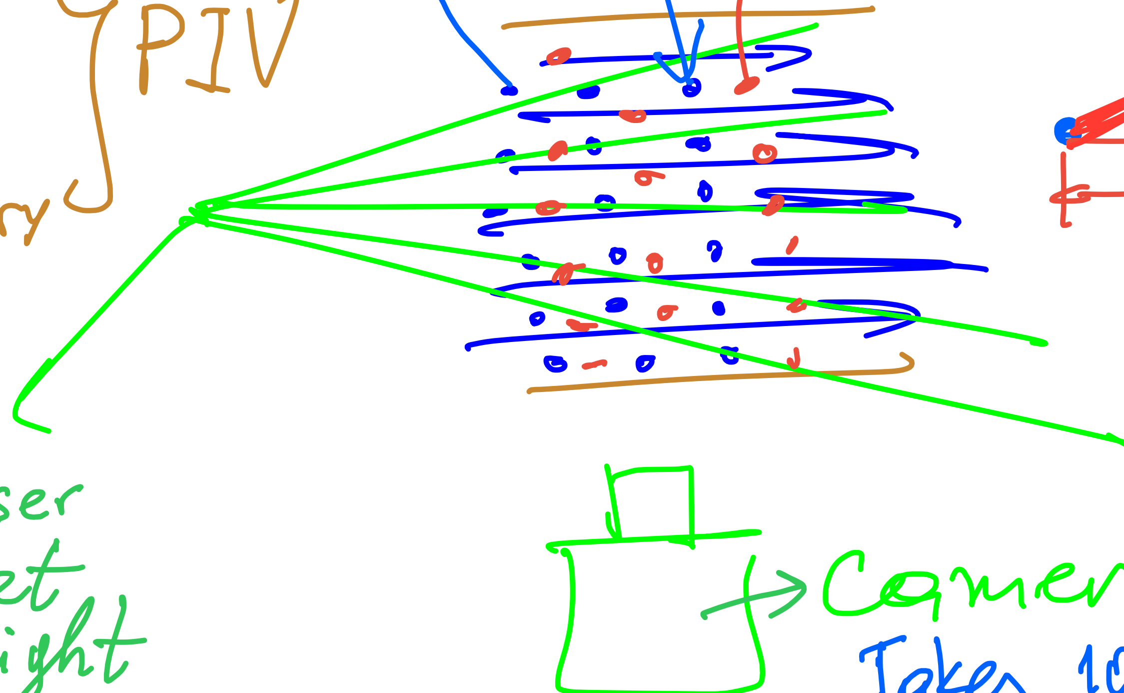
WHERE CAN WE ASSUME IRROTATIONAL FLOW?

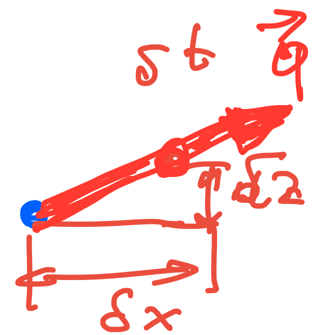
silver coated neutrally buoyant spheres ~ 50 μm wide

$t = 0$

$t = \delta t = 10^{-3}$ sec

Particle Image Velocimetry } PIV

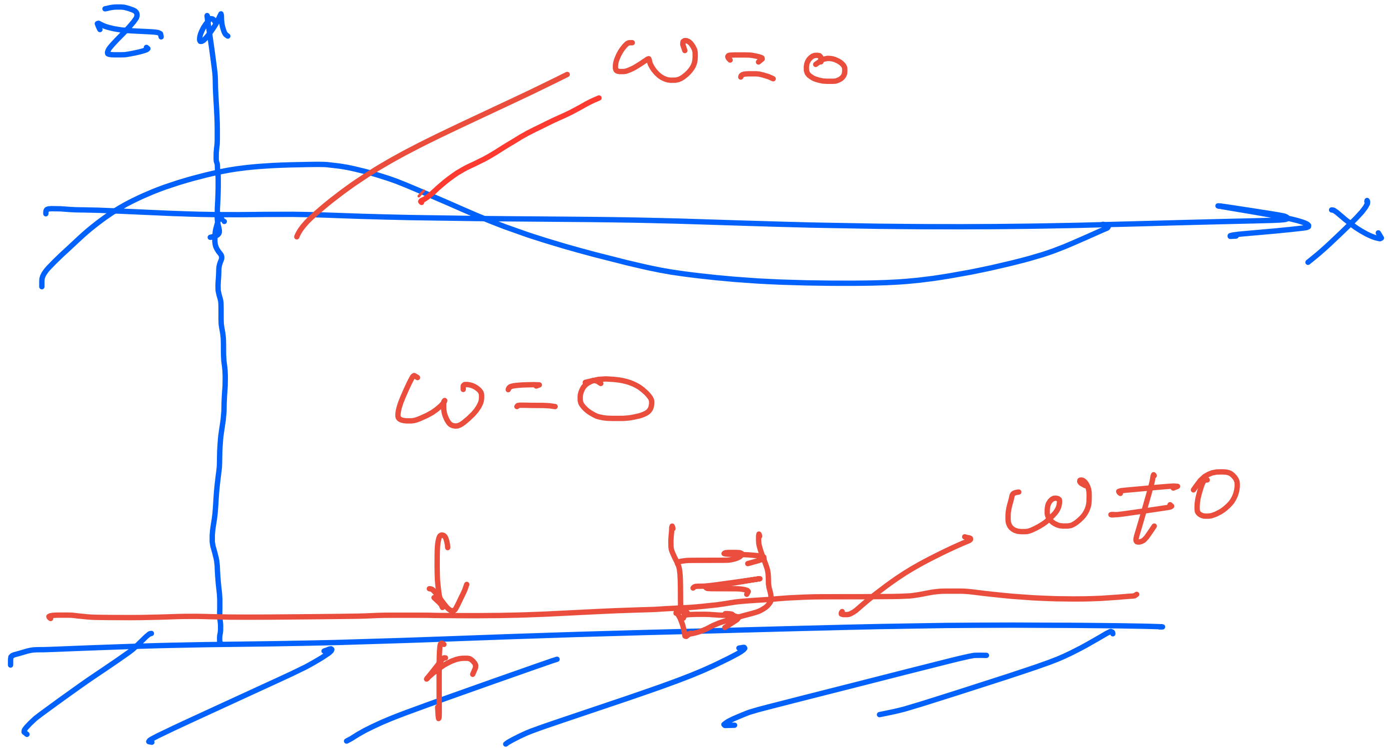




Laser sheet of light

Camera

Takes 1000 frames per sec



Wave mechanics, propagation can be modeled accurately by assuming irrotational flow ($\omega = 0$)

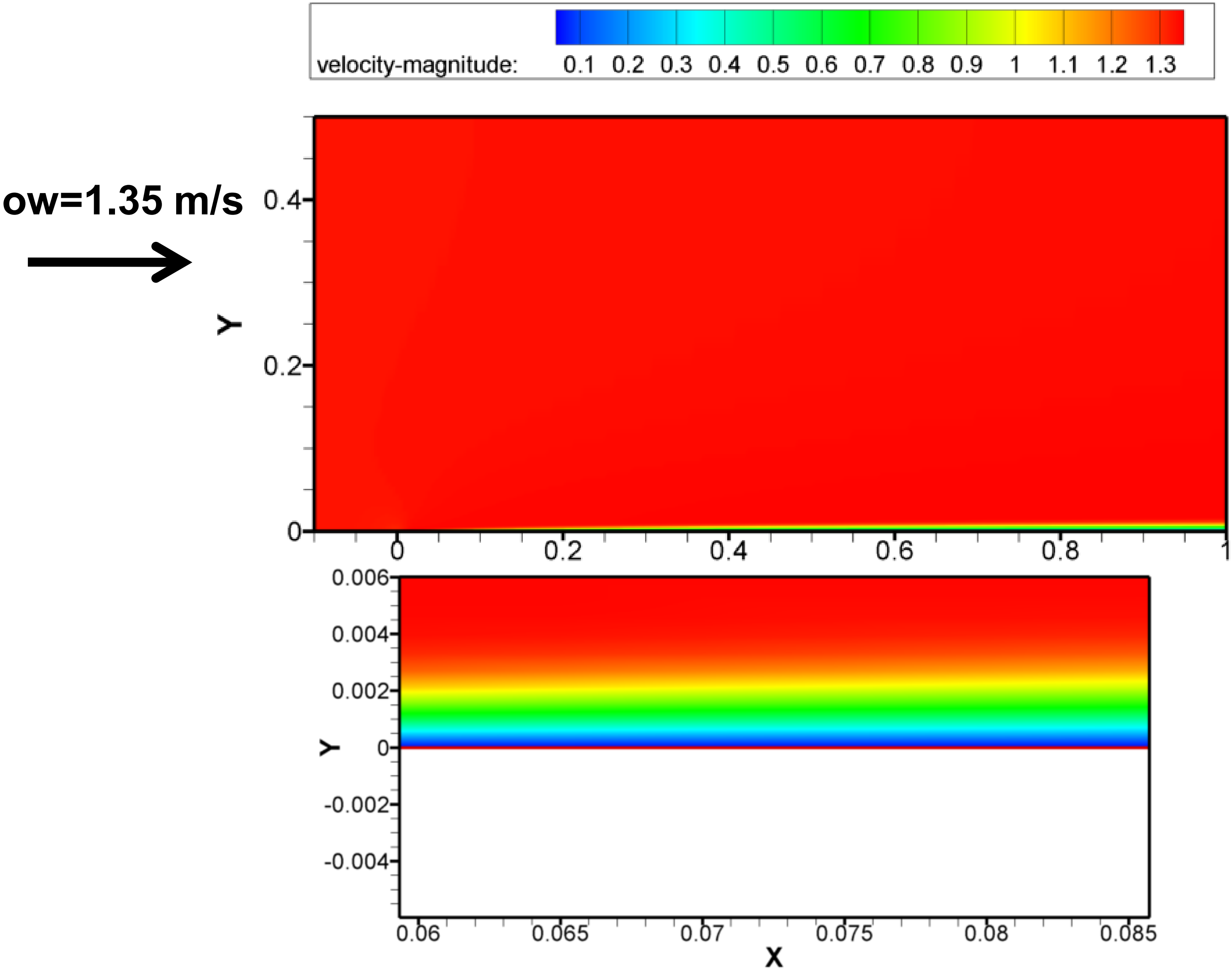
CE358 - Fall 2024

The following slides have been produced from running ANSYS/Fluent, a commercial Computational Fluid Dynamics, CFD, package, which solves for the viscous flow around an object.

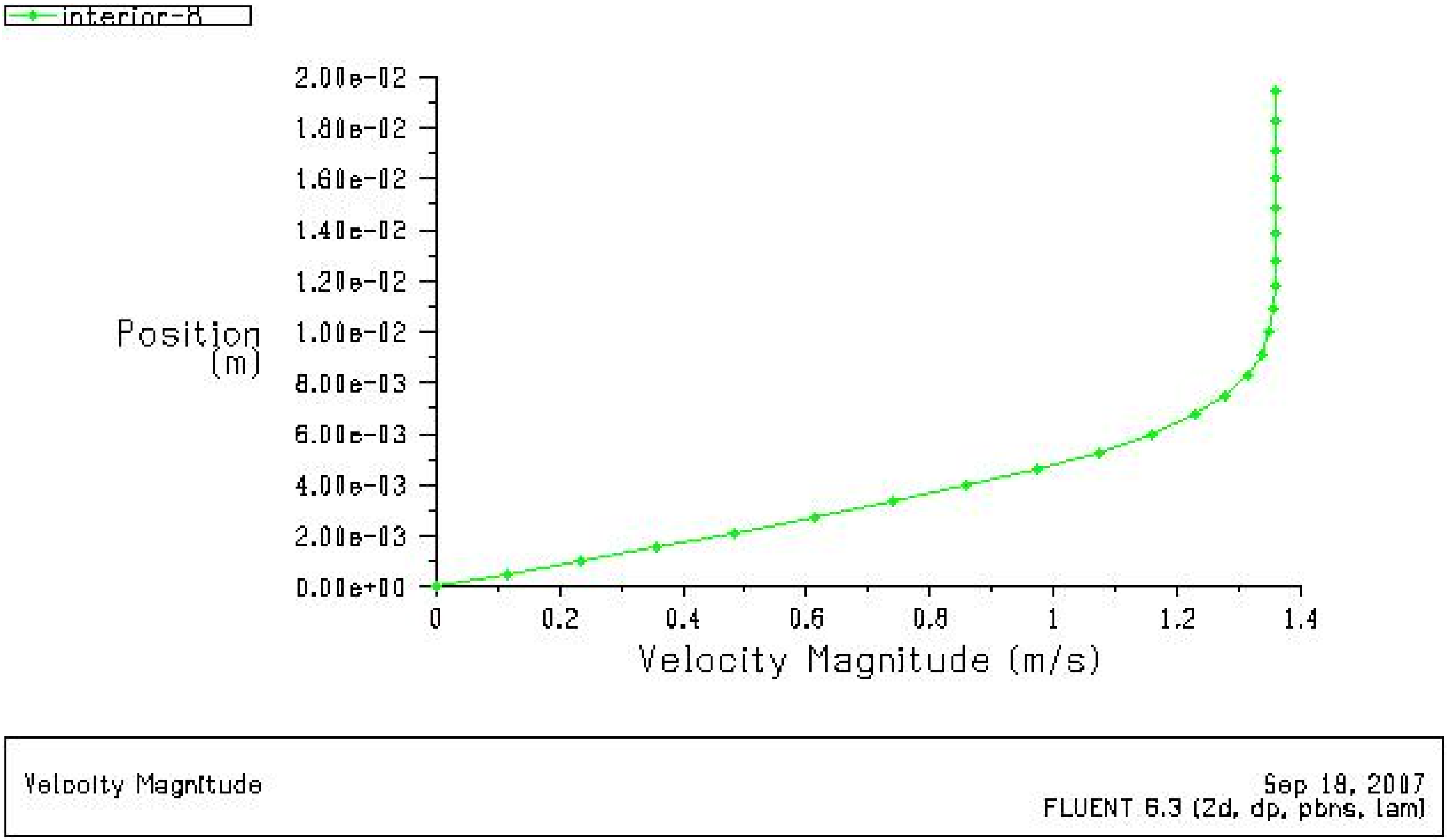
Results are shown to help you understand the concepts of flow velocity, streamlines, boundary layer, and vorticity.

Flow over flat plate (velocity magnitude) predicted by Fluent

Inflow=1.35 m/s



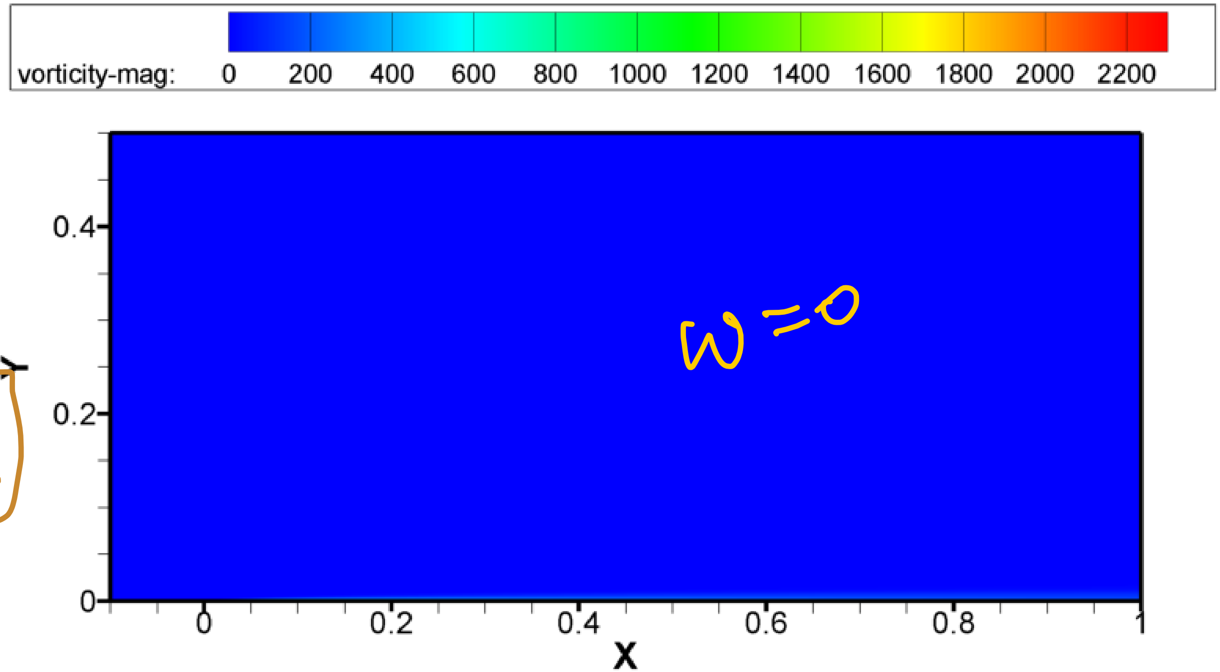
Flow over flat plate (velocity profile in boundary layer) predicted by Fluent

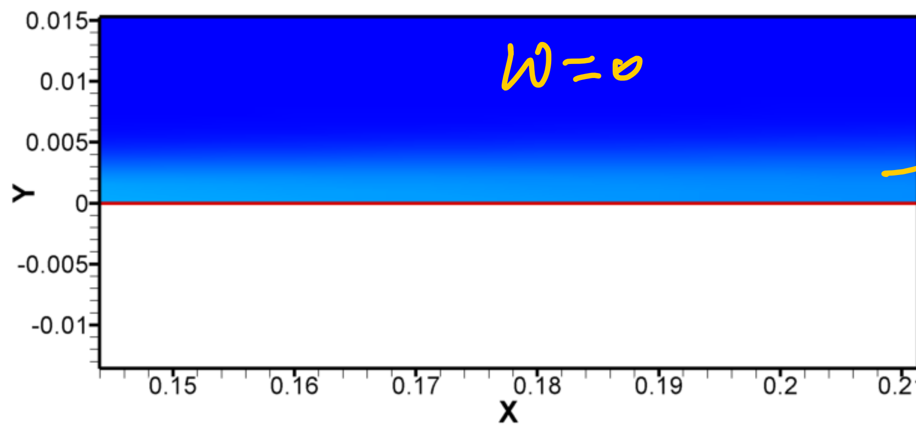


Flow over flat plate (vorticity magnitude) predicted by Fluent

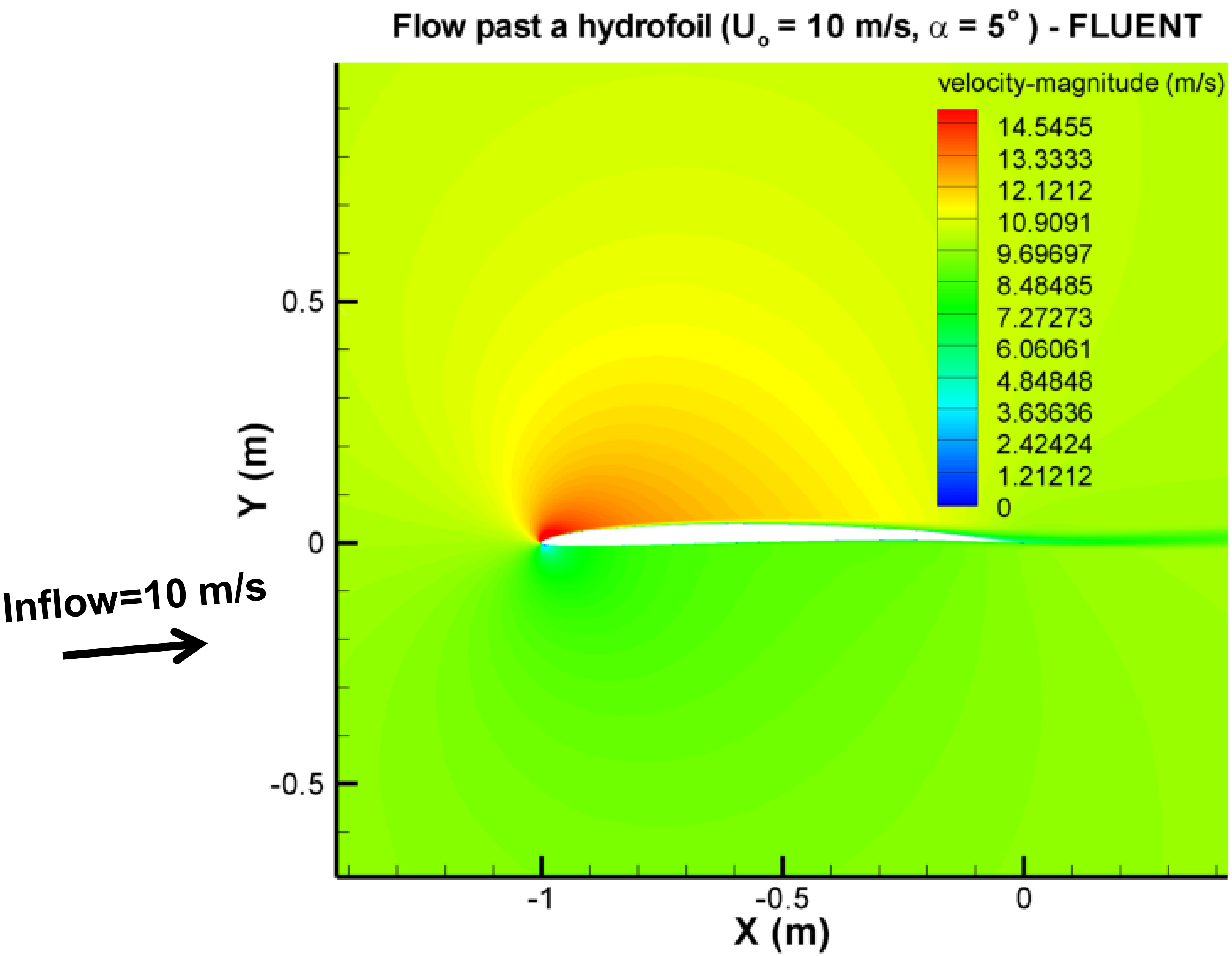
Inflow

$$\boxed{\omega = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}}$$

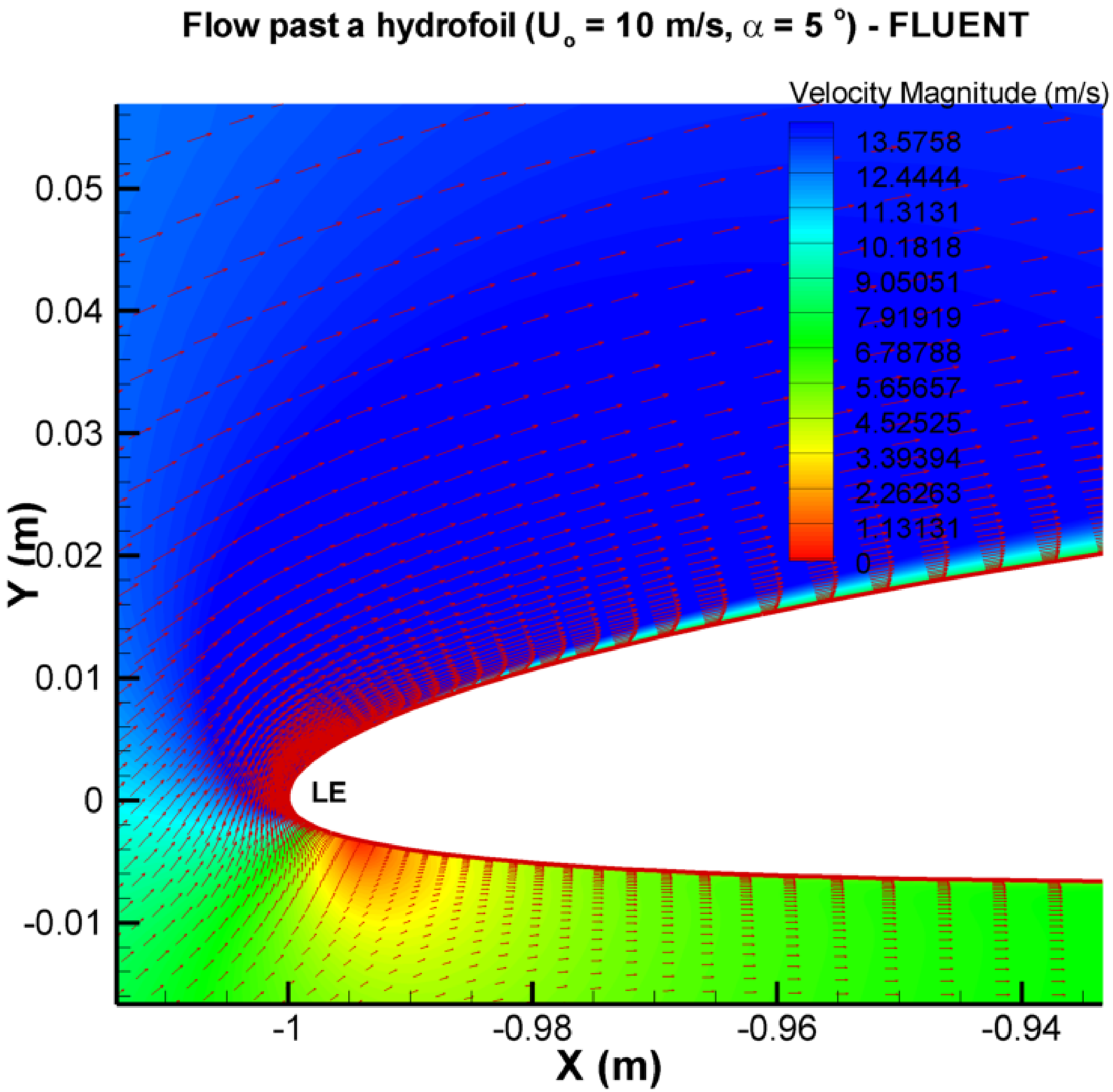




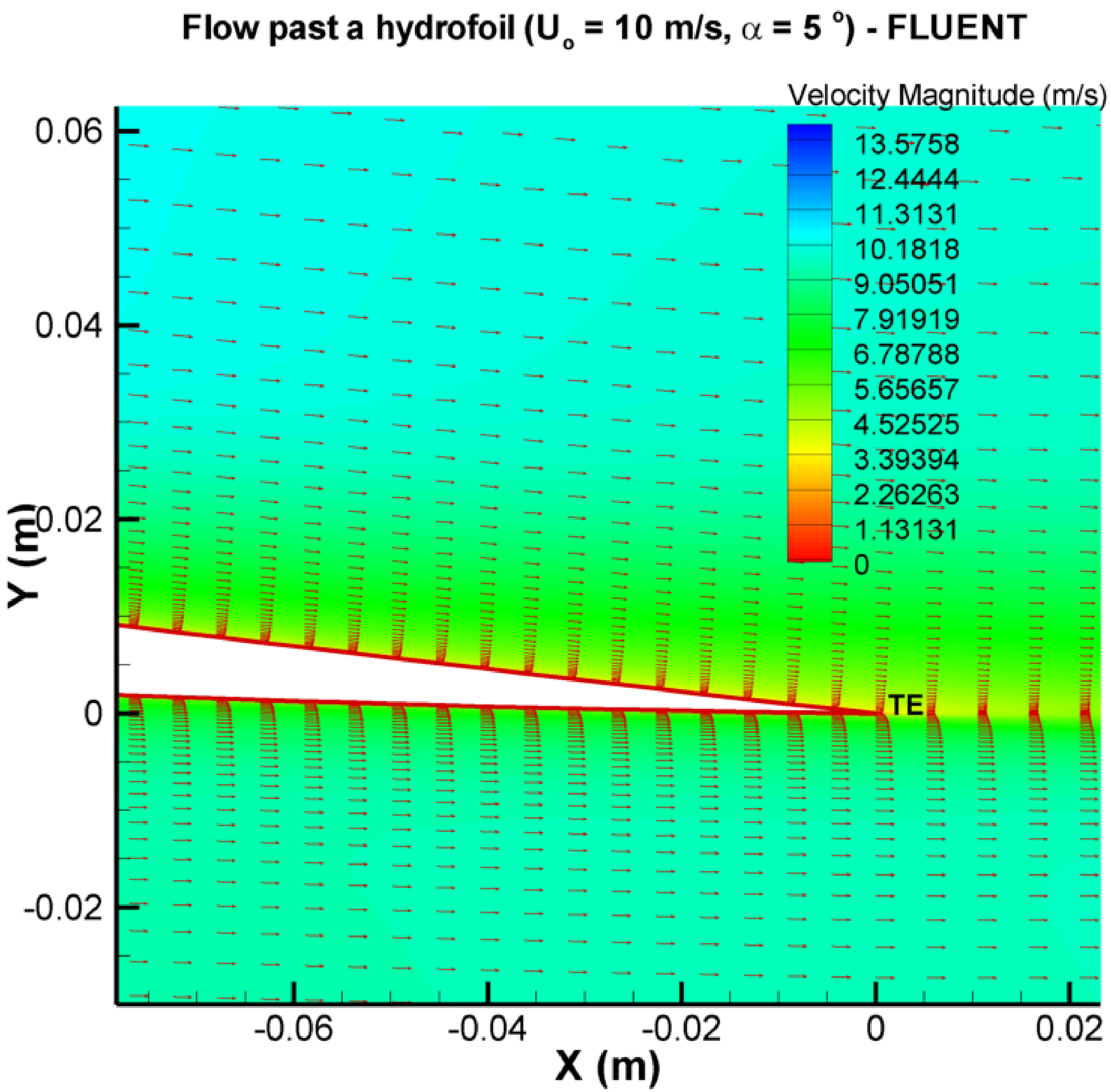
$\omega \neq 0$


Flow past a hydrofoil (U_o = 10 m/s, α = 5°) - FLUENT
velocity-magnitude (m/s)
14.5455
13.3333
12.1212
10.9091
9.69697
8.48485
7.27273
6.06061
4.84848
3.63636
2.42424
1.21212
0
0.5
0
-0.5
Y (m)
-1
-0.5
0
X (m)
Inflow=10 m/s

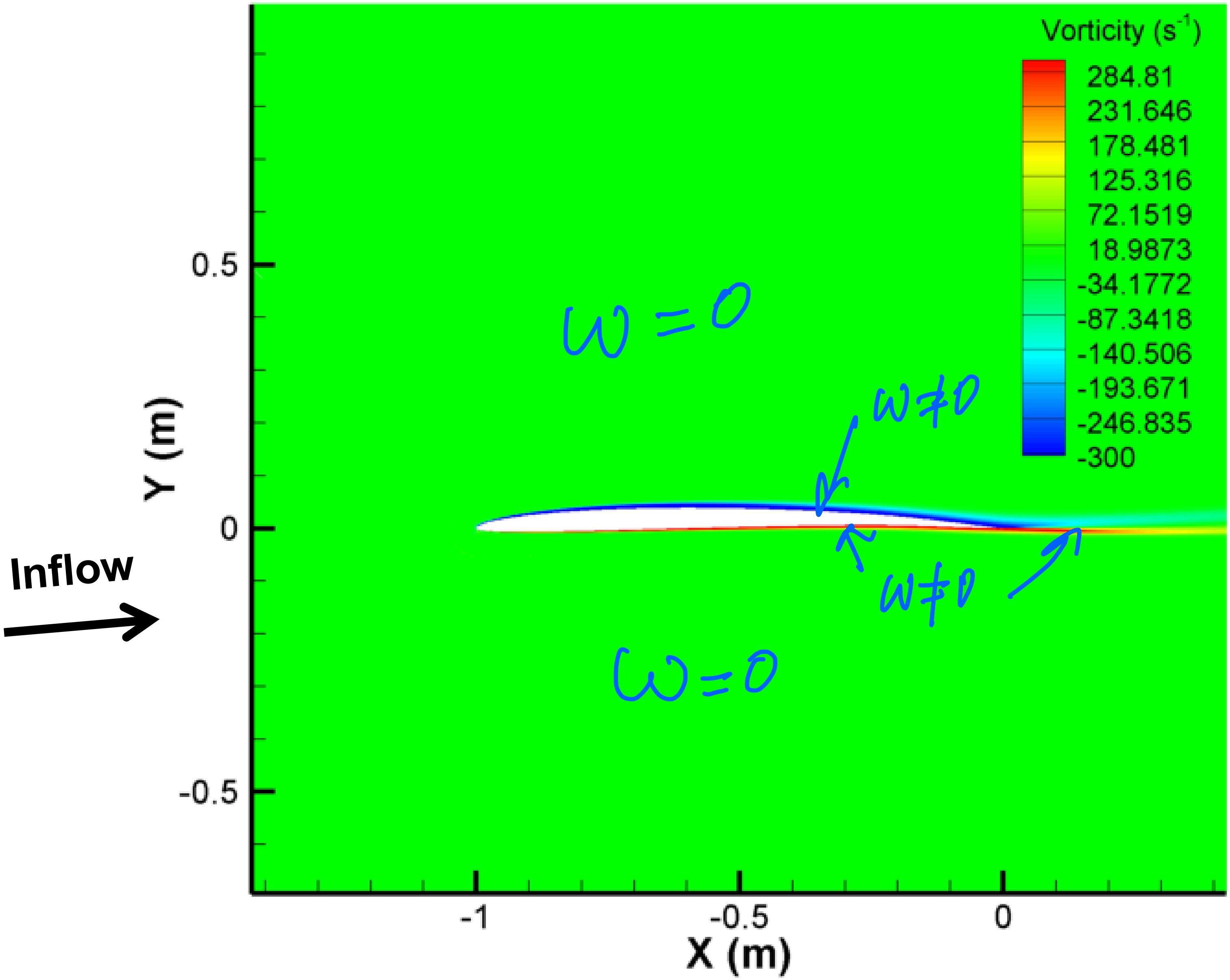
Flow detail at the Leading Edge (LE) of Hydrofoil



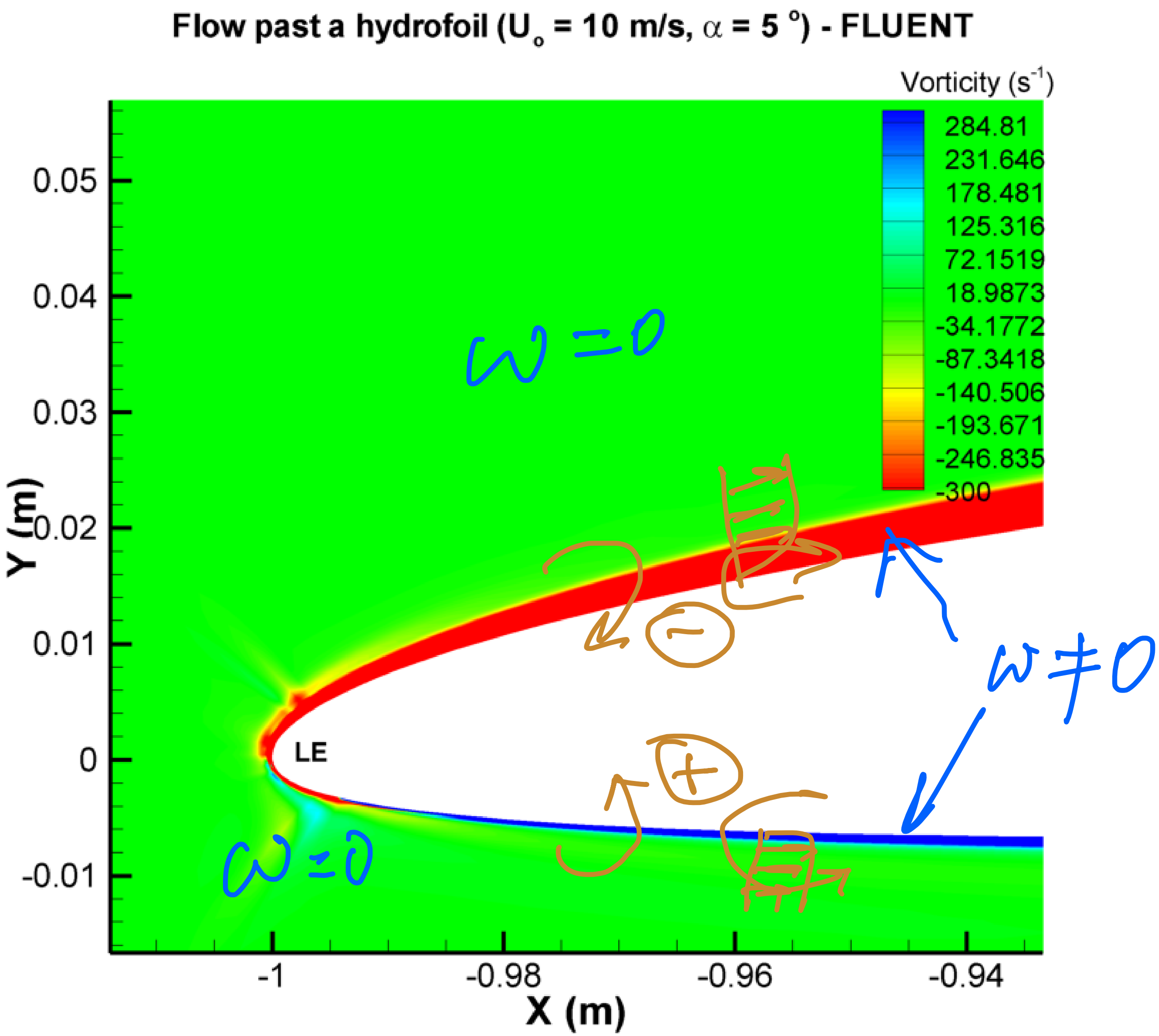
Flow detail at the Trailing Edge (TE) of the Hydrofoil



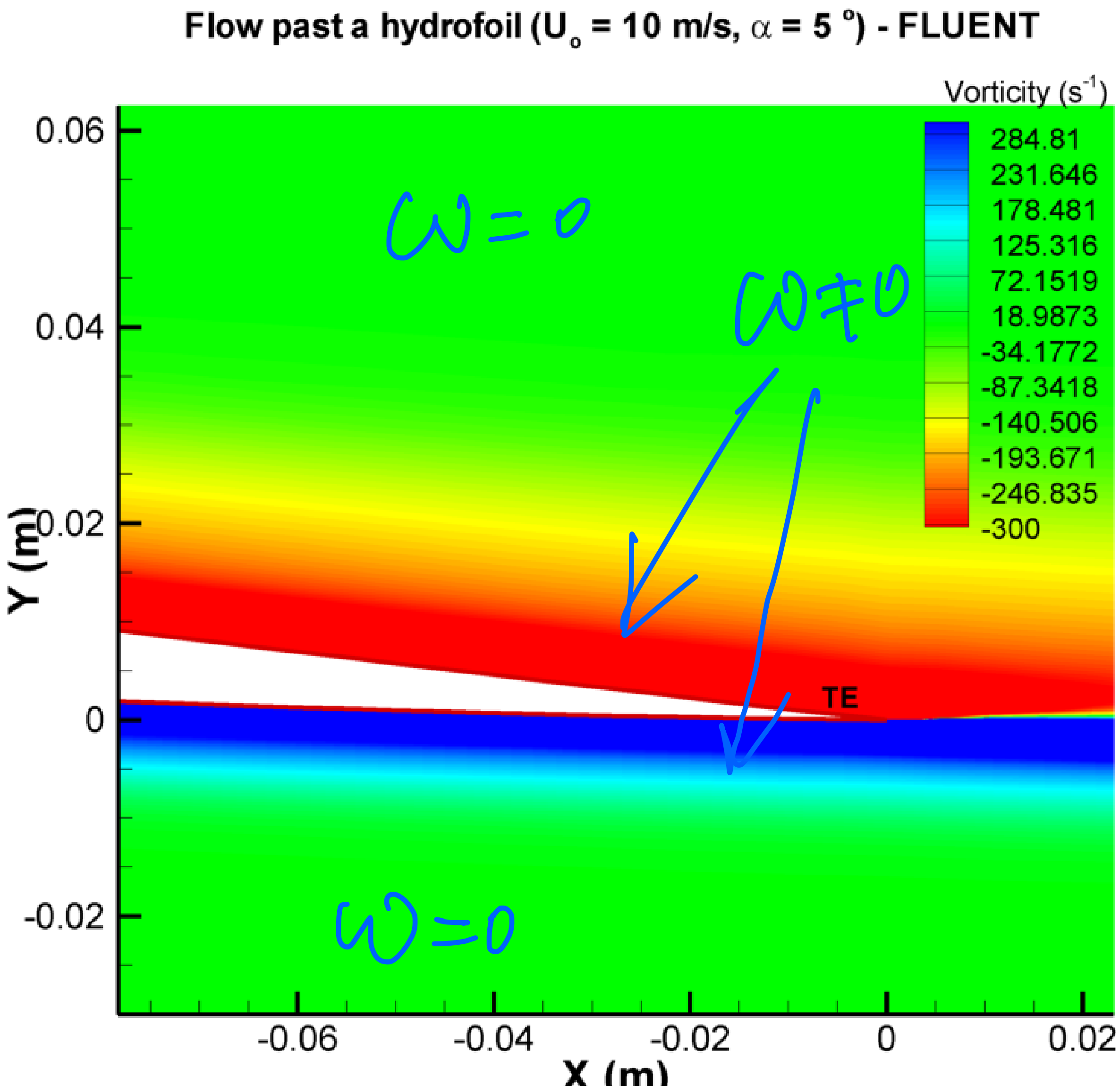
Flow past a hydrofoil (U_o = 10 m/s, α = 5°) - FLUENT



Vorticity detail at the Leading Edge (LE) of Hydrofoil

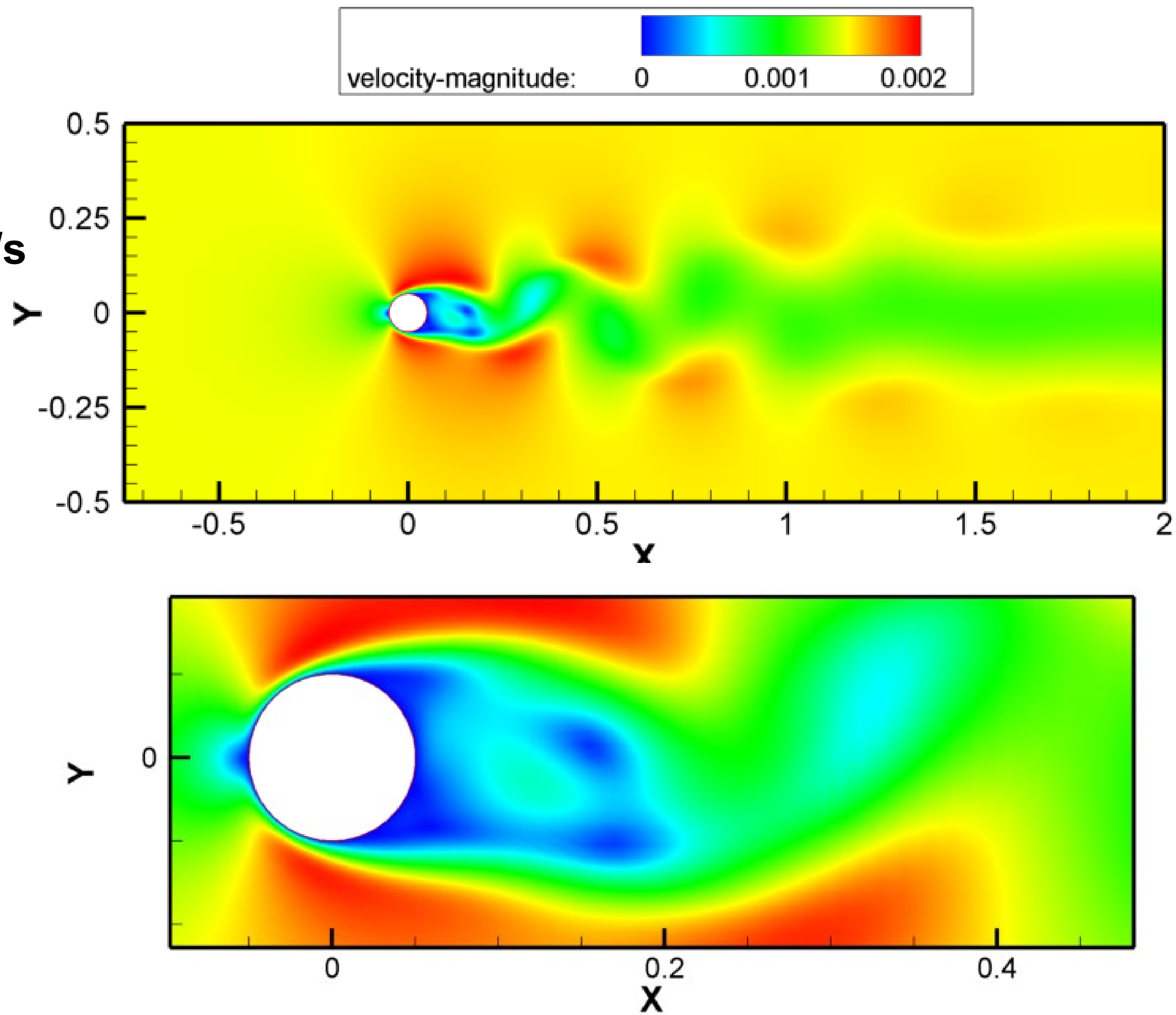


Vorticity detail at the Trailing Edge (TE) of the Hydrofoil

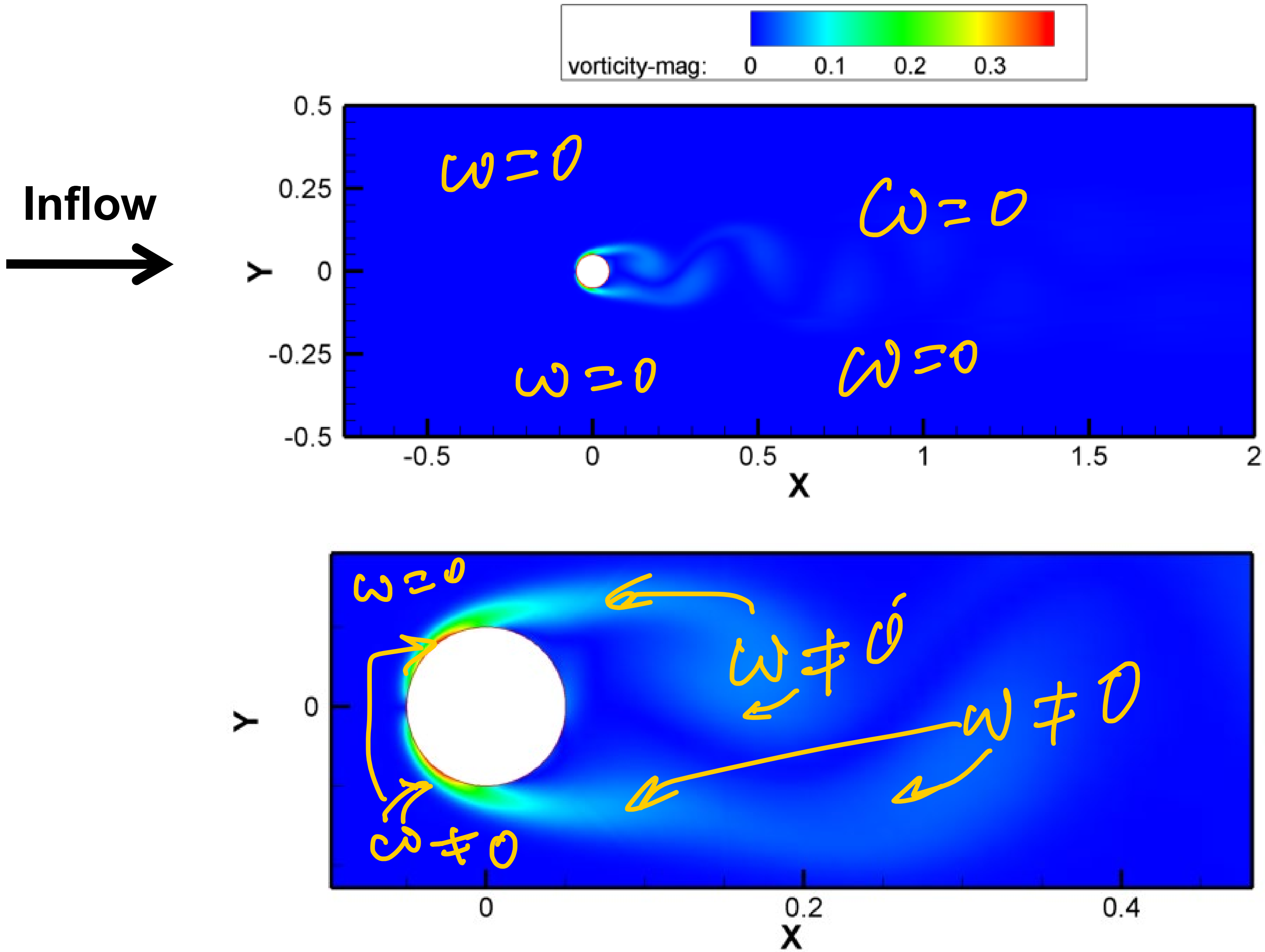


Flow around cylinder (velocity magnitude) predicted by Fluent

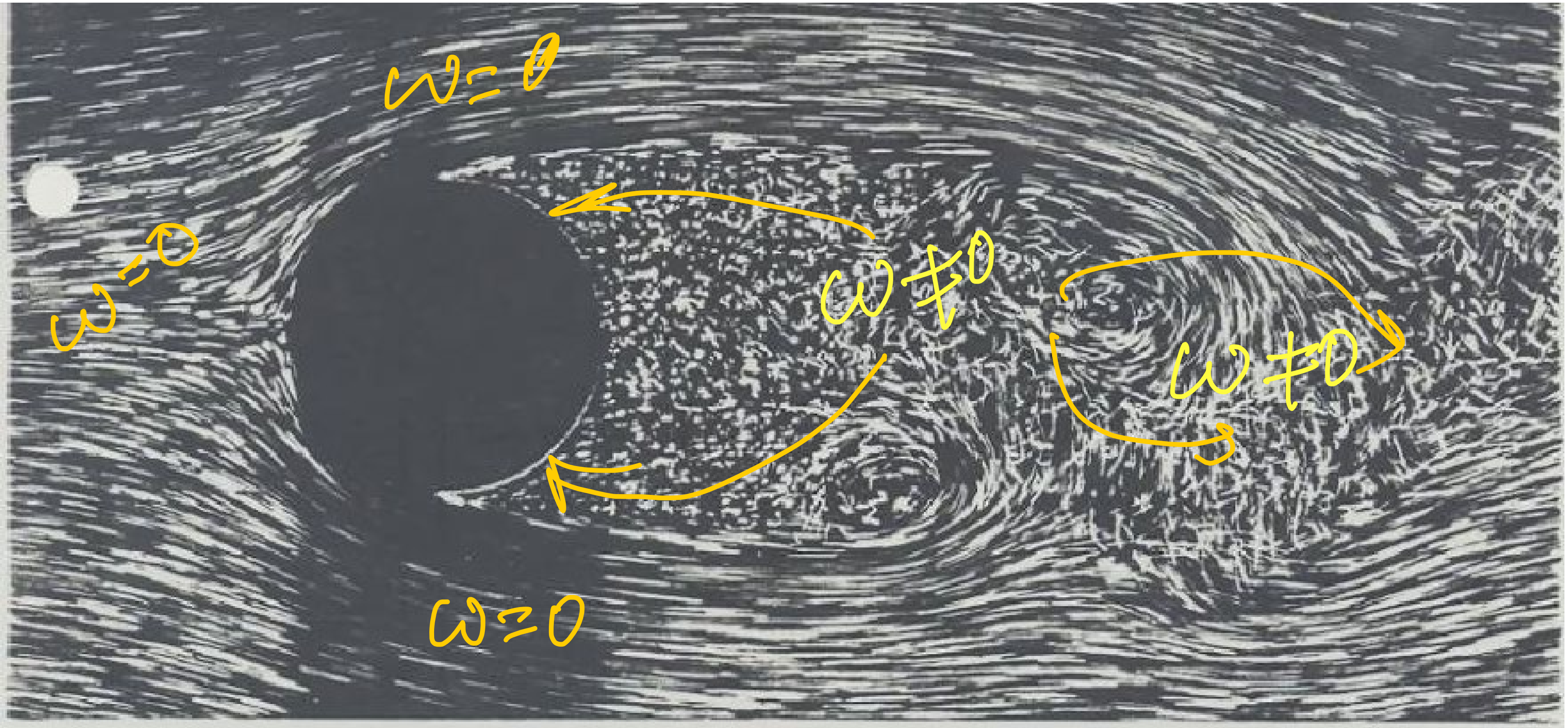
Inflow=0.0015 m/s



Flow around cylinder (vorticity magnitude) predicted by Fluent

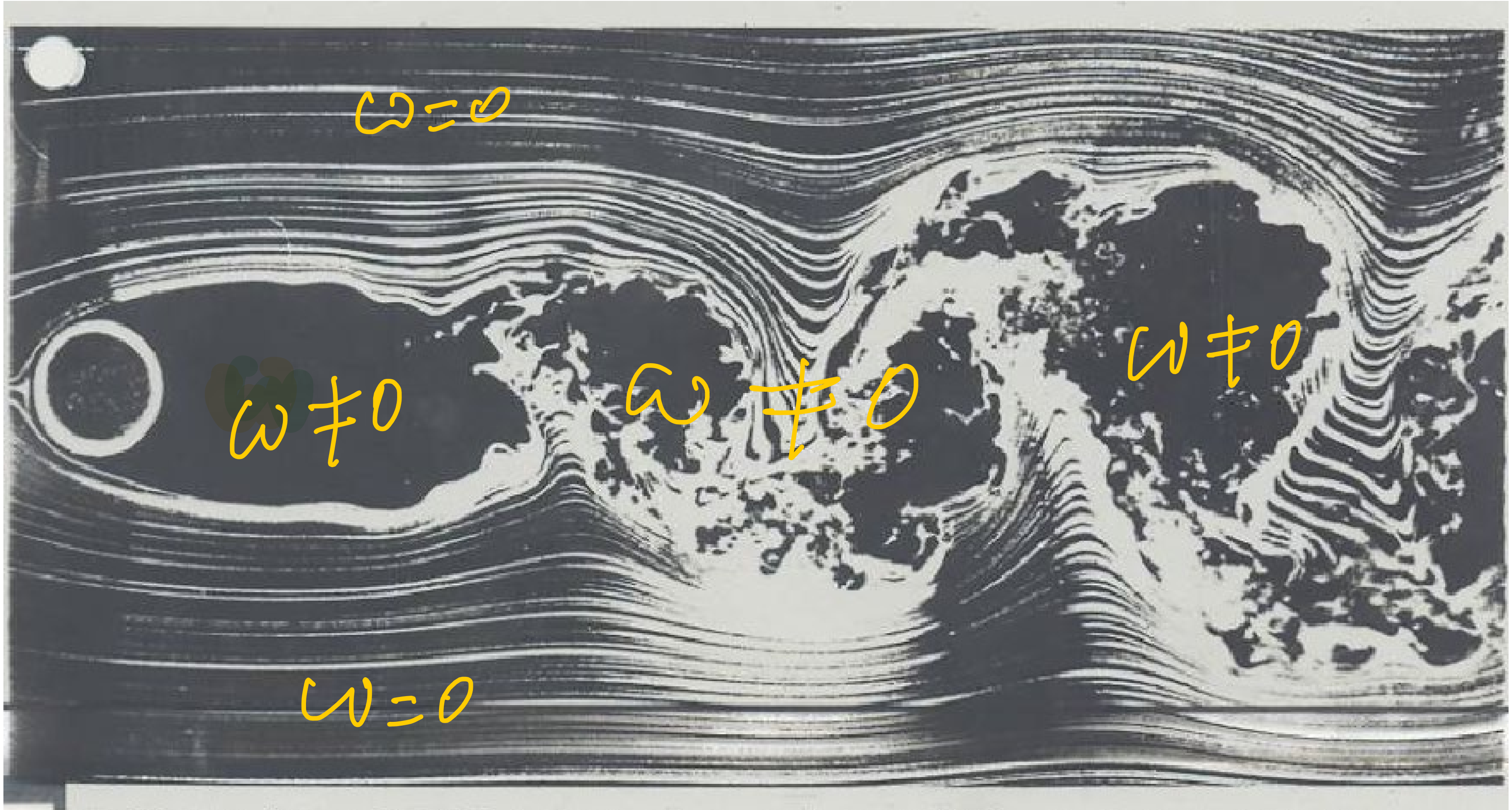


Flow around cylinder (from "An Album of Fluid Motion", by M. Van Dyke)



47. Circular cylinder at $R=2000$. At this Reynolds number one may properly speak of a boundary layer. It is laminar over the front, separates, and breaks up into a turbulent wake. The separation points, moving forward as the Reynolds number is increased, have now attained their upstream limit, ahead of maximum thickness. Visualization is by air bubbles in water. *ONERA photograph, Werlé & Gallon 1972*

Flow around cylinder (from "An Album of Fluid Motion", by M. Van Dyke)



48. **Circular cylinder at R=10,000.** At five times the speed of the photograph at the top of the page, the flow pattern is scarcely changed. The drag coefficient consequently remains almost constant in the range of Reynolds number spanned by these two photographs. It drops later when, as in figure 57, the boundary layer becomes turbulent at separation. *Photograph by Thomas Corke and Hassan Nagib*

EXAMPLE ON IRROTATIONAL FLOW

$$u(x,z) = \underline{A \cdot X + B \cdot Z} \tag{34}$$

$$w(x,z) = C \cdot X + D \cdot Z \tag{35}$$

$\omega = 0$

Continuity equ. $\underline{\dfrac{\partial u}{\partial x} + \dfrac{\partial w}{\partial z} = 0}$

$\dfrac{\partial u}{\partial x} = A$, $\dfrac{\partial w}{\partial z} = D$

$\underline{A + D = 0}$ $\rightsquigarrow$ $D = -A$

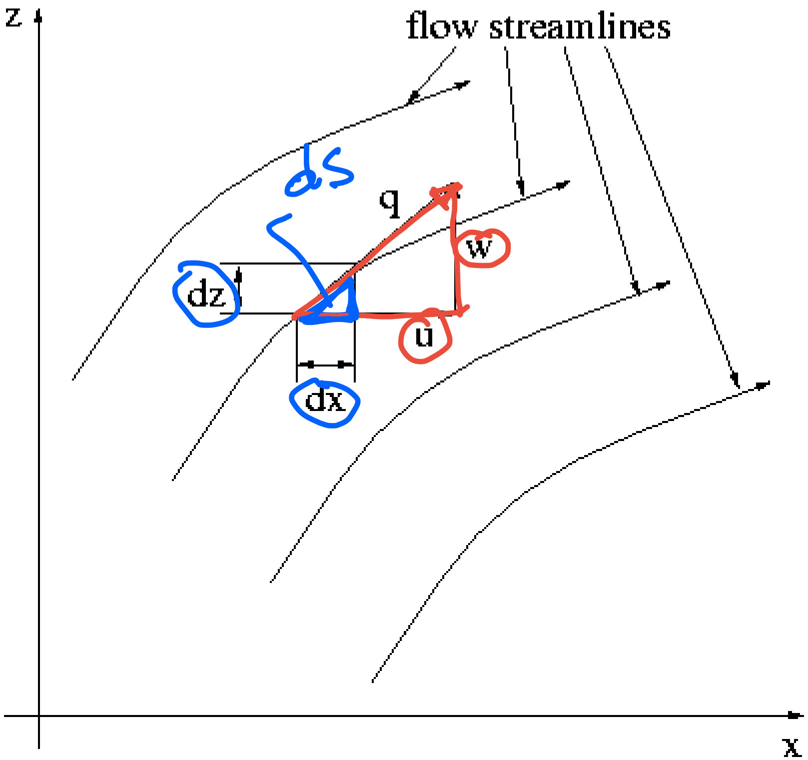
Irrotational flow: $\omega = 0$

$$\omega = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = 0$$

$\dfrac{\partial u}{\partial z} = B$, $\dfrac{\partial w}{\partial x} = C$ $\quad$ $\underline{B - C = 0}$ $\rightsquigarrow$ $C = B$

$$\boxed{\begin{aligned} u &= Ax + Bz \\ w &= Bx - Az \end{aligned}}$$

STREAMLINES - DEFINITION



$\vec{q}$ // streamlines

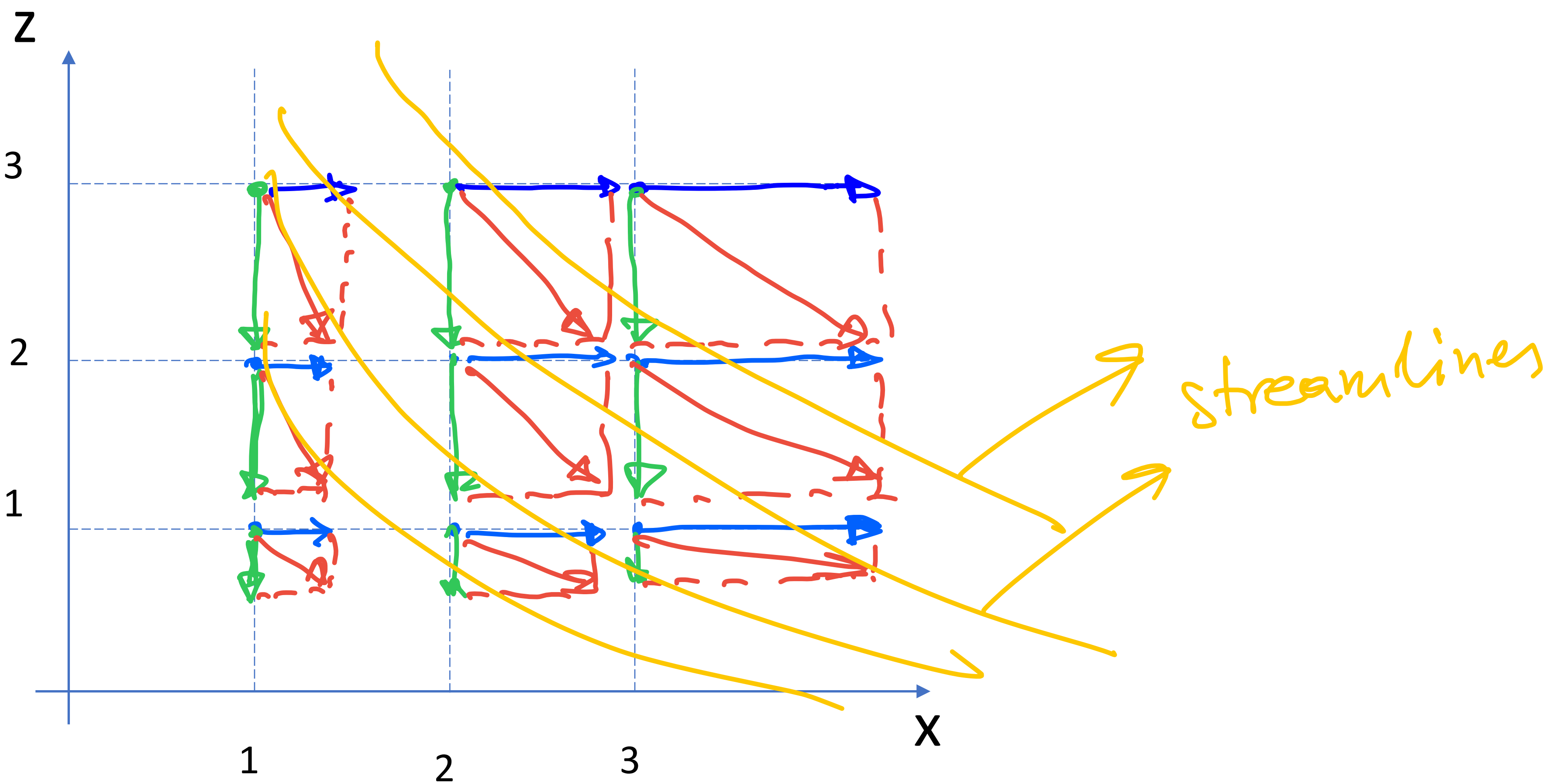
red $\triangle$ similar to blue $\triangle$

$$\boxed{\frac{w}{u} = \frac{dz}{dx}}$$

→ equation for the streamlines

STREAMLINES - EXAMPLE

$$\underline{u(x,z) = x} \qquad \underline{w(x,z) = -z} \qquad ; \quad x > 0, \quad z > 0 \tag{41}$$



STREAMLINES - EXAMPLE

$$u(x,z) = x \qquad w(x,z) = -z \qquad ; \quad x > 0, \quad z > 0 \tag{41}$$

Streamline equ': $\frac{w}{u} = \frac{dz}{dx} \rightsquigarrow \frac{-z}{x} = \frac{dz}{dx} \rightsquigarrow$

$\rightsquigarrow -\int \frac{dz}{z} = \int \frac{dx}{x} \rightsquigarrow -\ln z = \ln x + C \rightsquigarrow$

VERY IMPORTANT TO KEEP C!

$e^{-\ln z} = e^{\ln x + C} = e^{\ln x} \cdot e^{C}$

$e^{\ln x} = x$

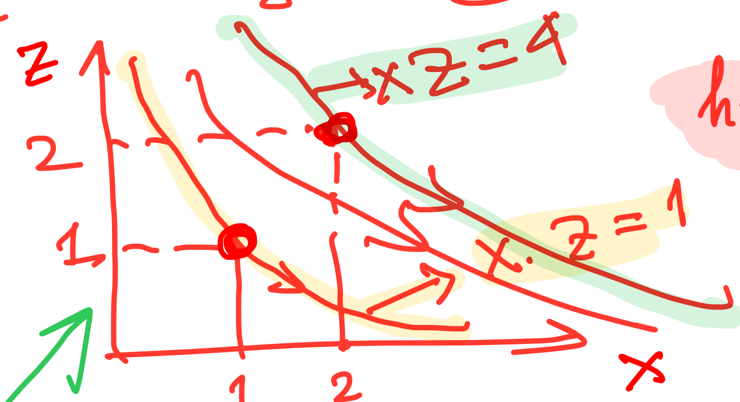
$e^{-\ln z} = \frac{1}{e^{\ln z}} = \frac{1}{z}$

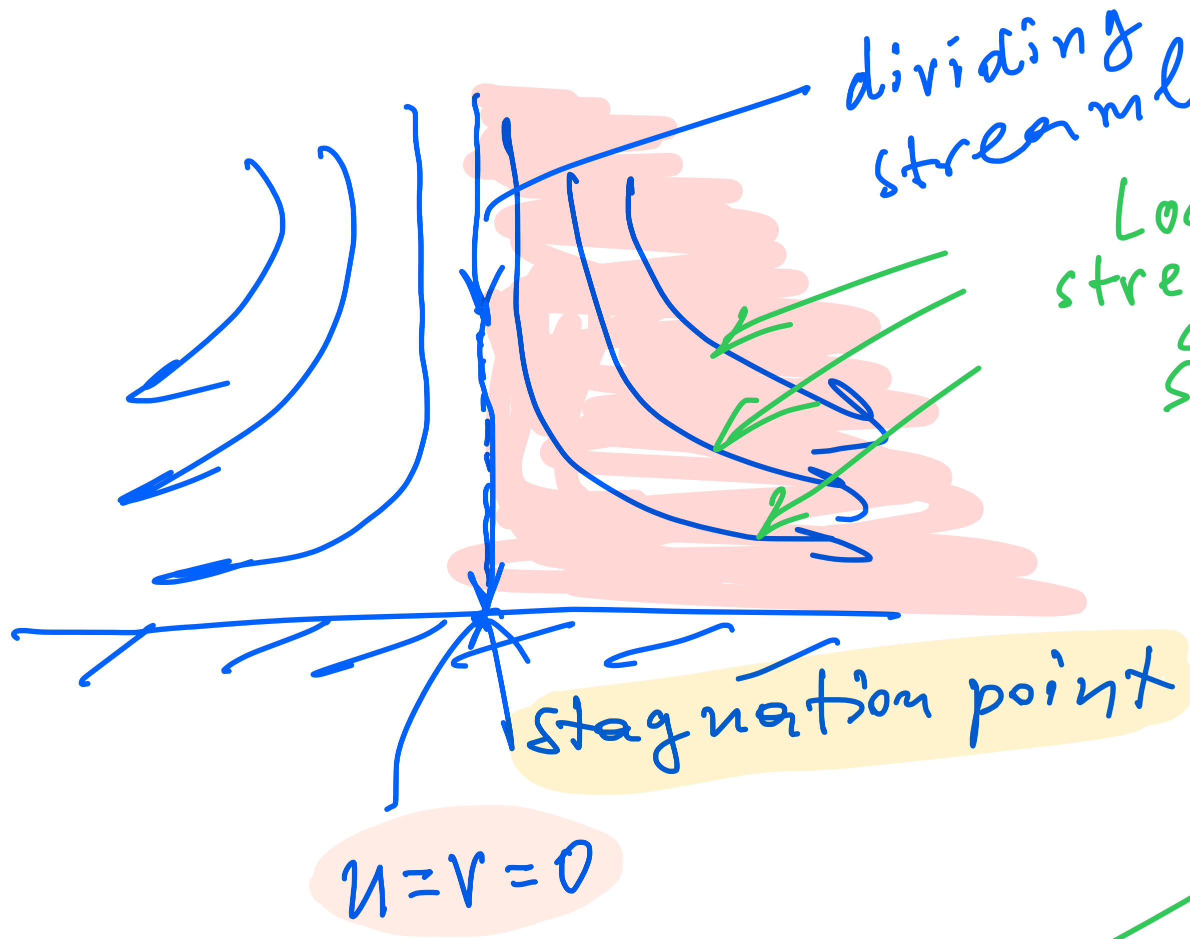
$\frac{1}{z} = x \cdot e^{C}$

$\rightsquigarrow \boxed{\underline{xz} = e^{-C} = C'}$

hyperbolae

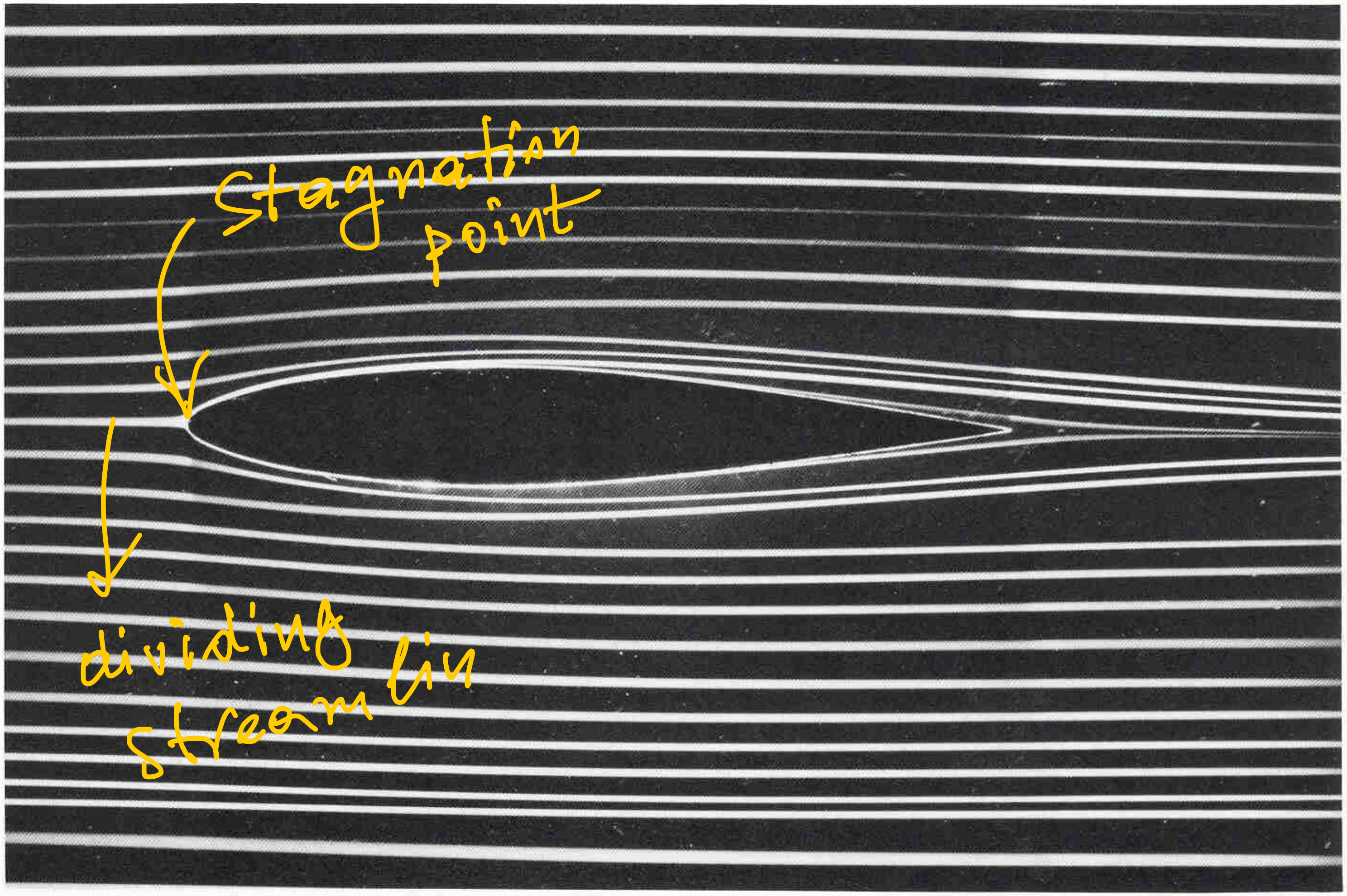
different C' provide different streamlines





Local flow, streamlines clos to stagnation point, are the same as

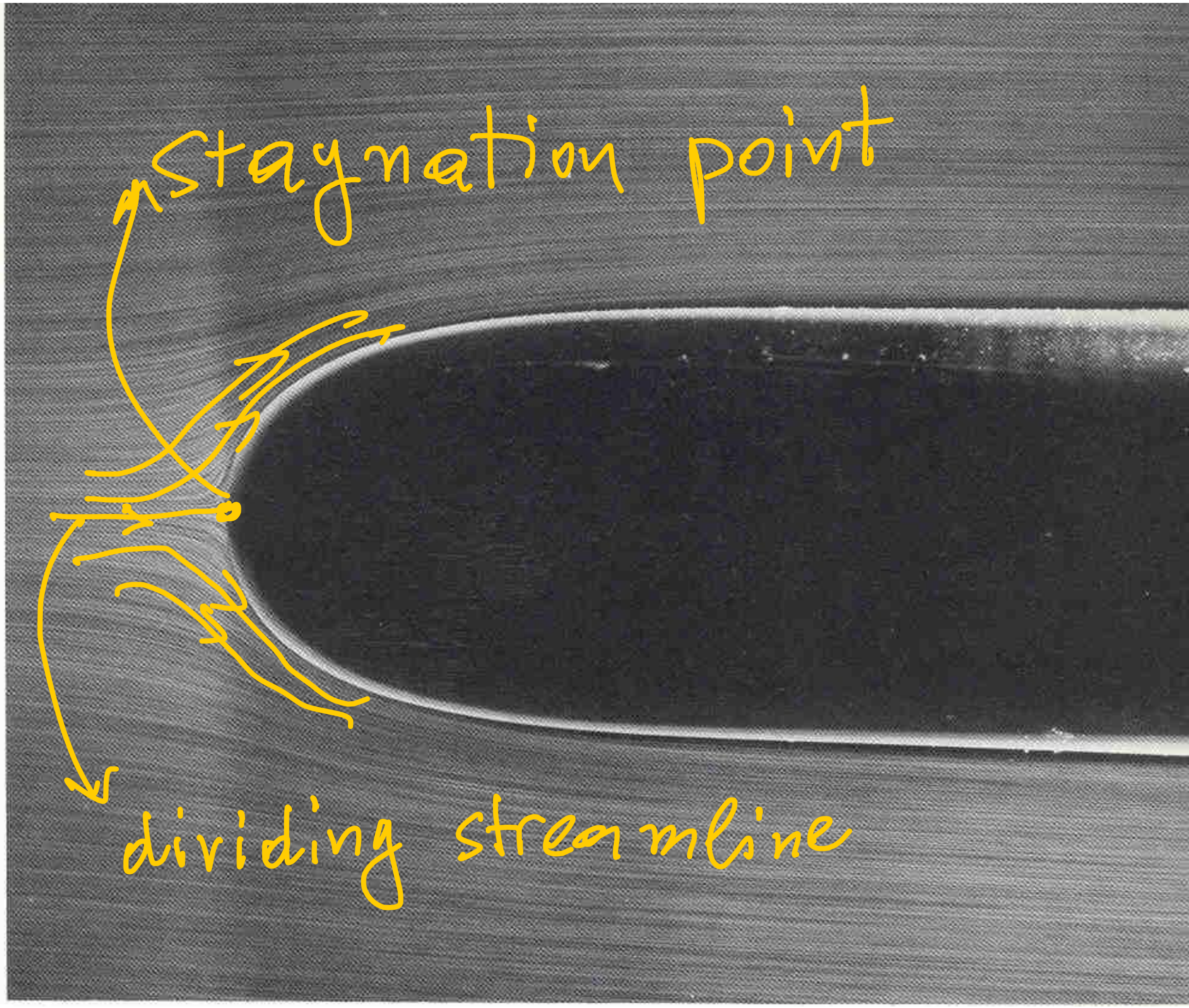
STREAMLINES AROUND AIRFOIL



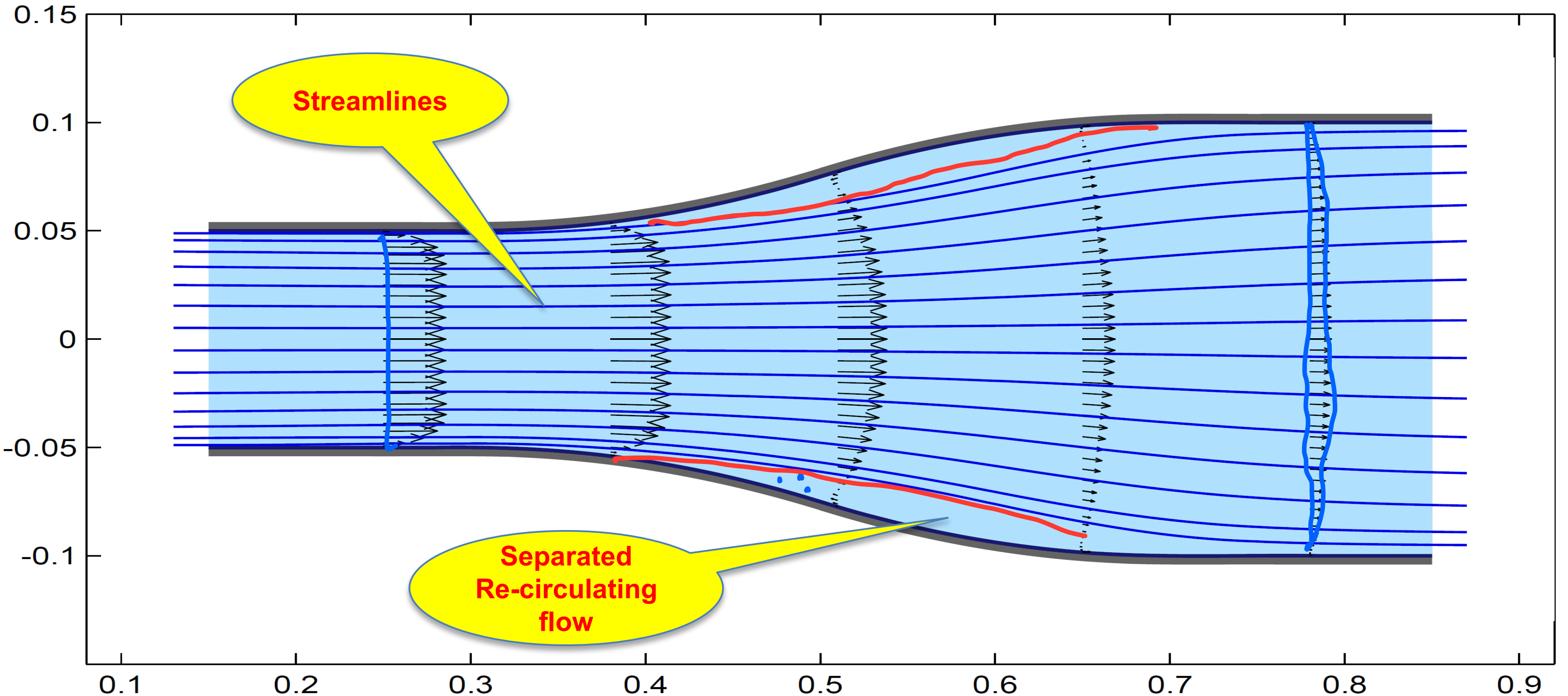
23. Symmetric plane flow past an airfoil. An NACA 64A015 profile is at zero incidence in a water tunnel. The Reynolds number is 7000 based on the chordlength. Streamlines are shown by colored fluid introduced upstream. The flow is evidently laminar and appears to be unseparated, though one might anticipate a small separated region near the trailing edge. *ONERA photograph, Werlé 1974*

STREAMLINES AROUND AXI-SYMMETRIC BODY

22. Axisymmetric flow past a Rankine ogive. This is the body of revolution that would be produced by a point potential source in a uniform stream—the axisymmetric counterpart of the plane half-body of figure 2. Its shape is so gentle that at zero incidence and a Reynolds number of 6000 based on diameter the flow remains attached and laminar. Streamlines are made visible by tiny air bubbles in water, illuminated by a sheet of light in the mid-plane. *ONERA photograph, Werlé 1962*



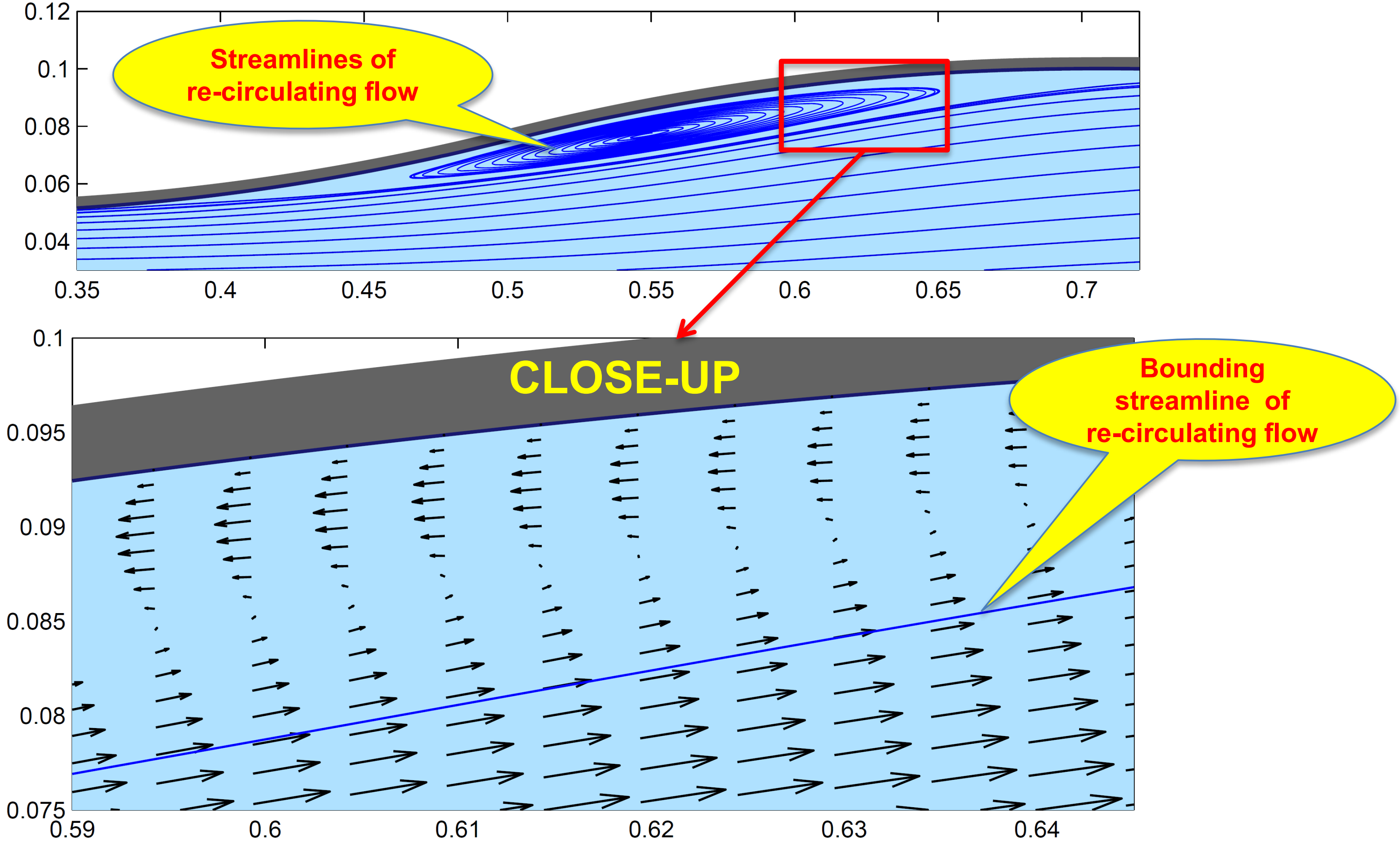
Velocities and Streamlines of Flow through a Pipe Expansion (computed via Fluent/turbulent)



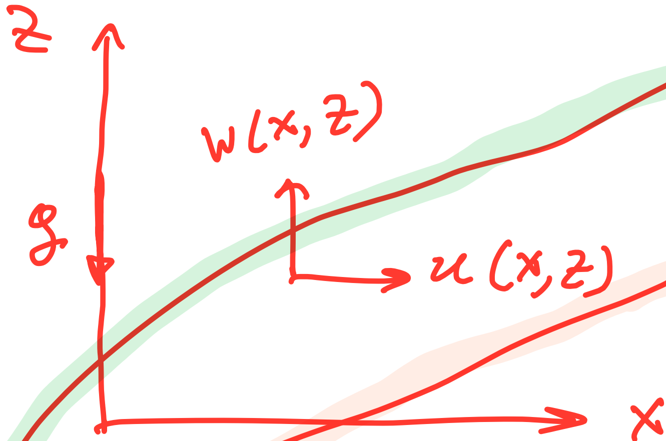
Consider fluid flowing through a circular pipe of expanding radius as illustrated above. Inlet pipe diameter D_1=0.1m; outlet pipe diameter D_2=0.2m; inlet velocity U_1=5m/s; fluid density ρ=100kg/m^3; dynamic viscosity $\mu=2\times10^{-2}$kg/(ms). The Reynolds number, Re, based on the pipe diameter, D_1, and velocity, U_1, at the inlet is:

$$Re = \frac{U_1 D_1}{\nu} = \frac{\rho U_1 D_1}{\mu} = 2500 \qquad \left(\text{Remember: } \nu = \frac{\mu}{\rho} \right)$$

Streamlines and Velocities inside Separated/Re-circulating Flow



DEFINITION OF VELOCITY POTENTIAL & LAPLACE EQUATION



Continuity equ.: $\dfrac{\partial u}{\partial x} + \dfrac{\partial w}{\partial z} = 0$

irrotationality: $\omega = \dfrac{\partial u}{\partial z} - \dfrac{\partial w}{\partial x} = 0$
$(\omega = 0)$

Velocity potential, $\phi(x,z)$

$u = \dfrac{\partial \phi}{\partial x}$ and $w = \dfrac{\partial \phi}{\partial z}$

$$\frac{\partial}{\partial z}\left(\frac{\partial \phi}{\partial x}\right) - \frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial z}\right) = 0$$

Always $\omega = 0$ since order of differentiation does not change result!

$$\frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\partial \phi}{\partial z}\right) = 0$$

Must be valid at ALL points and at ALL times

$$\boxed{\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0}$$

Laplace equation is equivalent to continuity equ.!

Laplace operator: $\nabla^2 \Box \equiv \frac{\partial^2 \Box}{\partial x^2} + \frac{\partial^2 \Box}{\partial z^2}$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad \text{or} \quad \nabla^2 \phi = 0$$

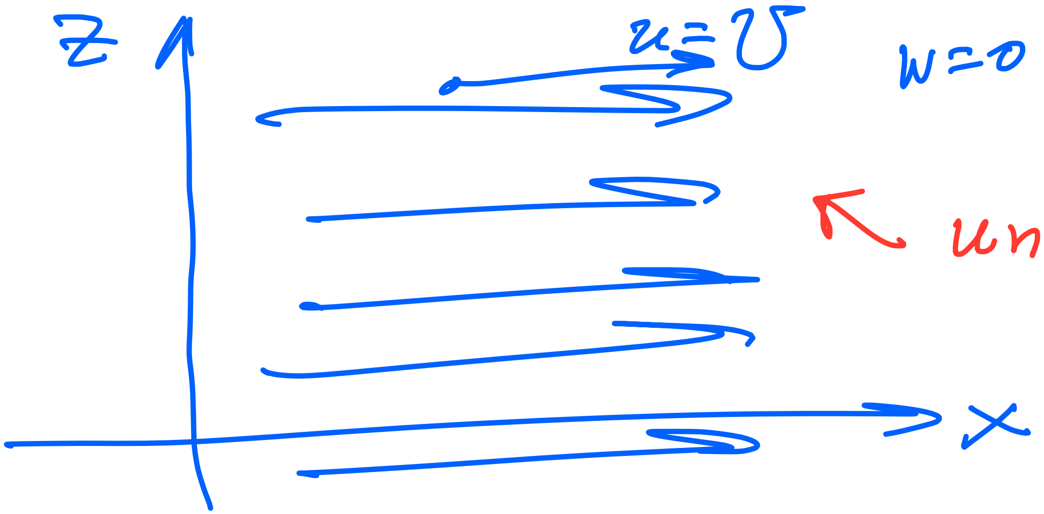
Note the velocity potential is defined (or can be used) ONLY in the case the flow is irrotational ($\omega = 0$)

VELOCITY POTENTIAL – Example 1

Determine the flow field if $\Phi = U x$

$u, w = ?$

$$u = \frac{\partial \Phi}{\partial x} = U \quad ; \quad w = \frac{\partial \Phi}{\partial z} = 0$$



uniform flow

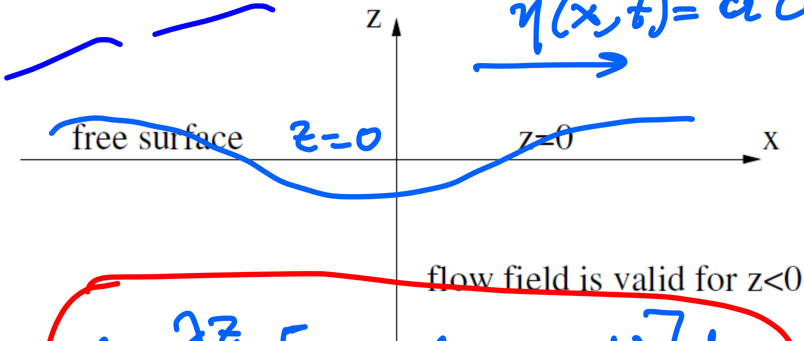
VELOCITY POTENTIAL – Example 2

The velocity potential, ϕ, for a wave in deep water can be expressed as follows ($-\infty < x < +\infty$, $z < 0$, $t > 0$):

$$\phi(x,z;t) = Ae^{\lambda z}\sin(kx - \omega t) \qquad (2)$$

Determine the constant λ so that this flow-field is *physically meaningful* at *all locations* and at *all times*.

$\eta(x,t) = a\cos(kx-\omega t)$



$$\frac{\partial \phi}{\partial x} = Ae^{\lambda z}\left[\cos(kx-\omega t)\right]k = u$$

$$\frac{\partial^2 \phi}{\partial x^2} = Ae^{\lambda z}\left[-\sin(kx-\omega t)\right]k\cdot k = -Ak^2e^{\lambda z}\sin(kx-\omega t)$$

$$\frac{\partial \phi}{\partial z} = Ae^{\lambda z}\cdot\lambda\sin(kx-\omega t) = w$$

$$\frac{\partial^2 \phi}{\partial z^2} = Ae^{\lambda z}\lambda\lambda\sin(kx-\omega t) = A\lambda^2e^{\lambda z}\sin(kx-\omega t)$$

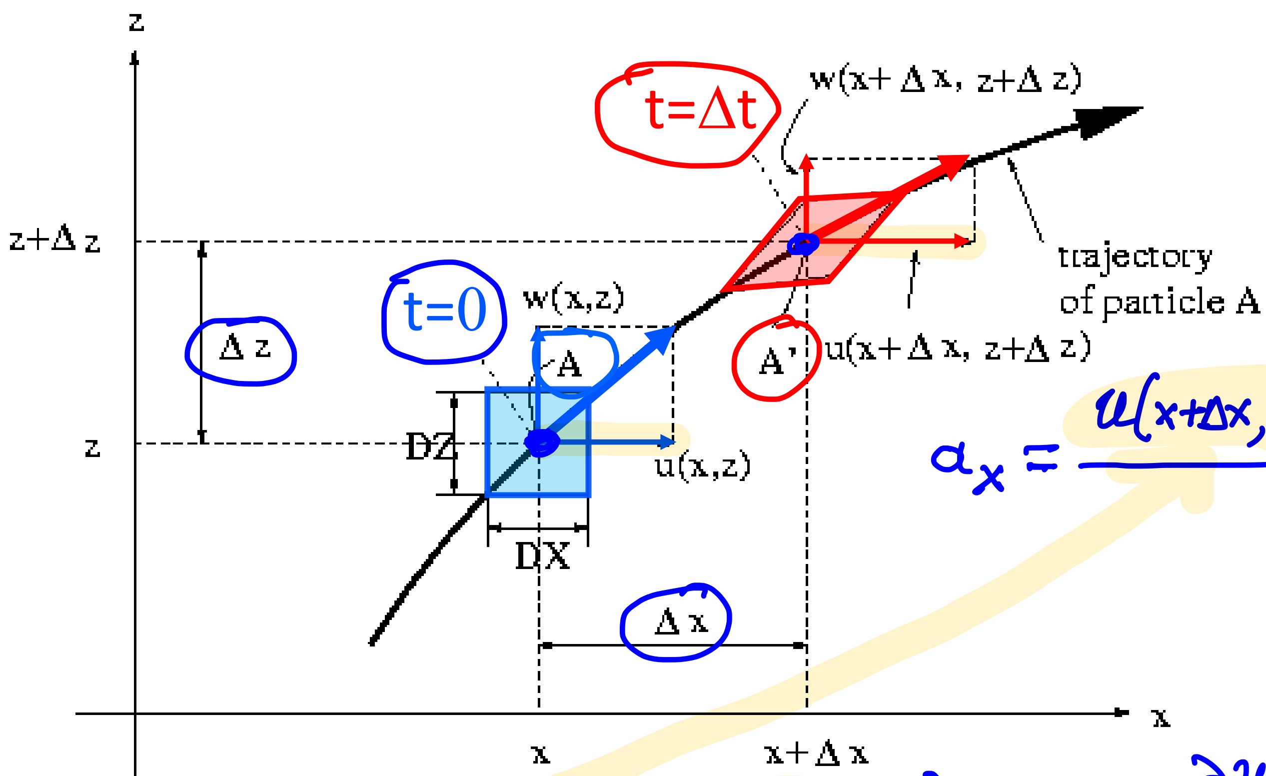
VELOCITY POTENTIAL – Example 2

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = \underbrace{A e^{\lambda z} \sin(kx - \omega t) \left[-k^2 + \lambda^2 \right] = 0}_{\text{at } \underline{\underline{\text{all}}} \ x, z \text{ and } \underline{\underline{\text{all}}} \text{ times } t}$$

$$\Rightarrow -k^2 + \lambda^2 = 0 \Rightarrow \lambda^2 = k^2 \Rightarrow \begin{cases} \lambda = +k \\ \cancel{\lambda = -k} \end{cases}$$

rejected since it provides <u>exponentially</u> growing flowfield as $z \to -\infty$

ACCELERATION OF FLUID PARTICLE



$$a_x = \frac{u(x+\Delta x, z+\Delta z) - u(x,z)}{\Delta t}$$

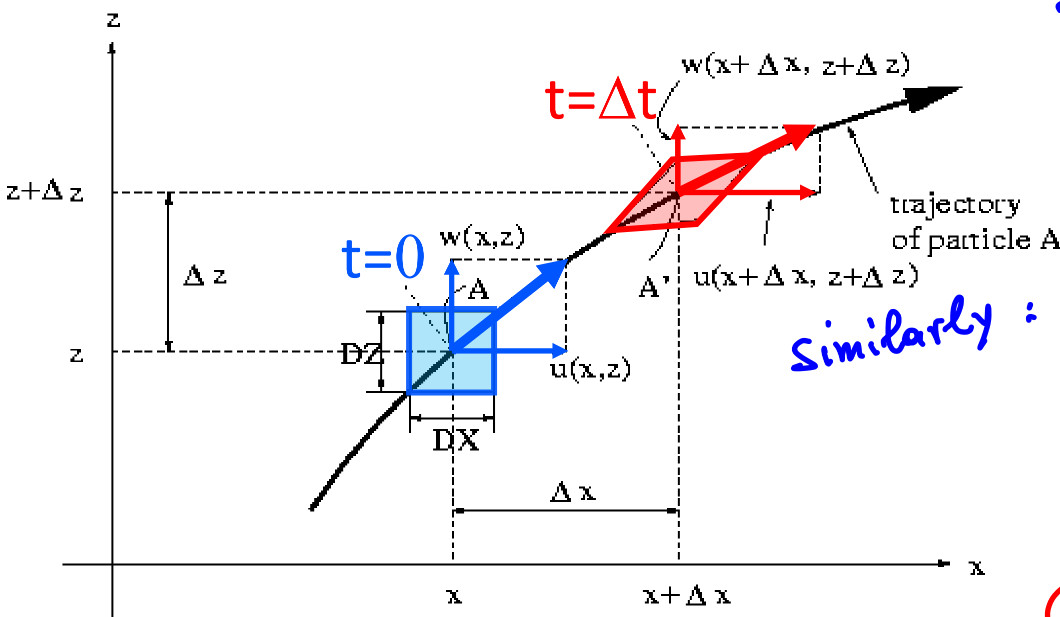
$$u(x+\Delta x, z+\Delta z) = u(x,z) + \frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial z}\cdot\Delta z$$

Taylor expansion (Linear)

$$\Rightarrow a_x = \frac{\frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial z}\Delta z}{\Delta t} = \frac{\partial u}{\partial x}\underbrace{\frac{\Delta x}{\Delta t}}_{= u} + \frac{\partial u}{\partial z}\underbrace{\frac{\Delta z}{\Delta t}}_{= w}$$

(as $\Delta t \to 0$)

ACCELERATION OF FLUID PARTICLE



$$a_x = u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z}$$

similarly :

$$a_z = u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z}$$

Convective terms of acceleration

total acceleration

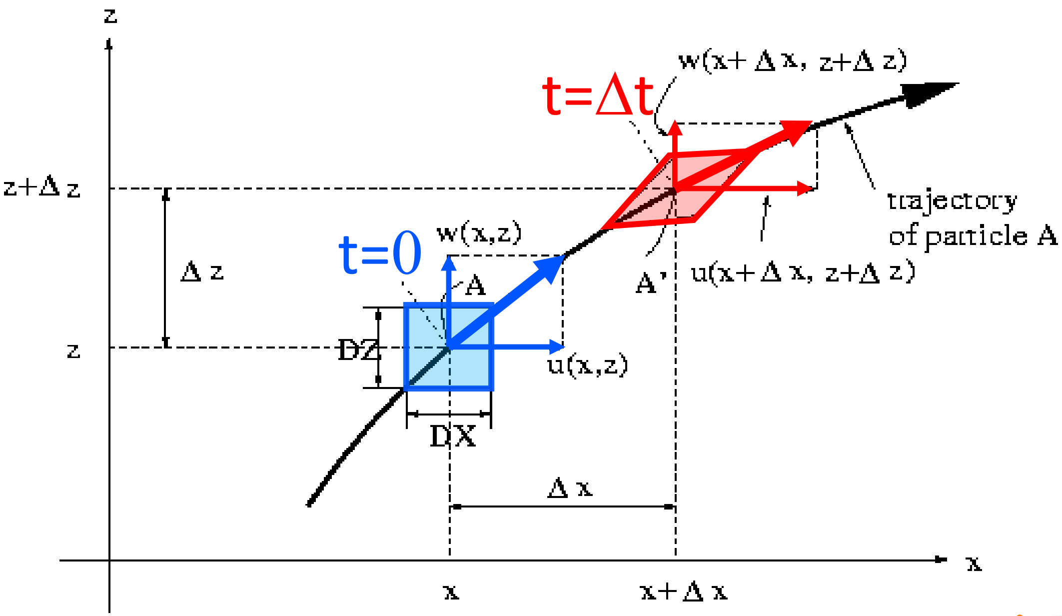
$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z}$$

Convective terms

local or unsteady terms

ACCELERATION OF FLUID PARTICLE



Define the substantial or total or material or substantive derivative:

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + w\frac{\partial}{\partial z}$$

$$a_x = \frac{Du}{Dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z}$$

$$a_z = \frac{Dw}{Dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z}$$

ACCELERATION OF FLUID PARTICLE - EXAMPLE

$$u(x,z)=x \qquad w(x,z)=-z \qquad ; \quad x>0, \quad z>0 \tag{41}$$

$$a_x = \underbrace{\cancel{\frac{\partial u}{\partial t}}}_{=0} + u\frac{\partial u}{\partial x} + \underbrace{\cancel{w\frac{\partial u}{\partial z}}}_{=0}$$

steady flow

$$\frac{\partial u}{\partial x} = 1$$

$$\frac{\partial u}{\partial z} = 0$$

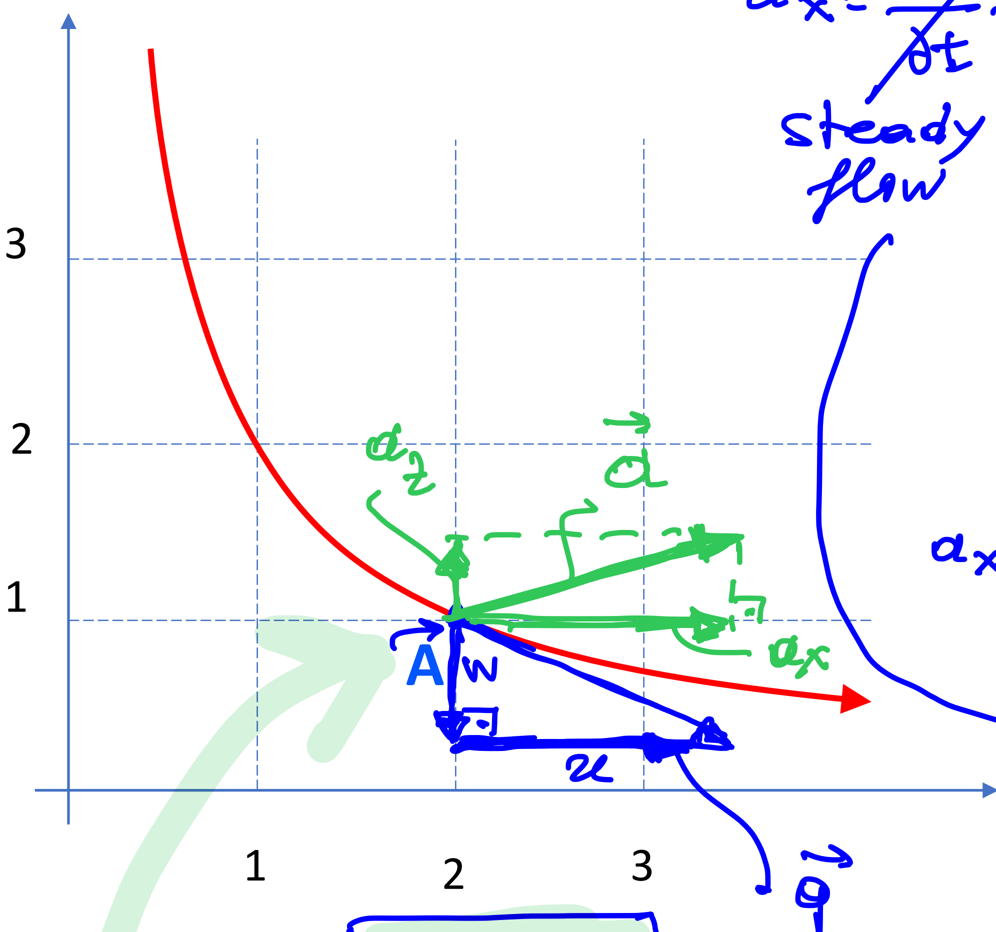
$$a_x = x \cdot 1$$

$$a_z = \underbrace{\cancel{\frac{\partial w}{\partial t}}}_{=0} + \underbrace{\cancel{u\frac{\partial w}{\partial x}}}_{=0} + w\frac{\partial w}{\partial z}$$

$$\frac{\partial w}{\partial x} = 0$$

$$\frac{\partial w}{\partial z} = -1$$

$$\Rightarrow a_z = (-z)(-1) = z$$



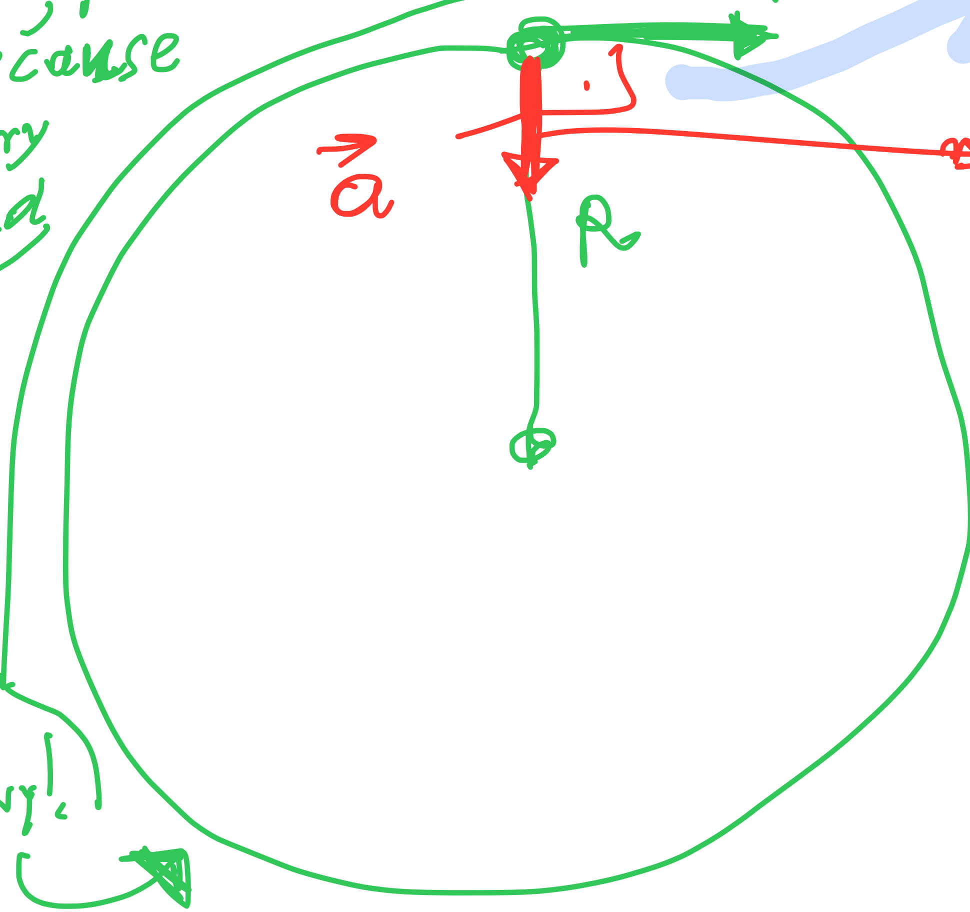
$$\boxed{\begin{aligned} a_x &= x \\ a_z &= z \end{aligned}}$$

At point A: $(x=2, z=1) \rightsquigarrow a_x = 2, \; a_z = 1$

Vecocity at A: $\longrightarrow u = 2, \; W = -1$

Q: Does it make sense that $\vec{a}, \vec{q}$ not aligned?

A: Yes because trajectory is curved like in the case of an object on a circular trajectory!



$\vec{a}$ & $\vec{q}$ are not aligned!

centripetal acceleration

$$|\vec{a}| = \frac{|\vec{q}|^2}{R}$$

$$|\vec{a}| = a$$

$$|\vec{q}| = q$$

$$\boxed{a = \frac{q^2}{R}}$$