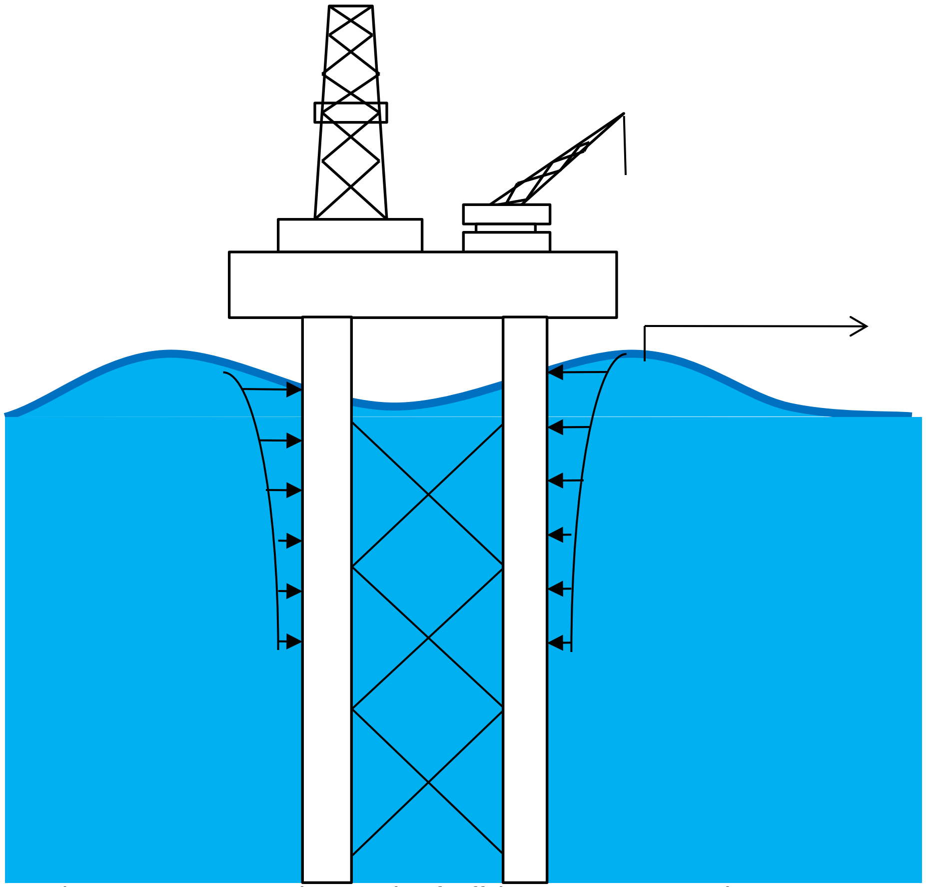
2013 Short Course on Fundamentals of Offshore Structures and Design of Fixed Offshore Platforms April 15-26, 2013, UT Austin



Wave Theory and Applications

by

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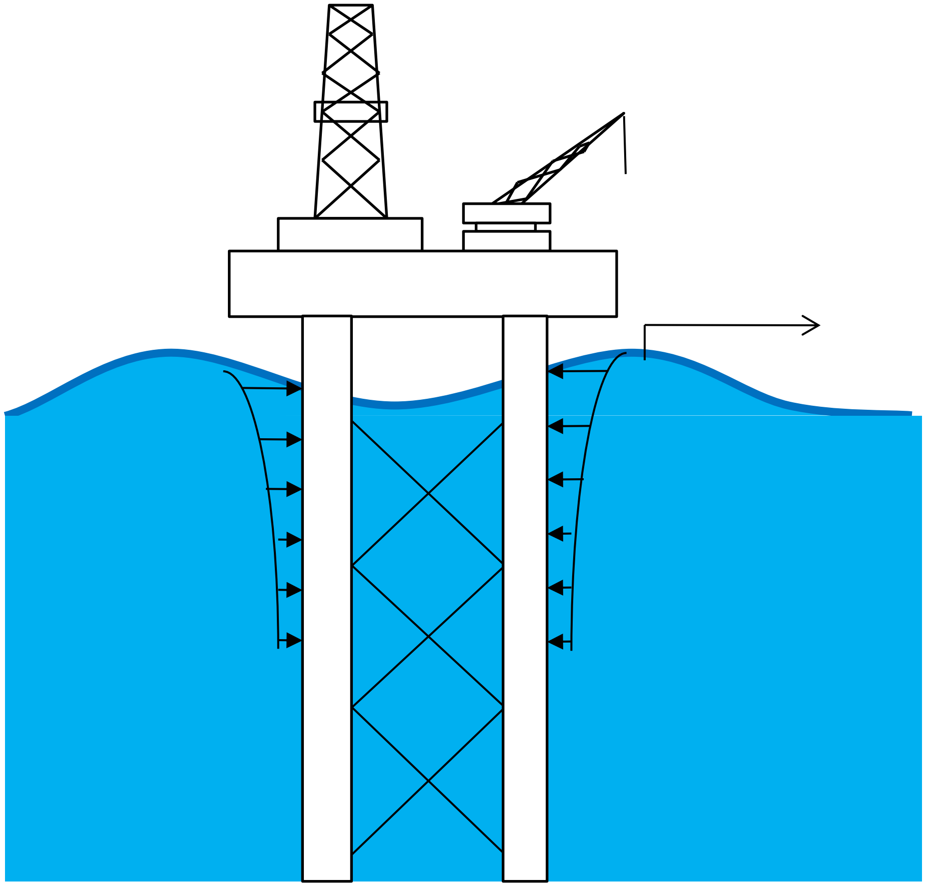
To determine wave forces on an offshore structure we need to know *fluid velocities & accelerations* under the wave surface



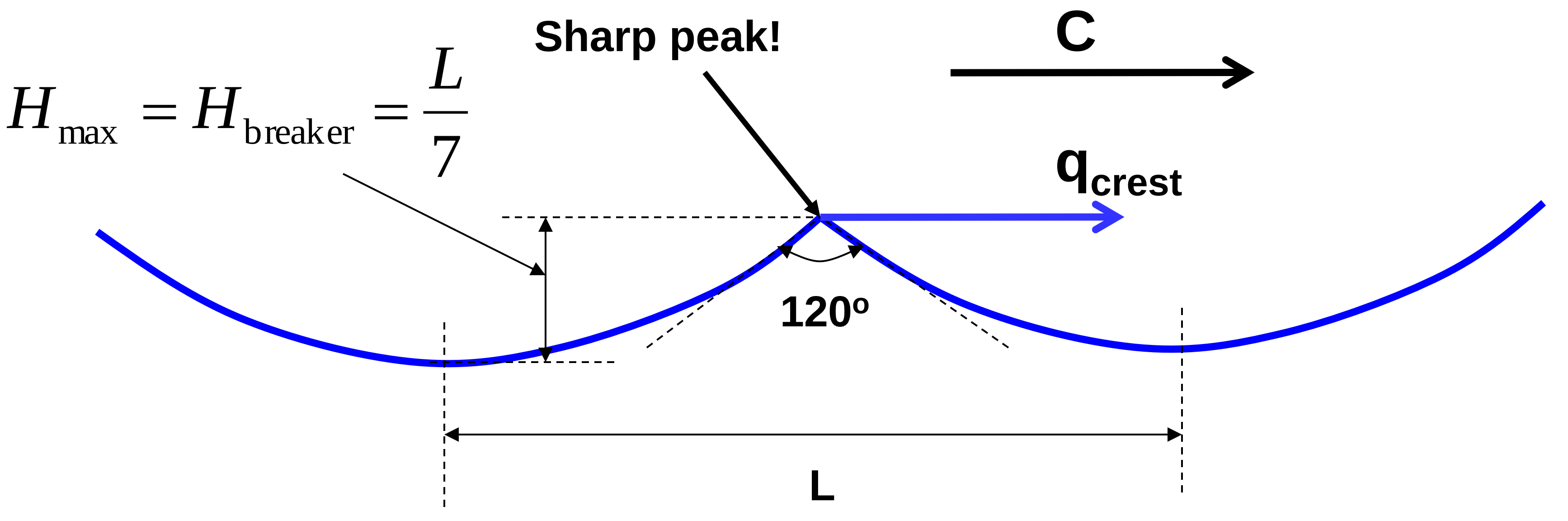
An offshore structure is designed to withstand the 100-year storm (wave/current/wind). A mono-chromatic wave of height H_{max} is assumed.

H_{max}/L is larger than required for linear wave theory to be valid.

Wave velocities and accelerations from linear wave theory, when used in Morison's equation, ***under-predict*** *wave forces!*

What is the maximum height a wave in deep water can have (before it breaks)?

Stokes' Criterion for wave breaking: $q_{crest}=C$



Q:How well could linear theory handle waves at breaking?
A: Pretty BAD! (predicts $H_{max}=L/\pi$ in deep water!)

Stokes' 2nd order non-linear wave theory (1/2)

$$\eta(x,t) = \frac{H_1}{2}\cos(kx-\omega t) + \frac{H_2}{2}\cos[2(kx-\omega t)]$$

1st order term

2nd order term

$$H_1 = H \quad \text{and} \quad H_2 = \frac{\pi}{4}\frac{H^2}{L}\frac{\cosh(2\pi d/L)}{\sinh^3(2\pi d/L)}[2+\cosh(4\pi d/L)]$$

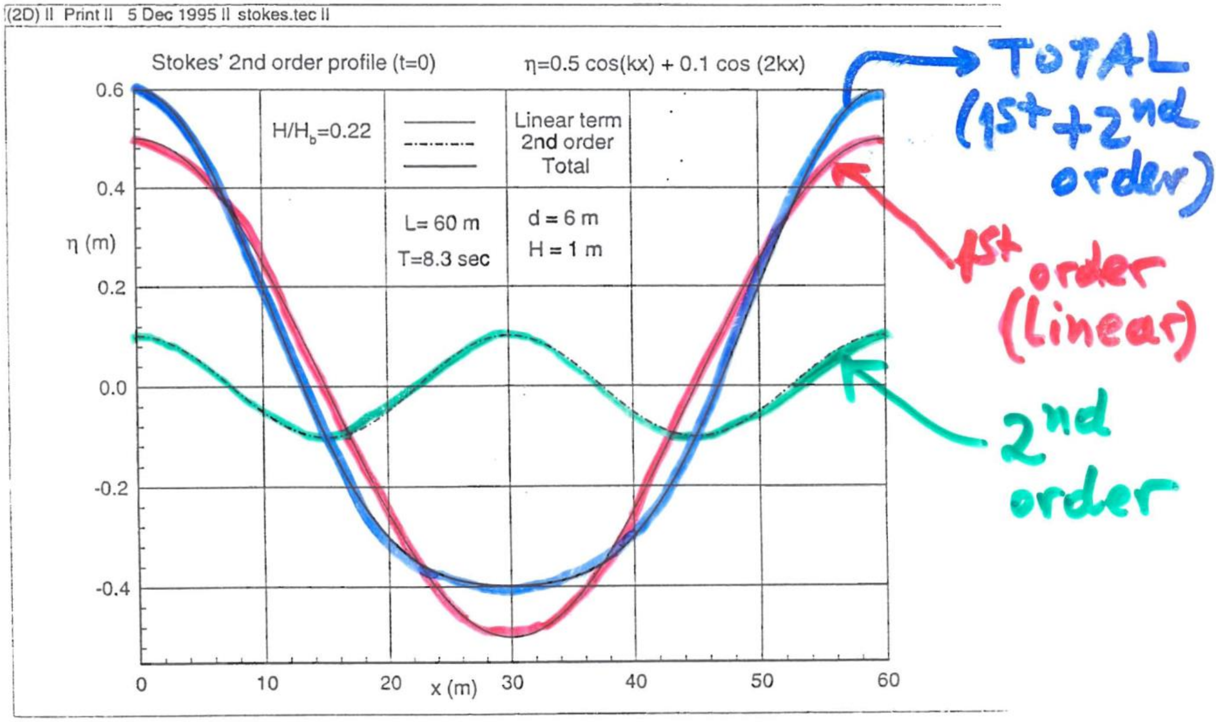
Case 1

$d = 6\text{ m}, H = 1\text{m}$

$\text{L} = 60\text{m}, \text{T} = 8.3\,\text{sec}$

$H/L = 1/60$

$d/L = 0.1$



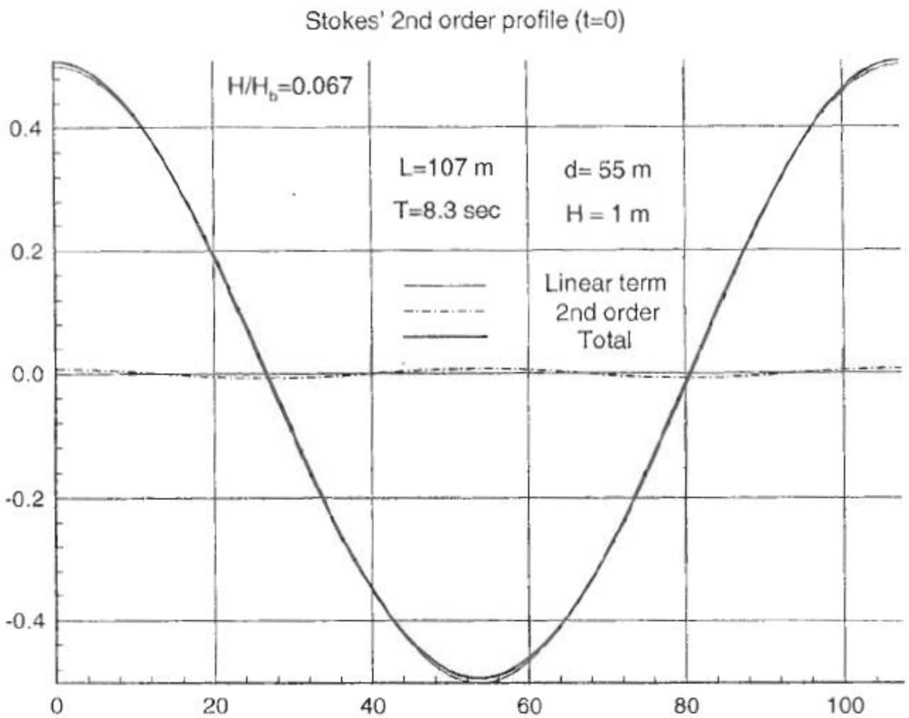
Stokes' 2nd order non-linear wave theory (2/2)

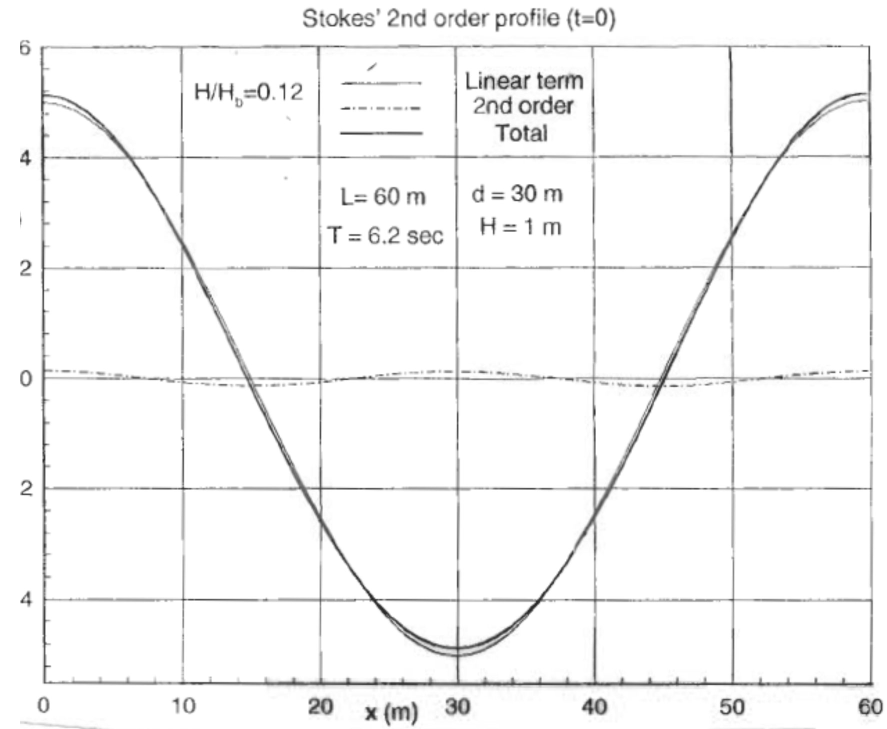
Case 2

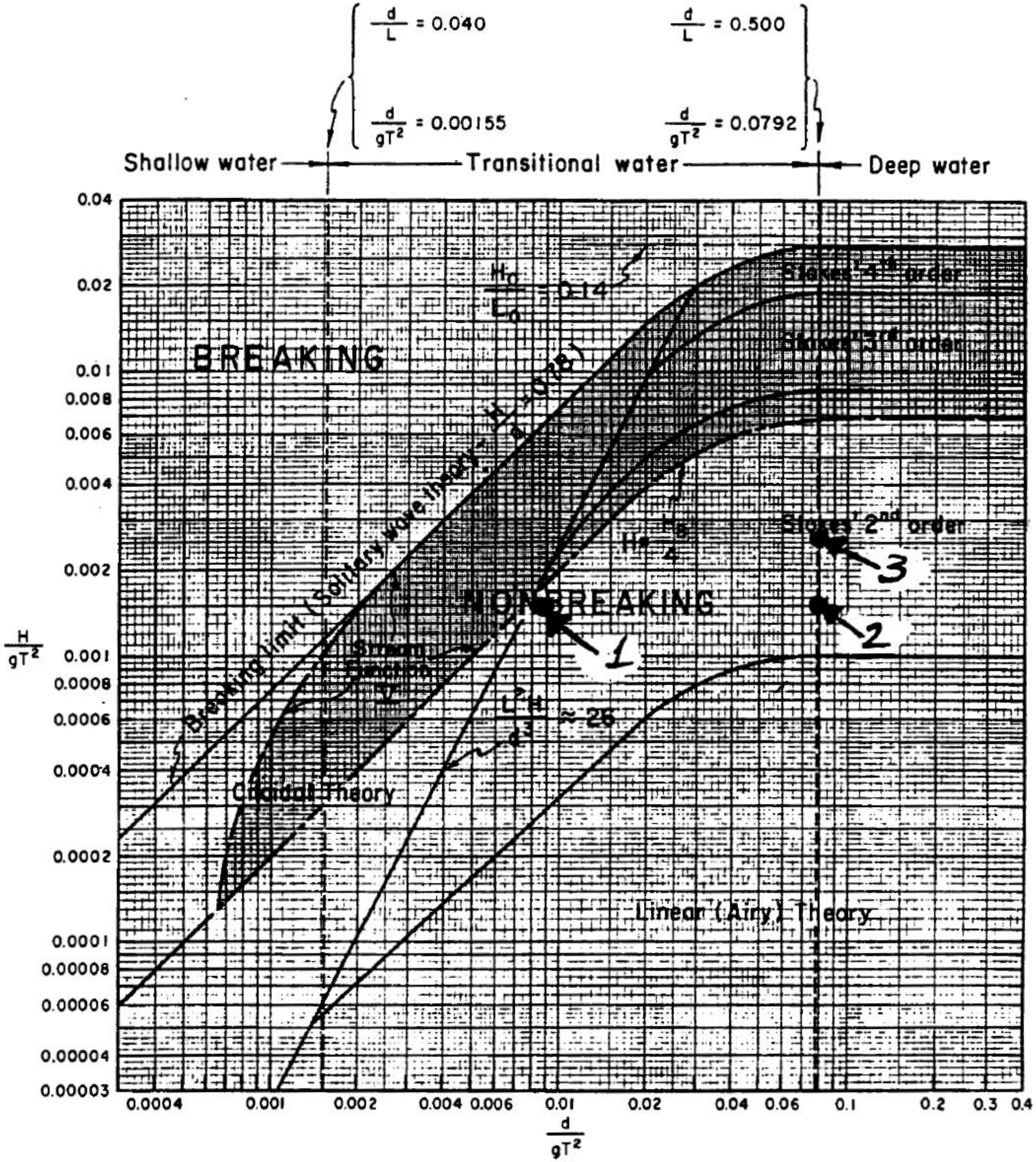
$$\text{deep water}, H = 1m$$
$$\mathrm{T} = 8.3 \ (\text{same as in Case 1})$$
$$\mathrm{L} = 107\mathrm{m}$$
$$H / L = 1/107$$

Case 3

$$\text{deep water}, H = 1m$$
$$L = 60\mathrm{m} \ (\text{same as Case 1})$$
$$\mathrm{T} = 6.2 \ \text{sec}$$
$$H / L = 1/60$$

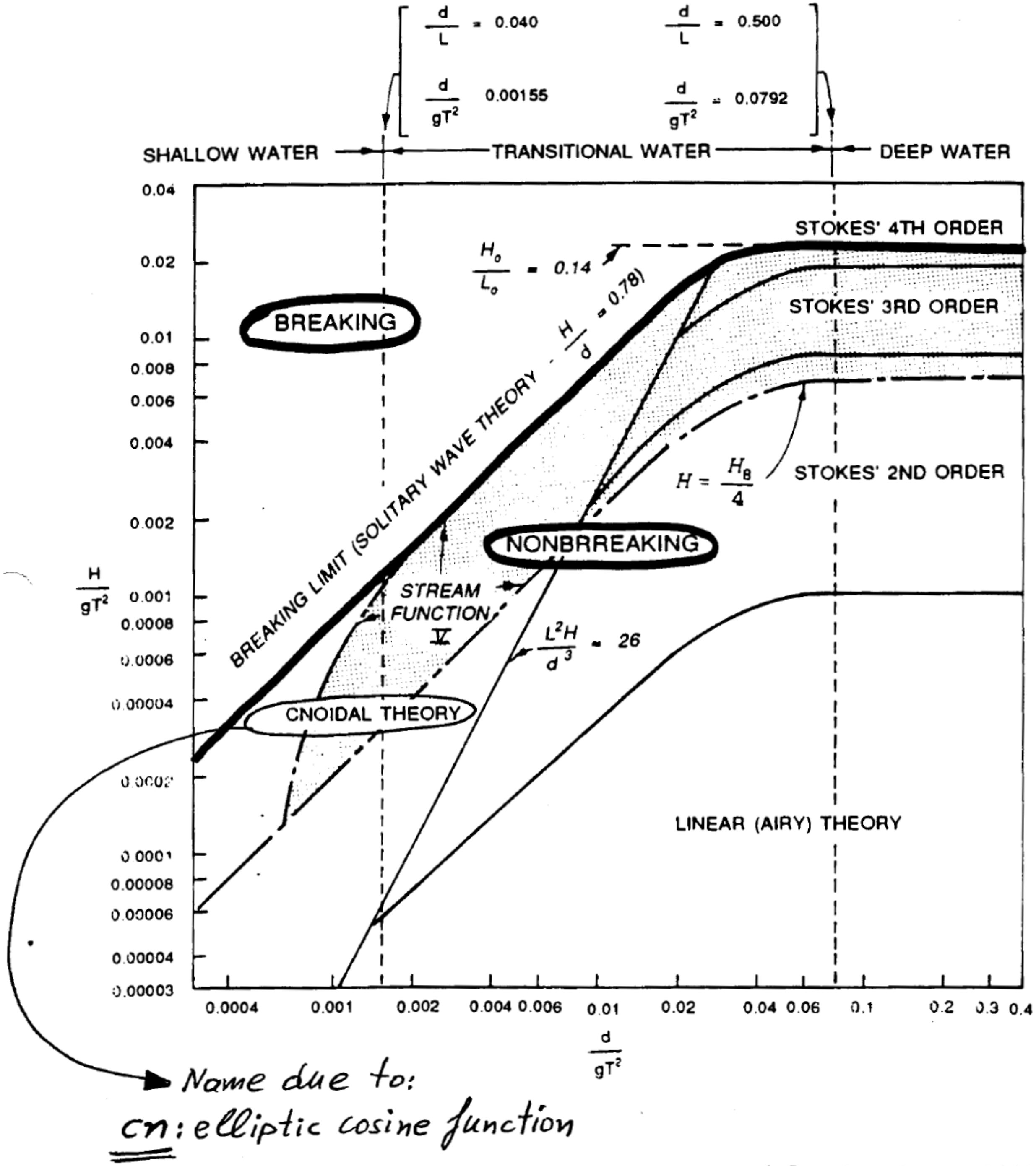






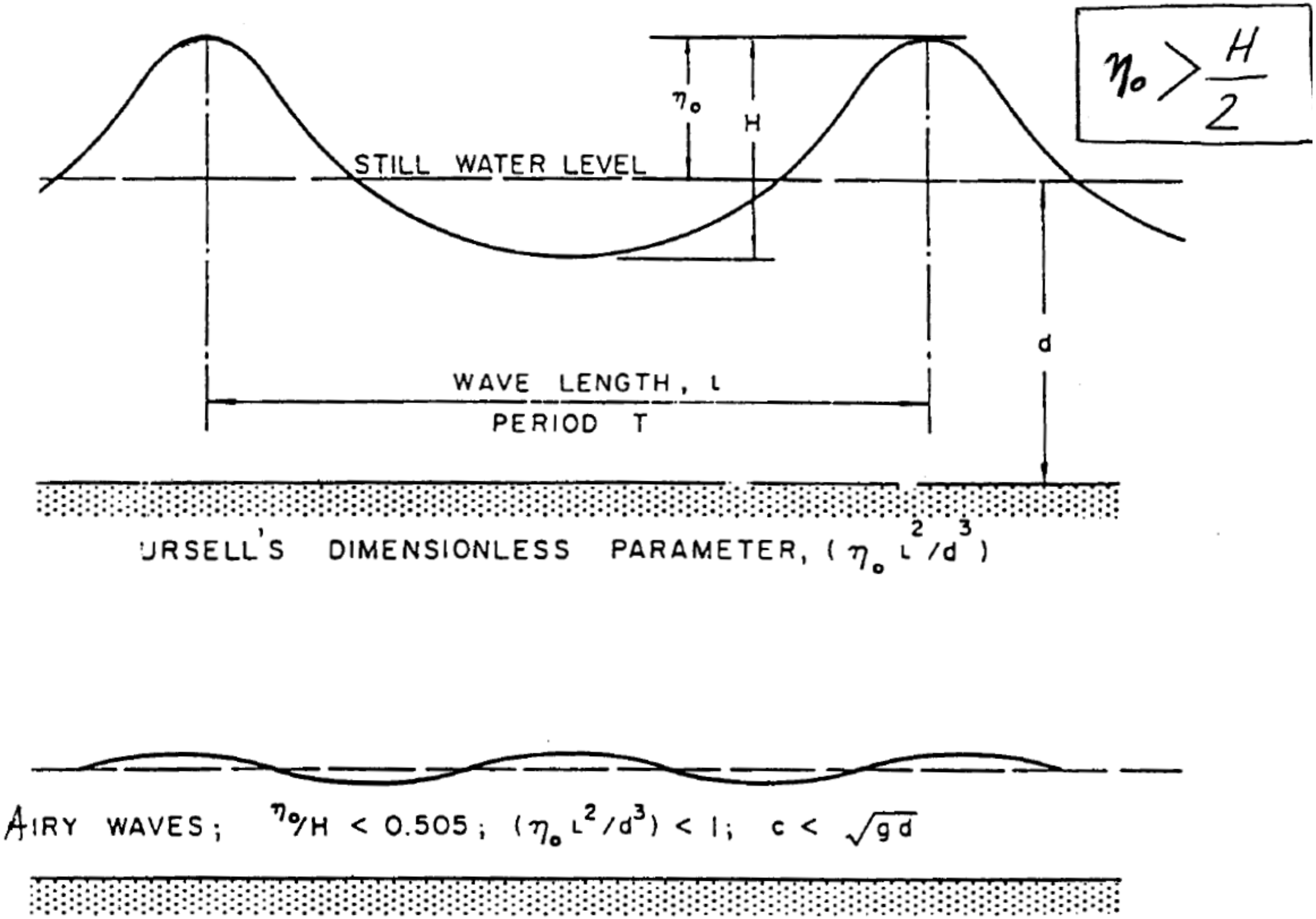
Cases 1, 2, & 3 on the graph by Le Mehaute

A cleaner version of the graph by Le Mehaute

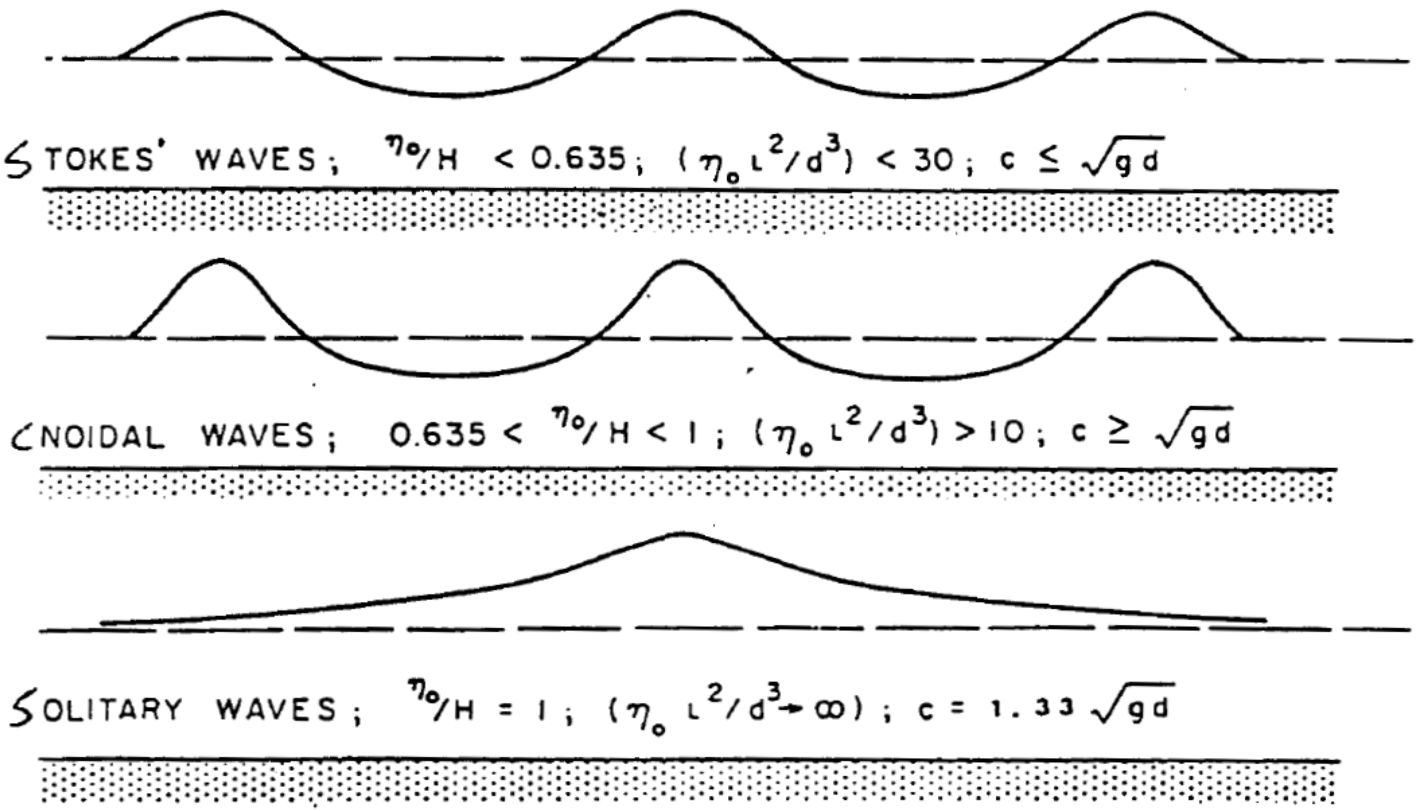


(after Le Mehaute, 1969)

Different types of waves (1/2):



Different types of waves (2/2):

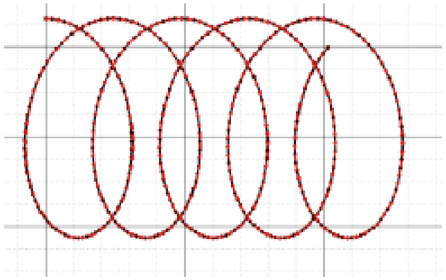


;. **3.18.** Profile shape of gravity water waves [Wilson (1963)]

Short-comings of linear wave theory:

- ➢*Does not produce accurate wave profiles for some combinations of d/gT^2 and H/gT^2, and cannot handle waves just before breaking*

- ➢***Does not allow for mass transfer*** *along the wave direction. Particle trajectories are evolving tight spirals, rather than closed, as shown here:*



- ➢*Provides inaccurate fluid velocities and accelerations which, when used in the Morison's equation (to be described later),* ***under-predict the forces*** *acting on the elements of an offshore structure*

Detailed formulas for the wave profile and the corresponding flow-field for a non-linear wave are given in the provided document:

NOTES ON FIFTH-ORDER GRAVITY WAVE THEORY