Formulas for $F_i(t)$ and $F_D(t)$ for "short" piles

$$F_i(t) = f_i \cdot \ell \quad \& \quad F_D(t) = f_D \cdot \ell \quad (1)$$

ℓ = length of column

(2a) $f_i = C_M \rho \frac{\pi D^2}{4} a$; a: acceleration

(2b) $f_D = C_D \frac{1}{2} \rho D u|u|$; u = velocity

As also mentioned in class, we will use the approximation that f_i and f_D are uniform over the pile with values those at midpoint of the pile.

Based on this assumption equ. (1) applies. Then we evaluate f_i & f_D at the midpoint of the pile by using formulas from Fig 2-6 of SPM: $a_x = a_{max} \sin\theta$ and $u = u_{max} \cos\theta$ (the expressions of a_{max}, u_{max} are given on Fig. 2-6)

Then: $\theta = -\omega t \Rightarrow a_x = -a_{max} \sin(\omega t)$ & $u = u_{max} \cos(\omega t)$

Replacing a_x and u in (2a) & (2b) and f_i & f_D in (1) we can get expressions for F_{im} and F_{Dm} (max values of $F_i(t)$ & $F_D(t)$) and then apply formulas to determine F_{Tm} (max total force).