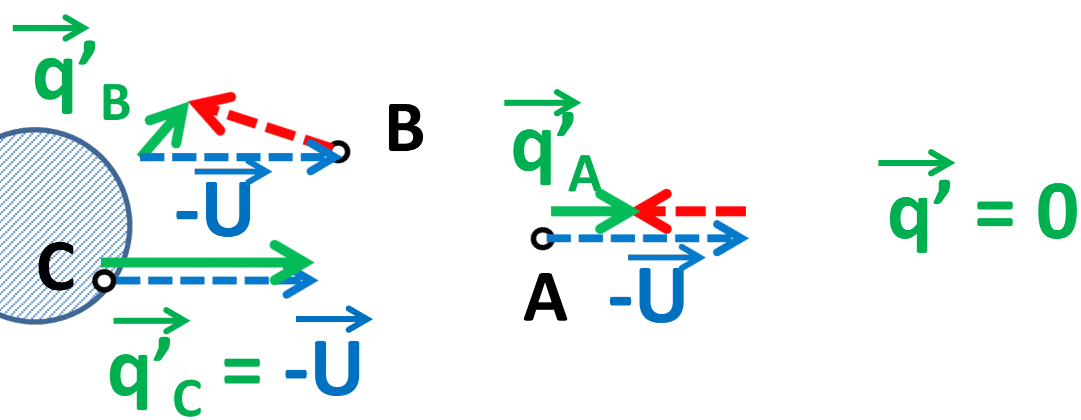
Two Different Frames of Reference (for the same problem!):

Body is stationary and flow is moving to the left with speed U



$$\vec{q}' = \vec{q} - \vec{U}$$

Flow is stationary (far upstream) and body is moving to the right with speed U



The concept of Added Mass (5/7)

$$F_{res} = (P''_R - P''_L)\, A = \sum m\, \dot{U}'_{iR} + \sum m\ \dot{U}'_{iL} = m_{added}\dot{U}$$

$$\dot{U}'_{iR} = -\dot{U}_{iR} - (-\dot{U}) = \dot{U} - \dot{U}_{iR}$$

$$\dot{U}'_{iL} = -\dot{U}_{iL} - (-\dot{U}) = \dot{U} - \dot{U}_{iL}$$

Thus:

$$\sum m\,(\dot{U} - \dot{U}_{iR}) + \sum m\ (\dot{U} - \dot{U}_{iL}) = m_{added}\dot{U}$$

$$F_{body} = m_{fluid}\ \dot{U} + \sum m\,(\dot{U} - \dot{U}_{iR}) + \sum m\ (\dot{U} - \dot{U}_{iL})$$

Thus:

$$\mathbf{F_{body} = m_{fluid}\ \dot{U} + m_{added}\dot{U} = = (m_{fluid} + m_{added})\ \dot{U}}$$

$$\mathbf{F_{body} = F_{inertial} = C_M\ m_{fluid}\ \dot{U}}$$

Inertia coefficient C_M

$$C_M = \frac{F_{body}}{F_{fluid}}$$

$$C_M = \frac{m_{fluid} + m_{added}}{m_{fluid}}$$

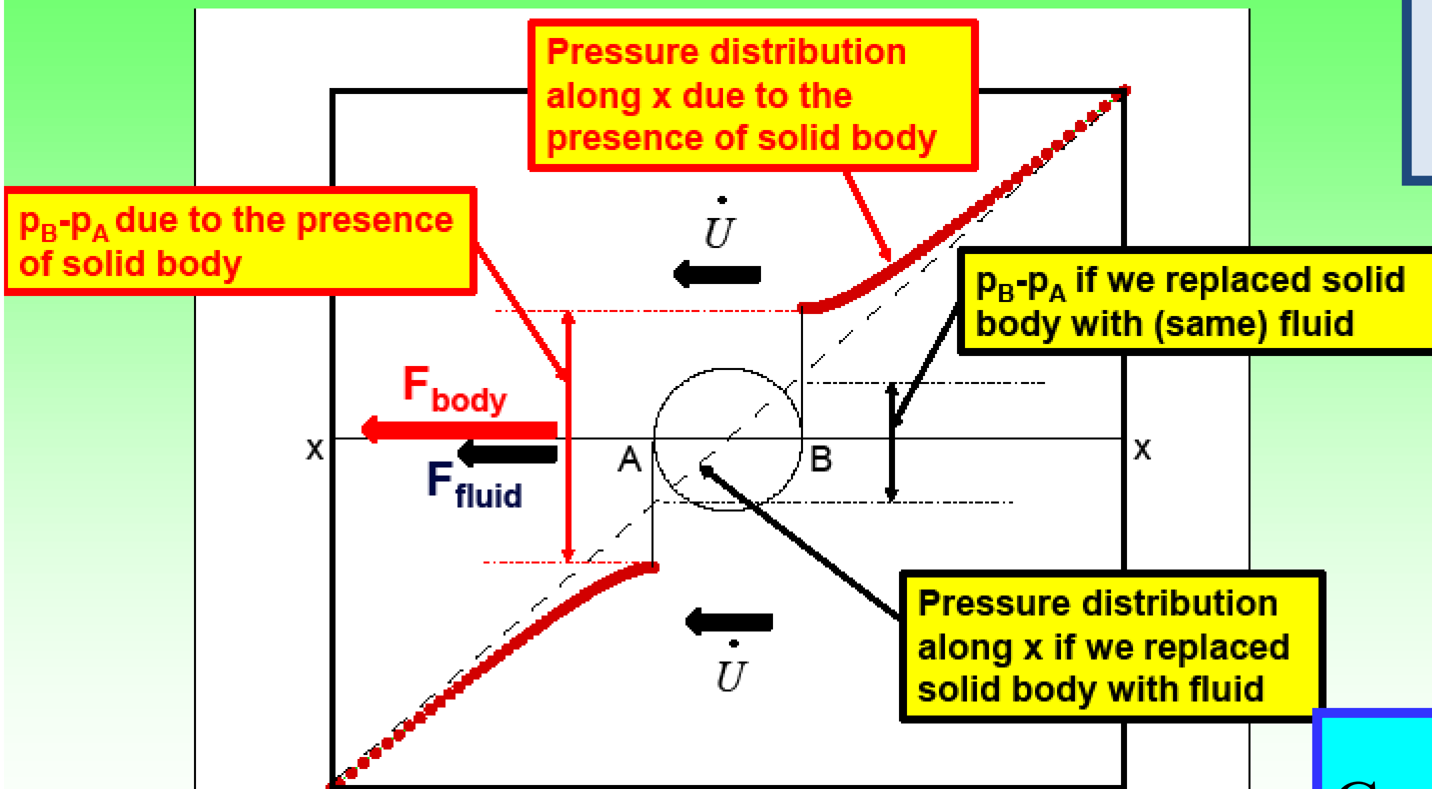
- **m_{fluid}=mass of fluid displaced by body**
- **m_{added}=added mass; function of body shape and inflow direction**

The concept of Added Mass (6/7)

Cylinder subject to accelerated inflow.

Results from Inviscid flow simulation

Definition of inertia coefficient C_M



$$F_{fluid} = \rho(\pi D^2/4)\dot{U}$$

$$F_{body} = C_M \rho(\pi D^2/4)\dot{U}$$

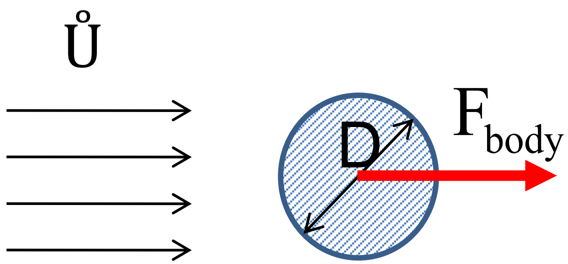
$$C_M = \frac{F_{body}}{F_{fluid}}$$

$$C_M = \frac{m_{fluid} + m_{added}}{m_{fluid}}$$

For inviscid flow around 2-D cylinder: C_M=2

The concept of Added Mass (7/7)

Summary: (all quantities are per unit width in 2-D)



$$F_{body} = C_M m_{fluid} \mathring{U}$$

$$m_{fluid} = \rho_{fluid} V_{fluid} = \text{mass of diplaced fluid}$$

$$V_{fluid} = \text{volume of displaced fluid}$$

$$C_M = \text{inertia coefficient} = \frac{m_{fluid} + m_{added}}{m_{fluid}} = 1 + \frac{m_{added}}{m_{fluid}} = 1 + a$$

$$a = \frac{m_{added}}{m_{fluid}} = \text{added mass coefficient (depends on shape + direction of flow)}$$

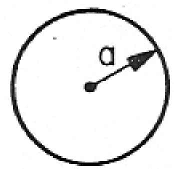
For a <u>cylinder</u> (circle in 2-D) V_{fluid}=area of cross section = $\pi D^2/4$

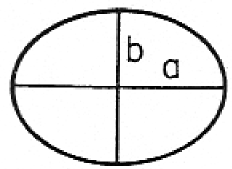
For <u>a cylinder</u> in **inviscid unbounded flow**: $C_M=2$ ($a=1$)

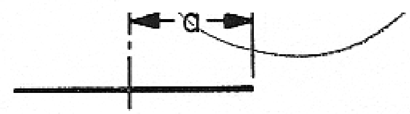
(Inviscid) Added Mass for other shapes:

m_{11} and **m_{22}** are the added masses when the flow is accelerated in the horizontal or the vertical axis, respectively. **m_{66}** is the added moment of inertial when the body rotates around an axis normal to the paper.

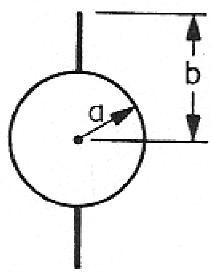
Table 4.3
Added-Mass Coefficients for Various Two-Dimensional Bodies.

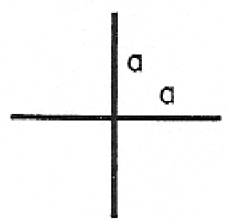


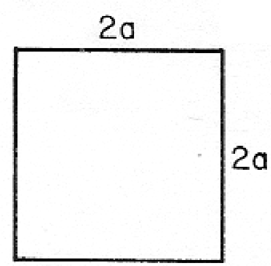




m_{11}:	$\pi\rho a^2$	$\pi\rho b^2$	0
m_{22}:	$\pi\rho a^2$	$\pi\rho a^2$	$\pi\rho a^2$
m_{66}:	0	$\frac{1}{8}\pi\rho(a^2 - b^2)^2$	$\frac{1}{8}\pi\rho a^4$







m_{11}:	$\pi\rho[a^2 + (b^2 - a^2)^2/b^2]$	$\pi\rho a^2$	$4.754\ \rho a^2$
m_{22}:	$\pi\rho a^2$	$\pi\rho a^2$	$4.754\ \rho a^2$
m_{66}:	*	$\frac{2}{\pi}\rho a^4$	$0.725\ \rho a^4$

*For the finned circle the added moment of inertia is given by the formula

$$m_{66} = \rho a^4(\pi^{-1}\csc^4\alpha\,[2\alpha^2 - \alpha\sin 4\alpha + \tfrac{1}{2}\sin^2 2\alpha] - \pi/2)$$

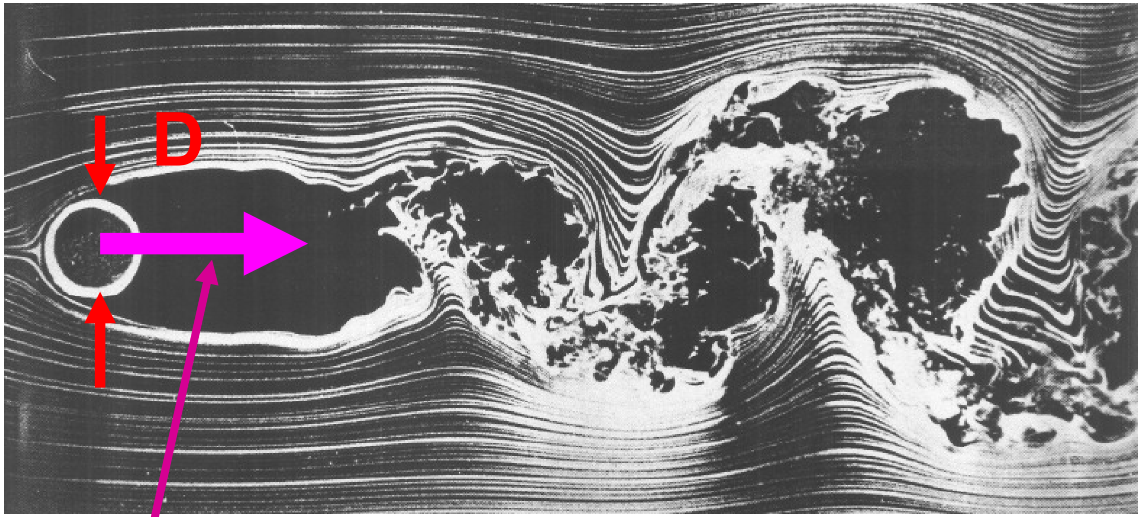
From Marine Hydrodynamics, Newman, J.N., 1977

Morison's equation for total force in the direction of wave propagation

[Morison, J. R.; O'Brien, M. P.; Johnson, J. W.; Schaaf, S. A. (1950), "The force exerted by surface waves on piles", Petroleum Transactions (American Institute of Mining Engineers) 189: 149–154]

Velocity, u

Acceleration, a



Total force = Viscous force + Inertial force

Total force (per unit width) →

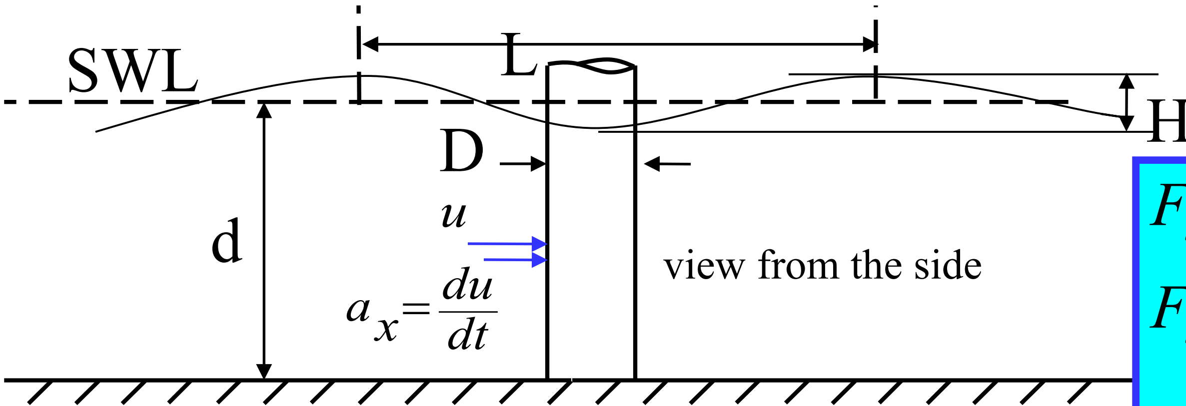
$$C_D \frac{1}{2} \rho D u\,|u| + C_M \rho \frac{\pi D^2}{4} a$$

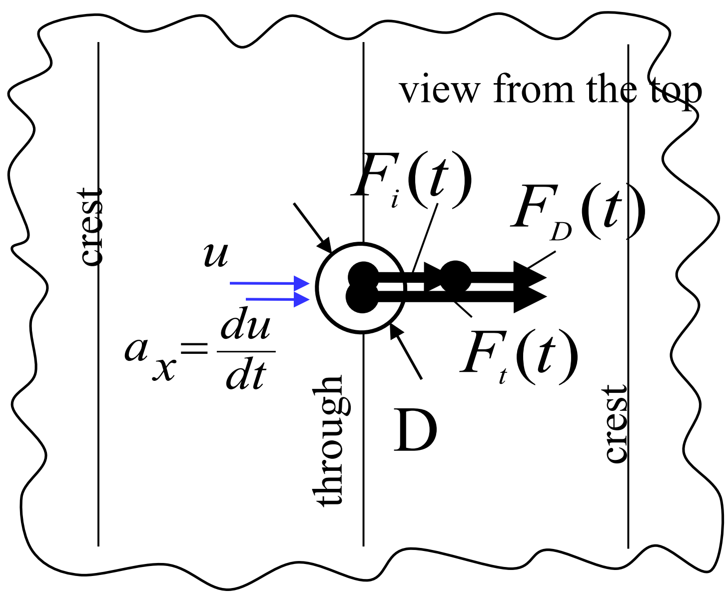
Drag coefficient (C_D)

Inertia coefficient (C_M)

Application of Morison's equation to determine forces on vertical piles

Morison's equation is integrated over the length of the pile, after the values for **u** and **a_x** have been determined by either using linear or **non-linear wave theories**





$$\vartheta = -\omega\, t$$

$$F_{total}(t) = F_i(t) + F_D(t)$$

$$F_i(t) = F_{im} \cdot \sin(\vartheta)$$

$$F_{im} = C_M \cdot \rho g \cdot \frac{\pi D^2}{4} H \cdot K_{im}$$

$$F_D(t) = F_{Dm} \cdot \left|\cos\vartheta\right| \cos\vartheta$$

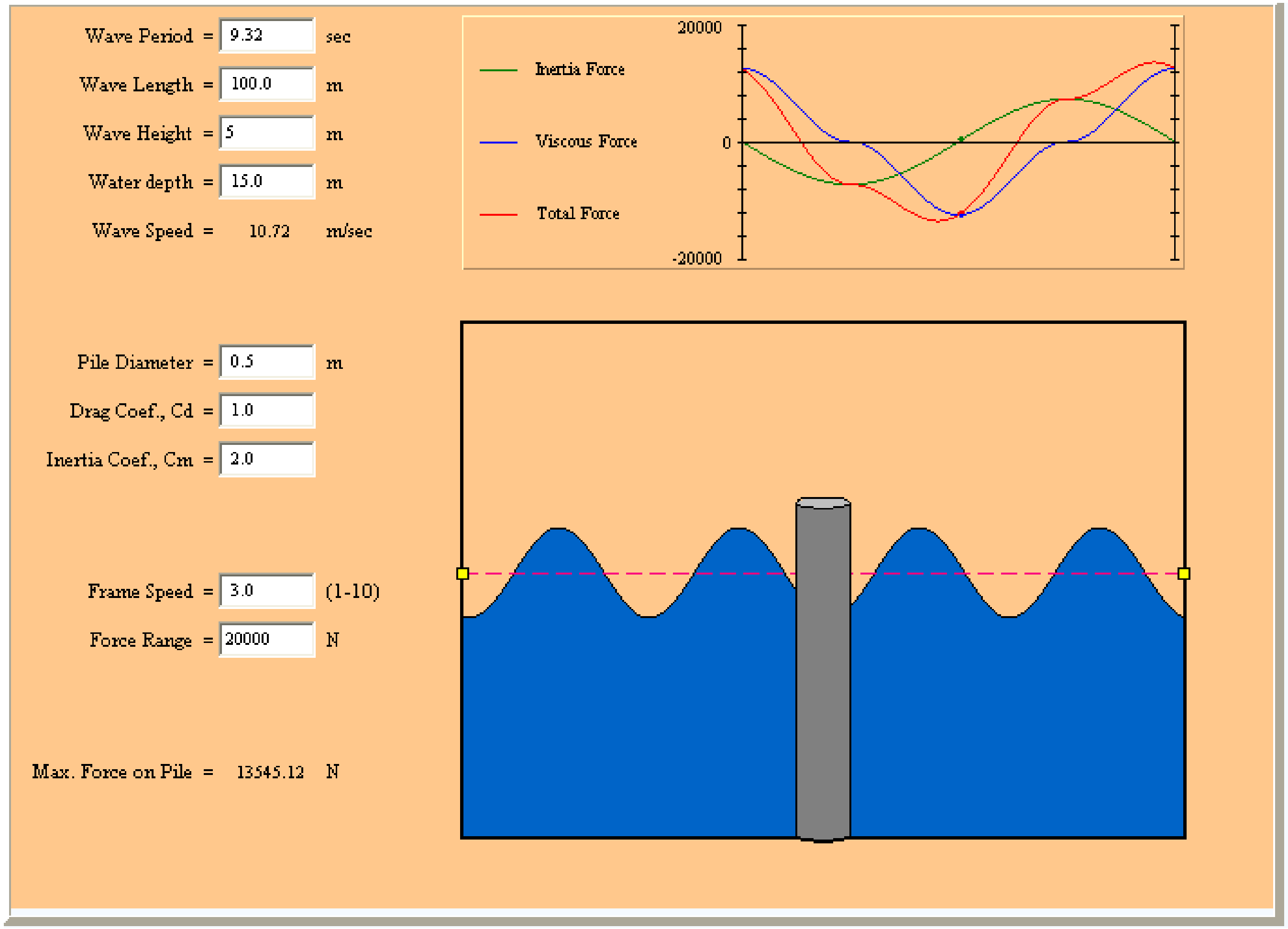
$$F_{Dm} = C_D \frac{1}{2} \rho g\ D H^2 \cdot K_{Dm}$$

$$K_{im} = \frac{1}{2} \tanh\left(\frac{2\pi d}{L}\right)$$

$$K_{DM} = \frac{1}{8}\left(1 + \frac{4\pi d / L}{\sinh\left[4\pi d / L\right]}\right)$$

The wave forces applet sums-up the forces per pile slice over the pile length

http://cavity.ce.utexas.edu/kinnas/wow/public_html/waveroom/Applet/WaveForces/WaveForces.html



Typical values of the drag and inertia coefficients
From API's (American Petroleum Institute) Recommended Practice 2A-WSD (Dec. 2000)

- **C_D = 0.65 and C_M=1.6 for smooth piles**
- **C_D = 1.05 and C_M=1.2 for rough piles (due to marine growth)**

Note: The diameter of the pile, D, also increases with marine growth

Total Force on Pile
(in the direction of wave propagation)

Total force = Viscous force + Inertial force

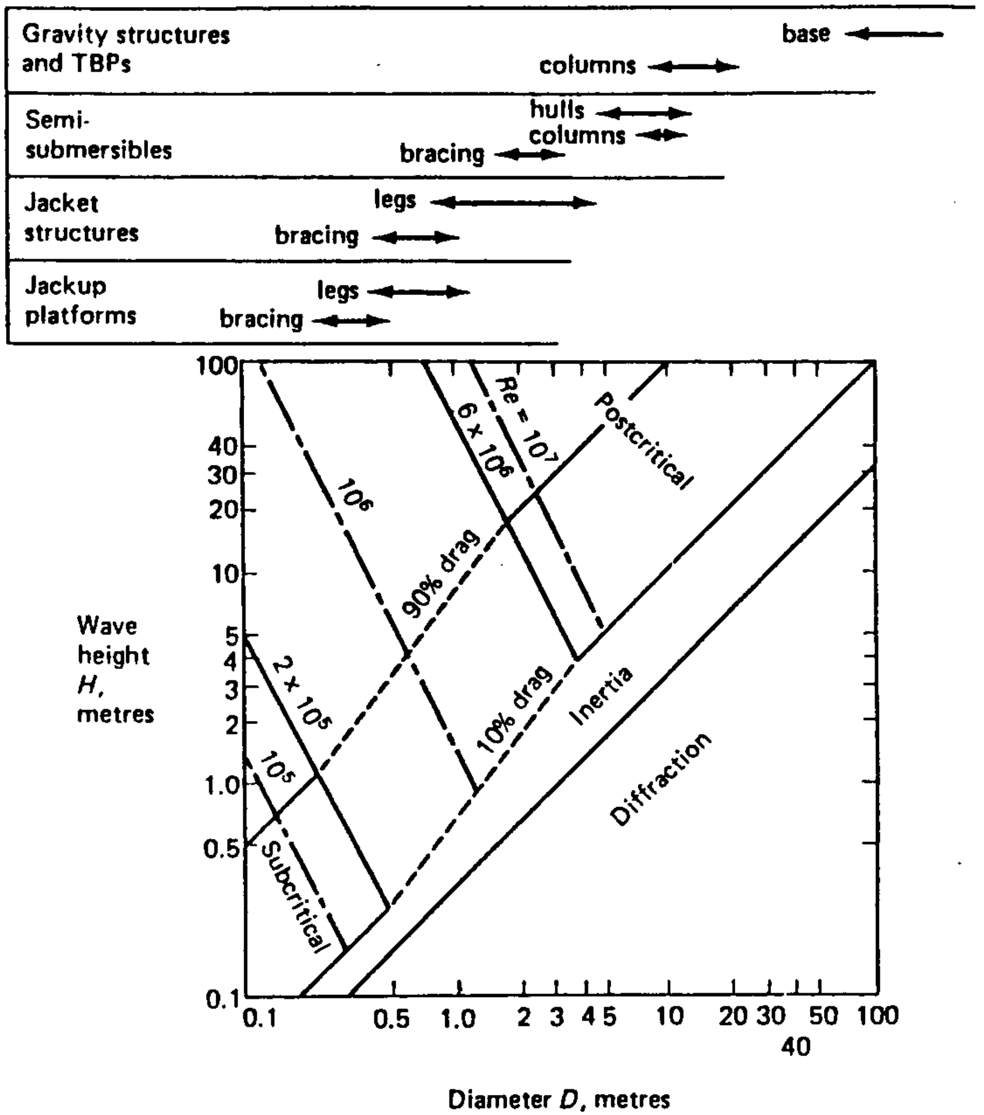
Morison's equation → $\sim C_D \rho D H^2$ + $\sim C_M \rho D^2 H$

Drag coefficient (C_D) **Inertia coefficient** (C_M)

$$\frac{\text{Viscous force}}{\text{Inertial force}} \sim \frac{C_D}{C_M} \frac{H}{D}$$

As H↑ or D↓ or H/D↑ the viscous forces become more important

Effect of wave height H and diameter of element D on importance of viscous forces

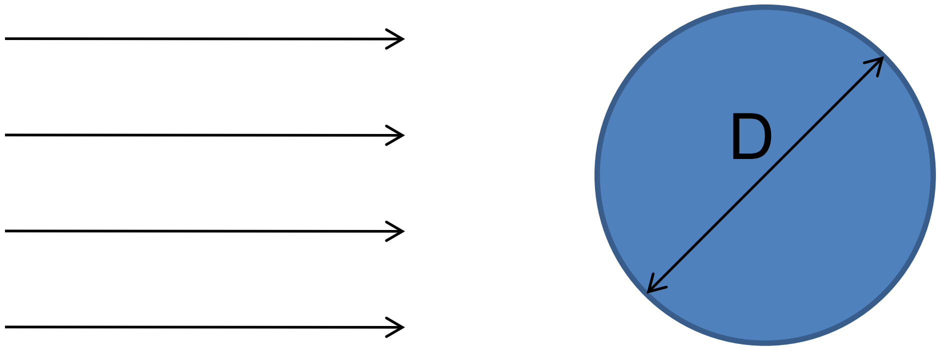


Loading regimes at still water level (from Hogben[4])

An assessment of Morison's equation using CFD (Computational Fluid Dynamics)

Two Dimensional Cylinder in Oscillatory Flow

$$U = U_m \cdot \cos(\omega t)$$



<u>Two important numbers:</u>

- **Re (Reynolds No)=$U_m D/\nu$**

- **KC (Keulegan-Carpenter No)=$U_m T/D$ ($T=2\pi/\omega$)**
  ~(distance the particles travel in T)/D

Morison's Equation

The inline force (force in the direction of the flow) is the sum of the drag force and the inertia force (per unit width)

$$F = \frac{1}{2}\rho C_D D|U|U + \frac{1}{4}\rho\pi D^2 C_M \frac{dU}{dt}$$

$$U = U_m \cdot \cos(\omega t)$$

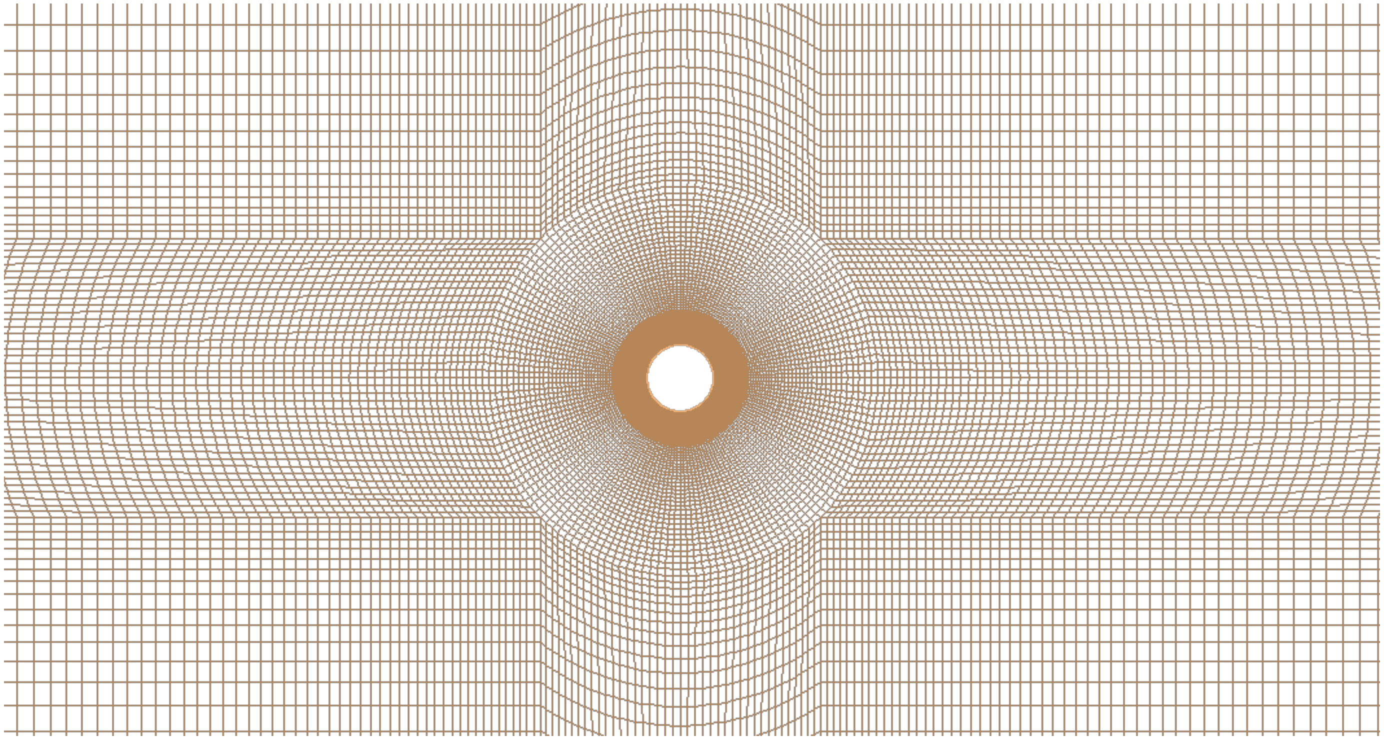
C_M is the inertia coefficient

C_D is the drag coefficient

- we also define: $C_x = \dfrac{F}{\frac{\rho}{2} {U_m}^2 D}$

Grid in Fluent

Structured mesh is used in the calculation domain



Mesh Info:

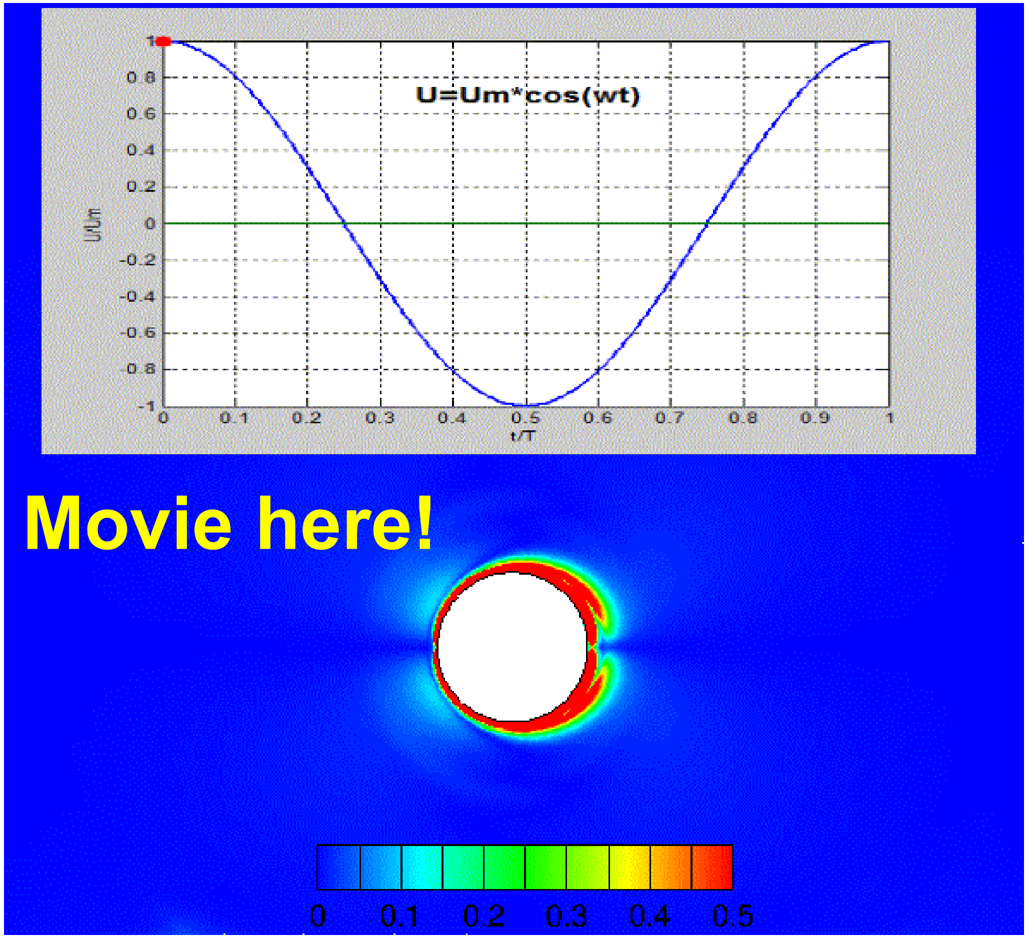
Cells: 76680

Faces: 154190

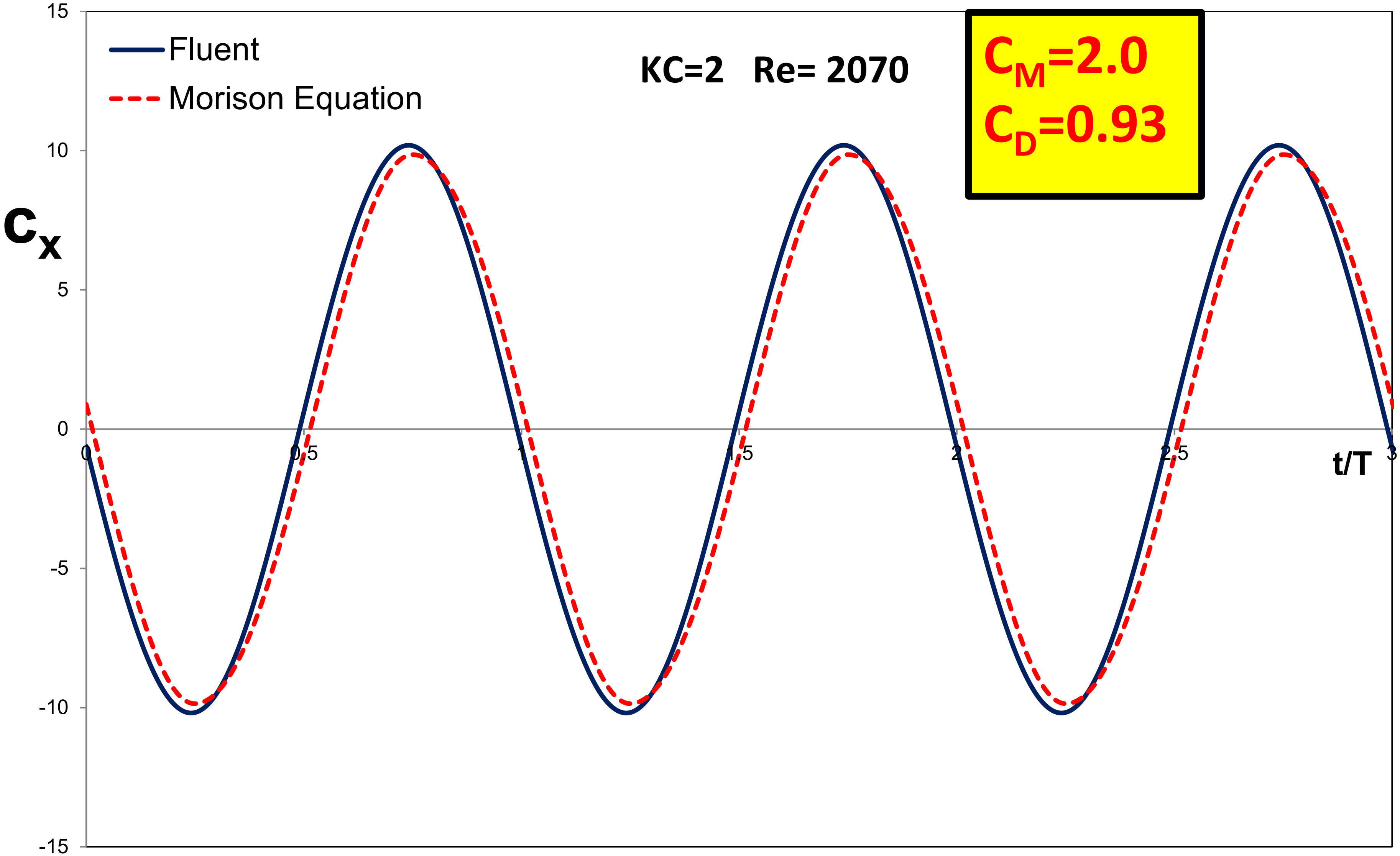
Nodes: 77510

Predicted flow (vorticity) by Fluent: KC=2, Re=1070

(click on the movie to play)

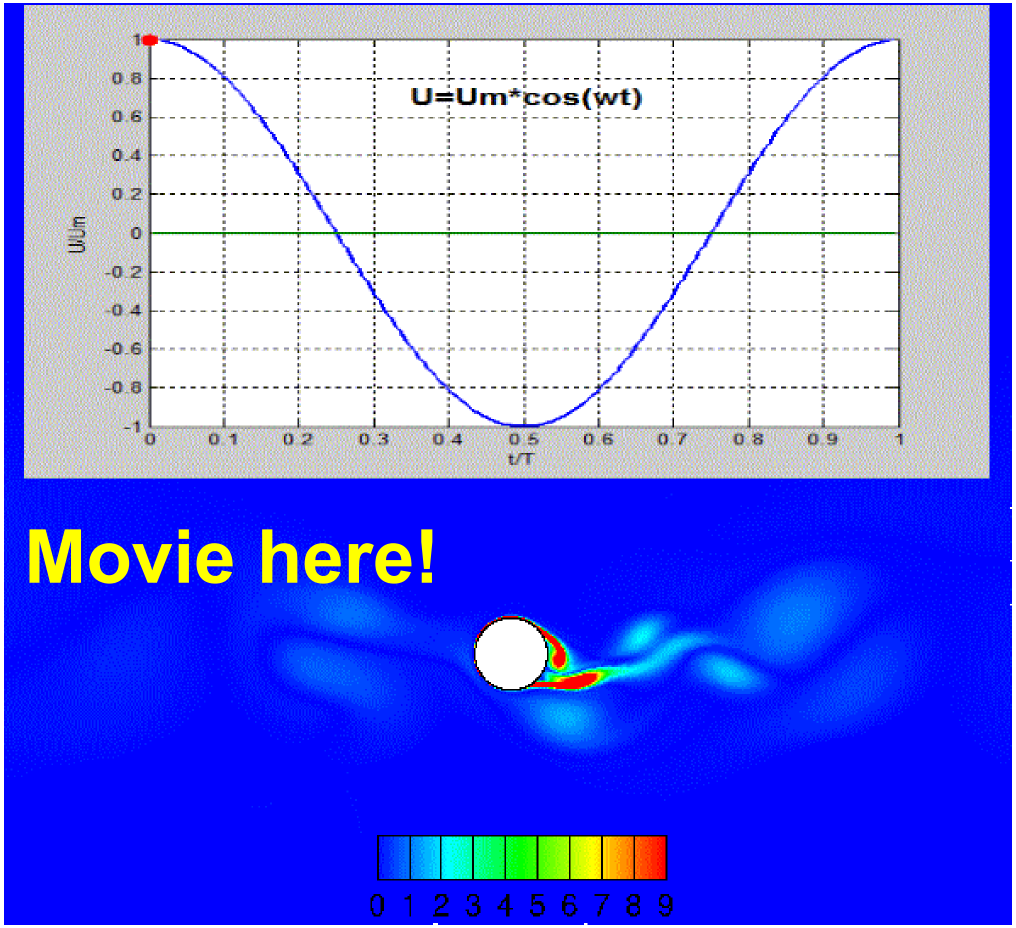


Case I: KC=2 Re=1070

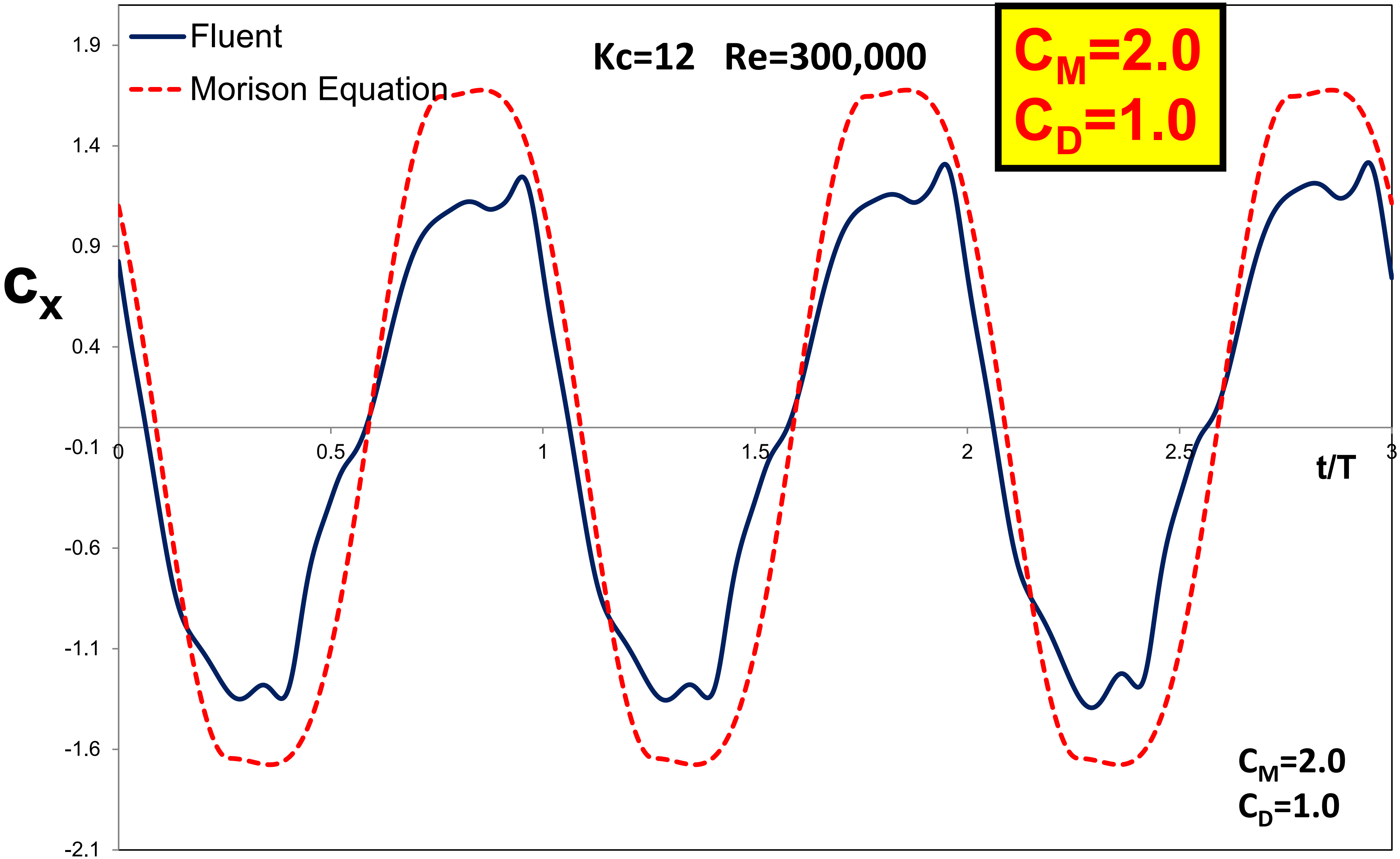


Predicted flow (vorticity) by Fluent: KC=12 Re=300,000

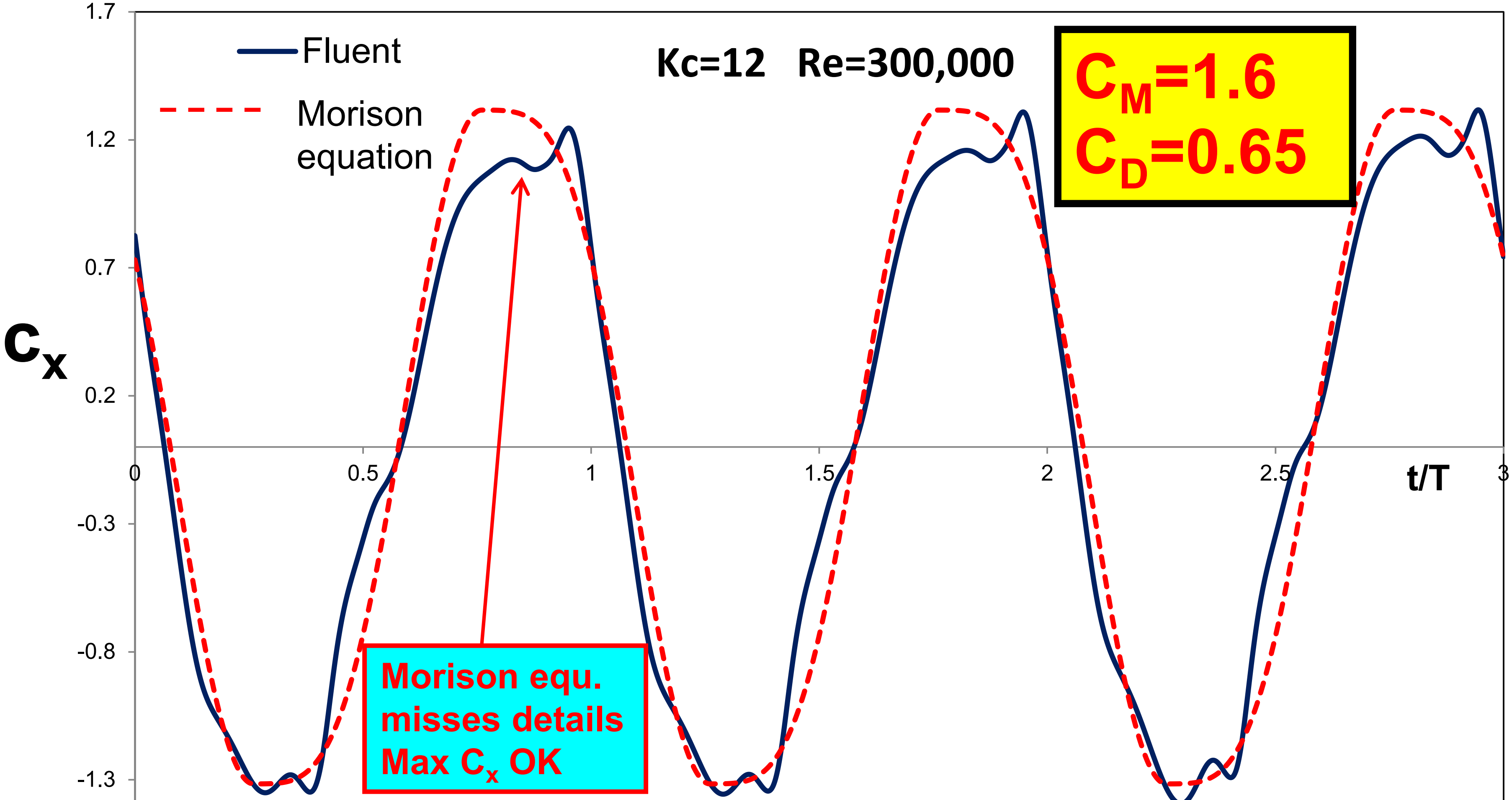
(click on the movie to play)



Case II: KC=12 Re=300,000



Case II: KC=12 Re=300,000



Maybe due to luck…more needs to be done!!! Some newer simulations will be shown in the lecture on Computational Hydrodynamics